

AN ANALYSIS OF MECHANISM FOR CUSTOMERS' PURCHASE AMOUNT AND NUMBER OF VISITS IN DEPARTMENT STORE

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Abstract The purpose of this study is to reveal how marketing affects customers' purchase amount and number of visits in Japanese department stores. We model purchase amounts by using a hierarchical Bayes regression model and number of visits by using a hierarchical Bayes Poisson regression model. Furthermore, we estimate the latent factor behind price as the purchase amount per month with a Type-1 Tobit model and the structural heterogeneity of each customer with a model for variable selection. Direct mail and events are used as marketing measures. The analytical results reveal marketing measures that raise customers' final purchase amounts.

Keywords: Marketing, hierarchical bayes model, tobit model, model for variable selection, MCMC

1. Introduction

In a mature society, changes in consumers' lifestyles and diversification of their needs are altering retail stores' strategies. According to statistics from the Japanese Department Store Association, net department store industry sales decreased from 9.7 trillion yen in 1991 to 6.3 trillion yen in 2010. Contrary to this trend, total department store floor space increased from 5.50 million m^2 in 1991 to 6.47 million m^2 in 2010 [28]. To increase customer share, department stores need to uncover marketing measures that precisely capture customers' needs. We define customer share as the ratio of the total purchase of a single company's product to the purchase amount by a single customer. If a consumer's total purchase amount remains the same, measures to increase these amounts must be considered. Recently, the concept of customer relationship management (CRM) has attracted attention as an effective marketing strategy for existing customers in the retail industry. However, the effectiveness of CRM activities at retail stores remains unclear. Moreover, CRM data in retail, such as a customer's purchase history and data, have not been effectively utilized, because they provide reward points and enhance customer relationships [5]. To increase customers' purchases, CRM data should be used more strategically and practically.

This study reveals how department stores' marketing measures the number of customer visits, which influence purchase amounts. Using simultaneous analysis, it also explains how marketing measures influence the number of customer visits. We model customers' purchase amounts and number of department store visits by using a hierarchical Bayes model that includes point-of-sale (POS) data (with IDs), as well as daily CRM information that focuses on customer attributes and records of marketing initiatives taken from department stores. Purchase amounts are estimated using a Tobit regression model, and the number of visits is calculated using a Poisson regression model. A Tobit model considers bias in the explained variable's distribution. This study adopts a Type-1 Tobit model and a purchase amount

(the explained variable) of zero. This purchase amount is assumed to be censored data, and the latent variable is assumed to be less than zero. A Markov chain Monte Carlo (MCMC) method is used for the estimation. We assume that department stores' marketing measures (direct mail [DM] and events) affect purchase amounts and the number of visits. For the explanatory variable, we use the number of DMs sent to announce new customer items and privileges and schedules for events or clearance sales. In addition, because we assume that the number of visits affects purchase amounts, we incorporate it into the purchase amount model. If the number of visits greatly affects purchase amounts, then information about marketing measures that affect it is important. Furthermore, incorporating customer attributes into a hierarchical model clarifies their relationship with marketing measures. Ultimately, we aim to uncover effective marketing measures that increase customers' department store purchases.

The rest of this article is organized as follows. Section 2 provides an overview of existing research, and Section 3 details the data used in the analysis and the proposed model. Section 4 applies the model proposed in Section 3 to actual data, and Section 5 suggests marketing measures. Section 6 offers a summary and suggests future work.

2. Review of Prior Research

2.1. Research on CRM

This section provides an overview of prior CRM research, research related to marketing measures conducted at department stores, and research about hierarchical Bayes models. Many existing CRM studies have verified the effectiveness of reward points. For example, [26] showed that supermarket reward points increased purchases. By contrast, [12] opined that reward points did not affect the number of visits or purchase amounts. However, CRM activities are not limited to reward points [5]. [27] used panel data from financial service companies and customers' self-reported survey data to improve the effectiveness of DM. He also analyzed how reward points affected existing customers' share and retention. The results, which were verified by regression and probit models, showed that reward points increased customer share and retention. However, DM did not improve customer retention. Notably, this study did not verify the effectiveness of DM using CRM data. Additionally, since detailed DM models for financial products were not made, it is difficult to build a strategy after evaluating these models. [5] used a customer questionnaire at supermarkets to determine how reward points and DM affected customer share, purchase amounts, and customer satisfaction; they found that reward points improved all three of these criteria. However, since this model did not use panel or POS data, it is doubtful whether the actual measures were effective. Furthermore, neither [27] nor [5] model considered customers' heterogeneity. In the current retail environment, where consumer preferences are diversifying, it is important to understand the purchasing behavior of each consumer. These are issues in CRM research. Many CRM studies target supermarkets ([5], [13], [11], [26], and [22]), but few target department stores. Products sold at department stores, such as clothing [7], are different from those sold at supermarkets, such as food. Each product group requires a different marketing strategy. For example, because department store items are purchased infrequently, they have to attract consumers from a wide geographic area. When considering marketing measures, department store managers must consider consumers' willingness to visit stores or to increase their purchase amounts while considering the number of visits. An accurate evaluation of a model is not possible without considering store-specific marketing measures.

2.2. Marketing measures at Japanese department stores

[18] showed that “new items” and “points/privileges” were the main motivations used by department stores. In retail stores, DM is often used to increase customer purchases. [10] suggested that paper DM created more responses than e-mail. He also indicated a high reading and retention rate for DM, which led to improved purchase rates. Based on these studies, DM that informs customers about “new items” and “points/privileges” is considered an effective way to motivate customers in a large geographic area. Department stores also hold regular and planned events at event halls on buildings’ upper floors, such as cultural events and product exhibitions. Unlike department stores in Europe and the United States, Japanese department stores sponsor cultural events that are not directly aimed at selling products [23]. For instance, product exhibitions at the Hokkaido Exhibition are held at least five times a year to attract customers from a large geographic area [29]. Regular events include summer and year-end gift centers, which differ slightly depending on the region. Summer gift centers open in July, and year-end gift centers usually open in December [29]. The main purpose of these events is to increase a department store’s capability to attract customers and to encourage customers to travel to lower floors. A customer buying from an upper floor is called a “shower effect” [31], which makes possible higher customer purchases during a single visit. Department stores also hold discount sales, namely, “clearance sales,” mainly in July and January. [15] showed that these events improve customer attendance and product turnover. In Japan, DM campaigns that promise “new items” and “points/privileges,” as well as clearance sales, are still held. While these marketing measures are effective in department stores, it is necessary to verify comprehensively and quantitatively whether DM, events, and clearance sales increase either purchases or the number of customer visits.

2.3. Hierarchical bayes model

As mentioned above, in a commercial environment where consumer preferences are diversifying, it is necessary to use a model that incorporates each consumer’s purchase behavior. There are two ways to assess consumer heterogeneity: a latent class model and a random coefficient model [19]. Unlike cluster analysis, a latent class model is statistical and uses a segmentation method that incorporates model-based verification. Representative studies include those by [9] and [17]. A random coefficient model assumes that parameters follow a continuous probability distribution. It is possible to estimate each consumer’s parameters. With the development of the MCMC method, a random coefficient model can be applied to the logit and probit models, and a considerable amount of marketing research incorporates this approach. Previous studies have adopted a brand selection model (i.e., [4], [2], [1], and [34]). Estimated parameters in a brand selection model are important for manufacturers because they directly relate to sales. For retailers, however, changes in brand selection have little to do with sales and therefore are less important [6]. Rather, retailers should use a purchase model that includes sales for an entire retail store.

