DYNAMIC PROGRAM MODELING FOR
A RETAIL SYSTEM UNDER TRADE CREDIT

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Abstract   Dynamic programming has been used to solve numerous complex problems in business and engineering. This study applies dynamic programming to a retail decision-making problem related to trade credit. A price, shelf-space, and time-dependent demand function is introduced to model the finite time horizon inventory. Trade credit was considered in the model because suppliers commonly provide retailers with credit periods. Consequently, the retailer is not required to pay for goods immediately upon receipt, and can instead earn interest on the retail price of the goods between the time the goods are sold and the end of the credit period. The objective of this paper is to determine the periodic retail price, shelf-space quantity, and ordering quantity that maximize total profit. The numerical examples explain the procedures of the solution approach and show that dynamic decision making is superior to fixed decision making regarding profit maximization.

Keywords: Dynamic programming, algorithm, retail management, trade credit

1.   Introduction

In operations research and management science, dynamic programming is a method used for solving complex problems by dividing them into simpler sub-problems. Because of changing and uncertain environments, dynamic decision making is becoming increasingly prevalent in practice. The benefits of dynamic decision-making techniques have long been considered in industries, such as airline, restaurant, hotel, fashion and high-tech product industries. For example, Boise Cascade Office Products sells numerous products online. Their prices on the 12,000 items that are ordered the most frequently online might change as often as daily.

Elmaghraby and Keskinocak [7] conducted a complete review of relevant literature and current practices in dynamic pricing. Xiao et al. [14] solved a semi-dynamic pricing and seat-inventory allocation problem by using the airline industry as an example. Tsao and Sheen [13] considered the dynamic pricing, promotion, and replenishment policies for a deteriorating item when payments were permissibly delayed. They assumed that demand is a linear function of price and time. Aziz et al. [1] proposed a hotel revenue-management model based on dynamic pricing for maximizing room revenue. Zhao et al. [16] studied a dynamic pricing problem by considering a monopolist firm selling perishable goods to consumers who may be influenced by inertia. Xu et al. [15] analyzed the dynamic pricing decision and compensation strategy of a firm that relies on a heterogeneous sales force to sell its product in two periods. IBM is investigating software that will enable it to adjust prices according to demand. Therefore, dynamic pricing is vital in businesses today.

Wal-Mart, the largest retailer worldwide, uses trade credit as a larger source of capital than that of its bank loans; Wal-Mart trade credit is eight times the amount of capital invested by shareholders (Chuledek [6]). In practice, vendors frequently provide forward financing (trade credit) to buyers. Numerous studies have been published regarding inventory problems occurring in trade credit situations. Recently, Tsao [10] determined two-phase pricing and inventory decisions regarding trade credit for deteriorating and fashion goods. Balkhi [2] considered an economic ordering policy of deteriorating items under different supplier trade credits for a finite horizon case. Tsao [11] discussed how to manage a retail-competition distribution channel involving a cash discount and credit period. Tsao et al. [12] discussed the effects of a maintenance policy on an imperfect production system under trade credit. Therefore, the issue of trade credit is extremely popular in this field of research.

Retail price and shelf space are two central factors that affect demand. Murray et al. [8] and Szmerekovsky et al. [9] considered a demand function that is decreasing in price and increasing in shelf-space allocation. Our paper extends the price- and shelf-space-sensitive demand function for considering a price, shelf-space, and a time-dependent demand function. The demand form is practical because, in a retail store, such as Wal-Mart or Target, retail price and shelf space are adjusted periodically. Chen and Chang [4] and Chen [3] stated that a truly efficient supply chain achieves other objectives in addition to reducing cost. In this study, we conducted dynamic retail-price, shelf-space, and ordering-quantity decisions concurrently under trade credit to maximize total profit. Both declining and increasing market-demand patterns were considered, which enhanced the model applications.

In this study, we determined optimal pricing, shelf-space, and ordering decisions. This paper introduces a price, shelf-space, and time-dependent demand function. The demand function is an exponentially decreasing function of the price, which exponentially increases according to shelf space and varies exponentially over time. Finding the closed form of the solution directly is impossible because of the complexity of the demand form. Instead, we solved the problem by applying an extreme value search method. The contributions of this paper to the literature and to practice are as follows. First, this is the first study to consider dynamic pricing, shelf-space, and ordering decisions concurrently in a trade credit situation. Second, a price, shelf-space, and time-dependent demand function was introduced into this model. Third, we demonstrated that dynamic decision making is superior to fixed decision making. The results of this study could be used as a reference by business managers or administrators.

