

DESIGNING NETWORK TOPOLOGY USING DATA ENVELOPMENT ANALYSIS

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Abstract The topology of a network seriously affects its cost, reliability, throughput, and traffic pattern, etc, so we need to simultaneously consider these multiple criteria, which have different units, when evaluating network topologies. However, ordinary methods of network topology design considered only a single criterion. DEA (data envelopment analysis) enables us to simultaneously evaluate multiple criteria, and it has been widely used when evaluating the efficiency of a business or a project. DEA derives the optimality of each candidate by emphasizing strong criteria in its evaluation. In this paper, we apply DEA to network topology design, and we numerically show that it enables us to effectively focus on a small number of desirable topology candidates. We also compare the results of DEA with those obtained by the generic algorithm (GA).

Keywords: DEA, network topology, design, multiple criteria

1. Introduction

The Internet consists of a variety of networks, i.e., campus networks, regional networks, metropolitan area networks (MANs), and backbone networks. Campus networks operated by universities, and regional networks operated by regional internet service providers (ISPs) directly connect with local area networks (LANs) or end-user terminals. MANs operated by regional ISPs accommodate some campus and regional networks and transfer data packets among these networks. Backbone networks operated by large-scale Tier-1 ISPs accommodate some MANs and regional networks, and they transfer data among MANs and regional networks. In this paper, we focus on the design problem of backbone networks. Many Tier-1 ISPs operate their backbone networks using IP directly over SONET (synchronous optical networks) or SDH (synchronous digital hierarchy) technologies in which IP packets are directly inserted into data frame of SONET/SDH [14]. Tier-1 ISPs deploy optical fibers and connect them to SONET/SDH cross connect systems.

For Tier-1 ISPs operating and managing physical network resources of backbone networks, one important problem is how to design the physical network topology. The physical topology determines many important factors, including the network equipment cost and network operating cost and the reliability against node or link failure. Moreover, the physical topology also affects the route that packets take and determines the quality perceived by the user, such as packet network delay or throughput. Therefore, it is important to investigate the optimum design method for the physical network topologies. In addition, the network virtualization technique in which network resources can be flexibly reserved for each network service has been widely investigated recently [19]. Using this technique, ISPs can flexibly design their network infrastructure for each service, so developing an optimal network topology design method has become more important for ISPs.

For the backbone network topology, we should carefully consider both the connectivity between any pair of edge nodes and the redundancy to maintain the connectivity in the case of node or link failure. To improve the redundancy, it is desirable to have more routes between each edge node pair by providing more intermediate nodes and links. However, the increase in nodes and links will also increase the equipment cost and the operating cost. For users, it is desirable to avoid congestion at intermediate nodes and have a shorter path length to reduce the packet network delay. From the viewpoint of efficient use of limited resources and reliability, traffic concentration on a specific link should be avoided, and the traffic load should be balanced among all links. However, if we decrease the number of nodes and links to reduce the network cost, the flexibility of path design is degraded and both balancing the traffic load among all links and suppressing the path length become difficult. Hence, when designing the network topology, we need to consider multiple incompatible criteria having different units.

Many studies on network topology design have been done. Tam et al. proposed a physical topology design that minimizes the total physical link count under the condition that connectivity between all pairs of nodes is maintained in the event of a single physical link failure [17]. Ramaswami et al. [15] and Krishnaswamy [11] proposed a logical topology design minimizing the maximum link load in a wavelength-routed optical network. A design method minimizing the average hop count of wavelength paths was proposed in [1], and another method maximizing the overall throughput in a wavelength-routed optical network was proposed in [21]. Chattopadhyay et al. [5] and Gersht et al. [7] presented heuristic approaches using a branch-and-bound method or a greedy method to solve the cost minimization problem with a constraint on the delay between nodes. Steiglitz et al. [16] presented a heuristic method using a local search that solves the cost minimization problem with the constraint that all node pairs have more than a specified number of disjoint routes. Wille et al. [20] depicted heuristic approaches using a tabu search and generic algorithm to solve the same problem with the constraint that the connectivity between any pair of nodes is maintained for any single-node failure. However, all these works consider only a single criterion as the optimization target. Although the optimized criterion of obtained topologies has a desirable value, the other criteria of obtained topologies are seriously degraded. When designing network topologies, it is ideal to select topologies with reasonable values in all the criteria according to the relative importance of each criterion.

