A DYNAMIC MODEL WITH RESOURCES PLACED ON SINGLE LINE IN REVENUE MANAGEMENT

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Abstract   Recently, a seating position can be often selected when a plane or bullet train ticket is reserved. Specially, for theater and stadium, it is important to decide how to assign reservations to seats. This paper proposes a dynamic model where seats' resources are located at a single line with considering seats position that have already been assigned. An analysis has been conducted and the results show that, 1) optimal policy for an arriving request is to allocate it to one side of the edges of the adjacent vacancies, 2) if all of the resources are vacant at beginning time for booking, then the model corresponds to a single-leg model with multiple seat bookings and single fare class in Lee and Hersh (1993), 3) it is not necessarily optimal that a request is allocated to the less adjacent seats' vacancy. Finally, this paper proposes an algorithm to solve the optimal policy using above results and conducts numerical examples.

Keywords: Decision making, revenue management, dynamic programming, Markov decision process, seating position

1. Introduction

The revenue management is based on how to decide price and how to allocate capacity to different customer segment to maximize revenue in the industries which have fixed capacity, perishable products and fixed costs. The traditional applications of the revenue management industries are airlines, hotels and rental cars.

A dynamic model in the revenue management that is formed by dynamic programming, such as Markov decision process, has been mentioned in Sec.2.5 in [6]. This dynamic model is used to decide whether a booking request should be accepted in a certain period to maximize revenue over booking time. The dynamic model for airline industry have been also extensively studied and summarized in [4]. McGill and Van Ryzin [4] itemized factors that complicate the revenue management models such as cancellation, overbooking, walk-in, no-show, upgrade, and etc. Many models which include some of the factors have been also intensively studied (for example, Lee and Hersh [9], Subramanian et al. [5] and etc.). Recently, Steinhardt and Gönsch [1] presented a dynamic model with upgrade and its structural properties.

The revenue management is applied to not only the traditional industries, but also hospitalities' services or entertainment industries, such as golf course, restaurant, casino, and etc. [10, 11].

When we book a seat on a plane or bullet train, we can often choose seat location. However, the assignment of seat location is more important if a facility is a theater or a stadium. These facilities are shown as applications of revenue management in Kimes and Wirtz [7]. Effects of the seating position and models with seats in a table have been studied in restaurant revenue management which is a field of revenue management. Kimes and Robson

[8] have been studied effects of type or position of tables for meal duration. Bertsimas and Shioda [2] showed some models: an integer programming, a stochastic programming, and an approximate dynamic programming model. Guerriero et al. [3] suggested a dynamic model with reservation and meal duration. Ogasawara [12] presented structural properties of a model in which the meal duration depends on exponential distribution. However, in the revenue management, there is no model which considers reserving adjacent seats together on a single line for group arrival, such as sushi bar.

This paper provides a dynamic model with batch requests, which are allocated to a position in resources placed on a single line. As a result from investigating the model, some features are obtained as the following 1) optimal policy for an arriving request is to allocate it to one side of the edges of the adjacent vacancies, 2) if all of the resources are vacant at beginning time for booking, then the model corresponds to a single-leg model with multiple seat bookings and single booking class, 3) it is not necessarily optimal that a request is allocated to the less adjacent seats' vacancy. This paper demonstrates these results and an algorithm to solve optimal policy using these results.

This paper is organized as follows, in Section 2, notation and formulation for the model are introduced. Section 3 indicates properties of the model and the algorithm. Some numerical examples are given and shown in Section 4.