2. MODEL FORMULATION

The following notations are used in this paper:

H**:** The length of the fixed planning horizon

n**:** The number of replenishment cycles during the planning horizon

p_j**:** The retail price per unit for cycle j

s_j**:** The shelf-space quantity for cycle j

q_j**:** The ordering quantity for cycle j

c**:** The cost per unit

A**:** The ordering cost per order

h**:** The inventory holding cost per unit per unit time

$I_j(t)$**:** The inventory level for cycle j at time t

m**:** The shelf-space maintenance cost per shelf per unit time

tc**:** The length of the credit period

$D(p_j, s_j, t)$**:** The basic demand rate, $D(p_j, s_j, t) = (\alpha p_j^{-\beta} s_j^{\gamma}) e^{\lambda t}$ where $\alpha > 0$, $\beta > 0$, $\gamma > 0$

Ie**:** The interest earned per dollar

Ic**:** The interest charged per dollar

π_j^i**:** The total profit in the time interval $[Z_{j-1}, Z_j]$; $i = 1$ for $tc \leq Z_j - Z_{j-1}$, $i = 2$ for $tc > Z_j - Z_{j-1}$

Π_j**:** The cumulated net profit over period $[0, Z_j]$

The mathematical model in this paper was developed based on the following assumptions.

1. A single item is considered over a known and finite planning horizon.
2. Replenishments occur instantaneously.
3. The basic demand rate $D(p_j, s_j, t) = (\alpha p_j^{-\beta} s_j^{\gamma}) e^{\lambda t}$ is an exponentially decreasing function of price that exponentially increases according to shelf space and decreases (increases) exponentially over time when $\lambda < 0$ ($\lambda > 0$). The demand function is a blend of the time-sensitive function proposed by Tsao and Sheen [13] and the price- and shelf-space-sensitive function provided in Chen et al. [5].
4. The unit selling price of products sold during the credit period is deposited in an interest-bearing account at rate Ie. At the end of this period, the credit is settled, and the retailer starts paying interest charges on items in stock at the rate Ic.

The variation of inventory level $I_j(t)$ in time interval $[Z_{j-1}, Z_j]$, in which the joint effect of demand and deterioration is considered, can be described by the following differential equation: $\dfrac{dI_j(t)}{dt} = -D(p_j, s_j, t)$, $Z_{j-1} \leq t \leq Z_j$, which contains the boundary condition $I_j(Z_j) = 0$, $j = 1, \ldots, n$ for cycle j. We can then obtain $I_j(t) = \dfrac{\alpha p_j^{-\beta} s_j^{\gamma}}{\lambda}(e^{\lambda Z_j} - e^{\lambda t})$, $Z_{j-1} \leq t \leq Z_j$ and the ordering quantity (for cycle j) $q_j = \displaystyle\int_{Z_{j-1}}^{Z_j} (\alpha p_j^{-\beta} s_j^{\gamma}) e^{\lambda t} \, dt$.

The net profit over cycle j consists of the following elements:

(1) *Sales revenue*:

$$SR_j = p_j \cdot \int_{Z_{j-1}}^{Z_j} (\alpha p_j^{-\beta} s_j^{\gamma}) e^{\lambda t} \, dt. \tag{2.1}$$

(2) *Purchasing cost*:

$$PC_j = c \cdot \int_{Z_{j-1}}^{Z_j} (\alpha p_j^{-\beta} s_j^{\gamma}) e^{\lambda t} \, dt. \tag{2.2}$$

(3) *Inventory holding cost*:

$$H_j = h \cdot \int_{Z_{j-1}}^{Z_j} I_j(t) \, dt = h \cdot \int_{Z_{j-1}}^{Z_j} \frac{\alpha p_j^{-\beta} s_j^{\gamma}}{\lambda}(e^{\lambda Z_j} - e^{\lambda t}) \, dt. \tag{2.3}$$

(4) *Ordering cost*: A.