To understand consumer purchasing behavior better, it is necessary to model the latent factors behind consumers’ behavior. Consumer purchasing behavior cannot be clarified by analyzing manifest variables based solely on POS data [20]. For example, using the purchase and consumption of milk, [21] modeled domestic inventory as a latent variable and incorporated it into their purchase model. [14] modeled the number of supermarket purchase points by incorporating “mental burden” (namely, any adverse effects on the mind) as a latent variable related to cumulative purchase amounts on paydays. In addition, [30] incorporated a variable selection model into the number of department store visits while

considering the efficacy of consumer marketing measures. By considering latent factors, these studies led to a deeper understanding of consumer purchasing behavior. This study refers to [30] and incorporates a variable selection model into a purchase amount model and a number of visits model. Furthermore, we adopt a Tobit model (primarily used in the field of economics) as the explained variable in the purchase amount model. A Tobit model considers distribution bias in the explained variable. Distribution bias includes censored, truncated, and incidental truncation data. Censored data limit thresholds so the minimum value is zero, but zero is obtained as zero. Truncated data exclude zero information. Incidental truncation data are disconnected from the selection. A Type-1 Tobit model is used for censored and truncated forms, and a Type-2 Tobit model is used for incidental truncation [16]. In a Type-1 Tobit model, the explained variable is observed when it exceeds a certain level, but it is censored to zero when below a certain level. In a Type-2 Tobit model, the explained variable is observed when a condition is satisfied, but nothing is observed when the condition is not satisfied [32]. This study adopts a Type-1 Tobit model, and if the purchase amount (the explained variable) is zero, then we consider latent factors lower than zero as censored data.

3. Model

3.1. Data

In this research, we used POS data with IDs, marketing data about DM for new items, and privileges, events, and customer attribute data in department store “A” in Nagoya, Japan. The data were collected between April, 2008, and March, 2009. The number of customers in department store “A” totaled 157,616. Randomly, 5,000 people were selected and analyzed as customers. Figure 1 shows these customers’ average purchase amount and average number of visits, and Figure 2 shows the retail mix (number of DMs sent and number of events). Average purchase amount and average number of visits indicate the amount of money spent and the number of visits per month for each customer. The average number of DMs is DM pieces per month for each customer, and the number of events is per month at department store “A”. Regarding average monthly purchase price, the highest was 32,003 yen in April, followed by 27,577 yen in December and 27,518 yen in July. The average number of visits was 1.7 in April and December, followed by 1.6 in July. A positive correlation exists between purchase amounts and the number of visits. Regarding the average number of DMs sent, the largest was 3.4 per person in November, followed by 2.9 in April and 2.6 in September. For the number of events, November, the most frequent, had four, followed by three in July and January and two in October and December.

Table 1: Customer attributes

Gender	Distance(km.)	Heavy
Age	Holiday visit ratio	Middle
Outside customer	Ratio of visits after 4 PM	Food purchase amount ratio

Customer attribute data in Table 1 include gender (male 0, female 1), age, outside customer, and distance (km.) between department store “A” and a customer’s home. Outside customer is a customer who has an individual employee and has products delivered at home. We assume that customers’ purchasing habits differ in the evening (after 4 PM). The highest-spending 30% of customers are “heavy,” and the next 30% are “middle.” We also include purchasing variables, such as holiday visit ratio and food purchase amount ratio. The holiday visit ratio, the ratio of visits after 4 PM, the spending categories of

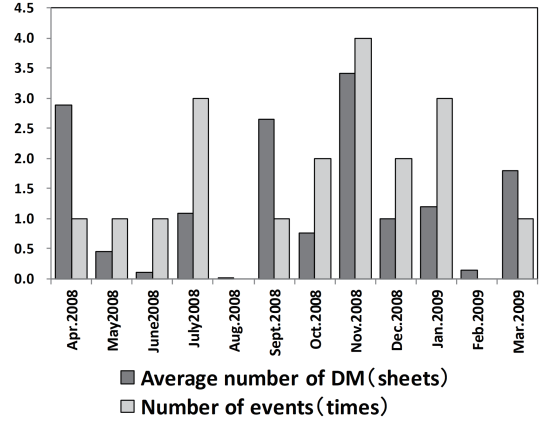
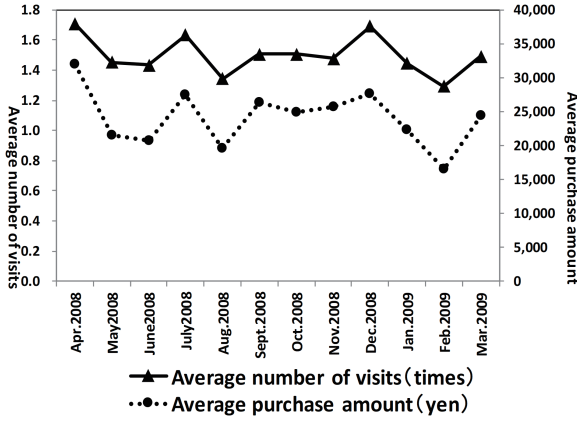


Figure 1: Average purchase amounts and average number of visits
 Figure 2: Average number of DM and average number of events

“heavy” and “middle”, and the food purchase amount ratio were collected from April, 2007, to March, 2008.

3.2. Individual model

In this study, we propose a simultaneous model of customer purchase amounts and store visits by using POS data with IDs, marketing data, and customer attribute data. In the following equations, the terms $i = (1, \dots, 5000)$ and $t = (1, \dots, 12)$ indicate the number of customers and the month, respectively. The simultaneous probability of the purchase amount y_{it1}^* and the number of visits y_{it2} is expressed by

$$f(y_{it1}^*, y_{it2} | \mathbf{x}_{it1}) = f_1(y_{it1}^* | y_{it2}, \mathbf{x}_{it1}) f_2(y_{it2} | \mathbf{x}_{it2}), \quad (3.1)$$

where

$$\mathbf{x}_{it1} = \left(1, DM_{it}^{(1)}, \dots, DM_{it}^{(8)}, EVE_t^{(1)}, \dots, EVE_t^{(7)}, y_{it2}\right), \text{ and}$$

$$\mathbf{x}_{it2} = \left(1, DM_{it}^{(1)}, \dots, DM_{it}^{(8)}, EVE_t^{(1)}, \dots, EVE_t^{(7)}\right).$$

$f_1(y_{it1}^* | y_{it2}, \mathbf{x}_{it1})$ is a Tobit regression model and $f_2(y_{it2} | \mathbf{x}_{it2})$ is a Poisson regression model.

3.2.1. Purchase amount model (Tobit regression model)

We adopt a Tobit model that considers bias in the distribution of purchase amounts (the explained variable). A Type-1 Tobit model is used, which assumes that the purchase amount zero, which is the explained variable, is censored data, and that there are latent variables behind zero. If the purchase price (in thousands of yen) y_{it1} exceeds zero, then it is observed as it is, but if y_{it1} is zero, then it is assumed that a negative value is censored to zero [32]. The situation is expressed by the following ([25], [33]):

$$y_{it1} = \begin{cases} 0, & (y_{it1}^* \leq 0), \\ y_{it1}^*, & (y_{it1}^* > 0). \end{cases} \quad (3.2)$$

Negative values censored to zero are generated based on the Appendix. We also assume that the explained variable y_{it1}^* can be determined by department store marketing activities (DM and events) and visits y_{it2} . The regression model is shown as follows (the superscript tp indicates transposition):

$$y_{it1}^* = \mathbf{x}_{it1}^{tp} \boldsymbol{\beta}_{1i} + u_{it}, \quad u_{it} | \mathbf{x}_{it1} \sim N(0, s^2). \quad (3.3)$$