2. Formulation

Batch request $p = 1, \cdots, P$ arrives for booking resources which are placed on a single line during booking horizon, where the size of request p has same number of the index. The resources can be regarded as seats at a counter in a restaurant, a part of seats on a plane or bullet train or a part of seats in a theater or stadium. Let C be the number of the resources. Suppose that the batch requests cannot be separated and need adjacent available resources which are equal in size when the requests are allocated to the resources. Let the booking horizon be sufficiently discrete into N time periods so that no more than one request arrives in each period $n = 1, \cdots, N$ (see Appendix A in [5]). The time period progresses from N to 1, and 0 is terminal time on the booking horizon. Suppose that there is a single fare class. The fare depends on the size of the request and the time period. Let r_p^n be a fare of the request p in time period n. Arrival rates of the requests also depend on the size of requests and the time period. Let λ_p^n be an arrival rate of the request p in time period n. Rate of no-event stands for $\lambda_0^n = (1 - \sum_{p=1}^{P} \lambda_p^n)$. Cancellation, overbooking and walk-in are ignored.

2.1. State space and policy space

To simplify the state of the resources, we do not distinguish between left and right side of the resources' state. Further, if we identify every position on the resources, then it is irrelevant because of generating various states to which the same requests can be accepted. However, this redundancy of the states can be removed, as shown next. Let $a_1, a_2, \cdots, a_C$ be states of resources with size C where $a_k = 1$ if the resource is booked and $a_k = 0$ if the resource is unbooked, $a_0 = 1$ and $a_{C+1} = 1$. When there are $a_k, a_{k+1}, \cdots, a_l (1 \leq k \leq l \leq C)$ and $a_{k-1} = 1, a_k = a_{k+1} = \cdots = a_l = 0, a_{l+1} = 1$, we call $a_k, a_{k+1}, \cdots, a_l$ a segment of size $l - k + 1$. Let X_n be the state space at n which is defined as

$$X_n = \left\{ (t_1, \cdots, t_C) | t_i \leq \lfloor \frac{C}{i} \rfloor, \sum_{i=1}^{C} i t_i \leq C - P(N - n) \right\}$$

for each $n = 0, \cdots, N$, where t_i is the number of segments of size i. Note that the state space is reduced by $P(N-n)$ because no more than one request arrives in a period n. Figure

1 shows variations of booked positions of the state $X = (0, 0, 1, 1, 0, 0, 0, 0, 0, 0)$. A shaded cell in the Figure 1 indicates a resource which is booked.
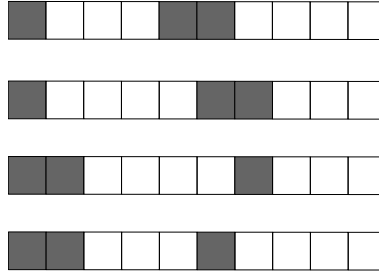

Figure 1: Variations of the state $X = (0, 0, 1, 1, 0, 0, 0, 0, 0, 0)$

Let $\boldsymbol{A}_p(X)$ be the policy space for a request p, $n = 1, \cdots, N$ and $X \in X_n$ which is defined as

$$\boldsymbol{A}_p(X) = \left\{ (i, a) | (a = 0) \vee (t_i > 0, p \geq i, 0 < a \leq \frac{i - p + 2}{2}) \right\} \tag{2.1}$$

where $\vee$ stands for "or". The condition $a > 0$ in the policy space indicates an index of a position in a segment i and $a = 0$ means to deny a request p. The condition $0 < a \leq \frac{i-p+2}{2}$ in the policy space is briefly explained as follows. We can easily recognize $0 < a < \frac{i}{2} + 1$ because the right side and the left side of the resources are indistinguishable. If $A_p = (i, a), a > 0$, then the range of index of the booked resources is from a to $a + p - 1$ and a relational expression $a + p - 1 \leq i - a + 1$ is established where $i - a + 1$ is an inverted edge of a.