(5) *Shelf-space maintenance cost*:

$$M_j = m \cdot \int_{Z_{j-1}}^{Z_j} s_j \, dt. \tag{2.4}$$

(6) *Interest earned*:

For each cycle, we must consider whether the length of the credit period is longer or shorter than the length of the cycle. For example, two cases must be discussed in Cycle 1. Case 1 is $tc \leq Z_j - Z_{j-1}$ and Case 2 is $tc > Z_j - Z_{j-1}$. Only one of these two cases occurs in each cycle of the planning horizon. IE_j^i represents the interest earned in the time interval $[Z_{j-1}, Z_j]$ of Cases i, $i = 1$, or 2, and $j = 1, \ldots, n$.

Case 1: When $tc \leq Z_j - Z_{j-1}$

$$IE_j^1 = Ie \cdot p_j \int_{Z_{j-1}}^{Z_{j-1}+tc} (Z_{j-1} + tc - t)(\alpha p_j^{-\beta} s_j^\gamma) e^{\lambda t}\, dt; \tag{2.5}$$

Case 2: When $tc > Z_j - Z_{j-1}$

$$IE_j^2 = Ie \cdot p_j \int_{Z_{j-1}}^{Z_j} (Z_j - t)(\alpha p_j^{-\beta} s_j^\gamma) e^{\lambda t}\, dt$$

$$+ Ie \cdot p_j \cdot [tc - (Z_j - Z_{j-1})] \int_{Z_{j-1}}^{Z_j} (\alpha p_j^{-\beta} s_j^\gamma) e^{\lambda t}\, dt. \tag{2.6}$$

(7) *Interest charged*:

Similarly, two interest-charged cases must be addressed and only one occurs in each cycle of the planning horizon.

Case 1: When $tc \leq Z_j - Z_{j-1}$

IC_j^1 is the interest charged in the time interval $[Z_{j-1}, Z_j]$, $j = 1, \ldots, n$.

$$IC_j^1 = Ic \cdot c \int_{Z_{j-1}+tc}^{Z_j} I_j(t)\, dt = Ic \cdot c \int_{Z_{j-1}+tc}^{Z_j} \frac{\alpha p_j^{-\beta} s_j^\gamma}{\lambda} (e^{\lambda Z_j} - e^{\lambda t})\, dt; \tag{2.7}$$

Case 2: When $tc > Z_j - Z_{j-1}$

No interest is charged in this case (i.e., $IC_j^2 = 0$).

Therefore, the net profit in cycle j using the two different cases can be expressed as follows:

Case 1: When $tc \leq Z_j - Z_{j-1}$

$$\pi_j^1(p_j, s_j) = SR_j - PC_j - H_j - A - M_j + IE_j^1 - IC_j^1; \tag{2.8}$$

Case 2: When $tc > Z_j - Z_{j-1}$

$$\pi_j^2(p_j, s_j) = SR_j - PC_j - H_j - A - M_j + IE_j^2 - IC_j^2. \tag{2.9}$$

Total profit in cycle j is

$$\pi_j(p_j, s_j) = \begin{cases} \pi_j^1(p_j, s_j) & \text{when } tc \leq Z_j - Z_{j-1} \\ \pi_j^2(p_j, s_j) & \text{when } tc > Z_j - Z_{j-1} \end{cases}$$

and the accumulated profit over period $[0, j]$ is calculated by using $\Pi_j = \sum_{g \leq j} \pi_g(p_g, s_g)$.

The total profit in cycle j, $\pi_j(p_j, s_j)$, is a two-branch nonlinear function containing two variables. In the case of $tc \leq Z_j - Z_{j-1}$, for a given cycle over $[Z_{j-1}, Z_j]$, the optimal retail price p_j and shelf-space quantity s_j in this cycle can be determined by differentiating Equation (2.8) according to p_j and s_j separately, and then setting these to zero. Similarly,

in the case of $tc > Z_j - Z_{j-1}$, for a given cycle over $[Z_{j-1}, Z_j]$, the optimal p_j and s_j in this cycle can be determined by differentiating Equation (2.9) according to p_j and s_j, and setting these to zero. We used the Hessian matrix to verify that the total profit function throughout each cycle is jointly concave according to retail price p_j and shelf-space quantity s_j. This means that, separately, the second partial derivatives of Equations (2.8) and (2.9) respective to p_j and s_j are strictly negative, and