Equation (3.3) can be written as

$$y_{it1}^* = \beta_i^{(0)} + \sum_{j=1}^8 \beta_i^{(j)} DM_{it}^{(j)} + \sum_{k=1}^7 \beta_i^{(k+8)} EVE_t^{(k)} + \beta_i^{(16)} y_{it2} + u_{it}, \quad (3.4)$$

where $DM_{it}^{(1)}, \dots, DM_{it}^{(8)}$ is the number of DMs sent to customer i in month t about a brand's goods, living, food, gentlemen, ladies, jewelry and watches, card holders' privileges, and storewide events. Card holders' privileges include special events, where customers can purchase items at a 10% discount. $EVE_t^{(1)}, \dots, EVE_t^{(7)}$ are summertime and year-end gifts, Hokkaido, foreign, and other exhibitions, cultural events, and clearance sales held by department store "A" in month t . Event variables are common to all customers. y_{it2} , the number of times customer i visited department store "A" in the month t , corresponds to the explained variable of the number of visits in the model, which will be described later. Incorporating number of visits into the purchase amount model clarifies the effect of store visits on purchase amounts. $\boldsymbol{\beta}_{1i} = (\beta_i^{(0)}, \dots, \beta_i^{(16)})^{tp}$ is the response parameter of customer i 's explanatory variables. The likelihood function is

$$L_i^{(1)}(\boldsymbol{\beta}_{1i}, \sigma^2 | \{\mathbf{y}_{it1}^*\}, \{\mathbf{x}_{it1}\}) = \prod_{t=1}^{12} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_{it1}^* - \mathbf{x}_{it1}^{tp} \boldsymbol{\beta}_{1i})^2}{2\sigma^2}\right). \quad (3.5)$$

3.2.2. Number of visits model (Poisson regression model)

We assume that the number of visits y_{it2} can be explained by department store marketing activities (DM and events), as described above. The mechanism of the Poisson regression model is expressed by

$$\Pr(Y_{it2} = y_{it2} | \lambda_{it}) = \frac{\lambda_{it}^{y_{it2}} \exp(-\lambda_{it})}{y_{it2}!}, \quad (3.6)$$

where $\lambda_{it} (> 0)$ is a parameter indicating mean and variance. The likelihood function is

$$L_i^{(2)}(\boldsymbol{\beta}_{2i} | \{\mathbf{y}_{it2}\}, \{\mathbf{x}_{it2}\}) = \prod_{t=1}^{12} \frac{\lambda_{it}^{y_{it2}} \exp(-\lambda_{it})}{y_{it2}!}. \quad (3.7)$$

$\boldsymbol{\beta}_{2i} = (\beta_i^{(17)}, \dots, \beta_i^{(32)})^{tp}$ is customer i 's response parameter. Similar to the purchase amount model, we assume that the logarithm of $\lambda_{it} (> 0)$ can be explained by the number of DMs sent to customer i in month t and the events held in month t at department store "A". The structure is shown as follows:

$$\log(\boldsymbol{\lambda}_{it}) = \beta_i^{(17)} + \sum_{j=1}^8 \beta_i^{(j+17)} DM_{it}^{(j)} + \sum_{k=1}^7 \beta_i^{(k+25)} EVE_t^{(k)}. \quad (3.8)$$

Equation (3.8) can be written as

$$\boldsymbol{\lambda}_{it} = \exp(\mathbf{x}_{it2} \boldsymbol{\beta}_{2i}). \quad (3.9)$$

In this study, we assume a common mechanism among customers behind the response parameter $\beta_i = \left(\beta_i^{(0)}, \dots, \beta_i^{(32)}\right)^{tp}$ and set it as a hierarchical model. Explanatory variables used in the model are customer attribute data $\mathbf{Z}_i = (z_{i1}, \dots, z_{i9})^{tp}$. The hierarchical model is expressed by the following:

$$\beta_i = \boldsymbol{\theta}^{tp} \mathbf{Z}_i + \boldsymbol{\varepsilon}_i, \quad \boldsymbol{\varepsilon}_i \sim N(\mathbf{0}, \boldsymbol{\Sigma}), \quad (3.10)$$

where $\boldsymbol{\theta}$ is the coefficient matrix (9 rows \times 33 columns), $\boldsymbol{\varepsilon}_i$ is the error term vector (33 rows \times 1 column), and $\boldsymbol{\Sigma}$ is the variance covariance matrix (33 rows \times 33 columns).

3.2.3. Model for variable selection

We use a model for variable selection that references [30]. We incorporate a structure that determines whether or not to include, for each customer, the constant term, $DM_{it}^{(1)}, \dots, EVE_t^{(7)}, y_{it2}$ in Equation (3.4) and the constant term, $DM_{it}^{(1)}, \dots, EVE_t^{(7)}$ in Equation (3.8). By incorporating a model for variable selection into the individual model, it is possible to grasp a structure where the explanatory variable affects the purchase amount and where the number of visits differs for each customer. Specifically, a variable selection vector $\mathbf{I}_i = \left(I_i^{(0)}, \dots, I_i^{(32)}\right)$ indicates whether or not the model includes explanatory variables for each customer. Each component of \mathbf{I}_i is 1 if the explanatory variable is included in the model and 1.0×10^{-4} otherwise [3]. By using \mathbf{I}_i , the purchase amount model can be expressed as

$$y_{it1}^* = \beta_i^{(0)} I_i^{(0)} + \sum_{j=1}^8 \beta_i^{(j)} I_i^{(j)} DM_{it}^{(j)} + \sum_{k=1}^7 \beta_i^{(k+8)} I_i^{(k+8)} EVE_t^{(k)} + \beta_i^{(16)} I_i^{(16)} y_{it2} + u_{it}. \quad (3.11)$$

And the number of visits model can be expressed as

$$\log(\lambda_{it}) = \beta_i^{(17)} I_i^{(17)} + \sum_{j=1}^8 \beta_i^{(j+17)} I_i^{(j+17)} DM_{it}^{(j)} + \sum_{k=1}^7 \beta_i^{(k+25)} I_i^{(k+25)} EVE_t^{(k)}. \quad (3.12)$$

In the above, the hierarchical model is shown in Equation (3.10). However, when the model for variable selection is incorporated, $\mathbf{C}_i = \text{diag}\left(I_i^{(0)}, \dots, I_i^{(32)}\right)$ is formed based on the variable selection vector \mathbf{I}_i and $\beta_i^* = \mathbf{C}_i^{-1} \beta_i$. Additionally, the hierarchical model is set for β_i^* instead of β_i . The hierarchical model of β_i^* is

$$\beta_i^* \sim N(\mathbf{C}_i \boldsymbol{\theta}^{tp} \mathbf{Z}_i, \mathbf{C}_i \boldsymbol{\Sigma} \mathbf{C}_i). \quad (3.13)$$

3.3. Model's illustration and estimation method

Figure 3 shows the proposed model's concept, as in the previous section. The purchase amount model is used when the purchase amount, which is the explained variable, exceeds zero, but it is censored to zero when negative. Moreover, it is characterized to include explanatory variables in the purchase amount model, and the number of visits model is determined by the variable selection vector \mathbf{I}_i . The proposed model is estimated by the MCMC method.

In addition, independent chain M-H sampling is used to sample β_{1i}, β_{2i} , and \mathbf{I}_i . Gibbs sampling is used for the parameter $\boldsymbol{\theta}, \boldsymbol{\Sigma}$ of the hierarchical model. The MCMC is repeated 15,000 times, with the first 10,000 times being the burn-in period. Details of the model are given in the Appendix.