Figure 2 shows policies on which a request $p = 1$ is admitted to $i = 4$ for a state $X = (0, 0, 1, 1, 0, 0, 0, 0, 0, 0)$. As shown the Figure 2, we can confirm that the state can be modified if the policy is changed.
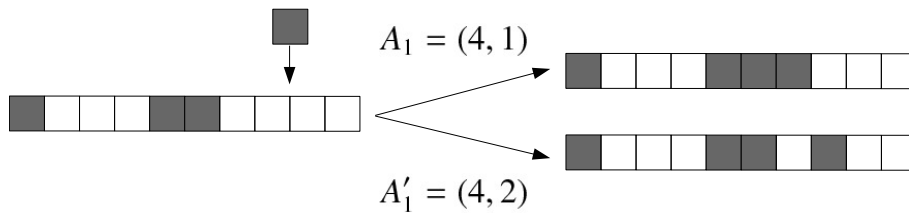

Figure 2: $A_1 = (4, 1)$ and $A_1' = (4, 2)$ for the state $X = (0, 0, 1, 1, 0, 0, 0, 0, 0, 0)$

2.2. Maximum expected revenue and optimal policy

Let $U_n(X)$ be maximum expected revenue which a facility with the resources on initial state $X \in X_n$ can be obtained from optimally operating over the period n to 0. The $U_n(X)$ is shown as following equation:

$$U_n(X) = \sum_{p=1}^{P} \lambda_p^n \max_{(i,a)\in\boldsymbol{A}_p(X)} \left\{ \sum_{a|a\neq 0} (r_p^n + U_{n-1}(X + \boldsymbol{e}_{a-1} + \boldsymbol{e}_{i-(p+a-1)} - \boldsymbol{e}_i)) + \sum_{a|a=0} U_{n-1}(X) \right\}$$
$$+ \lambda_0^n U_{n-1}(X),$$
$$X \in X_n, n = 1, \cdots, N$$
$$(2.2)$$

where $\boldsymbol{e}_i = (e_1^i, \cdots, e_C^i), i = 0, 1, \cdots, C, e_x^i = 1$ if $x = i$ and $1 \le i \le C$, $e_x^i = 0$ if $x \neq i$ and $1 \le i \le C$, and $e_1^0 = e_2^0 = \cdots = e_C^0 = 0$. Boundary conditions for the Equation (2.2) are $U_n(X) = -\infty, X \notin X_n$ and $U_0(X) = 0$.

3. Properties of the Model

It is difficult to solve optimal policies and the maximum expected revenue $U_n(X)$ due to the curse of dimensionality. However, we can reduce the search range of the policy space by using following Proposition 3.1.

Proposition 3.1. Given a request p and $X \in X_n$ for $n = 1, \cdots, N$, if there exists a policy $A_p = (i, a) \in \boldsymbol{A}_p(X)$ such that $a \ge 1$, then

$$\max_{(i,a)\in\boldsymbol{A}_p(X)} \sum_{a|a\neq 0} \left(r_p^n + U_{n-1}(X + \boldsymbol{e}_{a-1} + \boldsymbol{e}_{i-(p+a-1)} - \boldsymbol{e}_i) \right)$$
$$= \max_{(i,1)\in\boldsymbol{A}_p(X)} \left(r_p^n + U_{n-1}(X + \boldsymbol{e}_{i-p} - \boldsymbol{e}_i) \right). \tag{3.1}$$

Proof. Given a request p and $X \in X_n$ for $n = 0, \cdots, N-1$, for Proposition 3.1, it should be indicated that if there exists a policy $A_p = (i, a) \in \boldsymbol{A}_p(X)$ such that $a \ge 1$, then

$$U_n(X + \boldsymbol{e}_{\psi-1} + \boldsymbol{e}_{\delta-(p'+\psi-1)} - \boldsymbol{e}_\delta) \le U_n(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta)$$

by the inductive method. For case $n = 0$, it is obvious that $U_0(X + \boldsymbol{e}_{\psi-1} + \boldsymbol{e}_{\delta-(p'+\psi-1)} - \boldsymbol{e}_\delta) = U_0(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta) = 0$ from the boundary condition. Then, assume that $U_{n-1}(X + \boldsymbol{e}_{\psi-1} + \boldsymbol{e}_{\delta-(p'+\psi-1)} - \boldsymbol{e}_\delta) \le U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta)$. To simplify notation, set $X^{(\psi)} = X + \boldsymbol{e}_{\psi-1} + \boldsymbol{e}_{\delta-(p'+\psi-1)} - \boldsymbol{e}_\delta$ and $X^{(1)} = X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta$. It can be shown easily that