$$\frac{\partial^2 \pi_j^1(p_j, s_j)}{(\partial p_j)^2} \frac{\partial^2 \pi_j^1(p_j, s_j)}{(\partial s_j)^2} - \left[\frac{\partial^2 \pi_j^1(p_j, s_j)}{\partial p_j \partial s_j}\right]^2 \quad \text{or} \quad \frac{\partial^2 \pi_j^2(p_j, s_j)}{(\partial p_j)^2} \frac{\partial^2 \pi_j^2(p_j, s_j)}{(\partial s_j)^2} - \left[\frac{\partial^2 \pi_j^2(p_j, s_j)}{\partial p_j \partial s_j}\right]^2$$

are positive. The purpose of using this proposed model was to investigate the effect of dynamic decision making on total profit. This paper presents two developed solution procedures based on dynamic and fixed decision making in next section.

3. SOLUTION PROCEDURE

Regarding a dynamic pricing and shelf-space policy, the retailer wishes to dynamically determine the optimal retail price, shelf-space quantity, and ordering quantity of each replenishment cycle. Equations $\dfrac{\partial \pi_j^1(p_j, s_j)}{\partial p_j} = 0$ and $\dfrac{\partial \pi_j^1(p_j, s_j)}{\partial s_j} = 0$ are used to determine p_j and s_j when the considered ordering cycle is larger than or equal to the credit period. The optimal p_j^* and s_j^* in $\pi_j^1(p_j, s_j)$ are such that $\pi_j^1(p_j^*, s_j^*) = \max\limits_{p_j, s_j}\{\pi_j^1(p_j, s_j)\}$, which is the best of the local optimal solutions found. Similarly, when the considered replenishment cycle is smaller than the credit period, we used the same procedure to obtain the optimal p_j^* and s_j^* (by using $\dfrac{\partial \pi_j^2(p_j, s_j)}{\partial p_j} = 0$ and $\dfrac{\partial \pi_j^2(p_j, s_j)}{\partial s_j} = 0$). Therefore, for cycle j, the search procedure generates the optimal solutions of p_j^* and s_j^*. The accumulated net profit in cycle j is $\Pi_j = \Pi_{j-1} + \pi_j(p_j^*, s_j^*)$. The optimal retail price, shelf-space quantity, and sequence of replenishment time epochs can be determined by solving the dynamic programming models.

$$\Pi_{Z_i} = \max\big\{\Pi_{Z_{i-1}} + \pi_{Z_i}(p_j^*, s_j^*) : 0 \leq Z_{i-1} < Z_i \leq H\big\}. \tag{3.1}$$

Given boundary conditions $\Pi_0 = 0$ and $Z_0 = 0$, the recursive procedure proceeds in a forward manner to determine the maximal total profit over the time horizon; Π_{Z_n} is determined at the last stage of the procedure (i.e., the maximal net profit of the planning horizon). The optimal sequence of the replenishment time epochs Z_{j-1}, $j = 1, 2, \ldots, n$, and the associated optimal retail price p_j^*, shelf-space quantity s_j^*, and ordering quantity q_j^* can be determined.

Regarding a fixed pricing and shelf-space policy, if the company adopts fixed-pricing and shelf-space decisions, then the firm determines the non-adjustable retail price and shelf-space quantity at the beginning of the planning horizon by maximizing the relative profit function over $[0, H]$

$$\varpi(p, s) = (p_j - c) \cdot \int_0^H (\alpha p_j^{-\beta} s_j^{\gamma}) e^{\lambda t} \, dt - m \cdot \int_0^H s \, dt. \tag{3.2}$$

The optimal fixed retail price p^{**} and shelf-space quantity s^{**} can be determined by solving Equations (3.3) and (3.4) concurrently:

$$\frac{\partial \varpi(p, s)}{\partial p} = -\frac{(e^{H\lambda} - 1)\alpha p^{-\beta-1} s^{\gamma}[p(\beta - 1) - c\beta]}{\lambda} = 0, \tag{3.3}$$

$$\frac{\partial \varpi(p, s)}{\partial s} = \frac{(e^{H\lambda} - 1)\alpha \gamma p^{-\beta} s^{\gamma-1}(p - c)}{\lambda} - Hm = 0. \tag{3.4}$$

Table 1: Numerical results for dynamic decision-making with declining market demand

