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MINISUM AND MINIMAX LOCATION MODELS FOR HELICOPTER EMERGENCY MEDICAL SERVICE SYSTEMS

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Abstract This paper presents minisum and minimax location problems for helicopter emergency medical service (HEMS) systems. Given demand points (origins) and hospitals (destinations), the locations of rendezvous points and helicopter stations are selected to minimize the total demand-weighted transport time (minisum objective) and the maximum transport time (minimax objective) to a hospital. Rendezvous points are required for a helicopter to meet with an ambulance. In minimizing these objectives, each demand is allocated to either an already-available ground ambulance or a newly-introduced helicopter. We provide 0-1 integer formulations of the minisum and minimax problems, and develop a variable reduction procedure that reduces the size of the problem. Some optimal solutions of the proposed models tested for an idealized square city are analyzed. We also apply the models to the case study of Japan using geographical and population data, and the locations of the actual emergency medical centers. The proposed variable reduction procedure is shown to be effective for both examples. Results show that the proposed problems tend to focus on low-accessibility locations and that accessibility to a hospital is greatly improved.

Keywords: Facility location theory, emergency medical service, helicopter transportation system, minisum location model, minimax location model

1. Introduction

Recently, the introduction of helicopter transportation has received much attention in various countries. In Japan the number of ambulance calls has increased from 3.7 million calls in 1998 to 5.1 million calls in 2008 [18]. This increases the average response time of ground ambulances and the average arrival time at hospitals. Helicopters are much faster than ground ambulances, and thus can provide better service for patients in need of critical care or living in places with limited accessibility to hospitals. However, many parts of Japan are mountainous or densely populated areas in which landing is difficult. Therefore, constructing landing points is critical to the successful introduction of helicopter emergency medical services (HEMS).

The design of HEMS is complex because helicopters cannot always land directly at demand locations, so combined transportation with ambulances is often required. HEMS is typically operated as follows. First, emergency medical staff goes to the accident site by ground ambulance. When a patient is in critical condition and a helicopter would provide faster transportation to a hospital, the ambulance crew requests helicopter dispatch. Then they transport the patient to a location where a helicopter can land and depart (a rendezvous point). At the same time, a helicopter deployed at a station goes to the rendezvous point, picks up the patient, and transports the patient to a hospital. Therefore, the longer of these two travel times affects the total transportation time to the hospital. Because of this complicated structure, the design of effective HEMS systems requires much more complex decisions than a system with only ground ambulances.

To support such decision-making, this paper develops mathematical programming models of EMS helicopter deployment by focusing on locations of rendezvous points and helicopter stations. In many planning situations ground ambulances are already available, so improving service for low-accessibility locations is the primary goal when introducing helicopter emergency systems. Focusing on this aspect, we introduce emergency helicopters into an area where the service level of existing ground ambulance for each demand location is already given.

This paper considers two problems. First, given demand points (origins) and hospitals (destinations), the locations of rendezvous points and helicopter stations are selected from candidate locations so as to minimize the total demand-weighted transport time (minisum objective) to improve efficiency of the whole system in the region. Second, both locations are selected so as to minimize the maximum transport time (minimax objective) to a hospital to provide EMS for people living in areas of poor accessibility to hospitals by ground ambulance. In minimizing these objectives, each demand is allocated to either an already-available ground ambulance or a newly-introduced helicopter. We show in the numerical example section that optimal solutions for both minisum and minimax tend to provide helicopter service for those areas with poor accessibility to ground ambulance.

This paper is organized as follows. Section 2 reviews the related literature on optimization design models for emergency service systems in which helicopters are used. In Section 3, we describe the general situation of the proposed system, and examine how helicopter locations and rendezvous points affect the transport time of each location. In Section 4, formulations of minisum and minimax problems are given, and a variable reduction procedure that reduces the problem size is also developed. In Section 5, some optimal solutions obtained for a square city model are analyzed. We also apply our models to the case study of Japan using geographical and population data, and locations of emergency hospitals designated as transport destinations. In the final section, we give our conclusions and indicate future research directions.

2. Literature Review

The design and operation of medical service systems have been actively studied areas in operations research and management science. In particular, the facility location modeling approach [12, 14] for medical service systems has attracted many researchers [13], and studies focusing on ambulance location and deployment problems are abundant [10, 21, 24]. Despite widespread public concern for HEMS and the complexity of HEMS design, location models focusing on EMS helicopters are scarce and much less developed than ambulance location models involving only ground ambulances. In the following, we first review a few studies dealing with deployment of EMS helicopters, then literature related to the location problems of rendezvous points.

Schuurman et al. [26] develop a method that identifies where an additional EMS helicopter resource should be placed to cover the greatest population among those currently underserved according to their proximity to existing services in the region. They apply the model to analyze a real world situation in which two hospitals with helicopter medical service already exist. Their analysis uses five years of critical care data from the British Columbia Trauma Registry, along with population and travel time data. They then analyze how to introduce an additional helicopter over the existing two candidate hospitals.

Branas et al. [7] simultaneously consider locations for a trauma center and aero-medical stations. They also propose a heuristic algorithm for their problem [8], and apply the model to several regions in the United States [9]. The model is implemented and shared in an interactive web-site that uses web-based geographical information systems [9].

Bastian [2] proposes a multi-criteria decision analysis method using a scenario-based, stochastic optimization goal-programming model to help U.S. Army planners make strategic and tactical aeromedical evacuation asset plans, and to improve the current air evacuation system in Afghanistan. The model employs three objectives: minimizing the number of helicopters at each medical facility, maximizing the expected demands that can be served, and minimizing the maximal total vulnerability of evacuation sites to enemy attack. Fulton et al. [19] develop a stochastic optimization model for decision support in the relocation of medical evacuation assets, including medical evacuation helicopters and ground ambulances, during military stability operations. The model seeks air evacuation sites, hospitalization sites, and paths the evacuation assets should take so as to minimize expected travel time over all casualty scenarios.

Erdemir et al. [15] discuss aeromedical base locations in New Mexico considering that demands (e.g. automobile accidents) occur not only at nodes (e.g. intersections) but also on paths (e.g. roads). They use a model [16] for service facilities with requests originating from both nodes and paths to find optimal locations of helicopter stations to maximize the number of demands that can be served. Erdemir et al. [17] propose two covering models to determine the locations of ambulances, helicopters, and rendezvous points. They assume that a given demand is covered when it receives coverage from direct transportation by helicopter, transportation by two ground ambulances, or combined transportation by helicopter and ambulance. One of their models has a set-covering objective that seeks the minimum total cost required to locate three types of facilities when covering all demand points. The other has a maximum-covering objective that seeks to provide the maximum coverage of demands within a given total cost.

Furuta and Tanaka [20] develop location models for a “doctor-helicopter” system in which doctors are delivered to patients by helicopter for primary care. Their model aims to maximize the average survival rate by optimizing locations of helicopters and rendezvous points.

Location models of EMS helicopters except Erdemir et al. [17] and Furuta and Tanaka [20] assume that helicopters can land anywhere, and do not consider rendezvous points between air and ground ambulances. Our model explicitly assumes that helicopters require a rendezvous point. The objectives employed in [17, 20] are based on maximizing the demands covered within a specified transportation time. While this approach is important, we focus on different approaches. One is the average time taken to respond to demand calls for evaluating service quality. The other is a minimax approach that consider worst case performance of the system. This paper focuses on the time to transport a patient to a hospital, and proposes minisum and minimax location models for designing effective and equitable deployment of EMS helicopters.

There are some non-helicopter location models that are structurally similar to the proposed models.

Berman et al. [5] propose the transfer point location problem (TPLP), where the optimal location of a single transfer point needs to be found on a plane under the condition that the location of a single facility is given. The facility provides the same service to demands that can go to the facility via a transfer point or directly. Travel time from the transfer point to the facility is shorter than that from demand nodes to the transfer point or the facility due to a rapid transportation system. They also consider TPLP on a network taking into account the continuously varying travel time discount rate between the transfer point and the facility, and proposed a solution algorithm for the problems. Berman et al. [6] propose the multiple transfer points location problem (MTPLP) as a natural extension of TPLP, where the establishment of multiple transfer points is allowed. Furthermore, Berman et al. [4] consider generalized models called multiple location of transfer points (MLTP) and the facility and transfer points location problem (FTPLP), which make the location of multiple facilities possible. The location of facilities is given in MLTP, but an optimal location of both facilities and transfer points needs to be found in FTPLP. They proposed three heuristics to solve the minisum MLTP and FTPLP on a network, and reported comprehensive computational results using a benchmark data set provided by the OR-Library [3]. However, they did not suggest exact optimal solutions due to the size of the problems. Sasaki et al. [25] show another formulation of the minisum MLTP similar to the p-median problem. Their formulation allows a mathematical programming solver to obtain exact optimal solutions. They also present a new formulation of FTPLP and an enumeration-based approach to solve the problems with a single facility.