$$U_n(X^{(\psi)}) = \sum_{p=1}^{P} \lambda_p^n \max_{(i^{(\psi)},a^{(\psi)})\in\boldsymbol{A}_p(X^{(\psi)})} \left\{ \sum_{a^{(\psi)}|a^{(\psi)}\neq 0} (r_p^n + U_{n-1}(X^{(\psi)} + \boldsymbol{e}_{i^{(\psi)}-p} - \boldsymbol{e}_{i^{(\psi)}})) \right.$$
$$\left. + \sum_{a^{(\psi)}|a^{(\psi)}=0} U_{n-1}(X^{(\psi)}) \right\} + \lambda_0^n U_{n-1}(X^{(\psi)})$$

and

$$U_n(X^{(1)}) = \sum_{p=1}^{P} \lambda_p^n \max_{(i^{(1)},a^{(1)})\in\boldsymbol{A}_p(X^{(1)})} \left\{ \sum_{a^{(1)}|a^{(1)}\neq 0} (r_p^n + U_{n-1}(X^{(1)} + \boldsymbol{e}_{i^{(1)}-p} - \boldsymbol{e}_{i^{(1)}})) \right.$$
$$\left. + \sum_{a^{(1)}|a^{(1)}=0} U_{n-1}(X^{(1)}) \right\} + \lambda_0^n U_{n-1}(X^{(1)}).$$

Let $A_p^*(X^{(\psi)}) = (i^{(\psi)*}, a^{(\psi)*})$ be an optimal policy for a request p and $X^{(\psi)}$. Likewise, let $A_p^*(X^{(1)}) = (i^{(1)*}, a^{(1)*})$ be an optimal policy for a request p and $X^{(1)}$.

i) $a^{(\psi)*} \neq 0$, $a^{(1)*} \neq 0$:

We should make a comparison between $U_{n-1}(X + \boldsymbol{e}_{\psi-1} + \boldsymbol{e}_{\delta-(p'+\psi-1)} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(\psi)*}-p} - \boldsymbol{e}_{i^{(\psi)*}})$ and $U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(1)*}-p} - \boldsymbol{e}_{i^{(1)*}})$. Furthermore, this case is divided in two cases because the case $X^{(\psi)} + \boldsymbol{e}_{i^{(\psi)*}-p} - \boldsymbol{e}_{i^{(\psi)*}} \notin X_{n-1}$ exists if $\psi - 1$ and $\delta - (p' + \psi - 1)$ are changed in the case: $i^{(\psi)*} = \psi - 1 \vee i^{(\psi)*} = \delta - (p' + \psi - 1)$.

i-1) $i^{(\psi)*} \neq \psi - 1$, $i^{(\psi)*} \neq \delta - (p' + \psi - 1)$:

From the condition of this case and the inductive hypothesis, $U_{n-1}(X + \boldsymbol{e}_{\psi-1} + \boldsymbol{e}_{\delta-(p'+\psi-1)} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(\psi)*}-p} - \boldsymbol{e}_{i^{(\psi)*}}) \leq U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(\psi)*}-p} - \boldsymbol{e}_{i^{(\psi)*}}) \leq U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(1)}-p} - \boldsymbol{e}_{i^{(1)}})$.

i-2) $i^{(\psi)*} = \psi - 1 \vee i^{(\psi)*} = \delta - (p' + \psi - 1)$:

$U_{n-1}(X + \boldsymbol{e}_{\psi-1} + \boldsymbol{e}_{\delta-(p'+\psi-1)} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(\psi)*}-p} - \boldsymbol{e}_{i^{(\psi)*}})\big|_{i^{(\psi)*}=\psi-1 \vee i^{(\psi)*}=\delta-(p'+\psi-1)} \leq U_{n-1}(X + \boldsymbol{e}_{\delta-p'-g_p} - \boldsymbol{e}_\delta) = U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta + \boldsymbol{e}_{\delta-p'-g_p} - \boldsymbol{e}_{\delta-p'}) \leq U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(1)*}-g_p} - \boldsymbol{e}_{i^{(1)*}})$.