		Dynamic decision-making				*Fixed decision-making*			
j	Z_{j-1}	p_j^*	s_j^*	q_j^*	Π_j^*	Z_{j-1}	p^{**}	s^{**}	q_j^{**}	Π_j^*
1	0	24.5	5222	581	3222.08	0	24	3803	512	3108.31
2	1	24.5	4725	526	2725.13	1	24	3803	487	2673.69
3	2	28.5	3236	645	4472.98	2	24	3803	903	4254.73
$\sum$					10420.19					10036.73

The dynamic programming model can be used to determine the optimal sequence of replenishment time epochs Z_{j-1}, $j = 1, 2, \ldots, n$

$$\Pi_{Z_i} = \max\big\{\Pi_{Z_{i-1}} + \pi_{Z_i}(p^{**}, s^{**}) : 0 \leq Z_{i-1} < Z_i \leq H\big\}. \tag{3.5}$$

4. NUMERICAL EXAMPLES

To depict the solution procedure, we considered the following example: $H = 4$, $A = 200$, $h = 2$, $c = 8$, $Ie = 0.1$, $Ip = 0.15$, $tc = 1.5$, and $m = 1$. We first considered a case of declining market demand: $D(p_j, s_j, t) = (1000p_j^{-1.5}s_j^{0.5})e^{-0.05t}$. In practice, the parameters α, β, γ, and λ can be determined by conducting regression analysis using historical transaction data. Historical transaction data in this paper refer to the sales (representing demands), price, and shelf-space allocation information collected by observing the product during a period. The observed data can then be modeled by conducting regression analysis for estimating sales and demands by using a given amount of allocated shelf space and the retail price. Because of the powerful information technologies (e.g., POS system, ERP system, data mart, and warehouse) currently used in businesses, collecting relevant transaction data is easy.

Regarding dynamic decision making, the computed results are shown in Table 1 (left-hand side). The number of replenishments in the case of dynamic pricing and shelf space is $n = 3$, and the associated replenishment time epochs Z_{j-1}, $j = 1, 2, 3$, optimal retail price p_j^*, shelf-space quantity s_j^*, ordering quantity q_j^*, and accumulated net profit Π_j are all obtained. Figure 1 is a graphic representation of the concavity of π_1 in the dynamic decision-making case. The figure shows that the algorithm can be used to derive the optimal solution.

The number of replenishments in the case of fixed pricing and shelf space is $n = 3$, and the associated replenishment time epochs Z_{j-1}, $j = 1, 2, 3$, optimal retail price p^{**}, shelf-space quantity s^{**}, ordering quantity q_j^{**}, and accumulated net profit Π_j are shown in Table 1 (right-hand side). Figure 2 presents a graphic representation of the concavity of ϖ in a fixed decision-making case. A comparison of the results of the dynamic decisions and the fixed decisions shows that dynamic decision making is superior to fixed decision making. A 3.82% increase in total profit in the declining market demand case was predicted when fixed decision making was performed and dynamic pricing and shelf space were implemented.