Researchers have studied and improved many aspects of hub-and-spoke networks commonly used by many air carriers to capture widespread demand efficiently since the United States Airline Deregulation Act was enacted in 1978. A hub-and-spoke network consists of intermediate airports (hubs), special links between hubs (hub arcs), and links between non-hub airports and hubs (access arcs). Unit transportation costs on the hub arcs are lower than those on non-hub arcs due to economies of scale. The hub location problem is to seek optimal hub locations and allocations of non-hub airports to the hubs. Various hub location models have been studied so far and the literature is summarized in Alumur and Kara [1] and Campbell et al. [11].

Both transfer point location problems and hub location problems have two types of transportation speeds. However, the problems do not explicitly consider rendezvous between two types of movable units. To design effective HEMS, it is required to consider rendezvous between two mobile units (a helicopter and an ambulance) with different speeds, because helicopters cannot depart from the rendezvous point until the patient arrives in the ambulance.

3. Helicopter Emergency Medical Service System

Before introducing the mathematical programming models for our systems, we provide spatial characteristics of a system with two movable units with different speeds and a rendezvous between them. By using a simple city model, we can analyze the effect of location changes of a facility (a rendezvous point or station) on the average and maximum transportation time.

We now introduce the HEMS assumed in this paper. Consider the situation where an ambulance service is already well established, and an HEMS is to be introduced to upgrade the emergency medical system. We measure the travel time of both ambulance and helicopter modes as the time taken to transport a patient from the accident site to a hospital. We assume that the patient at the accident site needs critical care and should, therefore, be transported to a large hospital as quickly as possible. Hence, the patient should be transported by the mode with the shortest transportation time. An ambulance at the demand location requests helicopter dispatch when the patient can be provided with faster transportation to a hospital; otherwise, the ambulance transports the patient to a hospital. We assume helicopters can land directly at the hospital. The transportation time for each mode is discussed later.

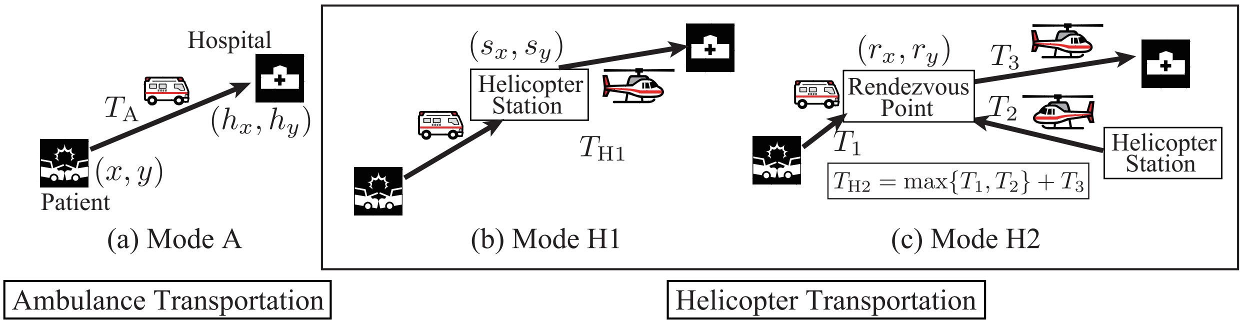


Figure 1: Three types of transportation modes in the helicopter emergency medical system

When an HEMS is introduced, we assume that three modes of transport, an ambulance mode and two helicopter modes, are available to move patients to a hospital (Figure 1). In ambulance mode (mode A), a patient is transported directly to a hospital by ambulance from the demand location. There are two helicopter modes. In mode H1, an ambulance first moves a patient to a helicopter station, then a helicopter deployed at the station carries the patient to a hospital. In mode H2, an ambulance and a helicopter meet at a rendezvous point, then the helicopter carries the patient to a hospital. Patients are assumed to be transported by the fastest of the three modes.

First, we analyze how locations of rendezvous points and helicopter stations affect each patient's travel time to a hospital using an idealized square city model. This analysis provides a basic framework to evaluate optimal solutions in Section 4. Let us consider a situation in which there is a single hospital and each demand has three modes to access the hospital. To determine the travel time for the three modes, consider a square city with sides of length L (km). We introduce a Cartesian coordinate system (x, y) with its origin at the square's center, and allow a demand location to be at an arbitrary point. The hospital, the rendezvous point, and station locations are denoted by (h_x, h_y), (r_x, r_y), and (s_x, s_y), respectively. and the helicopter and ambulance speeds are denoted by v and w. We assume that the travel distance between two points is the Euclidean distance, making it easy to evaluate spatial accessibility of various points in the targeted area. In mode A, the travel time by ambulance from a demand point to the destination is

$$T_{\text{A}} = \frac{\sqrt{(h_x - x)^2 + (h_y - y)^2}}{w}.$$

In mode H1, the travel time from a demand point to the destination, when using the station at (s_x, s_y), is

$$T_{\text{H1}} = \frac{\sqrt{(s_x - x)^2 + (s_y - y)^2}}{w} + \frac{\sqrt{(h_x - s_x)^2 + (h_y - s_y)^2}}{v}.$$

In mode H2, the travel time from a demand point to the destination, when using an rendezvous point at (r_x, r_y) and a station at (s_x, s_y), is

$$T_{\text{H2}} = \max\{T_1, T_2\} + T_3,$$

where

$$T_1 = \frac{\sqrt{(r_x - x)^2 + (r_y - y)^2}}{w},$$
$$T_2 = \frac{\sqrt{(r_x - s_x)^2 + (r_y - s_y)^2}}{v},$$
$$T_3 = \frac{\sqrt{(h_x - r_x)^2 + (h_y - r_y)^2}}{v}.$$

The max operator in the first term means that both the ambulance and the helicopter must arrive at the rendezvous point before the patient can depart for the hospital. The mode selected is the one that provides the shortest travel time to the hospital:

$$T = \min\{T_{\rm A},\ T_{\rm H1},\ T_{\rm H2}\}.$$

To evaluate the HEMS system, we also analyze each patient's travel time using the ambulance-only mode or the shorter of the two helicopter modes. The shortest of these three is the transportation time assumed in this paper. Figure 2 shows a contour plot and a three-dimensional plot for patient travel time to the hospital in helicopter mode, $\min\{T_{\rm H1},\ T_{\rm H2}\}$, when the station is located at $(-30, 30)$, the rendezvous point is located at $(10, -20)$, and the helicopter and ambulance speeds are $v = 200$ km/h and $w = 40$ km/h, respectively. The hospital is located at $(25, 0)$. The result is a circular area around the rendezvous point in which travel time to the hospital is constant. Within this area, patients (e.g., d1 in Figure 2(a)) arrive at the rendezvous point earlier than the helicopter: $T_1 < T_2$. As a result, travel time to the hospital is $T_{\rm H2} = T_2 + T_3$, a constant value. A patient located outside the area, for example d2 in Figure 2(a), arrives at the rendezvous point later than the helicopter, so the patient's travel time is $T_{\rm H2} = T_1 + T_3$, with T_1 increasing with the distance from the demand location to the rendezvous point. Demands around the station form a cone-like area in which patients are transported to the hospital by way of this station (Mode H1 in Figure 1).

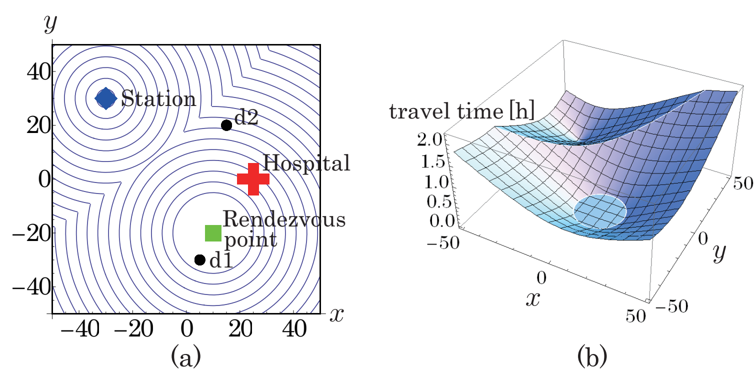


Figure 2: Travel time $\min\{T_{\rm H1},\ T_{\rm H2}\}$ using a helicopter

Next, we consider the case in which several rendezvous points exist. Figures 3(a) and 3(b) illustrate travel time to the hospital by ambulance ($T_{\rm A}$) and by helicopter ($\min\{T_{\rm H1},\ T_{\rm H2}\}$), respectively. In Figure 3(b) there are three rendezvous points, $(10, -20)$, $(15, 20)$, and $(-10, -30)$. Figures 4(a) and 4(b) represent the travel time from each demand location to the hospital when the fastest transportation mode is used. Travel time is clearly affected by the station and rendezvous point locations.