Therefore, $U_n(X^{(\psi)}) \leq U_n(X^{(1)})$ is obtained in the case i) from i-1) and i-2).

ii) $a^{(\psi)*} = 0$, $a^{(1)*} \neq 0$:

We should make a comparison between $U_{n-1}(X + \boldsymbol{e}_{\psi-1} + \boldsymbol{e}_{\delta-(p'+\psi-1)} - \boldsymbol{e}_\delta)$ and $r_p^n + U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(1)*}-p} - \boldsymbol{e}_{i^{(1)*}})$. From the fact $a^{(1)*} \neq 0$, $r_p^n + U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(1)*}-p} - \boldsymbol{e}_{i^{(1)*}}) \geq U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta) \geq U_{n-1}(X + \boldsymbol{e}_{\psi-1} + \boldsymbol{e}_{\delta-(p'+\psi-1)} - \boldsymbol{e}_\delta)$. Therefore, $U_n(X^{(\psi)}) \leq U_n(X^{(1)})$ is obtained in this case ii).

iii) $a^{(\psi)*} \neq 0$, $a^{(1)*} = 0$:

We should compare $r_p^n + U_{n-1}(X + \boldsymbol{e}_{\psi-1} + \boldsymbol{e}_{\delta-(p'+\psi-1)} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(\psi)*}-p} - \boldsymbol{e}_{i^{(\psi)*}})$ with $U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta)$. This case is divided in two cases from a reason which is the same to the one in the case i).

iii-1) $i^{(\psi)} \neq \psi - 1$, $i^{(\psi)} \neq \delta - (p' + \psi - 1)$:

$r_p^n + U_{n-1}(X + \boldsymbol{e}_{\psi-1} + \boldsymbol{e}_{\delta-(p'+\psi-1)} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(\psi)*}-p} - \boldsymbol{e}_{i^{(\psi)*}}) \leq r_p^n + U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(\psi)*}-p} - \boldsymbol{e}_{i^{(\psi)*}}) \leq U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta)$ is obtained.

iii-2) $i^{(\psi)} = \psi - 1 \vee i^{(\psi)} = \delta - (p' + \psi - 1)$:

$r_p^n + U_{n-1}(X + \boldsymbol{e}_{\psi-1} + \boldsymbol{e}_{\delta-(p'+\psi-1)} - \boldsymbol{e}_\delta + \boldsymbol{e}_{i^{(\psi)*}-p} - \boldsymbol{e}_{i^{(\psi)*}})\big|_{i^{(\psi)*}=\psi-1 \vee i^{(\psi)*}=\delta-(p'+\psi-1)} \leq r_p^n + U_{n-1}(X + \boldsymbol{e}_{\delta-p'-g_p} - \boldsymbol{e}_\delta) = U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta + \boldsymbol{e}_{\delta-p'-g_p} - \boldsymbol{e}_{\delta-p'}) \leq U_{n-1}(X + \boldsymbol{e}_{\delta-p'} - \boldsymbol{e}_\delta)$.

Therefore, $U_n(X^{(\psi)}) \leq U_n(X^{(1)})$ is obtained in the case iii) from the cases iii-1) and iii-2).

iv) $a^{(\psi)*} = 0$, $a^{(1)*} = 0$:

$U_n(X^{(\psi)}) \leq U_n(X^{(1)})$ is easily obtained from the inductive hypothesis.

Consequently, the Proposition 3.1 is indicated from the cases: i) - vi). $\square$

Note that Proposition 3.1 does not related to fares r_p^n. This fact shows that Proposition 3.1 is achieved in any orderings of the fares r_p^n among requests p. Proposition 3.1 also indicates that we should deal with only $(i, a) = A_p \in \boldsymbol{A}_p(X), a = 1 \vee a = 0$ for a request p and state $X \in X_n$ to solve the maximum expected revenue. This leads to a remarkable fact for the case $X_N = \{\boldsymbol{e}_C\}$ which is that all resources are not occupied with requests at the beginning of the booking horizon. Details of this fact are explained in the following Remark 3.1.