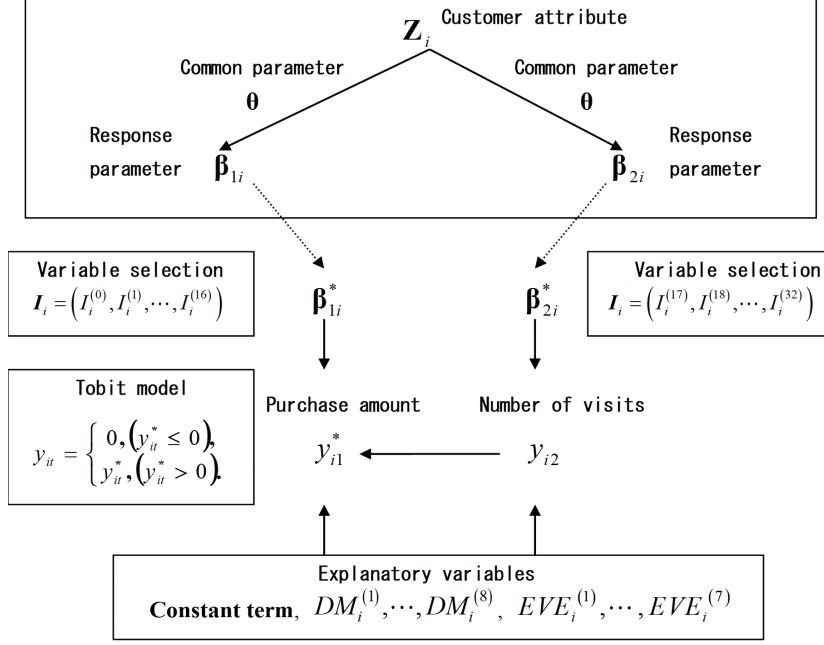


Figure 3: Conceptual model

4. Analytical Results

4.1. Comparing the models

This section verifies the description and prediction capability of the proposed model. Specifically, the accuracy of the three models (the constant term model, the model without variable selection, and the proposed model) is compared. Table 2 shows the results of DIC (deviance information criterion). Like AIC (Akaike information criterion), DIC is used as an index for evaluating the degree of fit to data [24]. The smaller the value, the better the fit of the model to the data. Since DIC can use the chain elements of Markov chain as they are, it has good compatibility with MCMC. Therefore, in this paper, DIC is used as a criterion for model comparison. The DIC was 240.790 for the constant term model, 64.069 for the model without variable selection, and 58.818 for the proposed model. This shows that the descriptive capability of the proposed model is relatively high for all three models. Next, we calculated the model's mean absolute percentage error (MAPE) without variable selection and the proposed model's prediction capability. MAPE, which uses verification data from April, 2009, to March, 2010, was calculated based on the procedures in Equations (4.1) to (4.3).

First, the prediction error at time t for customer i at iteration number r is given by

$$e_{it}^{(r)} = y_{it} - \hat{y}_{it}^{(r)}, \quad (4.1)$$

where y_{it} are the actual values of customers i at time t , and $\hat{y}_{it}^{(r)}$ is the predicted value for r iterations. The error rate $pe_{it}^{(r)}$ is expressed by

$$pe_{it}^{(r)} = \frac{e_{it}^{(r)}}{y_{it}} \times 100. \quad (4.2)$$

And MAPE at customer i and time t is calculated as

$$\text{MAPE}_{it} = \frac{1}{R} \sum_{r=1}^R |pe_{it}^{(r)}|, \quad (4.3)$$

where R is the total number of repetitions, excluding burn-in samples. Table 3 shows the average for customers in MAPE. The total MAPE average at customer and time points was 181.372 for the model without variable selection and 165.707 for the proposed model. The predictive capability of the proposed model seems to be higher than that of the model without variable selection. Figure 4 shows the distribution of MAPE for each customer and each time. The box frame is from the first (25%) quantile to the third (75%) quantile, and the center line of the box is the median. The closer the distribution to zero, the higher the prediction capability. In the proposed model, the distribution of MAPE is stable throughout the year. However, the model without variable selection is largely distributed in April and June, 2009. This difference increases the average MAPE value of the model without variable selection. From these results, we judged that the description and predictive capability of the proposed model was better than that of other models (the constant term model and the model without variable selection). In the following discussion, we explain the estimation results of the proposed model.

Table 2: DIC comparison

	Explanatory Variable	Variable Selection	DIC
Constant term model	1	No	240.790
Model without variable selection	33	No	64.069
Proposed model	33	Yes	58.818

Table 3: Average value of MAPE

	April 2009	May 2009	June 2009	July 2009	Aug.2009	Sept.2009
Model without variable selection	181.577	173.799	310.388	170.210	160.077	184.730
	Oct.2009 164.679	Nov.2009 163.653	Dec.2009 170.322	Jan.2010 172.418	Feb.2010 161.997	Mar.2010 162.620
Proposed model	April 2009 174.085	May 2009 162.524	June 2009 168.101	July 2009 170.451	Aug.2009 154.016	Sept.2009 171.566
	Oct.2009 167.769	Nov.2009 169.693	Dec.2009 167.106	Jan.2010 171.484	Feb.2010 152.890	Mar.2010 158.798

4.2. Evaluation of structural heterogeneity

To evaluate the model's structural heterogeneity, we incorporated a variable selection model into the individual model. In this section, we show the results of a variable selection pattern verification, performed after the model's estimation. For iterations between 10,001 and 15,000, 50% (2,500) or more were adopted for each element of the variable selection vector $\mathbf{I}_i = (I_i^{(0)}, \dots, I_i^{(32)})$, replaced by 1 for each customer. If less than 1, then it was replaced by zero. A pattern matrix for each customer was created. In addition, we conducted a correspondence analysis using a pattern matrix to confirm whether or not a similarity exists in the variable selection patterns of customers. Table 4 shows the results of a correspondence analysis. When the contribution ratio of the extracted 15 axes is confirmed, the features of the pattern matrix are not captured by the minority axis. This result suggests that the individual model has strong structural heterogeneity among customers. To implement marketing measures that consider customer heterogeneity from a CRM perspective, it is necessary to confirm the model's structure for each customer. There is no reason to

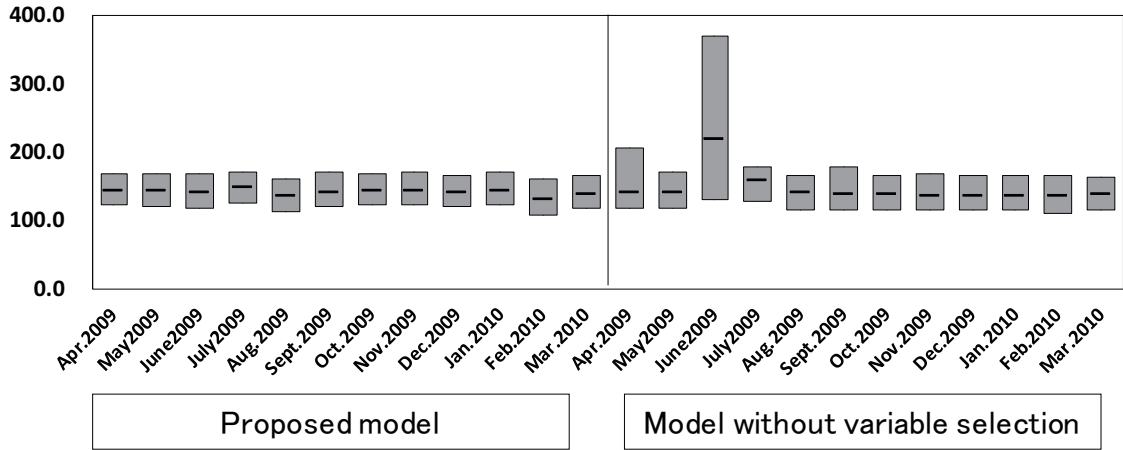


Figure 4: Distribution of MAPE for each customer

adopt 50% in creating the pattern matrix, because it is used for simple evaluation. In the following, we show verification results after evaluating the structural heterogeneity of the individual model.