As an approach that considers multiple criteria, the concept of the Pareto frontier is well known [18], and Huiban et al. applied this concept to logical topology design [8]. Assume that there are M criteria, V_1, \dots, V_M , and let $V_{m,x}$ denote the m -th criterion of candidate x . We can say that candidate x is better than candidate y in the Pareto sense only if $V_{m,x} \leq V_{m,y}$ for any m and there exists criterion m that satisfies $V_{m,x} < V_{m,y}$. (Assume that smaller values are desirable in all criteria.) All candidates that are surpassed by no other candidates are the optimum solution set, i.e., the Pareto frontier. However, a large number of candidates are regarded as the Pareto frontier, so it is difficult to effectively limit the optimum candidates and select one network topology to use.

As another evaluation method considering multiple criteria simultaneously, data envelopment analysis (DEA) is known [2, 4]. DEA has been developed mainly as a way of evaluating the efficiency of businesses or projects, and it derives the efficiency by emphasizing the strong criteria of each candidate. Using DEA, we can evaluate the optimality of topology candidates considering multiple criteria simultaneously. In this paper, we ap-

ply DEA to the network topology design problem*. The topology design method proposed in this manuscript will benefit Tier-1 ISPs operating and managing the physical backbone networks. Using the proposed design method, Tier-1 ISPs can effectively focus on a small number of desirable topology candidates. In Section 2, we summarize the problem of network topology design. In Section 3, we describe the topology design method using DEA. We present the results of numerical evaluation in Section 4. We conclude with a brief summary in Section 5.

2. Physical Network Topology Design

2.1. Definition of problem

As general components of a network, we consider three components: edge nodes, core nodes, and links. Edge nodes include source and destination nodes of traffic and accommodate end hosts either directly or through an access network. On the other hand, a core node provides just a relaying function and does not accommodate end hosts. A link connects any edge or core node. Assume that the traffic matrix between any pair of edge nodes is given. Tier-1 ISPs already have conduits and buildings, which were constructed considering the geographical conditions, so it is unrealistic to redesign these fixed properties. Hence, we assume that the positions of all edge nodes as well as the positions where we can put core nodes or links are given.

A realistic problem is how to select the positions for setting optical fibers or routers among the existing infrastructure, according to the network requirements. In this paper, therefore, we define the physical topology design as *selecting the locations of core nodes and links from the given candidate positions*. Edge nodes are put at all the candidate locations for edge nodes. Because the physical topology is designed before the actual network resources, such as optical fibers or routers, are provided, restrictions of network resources, e.g., link capacity, are not considered in the physical topology design.

2.2. Evaluation criteria

Now, let us enumerate criteria that should be considered when designing the physical topology. For the physical topology of backbone networks, we should consider both connectivity between any pair of edge nodes and redundancy to maintain the connectivity in the case of node or link failure. Hence, we consider two restrictions: (i) connectivity between any pair of edge nodes and (ii) maintaining the connectivity even when a single link failure occurs. Although there are various evaluation criteria that we could choose, we use the following four as examples, hereafter. (Let V_i denote the i -th criterion.)

1. Total node count

This is the total number of edge and core nodes. Letting N_e and N_c denote the numbers of edge nodes and deployed core nodes, respectively, we have

$$V_1 = N_e + N_c. \quad (2.1)$$

From the viewpoint of the network equipment cost and the network operating cost, smaller V_1 is desirable.

2. Total link length

This is the sum of all link lengths. Let us define d_l as the length of link l and \mathbf{E}_a as the deployed link set. We have

$$V_2 = \sum_{l \in \mathbf{E}_a} d_l. \quad (2.2)$$

*A shorter version of this manuscript was presented in [10].

From the viewpoint of the network equipment cost and the network operating cost, smaller V_2 is also desirable.

3. Sum of path lengths weighted by path traffic

Because the propagation delay between edge nodes is proportional to the path length, a shorter path length is desirable. Therefore, we can consider the sum of path lengths weighted by path traffic, $\zeta = \sum_{s,d \in N_e, s \neq d} D_{sd} T_{sd}$, as the third criterion, where T_{sd} is the amount of traffic on P_{sd} , P_{sd} is the path from edge node s to d , D_{sd} is the length of P_{sd} ($D_{sd} = \sum_{l \in \Phi_{sd}} d_l$), and Φ_{sd} is the link set on P_{sd} . Moreover, let t_l denote the total amount of traffic on link l . ζ can also be obtained from d_l and t_l as $\zeta = \sum_{l \in \mathbf{E}_a} d_l \times t_l$. So, we choose V_3 as

$$V_3 = \sum_{l \in \mathbf{E}_a} d_l \times t_l. \quad (2.3)$$

Smaller V_3 is also desirable.