We also compared the dynamic decision model with the following two models: (a) a model in which retail price is dynamic and shelf space is fixed, and (b) a model in which retail price is fixed and shelf space is dynamic. Table 2 shows that the total profit obtained using the dynamic-price and fixed-shelf-space policy was \$10,212.26. Compared with the \$10,036.73 obtained using the fixed price and shelf-space policy, a 2.03% increase in total profit was predicted in the declining market demand case when the dynamic-price and fixed-shelf-space policy was implemented. Table 2 also shows that the total profit obtained using the dynamic-shelf-space and fixed-price policy was \$10,258.58. A 1.58% increase of total profit was predicted in the declining market demand case when the dynamic-shelf-space and
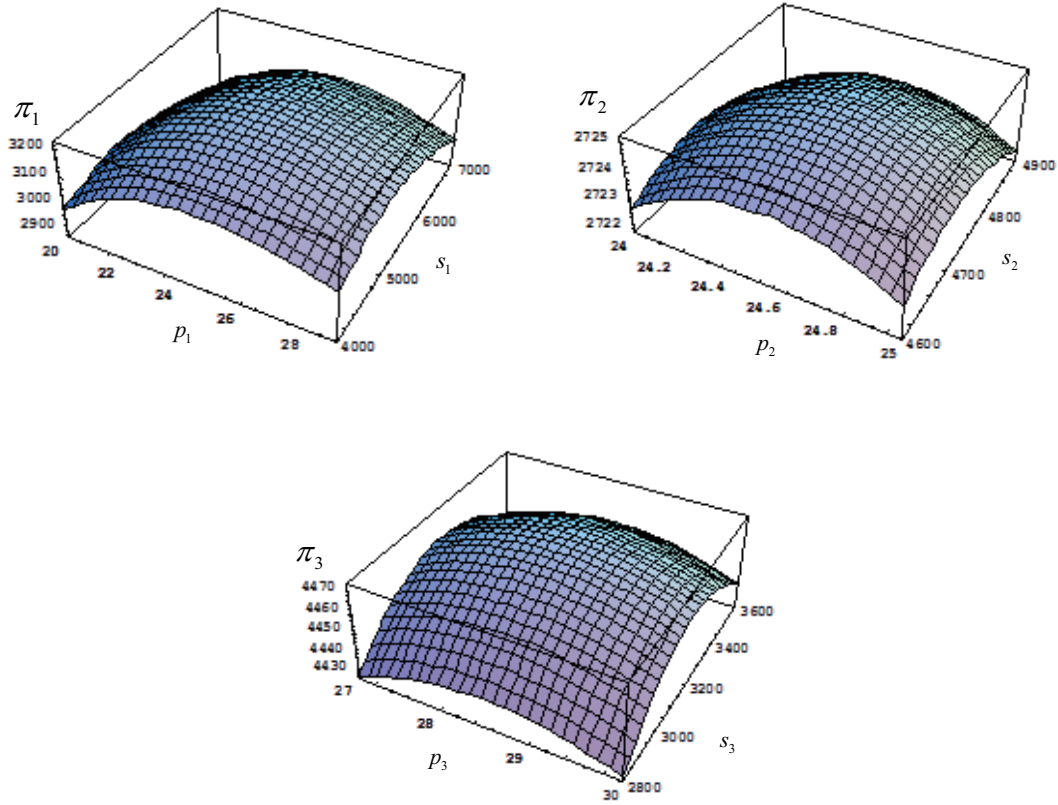
Figure 1: Graphical representation of π_i

Table 2: Dynamic price and fixed shelf-space v.s. dynamic shelf-space and fixed price

	Dynamic price and fixed shelf-space				Dynamic shelf-space and fixed price			
j	Z_{j-1}	p_j^*	s_j^*	q_j^*	Π_j^*	Z_{j-1}	p^{**}	s^{**}	q_j^{**}	Π_j^*
1	0	24.5	3803	496	3109.81	0	24	5220	599	3220.30
2	1	24.5	3803	471	2675.13	1	24	4724	542	2723.52
3	2	28.5	3803	699	4427.32	2	24	3157	823	4314.76
$\sum$					10212.26					10258.58

fixed-price policy was implemented.

Several numerical analyses were conducted to gain management insight into the structures of the proposed policies. The results presented in Tables 3 to 5 are as follows:

1. When the inventory holding cost h increases, the optimal shelf-space quantity s_j^*, the optimal ordering quantity q_j^*, and the total network profit decline, but the optimal retail price p_j^* increases. When the inventory holding cost increases, the retailer reduces the ordering quantity and shelf space to reduce inventory costs.

2. When the credit period tc increases, the optimal shelf-space quantity s_j^*, the optimal ordering quantity q_j^*, and total network profit rise, but the optimal retail price p_j^* declines. When the inventory holding cost increases, the retailer raises the ordering quantity to obtain additional benefits in the credit period. In turn, the inventory requires additional shelf space to stock the products.

3. When shelf-space maintenance cost m increases, the optimal shelf-space quantity s_j^*, the optimal ordering quantity q_j^*, and the total network profit decline. When the shelf-
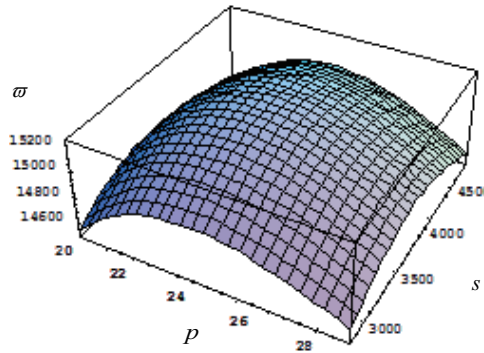
Figure 2: Graphical representation of ϖ