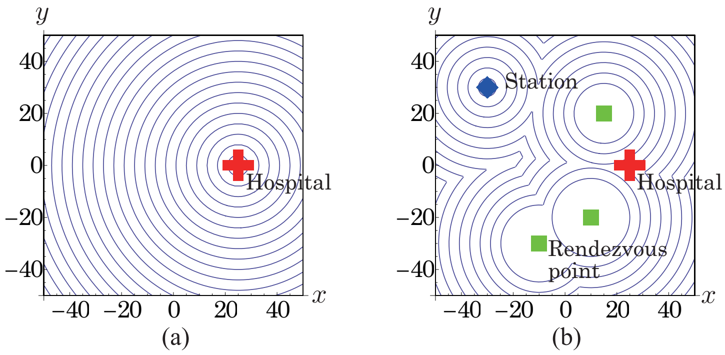


Figure 3: Travel time contours for each mode : (a) ambulance mode T_{A}; (b) helicopter mode $\min\{T_{\mathrm{H1}},\ T_{\mathrm{H2}}\}$

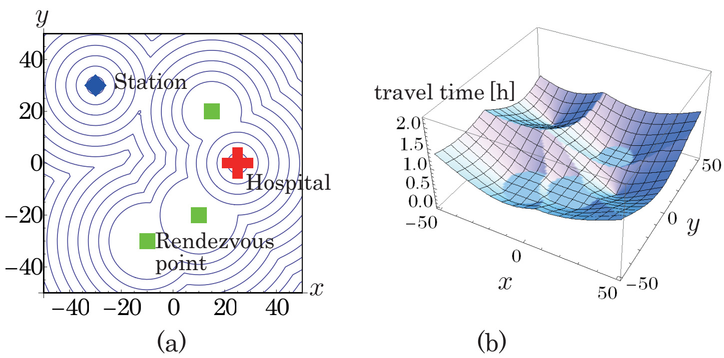


Figure 4: The shortest travel time of all three modes: $\min\{T_{\mathrm{A}},\ T_{\mathrm{H1}},\ T_{\mathrm{H2}}\}$

Next, we analyze the total travel time and the maximum travel time to the hospital for all demands, the two objective functions we use in our models. We assume that demands are regularly and uniformly distributed in a 20×20 grid in a square city.

Figures 5(a) and 5(b) show how the total travel time to the hospital, obtained by summing each patient's travel time, is affected by the location of the station. In the figures, two rendezvous points are located at $(0, 30)$ and $(-25, 10)$. In Figure 5(a), darker color means smaller total travel times. The figures show that the most desirable location for a station is some distance from rendezvous points and the hospital. Because of the high speed of helicopters, it is not advantageous to locate stations near rendezvous points. When using a helicopter, each demand is allocated to a rendezvous point or to the station, so the desirable station location is at a point that effectively addresses all demands. Next we analyze how rendezvous point location affects the total travel time when the station location is fixed. Figures 6(a) and 6(b) are the contour plot and the three-dimensional plot of the total travel time when a station is located at $(-30, 30)$. When the rendezvous point is located near the hospital at $(25, 0)$ or the station, the total travel time increases. In such cases, no demands receive sufficient benefit from using the rendezvous point because each patient has to be transported by a lower-speed ambulance to the rendezvous point near the hospital or the station. In fact, the benefit is exactly zero when the rendezvous point is located at $(25, 0)$ or $(-30, 30)$, which is the location of the hospital or the station.

Figures 7(a) and 7(b) show how the maximum travel time to the hospital, the minimax objective, is influenced by the location of the station. In the figures, two rendezvous points are fixed at $(0, 30)$ and $(-25, 10)$. A hospital is located at the right of the city center,

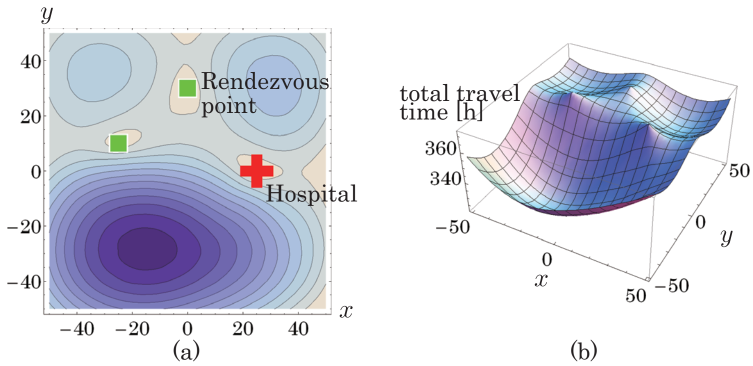


Figure 5: Total travel time as a function of station location with rendezvous points at $(-25, 10)$, $(0, 30)$

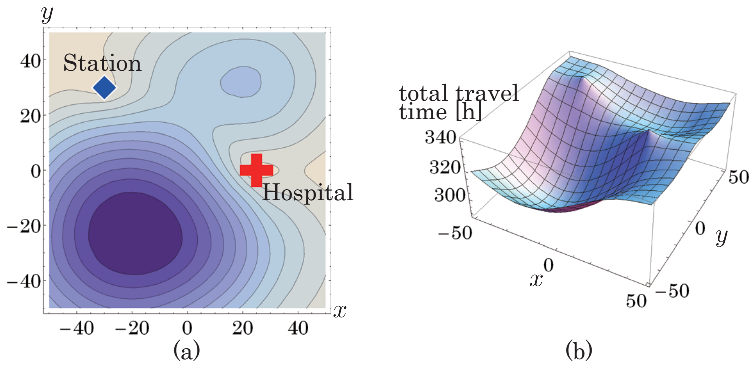


Figure 6: Total travel time as a function of rendezvous point location with station at $(-30, 30)$

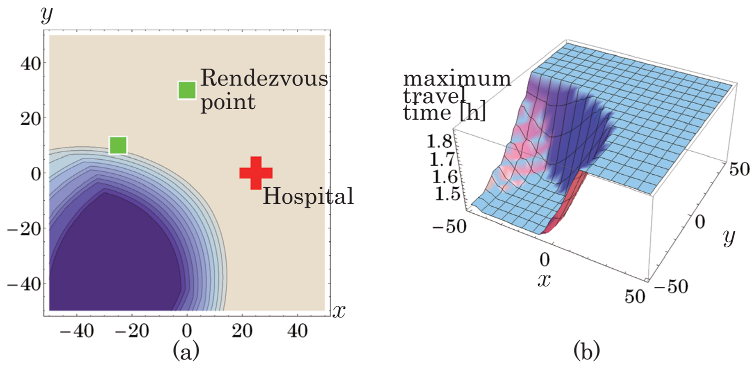


Figure 7: Maximum travel time as a function of station location with rendezvous points at $(-25, 10)$, $(0, 30)$

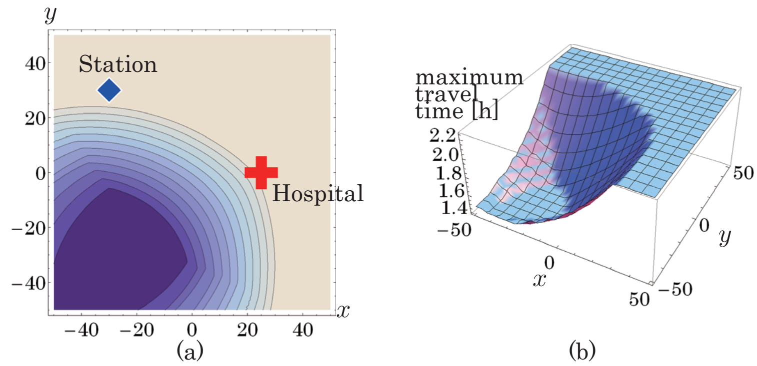


Figure 8: Maximum travel time as a function of rendezvous point location with station at $(-30, 30)$

at $(25, 0)$. Before the introduction of HEMS, the upper-left and lower-left corners require the maximum transportation time by ground ambulance. To reduce the minimax objective value, transportation from these points should be by helicopter. As illustrated in Figure 7, the maximum transportation time is reduced only when a station is located near the lower-left corner. A station near the lower-left corner means transportation from there is faster by mode H1, and transportation from the upper-left point is faster by mode H2, decreasing maximum transportation time. Next we analyze how rendezvous point location affects the maximum travel time when the station location is fixed. Figures 8(a) and 8(b) show how the maximum travel time is affected by the location of a rendezvous point. When a station is located at $(-30, 30)$, the upper-left corner point can obtain a faster transportation by mode H1. The transportation time of the lower-left corner point should be reduced to reduce the maximum transportation time. A rendezvous point near the lower-left corner means transportation is faster from the lower-left point using mode H2, decreasing the maximum transportation time.