Table 4: Correspondence analysis contribution rate

	1st Axis	2nd Axis	3rd Axis	4th Axis	5th Axis	6th Axis	7th Axis	8th Axis
Contribution rate	8.7%	7.8%	7.2%	7.1%	7.0%	6.8%	6.6%	6.5%
Cumulative contribution rate	8.7%	16.5%	23.7%	30.8%	37.8%	44.6%	51.2%	57.6%
	9th Axis	10th Axis	11th Axis	12th Axis	13th Axis	14th Axis	15th Axis	*
Contribution rate	6.3%	6.3%	6.2%	6.0%	6.0%	5.9%	5.8%	*
Cumulative contribution rate	64.0%	70.3%	76.4%	82.4%	88.3%	94.2%	100.0%	*

4.3. Results of the purchase amount model

Table 5 shows the estimation results of the posterior statistics of the response parameter β_i in the purchase amount model (Tobit regression model). The posterior average is the average value of each response parameter for each customer with a history of 10,001 to 15,000 iterations.

The highest number of visits is 13.187, which means that if the number of visits by customers increases, the purchase amount also can be increased. Among DMs, jewelry and watch are the highest (2.085), followed by privileges of card holders (1.885) and living (1.319). Among events, year-end gift is the highest (1.169), followed by cultural events (1.139), foreign exhibitions (1.123), and clearance sales (1.053). These events increase purchase amounts. Figure 5 illustrates the estimated values related to marketing measures among Table 5's response parameters, with the average posterior range for each customer in the distribution. In Figure 5, the box frame is from the first (25%) quantile to the third (75%) quantile. The center line in the box is the median, and the top and bottom are the 95% quantile and 5% quantile, respectively. As mentioned above, jewelry and watch DM, privileges of card holders DM, living DM, year-end gift, cultural events, foreign exhibitions, and clearance sales influence purchase amounts.

Nakayama and Tsurumi [18] showed that DM, which informs customers about “new items” and “points/privileges,” is an effective way to increase purchasing motivation at department stores. This study's results quantitatively substantiated Nakayama and Tsurumi's results. However, “new items” were limited to jewelry and watch DM and living DM. In

Table 5: Estimated response parameters (purchase amount model)

	Posterior Average	95% Quantile	3rd Quantile	Median	1st Quantile	5% Quantile
Constant term	0.868	2.183	1.151	0.683	0.391	0.134
Brand goods DM	0.787	1.492	1.018	0.725	0.500	0.261
Living DM	1.319	2.421	1.762	1.283	0.827	0.336
Food DM	-1.119	0.020	-0.601	-1.149	-1.626	-2.233
Gentlemen DM	-0.225	0.094	-0.084	-0.225	-0.370	-0.553
Ladies DM	0.378	0.899	0.569	0.311	0.162	-0.005
Jewelry and watch DM	2.085	3.732	2.757	2.040	1.353	0.587
Privileges DM	1.885	3.616	2.527	1.799	1.140	0.449
Storewide events DM	-1.806	-0.157	-1.030	-1.823	-2.535	-3.543
Summer gift	0.057	0.465	0.197	0.039	-0.106	-0.311
Year-end gift	1.169	2.134	1.548	1.122	0.753	0.331
Hokkaido exhibition	-1.431	-0.081	-0.829	-1.451	-2.014	-2.785
Other exhibition	0.472	1.022	0.706	0.417	0.239	0.034
Foreign exhibition	1.123	2.088	1.500	1.081	0.703	0.270
Culture event	1.139	2.254	1.504	1.048	0.694	0.314
Clearance sale	1.053	1.954	1.412	1.033	0.657	0.231
Number of visits	13.187	23.601	17.747	13.228	8.353	3.089

addition to events held at event halls, year-end gift, cultural event, foreign exhibition, and clearance sales also increase purchase amounts.

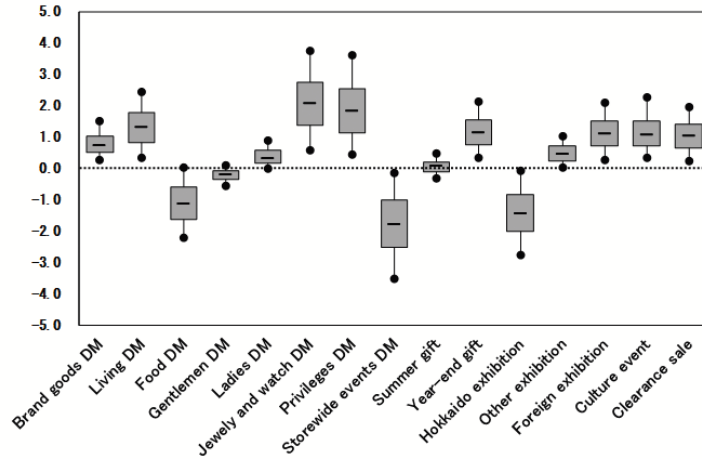


Figure 5: Distribution of posterior averages for response parameters (purchase amount model)

4.4. Number of visits model

Table 6 shows the results of posterior statistics for response parameter β_i in the store visit model (Poisson regression model). Similar to Section 4.3, this is an average value for each customer with 10,001 to 15,000 iterations. Clearance sales (0.052) are the highest, followed by living DM (0.034), year-end gift (0.020), and privileges of card holders DM (0.018). Clearance sales, living DM, year-end gift, and privileges of card holders DM have a positive impact on the number of visits.

Figure 6 shows the estimated values of the response parameters. The definitions of box frame, center line, top, and bottom are the same as in Figure 5. Privileges of card holders and living for DM events, as well as year-end gift and clearance sales, are distributed in a relatively positive area. However, some customers' reactions are different. Prior research has shown that DM informing customers about “product trends” and “points/privileges”

generally motivates store visits, but it decreases visits for some customers. It is necessary to grasp which plan attracts customers and to implement more individualized marketing measures.

Table 6: Estimated response parameters (number of visits model)

	Posterior Average	95% Quantile	3rd Quantile	Median	1st Quantile	5% Quantile
Constant term	0.039	1.347	0.419	-0.011	-0.380	-1.108
Brand goods DM	-0.072	0.193	0.037	-0.066	-0.186	-0.354
Living DM	0.034	0.222	0.111	0.038	-0.043	-0.175
Food DM	-0.045	0.101	0.019	-0.032	-0.102	-0.233
Gentlemen DM	-0.097	0.221	0.037	-0.096	-0.243	-0.436
Ladies DM	-0.021	0.173	0.054	-0.017	-0.100	-0.225
Jewelry and watch DM	-0.180	0.121	-0.040	-0.170	-0.314	-0.526
Privileges DM	0.018	0.190	0.080	0.016	-0.045	-0.159
Storewide events DM	-0.024	0.079	0.018	-0.012	-0.059	-0.164
Summer gift	-0.107	0.100	-0.006	-0.096	-0.202	-0.364
Year-end gift	0.020	0.191	0.085	0.022	-0.045	-0.158
Hokkaido exhibition	-0.022	0.167	0.055	-0.012	-0.093	0.250
Other exhibition	-0.210	0.009	-0.078	-0.173	-0.307	-0.570
Foreign exhibition	-0.033	0.112	0.031	-0.022	-0.089	-0.215
Culture event	-0.005	0.146	0.056	0.003	-0.061	-0.180
Clearance sale	0.052	0.283	0.135	0.048	-0.032	-0.164

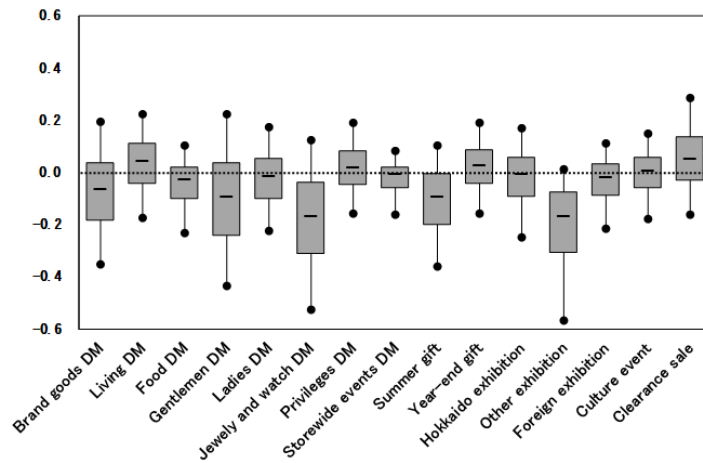


Figure 6: Distribution of posterior averages for response parameters (number of visits model)

When the response parameter is negative in the purchase amount and number of visits model, it indicates that marketing measures reduce purchase amounts and visits. If the parameters of the purchase amount model are negative, although DM and events effectively promote purchases, then the purchase price could not be increased because of low prices. If the parameters for the number of visits model are negative, then these measures possibly did not match customer preferences and therefore did not promote visits. However, this requires further analysis.