Table 3: Effects of h

		$h = 1$					$h = 2$			
j	Z_{j-1}	p_j^*	s_j^*	q_j^*	Π_j^*	Z_{j-1}	p_j^*	s_j^*	q_j^*	Π_j^*
1	0	23.2	5527	651	3526.85	0	24.5	5222	581	3222.08
2	1	23.2	5001	589	3000.90	1	24.5	4725	526	2725.13
3	2	25.7	3588	792	5175.90	2	28.5	3236	645	4472.98
$\sum$										10420.19

space maintenance cost decreases, the retailer increases the shelf-space quantity to meet demand, and the ordering quantity also increases.

For increasing market demand, we set $D(p_j, s_j, t) = (1000p_j^{-1.5}s_j^{0.5})e^{0.05t}$. The numerical results for dynamic decision-making and fixed decision-making are summarized in Table 6. It also shows that dynamic decision-making is superior to fixed decision-making. It predicts a 1.42% increase in total profit in the increasing market demand case if the dynamic pricing and shelf-space is implemented.

5. CONCLUSION

We developed a finite time horizon inventory model used for considering price, shelf-space, and time-dependent market demand under trade credit. Decisions regarding retail price, shelf space, and ordering quantity are arbitrarily adjusted upward or downward to respond to changes in market demand throughout the planning horizon. The objective of this study was to determine the periodic retail price, shelf-space quantity, and ordering quantity that maximize total profit. The demand function considered in this paper is an exponentially decreasing function of price that exponentially increases according to shelf space, and varies

Table 4: Effects of tc

		$tc = 1.2$					$tc = 1.5$			
j	Z_{j-1}	p_j^*	s_j^*	q_j^*	Π_j^*	Z_{j-1}	p_j^*	s_j^*	q_j^*	Π_j^*
1	0	25.2	4807	535	2806.52	0	24.5	5222	581	3222.08
2	1	25.2	4349	471	2349.11	1	24.5	4725	526	2725.13
3	2	29.4	3017	594	4043.54	2	28.5	3236	645	4472.98
$\sum$										10420.19

Table 5: Effects of m

		$m = 0.8$				$m = 1$			
j	Z_{j-1}	p_j^*	s_j^*	q_j^*	Π_j^*	Z_{j-1}	p_j^*	s_j^*	q_j^*	Π_j^*
1	0	24.5	8159	726	4527.60	0	24.5	5222	581	3222.08
2	1	24.5	7383	657	3906.41	1	24.5	4725	526	2725.13
3	2	28.5	5057	806	6091.23	2	28.5	3236	645	4472.98
$\sum$										10420.19

Table 6: Numerical results for fixed decision-making with increasing market demand

		Dynamic decision-making				*Fixed decision-making*			
j	Z_{j-1}	p_j^*	s_j^*	q_j^*	Π_j^*	Z_{j-1}	p^{**}	s^{**}	q^{**}	Π_j^*
1	0	24.58	5747	638	3747.53	0	24	5674	657	3744.83
2	1	24.58	6352	705	4352.01	1	24	5674	691	4330.26
3	2	24.58	7020	779	5020.05	2	24	5674	726	4945.71
4	3	24.58	7758	861	5758.36	3	24	5674	763	5592.71
$\sum$					18877.95					18613.51

exponentially over time. Directly determining the closed solution to the demand form is impossible because of its complexity. We solved the form by using an extreme value search method. The provided numerical examples illustrate the procedures performed in the solution approach and show that dynamic decision making is superior to fixed decision making regarding profit maximization.

This paper serves as a key starting point for dynamic decision-making and multivariable-demand research directions. Our model does not consider shortages, which is useful for products with high shortage costs (e.g., fashion items, such as clothes and jeans). If customers cannot find the clothes they want in stores, then they leave and the sale is lost. Retailers of this type of product attempt to avoid out-of-stock situations as much as possible. In addition, our model can be applied to products that do not deteriorate (e.g., toys and magazines). Products such as canned foods and detergents can also be regarded as non-deteriorating items. Therefore, further research on this topic could relax certain assumptions to match real-world scenarios, such as deteriorating items or shortages. Thus, the model can be modified and applied to other types of products.

Acknowledgements

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