Remark 3.1. Let $\hat{X} \in \hat{X}_n, n = 0, \cdots, N$ be the set of states which are obtained when the maximum expected revenue is solved in the case $X_N = \{\boldsymbol{e}_C\}$. From Proposition 3.1 and the

condition of X_N, $\hat{X}_n = \{e_x | 0 \le x \le C - P(N - n)\}, n = 0, \cdots, N$. The x can be regarded as the number of unbooked resources. Set $V_n(x) = U_n(e_x)$ and $\boldsymbol{A'}_p(x) = \{a | (p \le x, a = 1) \vee a = 0\}, x = 0, 1, \cdots, C$. Then, it follows that

$$V_n(x) = \sum_{p=1}^{P} \lambda_p^n \max \{r_p^n + V_{n-1}(x - p), V_{n-1}(x)\} + \lambda_0^n V_{n-1}(x),$$

$$0 \le x \le C, n = 1, \cdots, N. \qquad (3.2)$$

Boundary conditions are $V_0(x) = 0$ and $V_n(x) = -\infty, x < 0$.

The Equation (3.2) can be regarded as a single-leg model with single booking class and multiple seat booking in [9]. This fact indicates that the maximum expected revenue and the optimal policy which is obtained by the single-leg model is the same as the ones which is obtained by the model of this paper for the case $X_N = \{e_C\}$.

However, the Equation (3.2) cannot be applied to the case: $X_N \ne \{e_C\}$. From Proposition 3.1, the policy space can be rewritten in the following form because i in $\boldsymbol{A}_p(X)$ is only decided if a booking request p is accepted.

$$\hat{A}_p \in \hat{\boldsymbol{A}}_p(X) = \{d | (d = i, t_i \ge 0, i \ge p) \vee (d = 0)\} .$$

$d = 0$ in the policy space stands for denying a request. By using this $\hat{\boldsymbol{A}}_p(X)$, the Equation (2.2) can be rewritten as the following equation:

$$U_n(X) = \sum_{p=1}^{P} \lambda_p^n \max_{d \in \hat{\boldsymbol{A}}_p(X)} \left\{ \sum_{d | d \ne 0} (r_p^n + U_{n-1}(X + \boldsymbol{e}_{d-p} - \boldsymbol{e}_d)) + \sum_{d | d = 0} U_{n-1}(X) \right\} + \lambda_0^n U_{n-1}(X),$$

$$X \in X_n, n = 1, \cdots, N. \qquad (3.3)$$

Further, the Equation (3.3) is rewritten in the following equation

$$U_n(X) = \sum_{p=1}^{P} \lambda_p^n \left(r_p^n - \min_{d \in \hat{\boldsymbol{A}}_p(X)} \Delta_p^d U_{n-1}(X) \right)^+ + U_{n-1}(X),$$

$$X \in X_n, n = 1, \cdots, N. \qquad (3.4)$$

where $(k)^+ = \max\{k, 0\}$, $\Delta_p^d U_{n-1}(X) = U_{n-1}(X) - U_{n-1}(X + \boldsymbol{e}_{d-p} - \boldsymbol{e}_d)$ and $\Delta_p^0 U_n(X) = \infty$. $\min_{d \in \hat{\boldsymbol{A}}_p(X)} \Delta_p^d U_{n-1}(X)$ can be seen as threshold price so that a request is accepted if a price which is obtained from the request exceeds the threshold price and rejected if the price is less than the threshold price (referring pp.32-33 in [6]). From the the Equation (3.4), it is obvious to acquire the following Theorem 3.1.

Theorem 3.1. For a given $X \in X_0$, $U_n(X)$ is non-decreasing in n.