4. Minisum Model and Minimax Model for HEMS

4.1. Formulation

As described in the previous section, travel time to the hospital from a demand point is greatly affected by the rendezvous point and station locations. Assuming that demands are distributed discretely over the target region, this section develops the following problems and their integer programming formulations. Given demand points (origins) and hospitals (destinations), the locations of rendezvous points and stations are selected so as to minimize the total demand-weighted transport time (minisum objective) and the maximum transport time (minimax objective) to a hospital. In minimizing these objectives, each demand is allocated to either an already-available ground ambulance or a newly-introduced helicopter.

The minisum problem is how to locate p rendezvous points and q stations so as to minimize the total demand-weighted travel time to the hospital. The proposed minisum model is an extended version of the classical p-median model [22, 23]. The p-median problem seeks to simultaneously locate p new facilities over a targeted region so as to minimize the total demand-weighted distance. The p-median model tends to locate facilities near large demand locations since the impact of reducing distance between demand location and a facility increases with demand. The proposed minisum model focuses on introducing HEMS into an area where ground ambulance services already exist. In such situations, transport time by ground ambulance in a suburban area tends to take longer than that in a central area with large population. Thus, a larger reduction in transport time can be attained by improving the service level of low-accessibility suburban locations than high-accessibility locations. This is a distinct characteristic of the proposed minisum problem.

Furthermore, we propose a minimax model seeking locations for p rendezvous points and q stations that minimizes the maximum transportation time to the hospital for all demands. The minimax model is constructed as an extended version of the classical p-center model [22, 23], in which p facilities are located so as to minimize the maximum distance to the nearest facility. The proposed minisum model focuses on providing EMS for people living in an area where there is only poor accessibility to hospitals by ground ambulance.

To formulate the problems, the following notation is introduced:

Parameters

I : set of demand points, indexed by i

J : set of candidate locations for rendezvous points, indexed by j

K : set of candidate locations for stations, indexed by k

p : number of rendezvous points to be located

q : number of stations to be located

h_i : demand at demand point i

t_i^{A} : travel time from demand point i to the hospital by an ambulance (mode A)

t_{ik}^{H1} : travel time from demand point i to the hospital via the station k (mode H1)

t_{ijk}^{H2} : travel time from demand point i to the hospital by using the combination of rendezvous point j and station k (mode H2)

Decision variables

x_j : 0-1 location variable; 1 if an rendezvous point is constructed at node j, 0 otherwise

y_k : 0-1 location variable; 1 if a station is constructed at node k, 0 otherwise

u_i : 0-1 allocation variable; 1 if demand point i is served by an ambulance (mode A), 0 otherwise

v_{ik} : 0-1 allocation variable; 1 if demand point i is served by station k (mode H1), 0 otherwise

w_{ijk} : 0-1 allocation variable; 1 if demand point i is served by the combination of rendezvous point j and station k (mode H2), 0 otherwise

The minisum model for HEMS is then formulated as follows.

Minisum model for HEMS

$$
\begin{aligned}
\min \quad & \sum_{i \in I} h_i \left\{ t_i^{\mathrm{A}} u_i + \sum_{k \in K} t_{ik}^{\mathrm{H1}} v_{ik} + \sum_{j \in J} \sum_{k \in K} t_{ijk}^{\mathrm{H2}} w_{ijk} \right\}, && && (4.1)\\
\text{s. t.} \quad & v_{ik} \le y_k, && i \in I, k \in K, && (4.2)\\
& \sum_{k \in K} w_{ijk} \le x_j, && i \in I, j \in J, && (4.3)\\
& \sum_{j \in J} w_{ijk} \le y_k, && i \in I, k \in K, && (4.4)\\
& u_i + \sum_{k \in K} v_{ik} + \sum_{j \in J} \sum_{k \in K} w_{ijk} = 1, && i \in I, && (4.5)\\
& \sum_{j \in J} x_j = p, && && (4.6)\\
& \sum_{k \in K} y_k = q, && && (4.7)\\
& x_j \in \{0,1\}, && j \in J, && (4.8)\\
& y_k \in \{0,1\}, && k \in K, && (4.9)\\
& u_i \in \{0,1\}, && i \in I, && (4.10)\\
& v_{ik} \in \{0,1\}, && i \in I, k \in K, && (4.11)\\
& w_{ijk} \in \{0,1\}, && i \in I, j \in J, k \in K. && (4.12)
\end{aligned}
$$

The objective function (4.1) is the total demand-weighted transportation time to the hospital. Inequality (4.2) states that demand i can only be assigned to mode H1 at station k if a station is located at k. Inequalities (4.3) and (4.4) combine to mean that demand i can use a rendezvous point j and station k pair ($w_{ijk} = 1$) when a rendezvous point and a station are constructed in both locations, and demand i is assigned to at most one pair (j, k). Equation (4.5) means that each demand is allocated to only one transportation mode. Minimization of the objective function means each demand is allocated to the mode that provides the shortest transportation time among all possible alternatives. Equations (4.6) and (4.7) mean that the number of rendezvous points and stations is p and q, respectively. Equations (4.8), (4.9), (4.10), (4.11), and (4.12) are the standard binary constraints.

So far, we have assumed that there is only one hospital as the final destination. We can easily relax this assumption and can address the case with several hospitals without modifying the above formulation; if several hospitals exist, we select the hospital with minimum travel time from all hospitals when calculating the values of t_i^{A}, t_{ik}^{H1}, and t_{ijk}^{H2}. In Section 4.2, we deal with a real world example in which several hospitals exist.

Minimax model for HEMS

In a similar manner, we can formulate the minimax problem for an HEMS by introducing a new variable Z to represent the maximum transportation time between a demand point and the hospital.

$$\begin{aligned} &\min \quad Z, && (4.13)\\ &\text{s. t.} \quad t_i u_i + \sum_{k\in K} t_{ik} v_{ik} + \sum_{j\in J}\sum_{k\in K} t_{ijk} w_{ijk} \le Z, \qquad i \in I, && (4.14)\\ &\text{plus} \quad (4.2), (4.3), (4.4), (4.5), (4.6), (4.7), (4.8), (4.9), (4.10), (4.11), (4.12) \text{ above.} \end{aligned}$$

The objective function (4.13) represents the maximum transportation time for all demand locations. Constraint (4.14) stipulates that the maximum transportation time, Z, must be greater than the time to travel between any demand point and the hospital.

4.2. Variable reduction procedure

The number of variables in the minisum and minimax problems introduced in the previous section can easily become large because of the three subscripts of w_{ijk}. Therefore, it is desirable to develop a procedure to reduce the size of the problem. We present a variable reduction procedure that utilizes input data of transportation times in deciding which variables to eliminate. First, we consider how to reduce variables v_{ik} by comparing the travel times $t_i^{\rm A}$ and $t_{ik}^{\rm H1}$. When the transportation time of demand i by an ambulance, $t_i^{\rm A}$, is less than that of using station k, $t_{ik}^{\rm H1}$, as indicated in Figure 9(a), station k cannot be used because each demand is allocated to the shortest transportation mode. This means that v_{ik} must be 0 in the optimal solution, so we can eliminate v_{ik} in the above formulation. The set of variables v_{ik} that can be eliminated in advance is

$$V_{\rm A}^{\rm H1} = \bigcup_{i\in I}\{v_{ik},\ k \in K \mid t_i^{\rm A} \le t_{ik}^{\rm H1}\}.$$