4.5. Results of common parameters

Table 7 estimates the common parameter corresponding to response parameter β_{1i} in the purchase amount model, and Table 8 estimates the common parameter corresponding to response parameter β_{2i} in the number of visits model. By confirming the relationship between reaction and common parameters, we can understand the relationship between

marketing measures and customer attributes. The highest posterior density (HPD) interval is used to evaluate significance. For each θ , the 95% HPD interval (a, b) that satisfies $\Pr(a < \theta < b | \{\beta_i\}, \{Z_i\}) = 0.95$ is calculated. If the 95% HPD interval excludes zero, then the relevant explanatory variable significantly affects the explained variable (underlined) [21].

Table 7: Estimated common parameters(purchase amount model)

	Constant term	Brand Goods DM	Living DM	Food DM	Gentlemen DM	Ladies DM
Gender	<u>0.864</u>	<u>0.392</u>	<u>0.323</u>	<u>-0.381</u>	<u>-0.119</u>	<u>0.005</u>
Age	<u>0.029</u>	<u>0.007</u>	<u>0.020</u>	-0.019	-0.003	0.005
Outside customer	0.447	<u>0.327</u>	0.059	-0.099	0.107	-0.191
Distance	<u>0.057</u>	<u>0.019</u>	<u>0.032</u>	<u>-0.027</u>	<u>-0.003</u>	<u>0.011</u>
Holiday visit ratio	<u>1.723</u>	0.125	<u>0.950</u>	<u>-1.125</u>	<u>-0.263</u>	0.091
Ratio of visits after 4 PM	<u>0.309</u>	0.114	0.034	-0.082	<u>-0.095</u>	<u>0.194</u>
Heavy	1.384	0.575	0.690	<u>0.374</u>	0.137	<u>0.678</u>
Middle	0.068	-0.069	0.028	0.370	<u>0.044</u>	-0.009
Food purchase amount ratio	<u>-2.262</u>	<u>-0.433</u>	<u>-1.486</u>	<u>1.543</u>	<u>0.167</u>	<u>-0.309</u>
	Jewelry and Watch DM	Privileges DM	Storewide Events DM	Summer Gift	Year-End gift	Hokkaido Exhibition
Gender	<u>0.686</u>	<u>0.668</u>	-0.246	<u>0.227</u>	0.385	-0.473
Age	<u>0.027</u>	<u>-0.030</u>	<u>-0.031</u>	<u>-0.009</u>	0.014	-0.017
Outside customer	0.150	0.401	0.044	0.406	0.239	<u>-0.260</u>
Distance	<u>0.046</u>	<u>-0.055</u>	<u>-0.039</u>	-0.002	0.020	<u>-0.036</u>
Holiday visit ratio	<u>1.416</u>	1.178	<u>-1.335</u>	-0.167	0.954	<u>-1.434</u>
Ratio of visits after 4 PM	<u>0.443</u>	0.030	<u>-0.303</u>	<u>0.120</u>	0.177	<u>-0.302</u>
Heavy	0.971	<u>0.777</u>	0.339	0.403	<u>0.723</u>	0.427
Middle	-0.115	-0.100	<u>0.375</u>	<u>0.138</u>	0.094	0.233
Food purchase amount ratio	<u>-2.116</u>	<u>-2.070</u>	2.005	0.497	<u>-1.010</u>	<u>1.562</u>
	Other Exhibition	Foreign Exhibition	Culture Event	Clearance Sale	Number of Visits	*
Gender	0.133	<u>0.601</u>	<u>0.520</u>	<u>0.379</u>	<u>2.653</u>	*
Age	0.008	<u>0.014</u>	<u>0.012</u>	<u>0.018</u>	0.126	*
Outside customer	<u>-0.180</u>	<u>0.134</u>	0.227	0.110	1.022	*
Distance	<u>0.003</u>	0.028	<u>0.028</u>	<u>0.026</u>	<u>0.205</u>	*
Holiday visit ratio	0.135	<u>0.594</u>	<u>0.575</u>	<u>0.747</u>	<u>6.523</u>	*
Ratio of visits after 4 PM	<u>-0.126</u>	0.226	0.320	0.161	1.055	*
Heavy	<u>0.907</u>	0.661	1.273	0.391	2.718	*
Middle	<u>0.302</u>	0.082	0.202	<u>-0.107</u>	-0.360	*
Food purchase amount ratio	<u>-0.494</u>	<u>-1.479</u>	<u>-1.035</u>	<u>-1.560</u>	<u>-9.415</u>	*

Notice the common parameters of the purchase amount model. We confirm the relationship with customer attributes by focusing on living DM, jewelry and watch DM, privileges of card holders DM, year-end gift, foreign exhibitions, cultural events, and clearance sales. Women and the elderly respond positively to living DM, jewelry and watch DM, year-end gift, foreign exhibitions, cultural events, and clearance sales. If the “distance between stores” is relatively large, then responses to living DM, jewelry and watch DM, cultural events, and clearance sales will increase. If the “holiday visit ratio” is high, then customers are more likely to respond to living DM, jewelry and watch DM, foreign exhibitions, cultural events, and clearance sales. Customers who come to the store after 4 PM respond well to jewelry and watch DM. If the “food purchasing amount ratio” is low, the responses to living DM, jewelry and watch DM, privileges of card holders DM, year-end gift, foreign exhibitions, cultural events, and clearance sales increase. Next, we confirm the common parameter of the number of visits model. In DM, we see a relationship between the living DM or privileges of card holders DM and customer attributes. During events, we see a relationship between year-end gift or clearance sale and customer attributes. In “gender,” males respond more positively to living DM and privileges of card holders DM. As “age” rises, privileges of card holders DM and clearance sale responses increase. The longer the

Table 8: Estimated common parameter(number of visits model)