Then, an algorithm is shown as below. Algorithm 1 is to solve an optimal policy d^* for a given p, n and $X \in X_n$ by using the threshold price. $\hat{\boldsymbol{A}}_p(X)\backslash 0$ in Algorithm 1 stands for $\hat{\boldsymbol{A}}_p(X)\backslash\{0\}$.

If $\arg\min_{d\in\hat{\boldsymbol{A}}_p(X)\backslash 0}\Delta_p^d U_{n-1}(X)$ is not unique, then without loss of generality, the smallest d is selected in Algorithm 1. Note that we cannot reduce the search range for segments i of $\boldsymbol{A}_p(X)$ in Algorithm 1.

We might expect that there is a structural property of upgrade model which is seen in Steinhardt and Gönsch [1] and Ogasawara [12]. This means that, for all n and $X \in X_n$, if there exist $d \in \hat{\boldsymbol{A}}_p(X)$ and $d' \in \hat{\boldsymbol{A}}_p(X)$ such that $d \neq 0, d' \neq 0$ and $d \leq d'$, then $\Delta_p^d U_n(X) \leq \Delta_p^{d'} U_n(X)$. Indeed, the monotonicity does not exist. To indicate the non-monotonicity, a counter-example is shown in next section.

Algorithm 1

Input $n, X \in X_n$ and p.

$d^* \leftarrow 0$

if $\hat{\boldsymbol{A}}_p(X)\backslash 0 \neq \phi$ **then**

$\quad$ calculate $\min_{d\in\hat{\boldsymbol{A}}_p(X)\backslash 0}\Delta_p^d U_{n-1}(X)$ and

$\quad \min_d\{\arg\min_{d\in\hat{\boldsymbol{A}}_p(X)\backslash 0}\Delta_p^d U_{n-1}(X)\}$.

$\quad \overline{d}^* \leftarrow \min_d\{\arg\min_{d\in\hat{\boldsymbol{A}}_p(X)\backslash 0}\Delta_p^d U_{n-1}(X)\}$

$\quad$ **if** $r_p^n \geq \min_{d\in\hat{\boldsymbol{A}}_p(X)\backslash 0}\Delta_p^d U_{n-1}(X)$ **then**

$\quad\quad d^* \leftarrow \overline{d}^*$

$\quad$ **end if**

end if

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4. Numerical Examples

This section shows numerical examples in which maximum expected revenues and optimal policies are calculated by using Algorithm 1 on a small scale. One of the examples is the counter-example which was mentioned in the previous section.

4.1. Numerical example using Algorithm 1

A data-set is $C = 6, N = 4, P = 3, X_4 = \{(0, 1, 1, 0, 0, 0)\}$. Arrival rates and fares are shown in Table 1. From $\max_n |X_n| = 9$, to simplify notation, elements of the state space are $X^1 = (0, 1, 1, 0, 0, 0), X^2 = (1, 0, 1, 0, 0, 0), X^3 = (0, 0, 1, 0, 0, 0), X^4 = (0, 2, 0, 0, 0, 0), X^5 = (1, 1, 0, 0, 0, 0), X^6 = (2, 0, 0, 0, 0, 0), X^7 = (0, 1, 0, 0, 0, 0), X^8 = (1, 0, 0, 0, 0, 0), X^9 = (0, 0, 0, 0, 0, 0)$.

The maximum expected revenues and the optimal policies which are calculated from the data-set are shown in Table 2 and Table 3, respectively. Specifically, a calculation process for $p = 1$ and $X^2 \in X_3$ in the Table 3 is explained as follows.