Similarly, w_{ijk} can be eliminated when $t_i^{\rm A} \le t_{ijk}^{\rm H2}$ (Figure 9(b)), that is, when a patient at demand point i can be transported to the hospital by an ambulance faster than using the combination of rendezvous point j and station k. In this case, the set of variables w_{ijk} that can be eliminated in advance is

$$W_{\rm A}^{\rm H2} = \bigcup_{i\in I}\{w_{ijk},\ j \in J,\ k \in K \mid t_i^{\rm A} \le t_{ijk}^{\rm H2}\}.$$

This elimination procedure can similarly be continued by comparing the transportation times for mode H1 and mode H2 (Figure 9(c)). In this case, the set of variables w_{ijk} that can be eliminated in advance is

$$W_{\rm H1}^{\rm H2} = \bigcup_{i\in I,\ k\in K}\{w_{ijk},\ j \in J \mid t_{ik}^{\rm H1} \le t_{ijk}^{\rm H2},\}.$$

There is another less common situation in which the number of variables can be reduced. First, consider the case when we can remove some of the variables u_i. This occurs when the transportation time by an ambulance for demand i is larger than the transportation times of all the possibilities of mode H1 and mode H2. That is, when $t_i^{\rm A} > t_{ik}^{\rm H1}$ for all k and $t_i^{\rm A} > t_{ijk}^{\rm H2}$ for all (j, k) pairs, we can eliminate u_i. The set of variables u_i that can be eliminated is

$$U_{\rm H1,H2}^{\rm A} = \{u_i,\ i \in I \mid (t_{ik}^{\rm H1} < t_i^{\rm A}) \wedge (t_{ijk}^{\rm H2} < t_i^{\rm A}),\ \forall j \in J,\ \forall k \in K\}.$$

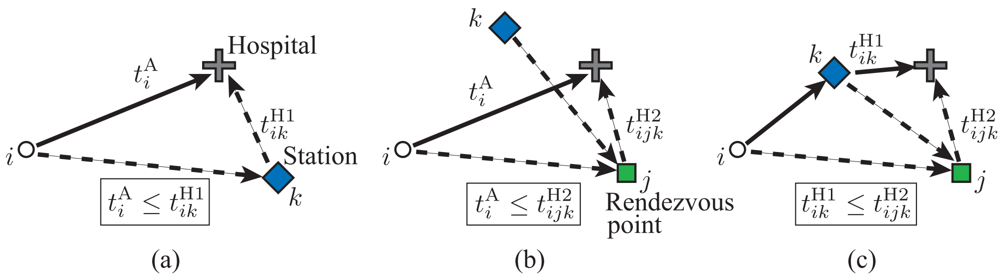


Figure 9: Comparing transportation times to the hospital between different modes for eliminating variables

Similarly, we can eliminate some of the variables v_{ik} when $t_{ik} > t_{ijk}$ for all j. Then, the set of variables v_{ik} that can be eliminated can be described as

$$V_{\mathrm{H2}}^{\mathrm{H1}} = \{v_{ik},\ i \in I,\ k \in K \mid t_{ijk}^{\mathrm{H2}} < t_{ik}^{\mathrm{H1}},\ \forall j \in J\}.$$

One or more of these eliminations can frequently be made, and dramatically reducing the problem size.

5. Computational Results

5.1. Analysis of optimal solutions using an idealized square city

This section presents some optimal solutions of the proposed models for an idealized square city model and discusses some insights obtained during this analysis. Optimal solutions were obtained with the commercial mathematical programming software IBM ILOG CPLEX 12.2 on a computer with a Intel Core i5 (2.53 GHz) and 4 GB of RAM. We consider a square target area with 100 km sides (Figure 10). A hospital is located at the center, $(x, y) = (0, 0)$. There are 49 (7×7) candidate rendezvous point locations (squares in Figure 10) and 25 (5×5) candidate station locations (diamonds in Figure 10). We use a regularly and uniformly distributed demand pattern in which each demand (circles in Figure 10) is located at a grid mesh center. The number of demands is 225 (15×15). The number of variables for u_i, v_{ik} and w_{ijk} in this problem example are 225, $5,625$, and $275,625$, respectively. Travel time is calculated by the Euclidean distance divided by the helicopter and ambulance speed: 200 km/h and 40 km/h.

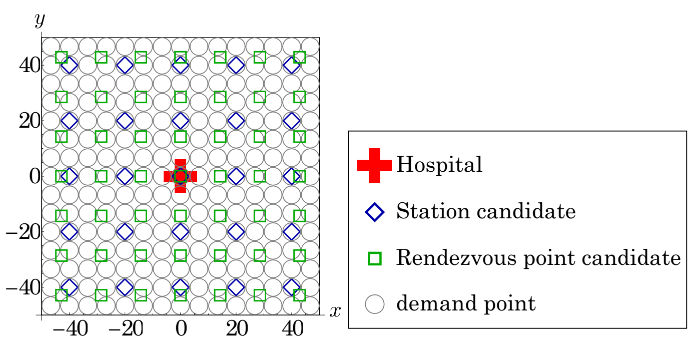


Figure 10: A square target area: $|I| = 225$, $|J| = 49$, $|K| = 25$

We first obtain exact optimal solutions and show the effect of the variable reduction procedure. We then examine spatial patterns of rendezvous points and stations for several (p, q)

Table 1: Computational times (s.) with and without variable reduction procedure (VRP)

	p	q	with VRP	without VRP
minisum model	4	1	70.14	131.93
	6	2	10.92	37.00
	10	3	481.40	1026.10
minimax model	4	1	134.86	392.08
	6	2	67.21	260.46
	10	3	228.67	249.29

Table 2: Average and maximum transportation times in the optimal solutions (min)

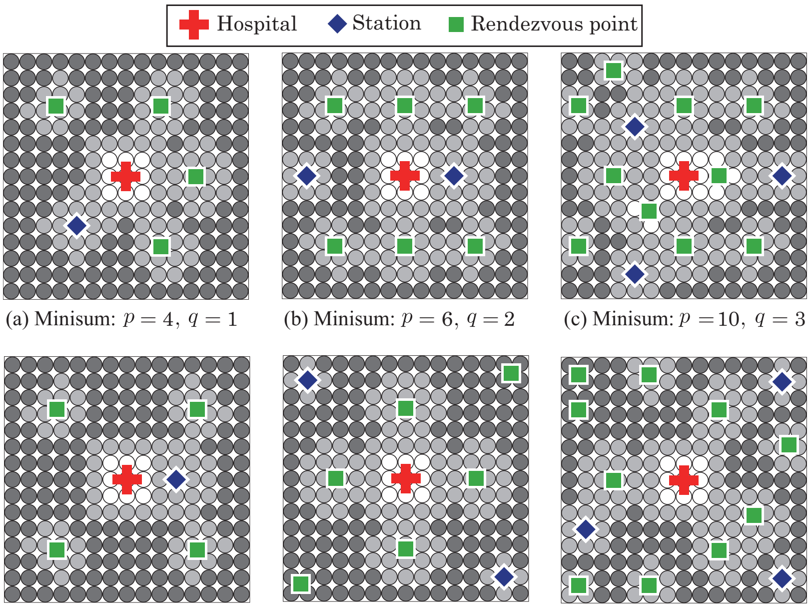
	p	q	average	maximum
without HEMS			57.27	98.98
minisum model	4	1	34.90	65.22
	6	2	29.90	50.50
	10	3	27.03	50.50
minimax model	4	1	34.95	62.85
	6	2	32.69	49.02
	10	3	28.39	41.20

pairs. We also analyze how each patient's transportation time is reduced by introducing an HEMS.

We applied the variable reduction procedure explained in the previous section. The number of variables eliminated from each set are $|V_{\mathrm{A}}^{\mathrm{H1}}| = 4,447$, $|W_{\mathrm{A}}^{\mathrm{H2}}| = 214,959$, $|W_{\mathrm{H1}}^{\mathrm{H2}}| = 139,447$ ($|W_{\mathrm{A}}^{\mathrm{H2}} \cap W_{\mathrm{H1}}^{\mathrm{H2}}| = 131,415$), $|U_{\mathrm{H1,H2}}^{\mathrm{A}}| = 0$, and $|V_{\mathrm{H2}}^{\mathrm{H1}}| = 0$. In total, the number of variables in u_i, v_{ik}, and w_{ijk} were reduced by 0 (0 %), 4,447 (79.1 %), and 222,991 (80.9 %), respectively. Computation time for obtaining optimal solutions by CPLEX with and without the variable reduction procedure are shown in Table 1. The table indicates that our procedure is sufficiently effective.