	Constant term	Brand Goods DM	Living DM	Food DM	Gentlemen DM	Ladies DM
Gender	<u>0.163</u>	<u>0.143</u>	<u>-0.074</u>	-0.043	0.044	<u>0.082</u>
Age	<u>-0.008</u>	-0.002	0.002	-0.001	<u>-0.007</u>	<u>0.002</u>
Outside customer	0.020	0.047	<u>-0.101</u>	0.039	-0.050	<u>-0.088</u>
Distance	<u>-0.023</u>	0.000	0.001	0.000	<u>-0.003</u>	-0.002
Holiday visit ratio	<u>-0.541</u>	<u>-0.394</u>	<u>-0.126</u>	<u>-0.055</u>	<u>-0.199</u>	-0.069
Ratio of visits after 4 PM	<u>0.097</u>	<u>-0.224</u>	0.068	<u>-0.081</u>	<u>0.179</u>	-0.067
Heavy	0.962	<u>0.130</u>	<u>0.183</u>	<u>0.107</u>	<u>0.276</u>	-0.036
Middle	<u>0.537</u>	0.036	<u>0.116</u>	<u>0.048</u>	<u>0.119</u>	<u>0.103</u>
Food purchase amount ratio	1.398	<u>0.353</u>	-0.019	<u>-0.094</u>	<u>0.452</u>	<u>0.156</u>
	Jewelry and Watch DM	Privileges DM	Storewide Events DM	Summer Gift	Year-End gift	Hokkaido Exhibition
Gender	<u>0.097</u>	<u>-0.062</u>	<u>0.061</u>	<u>-0.086</u>	-0.001	<u>0.084</u>
Age	-0.002	<u>0.002</u>	<u>-0.002</u>	<u>-0.003</u>	0.001	0.000
Outside customer	-0.042	-0.049	0.032	0.023	0.045	-0.030
Distance	<u>-0.003</u>	<u>0.003</u>	0.001	-0.001	<u>0.003</u>	0.002
Holiday visit ratio	<u>-0.590</u>	-0.011	<u>-0.117</u>	<u>-0.216</u>	-0.051	<u>-0.170</u>
Ratio of visits after 4 PM	<u>-0.122</u>	<u>0.069</u>	0.021	0.016	<u>-0.138</u>	<u>-0.107</u>
Heavy	0.036	<u>-0.078</u>	0.032	<u>0.176</u>	-0.009	0.012
Middle	0.032	-0.045	0.036	<u>0.125</u>	0.020	-0.017
Food purchase amount ratio	<u>0.262</u>	<u>-0.131</u>	-0.081	<u>0.169</u>	-0.035	<u>-0.108</u>
	Other Exhibition	Foreign Exhibition	Culture Event	Clearance Sale	*	*
Gender	<u>0.136</u>	-0.030	<u>-0.098</u>	0.067	*	*
Age	<u>-0.009</u>	0.001	0.001	<u>0.004</u>	*	*
Outside customer	0.007	-0.015	<u>-0.093</u>	-0.035	*	*
Distance	<u>-0.006</u>	<u>0.002</u>	-0.001	<u>0.006</u>	*	*
Holiday visit ratio	-0.108	<u>-0.247</u>	0.124	<u>-0.170</u>	*	*
Ratio of visits after 4 PM	<u>-0.155</u>	-0.037	0.008	-0.025	*	*
Heavy	0.022	0.045	0.030	<u>-0.102</u>	*	*
Middle	0.017	<u>0.088</u>	0.014	-0.110	*	*
Food purchase amount ratio	0.218	-0.078	0.052	<u>-0.501</u>	*	*

“distance between stores,” the higher the responses of privileges of card holders DM, year-end gift, and clearance sales. Customers with a low “holiday visit ratio” respond well to living DM and clearance sales. Customers who come to the store after 4 PM have a higher response to privileges of card holders DM but a lower response to year-end gift. Customers with a low “food purchasing amount ratio” are more likely to respond to privileges of card holders DM and clearance sales. In addition, in the purchase amount model, the number of visits by women who travel a long distance to a store and who purchase mainly nonfood items heightens purchase prices. If the attribute affecting the number of visits in the purchase amount model and the attribute affecting marketing in the number of visits model are the same, then the number of visits and purchased amounts increase. For example, if a customer travels to a store, purchases nonfood items, and is informed of privileges of card holders, then he/she will be encouraged to visit department store “A”, whose purchase prices then increase.

5. Marketing Suggestions

Figure 7 shows how effective each measure is for purchase amounts and number of visits, combined with response parameters from the purchase amount and number of visits models in living DM, privileges of card holders DM, year-end gift, and clearance sales. In Figure 7, “significant+” means that the response parameter is significant and positive, and “other” means that the response parameter is either insignificant or negative. To verify significance, we use the 95% HPD interval shown in Section 4.5. We do not evaluate customers whose response parameter for number of visits in the purchase amount model is insignificant or

negative, even if measures, such as living DM, are affected in the store visit model. Department stores cannot expect sustained growth unless they implement marketing measures that increase purchase amounts. The main way to accomplish this is to cross-sell and upsell [8]. Cross-selling, which encourages customers to make purchases across different categories, aims to increase customers' shares by promoting additional buying. The goal in upselling is for customers to purchase more expensive products by stimulating demand for high-quality products and high-end services.

In this section, we focus on customers and marketing measures that influence purchase amounts. In the verification results of living DM, 78 customers (1.6%) have a significant impact on the purchase amount and the number of visits. By sending DM to such customers, purchase amounts and number of visits can be increased. Mailing a living room catalog and using DM to encourage purchases of high-priced products are effective strategies. Proposing related or high-priced products through customer service is also helpful. A total of 150 customers significantly affect purchase amounts (3.0%). These efforts, which are conducted on the Internet or by DM, do not require customers to enter a store. In the verification results of the card preferential treatment DM, 72 customers (1.4%) are affected by the purchase price and the number of visits. For living DM, sending catalogs and providing customer service effectively induce purchases of high-priced products. A total of 202 customers (4.0%) sensitive to purchase amounts need to be notified of targeted sales on the Internet. For year-end gift DM, 63 customers (1.3%) are affected by the purchase amount and the number of visits. These customers' purchase amounts and number of visits can be increased by holding year-end gift events. At storefronts, customer sales and coupons can be used to induce buying of higher-priced products. Internet orders for year-end gifts are effective for 142 customers (2.8%), who also impact the purchase amount. Regarding clearance sales, 64 customers (1.3%) have an impact on the purchase amount and the number of visits. Since clearance sales increase the purchase price and the number of visits, it is necessary to distribute coupons and to promote a wide assortment of products. Internet clearance sales are effective for 142 customers (2.8%), who also influence the purchase amount.

		Living DM						Privileges DM			
		Purchase amount						Purchase amount			
Number of visits		Other		Significant+		Number of visits		Other		Significant+	
		Number	Ratio	Number	Ratio		Number	Ratio	Number	Ratio	
	Other	4,754	95.1%	150	3.0%		Other	4,709	94.2%	202	4.0%
	Significant+	18	0.4%	78	1.6%		Significant+	17	0.3%	72	1.4%

		Year-end gift						Clearance sale			
		Purchase amount						Purchase amount			
Number of visits		Other		Significant+		Number of visits		Other		Significant+	
		Number	Ratio	Number	Ratio		Number	Ratio	Number	Ratio	
	Other	4,781	95.6%	142	2.8%		Other	4,782	95.6%	142	2.8%
	Significant+	14	0.3%	63	1.3%		Significant+	12	0.2%	64	1.3%

Figure 7: DM and events that affect purchase amounts and number of visits

6. Summary and Future Work

In this study, we use POS data with IDs in department stores, marketing data, and customer attributes' data. We analyze them simultaneously, using a hierarchical Bayes model, to elucidate the mechanisms behind customer purchase amounts and store visits. To create

an individual model, we assume that a department store's marketing measures impact the purchase amounts and number of visits. Number of visits, which are assumed to affect the purchase amounts, are included in the explanatory variable. The purchase amount model uses a Tobit regression model, and the number of visits model uses a Poisson regression model. Each individual model incorporates variable selection to evaluate structural heterogeneity. These are academic contributions to the field of marketing. In marketing practice, it is important to continuously understand the customer's purchasing behavior and take measures. This is because customer behavior changes [29]. The proposed model of this paper can grasp the purchasing behavior of each customer by using the POS data accumulated every day and consider the marketing measures to increase the purchase amount and the number of store visits. This is a contribution part of this paper to marketing practice.

We note here two issues that require additional study. The first is the one-year time span of the model's data (April, 2008, to March, 2009). Customer share is better evaluated over longer periods of time. Lifetime customer data that accumulate sales and profit information are optimal. For this reason, a model with only one year of data is insufficient. The second issue is that the data were obtained during the 2008 global economic crisis (after September, 2008). In this period, the effectiveness of marketing measures may have been lower than usual. Therefore, additional research is needed.