Suppose that calculations are terminated at $n = 2$ by backward induction. We calculate for $d = 1$ because of $\hat{\boldsymbol{A}}_1(X^2)\backslash 0 \neq \phi$. For a policy $\hat{\boldsymbol{A}}_1(X^2) \ni d = 1$, $\Delta_1^1 U_2(X^2) = U_2(X^2) - U_2(X^3) = 28.8 - 24.4 = 4.4$. Applying $\hat{\boldsymbol{A}}_1(X^2) \ni d = 3$ to $\Delta_1^d U_2(X^2)$, $\Delta_1^3 U_2(X^2) = U_2(X^2) - U_2(X^5) = 28.8 - 16.2 = 12.6$. Therefore, from $\min_{d\in\hat{\boldsymbol{A}}_1(X^2)\backslash 0}\Delta_1^d U_2(X^2) = 4.4 \leq r_1^3 = 10$, $d^* = 1$ is indicated. Accordingly, the maximum expected revenues and the optimal policies for all states in all time periods can be calculated.

Table 1: Arrival rate and fare for p in n

$n\backslash p$	λ_p^n 1	2	3	r_p^n 1	2	3
1	0.2	0.3	0.5	10	20	30
2	0.4	0.3	0.2	10	20	30
3	0.4	0.3	0.2	10	20	30
4	0.3	0.2	0.1	10	20	30

Table 2: The maximum expected revenues $U_n(X^k)$ for n and k

$k\backslash n$	4	3	2	1	0
1	41.61	41.08	36.00	23.00	0.00
2	-	32.32	28.80	23.00	0.00
3	-	19.40	24.40	23.00	0.00
4	-	25.84	18.00	8.00	0.00
5	-	21.70	16.20	8.00	0.00
6	-	-	6.00	2.00	0.00
7	-	16.04	13.20	8.00	0.00
8	-	-	5.20	2.00	0.00
9	0.00	0.00	0.00	0.00	0.00

Table 3: The optimal policies for n and X^k

$k\backslash n$	$p=1$ 4	3	2	1	$p=2$ 4	3	2	1	$p=3$ 4	3	2	1
1	2	2	2	2	2	2	2	2	3	3	3	3
2	-	1	1	1	-	0	0	3	-	3	3	3
3	-	0	0	3	-	3	0	3	-	3	3	3
4	-	2	2	2	-	2	2	2	-	0	0	0
5	-	1	1	1	-	2	2	2	-	0	0	0
6	-	-	1	1	-	-	0	0	-	-	0	0
7	-	2	2	2	-	2	2	2	-	0	0	0
8	-	-	1	1	-	-	0	0	-	-	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0

4.2. Counter-example

Note that if request p is accepted, then all optimal policies in Table 3 take the smallest d for each p. However, this feature is not held in the counter-example which is shown in this subsection. A data-set of this counter-example is $C = 6, N = 3, P = 3$ and $X_4 = \{(0,1,1,0,0,0)\}$. Arrival rates and fares of this counter-example are shown in Table 4. Notations of elements of the state space are the same to the ones that was mentioned in the previous example.

Notice that the booking request $p = 2$ is accepted for all states in $n = 1, 2$ because the request $p = 2$ must arrive. Therefore, the maximum expected revenues can be easily

calculated until $n = 2$. Since $\Delta_1^2 U_2(X^1) = U_2(X^1) - U_2(X^2) = 40 - 20 = 20$ and $\Delta_1^3 U_2(X^1) = U_2(X^1) - U_2(X^4) = 40 - 40 = 0$, the optimal policy for $p = 1$ and $X^1 \in X_3$ is $d^* = 3$. Thus, it shows non-monotonicity of $\Delta_p^d U_n(X)$ in $d(\neq 0)$. This counter-example indicates that Algorithm 1 is needed for solving optimal policies and maximum expected revenues.

Table 4: Arrival rate and fare for p in n

$n \backslash p$	λ_p^n			r_p^n		
	1	2	3	1	2	3
1	0.0	1.0	0.0	10	20	30
2	0.0	1.0	0.0	10	20	30
3	0.4	0.3	0.2	10	20	30

5. Conclusion

This paper proposed a dynamic model with batch requests which are required to be placed together on a single line's resources. For the model, this study presented an algorithm by which the optimal policies can be effectively solved. Our future issues are to extended this model for multiple fare classes, to consider group-reservation which can be set across multiple lines and take into account of a reservation which selects its position of resources.

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