Table 2 lists two types of objective values in the optimal solutions of the minisum model and the minimax model. The minimax model usually has several optimal solutions. In that case, we select the one with the smallest total transportation time, the minisum objective, among the set of optimal solutions of the minimax model. Table 2 gives minisum objective values in terms of the average transportation time, measured in minutes for ease of comparison. Figure 11 shows the optimal solutions; squares and diamonds denote the selected optimal locations of rendezvous points and stations. Colors of demand points show the travel time to the hospital: black points mean the time is more than 30 minutes, gray points mean from 15 minutes to 30 minutes, and white means within 15 minutes. Demands farther from the hospital tend to have larger travel times but patients near rendezvous points or stations can be transported to the hospital in a relatively short time.

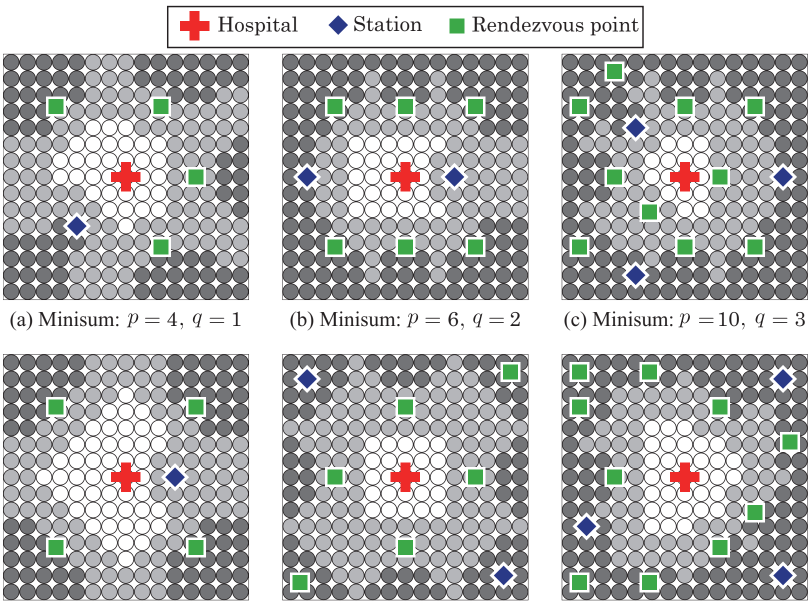
Figure 12 shows the reduction of each demands' transportation time after introducing an optimal HEMS. Demands colored white represent ambulance users whose reduction in the transportation time is 0. Demands colored light gray and dark gray represent helicopter users with reduction in the transportation time less than 30 minutes and more than 30 minutes, respectively. Figure 12 shows that the transportation time of demand points having a rendezvous point on the way to the destination is a greatly reduced.



(a) Minisum: $p = 4,\ q = 1$ (b) Minisum: $p = 6,\ q = 2$ (c) Minisum: $p = 10,\ q = 3$

(d) Minimax: $p = 4,\ q = 1$ (e) Minimax: $p = 6,\ q = 2$ (f) Minimax: $p = 10,\ q = 3$

Figure 11: Optimal solutions and each demand's transportation time



(a) Minisum: $p = 4,\ q = 1$ (b) Minisum: $p = 6,\ q = 2$ (c) Minisum: $p = 10,\ q = 3$

(d) Minimax: $p = 4,\ q = 1$ (e) Minimax: $p = 6,\ q = 2$ (f) Minimax: $p = 10,\ q = 3$

Figure 12: Optimal solutions and the reduction of each demand's transportation time

Minisum solutions have a larger number of demand points achieving a reduction of 30 minutes or more than minimax solutions, resulting in a smaller total transportation time. On the other hand, minimax solutions tend to have more rendezvous points in the peripheral areas of cities in comparison with minisum solutions, which provide a large benefit for people living in places far from the hospital.

5.2. Case study using actual cities

We apply the minisum and minimax problems to the location of rendezvous points and stations using actual population data for Japan. The target area comprises three major prefectures in the Kanto area: Tokyo, Saitama, and Kanagawa (Figure 13). The total population of this area in 2005 was 28,508,070. In this analysis, hospitals having an emergency medical center are selected as destinations and shown by cross marks in Figure 13. There are 46 such hospitals in the area. We assume that demand points and the candidate locations for rendezvous points are the 191 cities in the target area, shown by small dots in Figure 13. The population of each city is shown in gray-scale. For the candidate locations of stations, we select the 50 cities shown by diamonds in Figure 13. The central part of the area is densely inhabited and has a large number of emergency hospitals, whereas peripheral areas, especially in the northern part, are less inhabited and have only a few emergency hospitals. It is interesting to compare the optimal solutions of the minisum and minimax problems using actual city data, especially when populations are unevenly distributed. In this case, the numbers of 0-1 variables u_i, v_{ik}, and w_{ijk} are 191, $9,550$, and $1,824,050$, respectively. We apply the variable reduction procedure as illustrated in the previous section. The numbers of variables eliminated for each set are $|V_{\rm A}^{\rm H1}| = 9,302$, $|W_{\rm A}^{\rm H2}| = 1,797,140$, $|W_{\rm H1}^{\rm H2}| = 854,649$ ($|W_{\rm A}^{\rm H2} \cap W_{\rm H1}^{\rm H2}| = 852,941$), $|U_{\rm H1,H2}^{\rm A}| = 0$, and $|V_{\rm H2}^{\rm H1}| = 0$, respectively. In total, the reduction procedure reduced variables u_i, v_{ik}, and w_{ijk} by 0 (0 %), $9,302$ (97.4 %), and $1,798,848$ (98.6 %), respectively.

Figure 14(a) shows an optimal solution of the minisum problem with $p = 10$ and $q = 2$. Filled squares represent selected rendezvous points and filled diamonds represent selected stations. Cities shown in Figure 14(a) are allocated to helicopters. In the minisum solution, one station is located near the central area and the other is located some distance away from the central area. As to the location of rendezvous points, some are located near the central area while others are located in the northern part of the area. Also, Figure 14(a) indicates that rendezvous points are selected from cities where the distance to the nearest emergency hospital is relatively large. The objective function of the minisum problem is the total demand-weighted transportation time, and thus two types of approach exist for reducing the value of the objective function. One is reducing the transportation time by a small amount in an area having a large population. In this case, even though the time reduction is small, it can have a great effect on the objective value if the population is sufficiently large. The other approach is to reduce the transportation time by a large amount. In this case, there can be a large reduction of the objective value, even if the population is small. We can observe both types of reduction and the latter reduction more than the former one in Figure 14(a). Because the latter reduction is important in providing EMS to people living in areas with poor accessibility to a hospital, the minisum model is useful in designing medical service systems with helicopters.

Figure 14(b) shows an optimal solution of a minimax problem with $p = 10$ and $q = 2$. The locations of rendezvous points and stations in this solution have a very distinct pattern, different from that of the minisum model. Both stations are located in suburban areas, where accessibility to hospitals is poor. Also, rendezvous points are located in both the northern

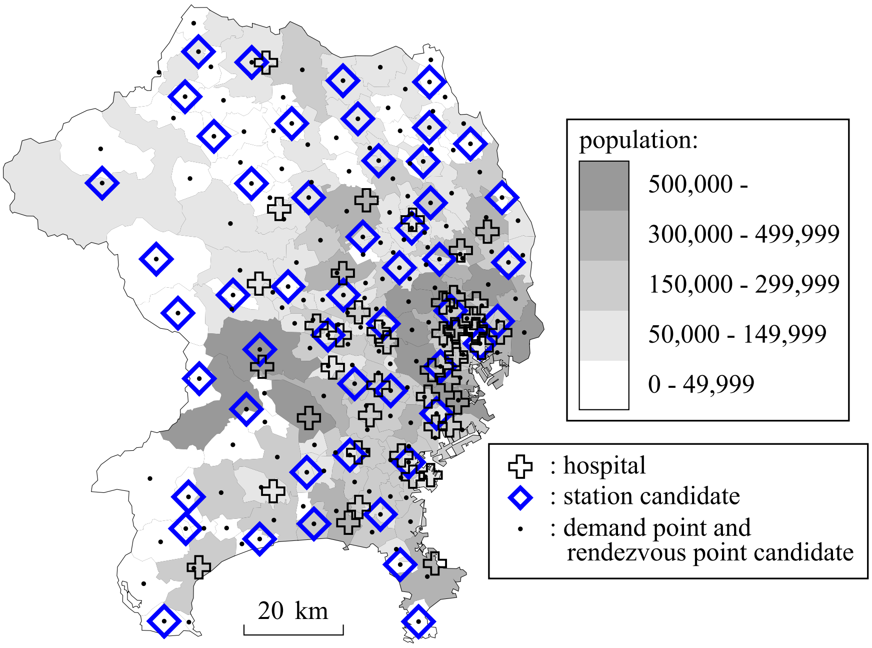


Figure 13: Target area (Tokyo, Saitama, and Kanagawa)

and western peripheral areas, greatly reducing the travel time to hospitals in these areas by providing helicopter transportation. One important role for an HEMS is to provide faster transportation for people living in areas with lower accessibility to hospitals. The minimax solution exhibits a suitable structure for achieving this goal: people living in areas with high accessibility to hospitals are served by ambulances while those living in low-accessibility areas benefit by being served by helicopters.