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Appendix

A.1. Algorithms Used for Estimations

In the proposed model, Equation (3.1) in Section 3 can be expressed as

$$\begin{aligned}
 f(y_{it1}^*, y_{it2} | \mathbf{x}_{it1}) &= f_1(y_{it1}^* | y_{it2}, \mathbf{x}_{it1}) f_2(y_{it2} | \mathbf{x}_{it2}) \\
 &= \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(- \frac{\left(y_{it1}^* - \left(\beta_i^{(0)} I_i^{(0)} + \sum_{j=1}^8 \beta_i^{(j)} I_i^{(j)} + \sum_{k=1}^7 \beta_i^{(k+8)} I_i^{(k+8)} EV E_t^{(k)} + \beta_i^{(16)} I_i^{(16)} y_{it2} \right) \right)^2}{2\sigma^2} \right) \right) \\
 &\times \left(\exp \left(\beta_i^{(17)} I_i^{(17)} + \sum_{j=1}^8 \beta_i^{(j+17)} I_i^{(j+17)} DM_{it}^{(j)} + \sum_{k=1}^7 \beta_i^{(k+25)} I_i^{(k+25)} EV E_t^{(k)} \right) \right)^{y_{it2}} \\
 &\times \exp \left(- \exp \left(\beta_i^{(17)} I_i^{(17)} + \sum_{j=1}^8 \beta_i^{(j+17)} I_i^{(j+17)} DM_{it}^{(j)} + \sum_{k=1}^7 \beta_i^{(k+25)} I_i^{(k+25)} EV E_t^{(k)} \right) \right) / y_{it2}!,
 \end{aligned}$$

where the first formula is the purchase amount model and the latter two are the number of visits models.

The prior distribution of σ is set as follows:

$$\sigma^2 \sim IG(r_0/2, \sigma_0/2), r_0 = 0.001, \sigma_0 = 0.001.$$

The prior distribution of θ, Σ , and $s^{(j)}$ is set as follows :

$$\theta \sim N(\boldsymbol{\mu}_{01}, \mathbf{M}_{01}), \boldsymbol{\mu}_{01} = \mathbf{0}, \mathbf{M}_{01} = f_{01} \mathbf{E}_q,$$

$$\Sigma \sim IW(f_{01}, \mathbf{F}_{01}), f_{01} = K + 3, \mathbf{F}_{01} = 0.001 \mathbf{E}_K, K = 33, \text{ and}$$

$$s^{(j)} \sim Beta(c, d), c = 800, d = 400.$$

A.2.Explained Variables in a Tobit Model

If y_{it1} is observed as zero, then y_{it1}^* is generated as follows :

$$y_{it1}^* | \beta_i, \sigma^2 \sim N_{(-\infty, 0]}(x_{it}^{tp} \beta_i, \sigma^2), \text{ if } y_{it1} = 0.$$

A.3.Sampling β_i, \mathbf{I}_i

$\beta_i, \mathbf{I}_i | \bullet$ is sampled by the independent chain MH algorithm. \bullet is a symbol that omits other parameters and data. In particular, sampling is performed based on the following :

(1) $I_i^{(j)(*)} = 1$, if $u_i^{[1](j)} \leq s^{(j)}$: $I_i^{(j)(*)} = 1.0 \times 10^{-4}$, otherwise generate $I_{ij}^{(*)}$, and set $\mathbf{I}_i^{(*)}$. $u_i^{[1](j)}$ is generated based on uniform random numbers, and $s^{(j)}$ is generated based on A.6.

(2) Based on $\mathbf{I}_i^{(*)}$, set $\mathbf{C}_i^{(*)} = \text{diag}(I_i^{(0)(*)}, \dots, I_i^{(32)(*)})$.

(3) $\beta_i^{(*)}$ is generated from $N(\mathbf{C}_i^{(*)} \boldsymbol{\theta} \mathbf{Z}_i, \mathbf{C}_i^{(*)} \boldsymbol{\Sigma} \mathbf{C}_i^{(*)})$.

(4) Generate a uniform random number $u_i^{[2]}$.

(5) Based on the adoption probability of α , and $u_i^{[2]}$, $\beta_i^{(*)}$, and $\mathbf{I}_i^{(*)}$ is performed as the following:

$$\begin{cases} \beta_i^{(n)} = \beta_i^{(*)}, \mathbf{I}_i^{(n)} = \mathbf{I}_i^{(*)}, & \text{if } u_i^{[2]} \leq \alpha \left(\left(\beta_i^{(n-1)}, \mathbf{I}_i^{(n-1)}, \beta_i^{(*)}, \mathbf{I}_i^{(*)} \right) \right), \text{ and} \\ \beta_i^{(n)} = \beta_i^{(n-1)}, \mathbf{I}_i^{(n)} = \mathbf{I}_i^{(n-1)}, & \text{otherwise,} \end{cases}$$

$$\alpha \left(\left(\beta_i^{(n-1)}, \mathbf{I}_i^{(n-1)} \right), \left(\beta_i^{(*)}, \mathbf{I}_i^{(*)} \right) \right) = \min \left(\frac{L_i^{(1)}(\beta_i^{(*)}, \mathbf{I}_i^{(*)}) \times L_i^{(2)}(\beta_i^{(*)}, \mathbf{I}_i^{(*)})}{L_i^{(1)}(\beta_i^{(n-1)}, \mathbf{I}_i^{(n-1)}) \times L_i^{(2)}(\beta_i^{(n-1)}, \mathbf{I}_i^{(n-1)})}, 1 \right).$$

$L_i^{(1)}(\bullet)$ is a Tobit regression likelihood of customer i , and $L_i^{(2)}(\bullet)$ represents a Poisson likelihood of customer i .

A.4.Sampling σ

σ is sampled via Gibbs sampling as

$$\sigma^2 \sim IG((v_0 + n) / 2, (\sigma_0 + M) / 2), \text{ and}$$

$$n = 12, M = (y_{it}^* - \mathbf{x}_{it}^{tp} \beta_i)^{tp} (y_{it}^* - \mathbf{x}_{it}^{tp} \beta_i).$$

A.5.Sampling $\boldsymbol{\theta}$, and $\boldsymbol{\Sigma}$

$\boldsymbol{\theta}$, and $\boldsymbol{\Sigma}$ is sampled via Gibbs sampling as

$$\text{vec}(\boldsymbol{\theta}) \sim N(\tilde{\mathbf{q}}, \mathbf{V} \otimes (\mathbf{Z}^{tp} \mathbf{Z} + \mathbf{A}_q)^{-1}),$$

$$\tilde{\mathbf{q}} = \text{vec}(\tilde{\mathbf{Q}}), \tilde{\mathbf{Q}} = (\mathbf{Z}^{tp} \mathbf{Z} + \mathbf{A}_q)^{-1} (\mathbf{Z}^{tp} \mathbf{Z} \hat{\mathbf{Q}} + \mathbf{A}_q \bar{\mathbf{Q}})^{-1}, \hat{\mathbf{Q}} = (\mathbf{Z}^{tp} \mathbf{Z})^{-1} \mathbf{Z}^{tp} \mathbf{B}, \text{ and}$$

$$\boldsymbol{\Sigma} \sim IW(\mathbf{f}_{01} + \mathbf{H}, (\mathbf{F}_{01} + \mathbf{S}^{tp})^{-1}),$$

$$\mathbf{S}^{tp} = \sum_{i=1}^H (\beta_i - \boldsymbol{\theta}^{tp} \mathbf{z}_i) (\beta_i - \boldsymbol{\theta}^{tp} \mathbf{z}_i)^{tp},$$

where \mathbf{Z} transposes \mathbf{z}_i , and \mathbf{B} transposes β_i for the entire customer.

A.6.Sampling s

$s^{(j)}$ considers $0 < s^{(j)} < 1$ and follows β distribution as

$$s^{(j)} \sim \text{Beta} \left(c + \sum_{i=1}^H IC_i^{(j)}, H - \sum_{i=1}^H IC_i^{(j)} + d \right).$$

The value of $IC_i^{(j)}$ is determined by the following:

$$IC_i^{(j)} = 1, \text{ if } I_i^{(j)} = 1; IC_i^{(j)} = 0, \text{ otherwise.}$$

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