Table 3: Average and maximum transportation times for three different solutions (min)

	average	maximum
without HEMS	7.4	56.3
minisum solution for $p = 10,\ q = 2$	6.2	32.7
minimax solution for $p = 10,\ q = 2$	6.7	19.3

Table 3 lists the average and maximum transportation times for three different solutions (measured in minutes): the solution without HEMS (in which all demands are served by ambulances), the minisum solution for Figure 14(a) and the minimax solution for Figure 14(b). The average transportation time is the minisum objective value divided by the total demand, and the maximum transportation time is the minimax objective value. The minisum solution reduces the average transportation time by more than one minute, while the minimax solution reduces it by more than 30 s. In terms of the maximum transportation time, the minimax solution performs very well: the value is reduced by roughly two-third. The minisum solution also performs fairly well, reducing the maximum transportation time roughly by half. The overall picture of the effect of introducing the system is illustrated in Figure 15, in which we compare histograms of transportation time for the three solutions.

Figure 16 illustrates the transportation time of each demand before and after the introduction of the HEMS, t_i^{A} and t_i^{new}, in the minisum and minimax solutions. Points represent demands and are divided into four classes according to the population size. Demands on

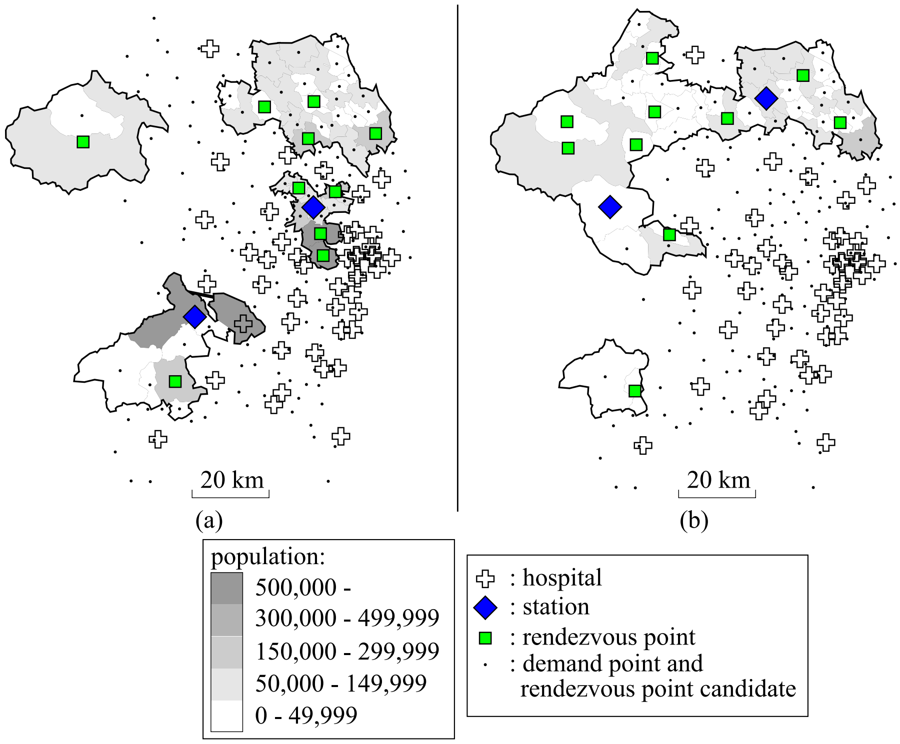


Figure 14: Obtained solutions for (a) the minisum model and (b) the minimax model

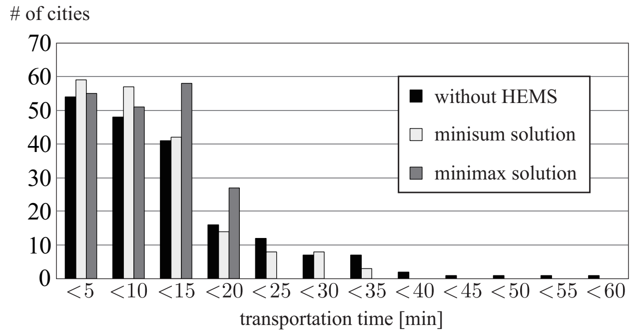


Figure 15: Histogram of transportation time

the diagonal remain ambulance users after the introduction of the system, so their transportation time does not change. Points located under the diagonal are helicopter users. The time reduction for these points is measured by the vertical distance from the diagonal. In the minisum solution, the ambulance transportation time of new helicopter users takes a wide range of values, while that in the minimax solution mainly takes values greater than 20 minutes. Some of the helicopter users in the minimax solution have values of transportation time that are close to the maximum (see Figure 16(b)).

Figure 17(a) shows the average transportation time for the minisum solutions for various values of p and q. Figure 17(b) is the maximum transportation time for the minimax solutions for various values of p and q. In the case of the minisum solution, the transportation time decreases smoothly and steady with the increase in the numbers of rendezvous points and stations. This is because the extra rendezvous point or station (1) allows some ambulance users to be served by helicopters, or (2) reduces the transportation time further for helicopter users. Both of these lead to a decrease in the average transportation time.

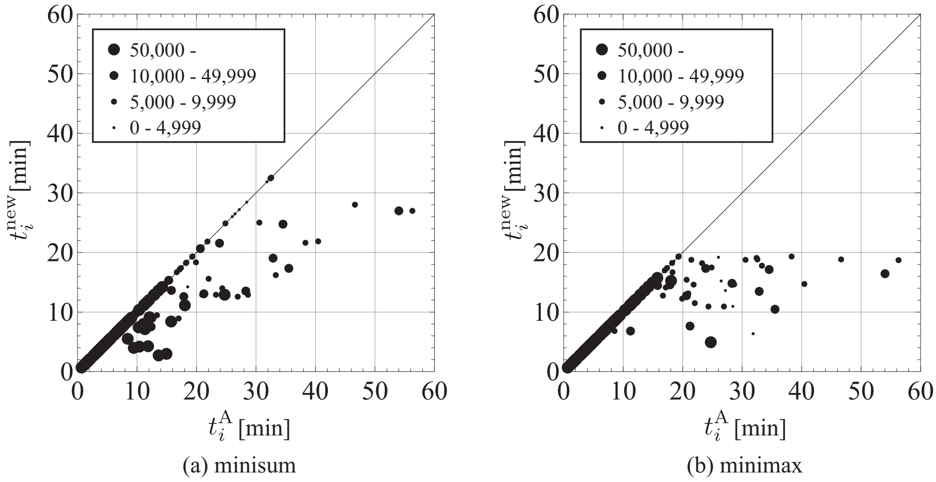


Figure 16: t_i^{A} vs. t_i^{new} for the minisum and the minimax solutions

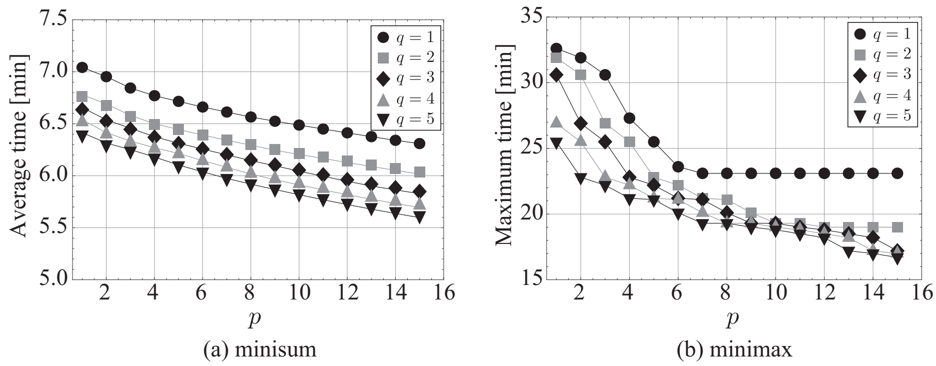


Figure 17: Optimal objective values for various values of (p, q)

On the other hand, the rate of decrease of the objective value for the minimax solutions is different from that of the minisum solutions: some solutions do not exhibit a significant decrease in the objective value. This occurs when there are several demands having almost the same transportation time as the maximum value, as we have already seen in Figure 16(b). In this case, to reduce the maximum value significantly, the transportation time for all of these demands points should be reduced, which is not always possible for just one rendezvous point or one station. This situation occurs especially when demand points are spatially dispersed. In this example, because ambulances take more than 25 minutes to travel to a hospital only for a limited number of patients, the addition of one rendezvous point or one station reduces the maximum transportation time accordingly. However, many rendezvous points and stations are required to reduce the maximum transportation time to less than 15 minutes, because many demand points have travel times of more than 15 minutes before the HEMS is introduced, as can be seen from Figure 15. This can be seen, for example, in Figure 17(b) at $p = 9$ and $q = 3$, where the addition of a few rendezvous points or stations does not decrease the objective value very much.

6. Summary and Future Work

This paper presented minisum and minimax location models for helicopter emergency medical systems. While helicopters are much faster than ground ambulances, they require landing and takeoff locations, making transportation decisions complex. Therefore, when designing a helicopter transportation system the locations of rendezvous points and stations are important determinants for the effective operation of the system. We constructed models assuming that there are three modes for transporting each demand to a hospital: by ambulance to a hospital (mode A), by ambulance to a station, then by helicopter to a hospital (mode H1) by ambulance to a rendezvous point, then by helicopter to a hospital (mode H2).

We first compared transportation times for the three modes at various locations in an idealized square region. Then, minisum and minimax problems that determine the locations of both rendezvous points and stations were presented, together with their 0-1 integer programming formulations. A variable reduction procedure for the problems was also presented. We derived some optimal solutions of the proposed models using a square city model, and compared desirable patterns of these solutions.

We then applied our models to an area in Japan using actual population data and hospital locations. The objective function of the minisum problem is the total demand-weighted transportation time, and we observed two types of reduction in transportation time. One is the result of having a large population: even when the amount of time reduction is small, it can greatly affect the objective value when the population is sufficiently large. The other is the result of reducing the transportation time by a large amount. In this case, even if the population is small, a great reduction in transportation time can have a large impact on the objective value. In the solution of the minimax model, stations and rendezvous points are located mainly in suburban areas of the region, in which the level of accessibility to hospitals by ambulance is relatively low. The minimax solution greatly reduced the maximum transportation time. The proposed variable reduction procedure was effective in this example.

Our proposed models will serve as a basis for evaluating the quality of HEMS as a public services that requires efficiency in operating the system and fairness between potential demands. Many researches employ maximal coverage objectives in dealing with EMS. This approach is useful when there are clear coverage criteria. However, the maximal covering approach treats services as binary; demand is either fully covered or not covered at all, and the uncovered demand is not taken much into consideration. This represents a difficult sacrifice for some people who are not covered. In many planning situations, existing ground ambulances are already available, and thus, improving the service levels for low-accessibility demands is the primary goal when introducing helicopter emergency systems. The proposed minisum and minimax models can provide optimal locations considering with the low-accessibility demands.

Various topics remain as subjects for future research work. One important one is to construct models that consider the possibility that the nearest helicopter is in use. Another goal is to devise sophisticated solution algorithms. Our model has many binary variables, and efficient algorithms are required to solve such large problems.

References

[1] S. Alumur and B.Y. Kara: Network hub location problems: The state of the art. *European Journal of Operational Research*, **190-1** (2008), 1–21.

[2] N.D. Bastian: A robust, multi-criteria modeling approach for optimizing aeromedical evacuation asset emplacement. *Journal of Defense Modeling and Simulation: Applications, Methodology, Technology*, **7-1** (2010), 5–23.

[3] J.E. Beasley: OR-Library: distributing test problems by electronic mail. *Journal of the*

Operational Research Society **41** (1990), 1069–1072.

[4] O. Berman, Z. Drezner, and G.O. Wesolowsky: The facility and transfer points location problem. *International Transactions in Operational Research* **12** (2005), 387–402.

[5] O. Berman, Z. Drezner, and G.O. Wesolowsky: The transfer point location problem. *European Journal of Operational Research*, **179** (2007), 978–989.

[6] O. Berman, Z. Drezner, and G.O. Wesolowsky: The multiple location of transfer points. *Journal of the Operational Research Society*, **59** (2008), 805–811.

[7] C.C. Branas, E.J. MacKenzie, and C.S. ReVelle: A trauma resource allocation model for ambulances and hospitals. *Health Services Research*, **35-2** (2000), 489–507.

[8] C.C. Branas and C.S. ReVelle: An iterative switching heuristic to locate hospitals and helicopters. *Socio-Economic Planning Sciences*, **35** (2001), 11–30.

[9] C.C. Branas, B.G. Carr, M. Heckert, and R.C. Cheetham: Trauma center siting, optimization modeling, and GIS. In R. Skinner (ed.): *GIS in Hospital and Healthcare Emergency Management* (CRC Press, 2010), 75–86.

[10] L. Brotcorne, G. Laporte, and F. Semet: Ambulance location and relocation models. *European Journal of Operational Research*, **147-3** (2003), 451–463.

[11] J.F. Campbell, A.T. Ernst, and K. Mohan: Hub location problems. In Z. Drezner and H.W. Hamacher (eds.): *Facility Location - Applications and Theory* (Springer, 2002), 373–408.

[12] M.S. Daskin: *Network and Discrete Location* (John Wiley & Sons, 1995).

[13] M.S. Daskin and L.K. Dean: Location of health care facilities. In M.L. Brandeau, F. Sainfort, and W.P. Pierskalla (eds.): *Operations Research and Health Care: A Handbook of Methods and Applications* (Kluwer, 2004), 43–76.

[14] Z. Drezner: *Facility Location - A Survey of Applications and Methods* (New York, Springer, 1994).

[15] E.T. Erdemir, R. Batta, S. Spielman, P.A. Rogerson, A. Blatt, and M. Flanigan: Optimization of aeromedical base locations in New Mexico using a model that considers crash nodes and paths. *Accident Analysis and Prevention*, **40** (2008), 1105–1114.

[16] E.T. Erdemir, R. Batta, S. Spielman, P.A. Rogerson, A. Blatt, and M. Flanigan: Location coverage models with demand originating from nodes and paths: application to cellular network design. *European Journal of Operational Research*, **190** (2008), 610–632.

[17] E.T. Erdemir, R. Batta, P.A. Rogerson, A. Blatt, and M. Flanigan: Joint ground and air emergency medical services coverage models: A greedy heuristic solution approach. *European Journal of Operational Research*, **207-2** (2010), 736–749.

[18] Fire and Disaster Management Agency: White paper on fire service in 2010. <http://www.fdma.go.jp/html/hakusho/h22/index.html> (in Japanese).

[19] L.V. Fulton, L.S. Lasdon, R.R. McDaniel, Jr., and M.N. Coppola: Two-stage stochastic optimization for the allocation of medical assets in steady-state combat operations. *Journal of Defense Modeling and Simulation: Applications, Methodology, Technology*, **7-2** (2010), 89–102.

[20] T. Furuta and K. Tanaka: Location models for maximizing the average survival rate in "doctor-helicopter" system. *Journal of the City Planning Institute of Japan*, **46-3** (2011), 823–828 (in Japanese).

[21] J.B. Goldberg: Operations research models for the deployment of emergency services vehicles. *EMS Management Journal*, **1-1** (2004), 20–39.

[22] S.L. Hakimi: Optimum locations of switching centers and the absolute centers and medians of a graph. *Operations Research*, **12** (1964), 450–459.

[23] S.L. Hakimi: Optimum distribution of switching centers in a communication network and some related graph theoretic problems. *Operations Research*, **13** (1965), 462–475.

[24] V. Marianov and C.S. ReVelle: Siting emergency services. In Z. Drezner (ed.): *Facility Location: A Survey of Application and Methods* (Springer-Verlag, New York, 1994), 199–223.

[25] M. Sasaki, T. Furuta, and A. Suzuki: Exact optimal solutions of the minisum facility and transfer points location problems on a network. *International Transactions in Operational Research*, **15** (2008), 295–306.

[26] N. Schuurman, N.J. Bell, R. L'Heureux, and S.M. Hameed: Modelling optimal location for pre-hospital helicopter emergency medical services. *BMC Emergency Medicine*, **9-6** (2009), doi:10.1186/1471-227X-9-6.

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