4. Amount of traffic on maximally loaded link

When traffic concentrates on a specific link, the quality of paths on the bottleneck link is seriously degraded. Because the link transmission capacity can be adjusted in discrete units, i.e., optical fiber or wavelength, the link load should be balanced among all links from the viewpoint of effectively utilizing the network resources as well. We can consider the variance of link load or the amount of traffic on the maximally loaded link as the criterion indicating the degree of load balancing. Here, we choose the latter as the fourth criterion and set

$$V_4 = \max_{l \in \mathbf{E}_a} t_l. \quad (2.4)$$

Smaller V_4 is also desirable.

2.3. Making topology candidates

If we freely select the locations to put core nodes and links among the given positions, the obtained physical topology set includes ones that do not satisfy the restrictions mentioned in Section 2.2. Therefore, we first construct an initial physical topology that does satisfy the restrictions and then make the physical topology candidates by adding core nodes or links to it. As a result, we can choose any physical topology from the candidate set without considering the restrictions because they have already been satisfied at the stage of initial topology construction.

Let z denote the number of positions where we can put a link, except the ones where links are deployed in the initial physical topology. The number of topologies obtained by putting links at any possible position is 2^z . For each obtained physical topology, we put core nodes at the possible locations with degree of three or more. However, no switching function is required at the possible locations with degree of two, so we do not put core nodes at these positions; instead, the two links are directly connected. All topologies having core node positions with degree of one are invalid topologies, so we eliminate them from the candidate set.

Next, for each topology candidate, we set logical paths between each pair of edge nodes using the given traffic matrix. Here, we assume that a single logical path P_{sd} is provided from edge node s to d , and logical paths are deployed using a greedy algorithm in descending order of path traffic T_{sd} . The path design method affects the path length and the degree of load balancing among the four criteria mentioned in Section 2.2. Hence, the route having the minimum value of $\sum_{l \in \Phi_{sd}} d_l \times t'_l$ is selected when each path route is allocated. Here, t'_l is the amount of traffic on link l at the stage of target path accommodation. After setting

all the paths, we also eliminate from the candidate set all the topologies with links that do not accommodate any paths.

3. Network Topology Design using DEA

3.1. Overview of DEA

Each candidate to have its efficiency evaluated is called a decision making unit (DMU), and the efficiency is defined as the ratio of the linear combination of s output variables against the linear combination of r input variables. Here, the input variables represent what is consumed to obtain the outcome, i.e., the output variables. When a DMU is a business office, for example, we can regard the number of sales staff as the input variable and the sales volume as the output variable [2, 4]. Letting N denote the number of DMUs, we define the input and output variables of DMU $_j$ ($j = 1, \dots, N$) as

$$\mathbf{x}_j = (x_{j1}, x_{j2}, \dots, x_{jr}), \quad j = 1, \dots, N \quad (3.1)$$

$$\mathbf{y}_j = (y_{j1}, y_{j2}, \dots, y_{js}), \quad j = 1, \dots, N \quad (3.2)$$

respectively. We also define the weight parameters of \mathbf{x}_j and \mathbf{y}_j as

$$\mathbf{v} = (v_1, v_2, \dots, v_r), \quad (3.3)$$

$$\mathbf{u} = (u_1, u_2, \dots, u_s), \quad (3.4)$$

respectively. The efficiency score of DMU $_o$ is given by $\theta = \mathbf{u}\mathbf{y}_o/\mathbf{v}\mathbf{x}_o$. Parameters \mathbf{v} and \mathbf{u} are set to maximize θ in the range of $0 < \theta \leq 1$, so the parameters have different values for each DMU. By determining the weight parameters independently for each DMU, DEA emphasizes strong criteria of each DMU. DEA derives the weight parameters that maximize θ by solving the following fractional linear program.

$$\max \theta = \mathbf{u}\mathbf{y}_o/\mathbf{v}\mathbf{x}_o \quad (3.5)$$

$$\begin{aligned} \text{subject to } & \mathbf{u}\mathbf{y}_j/\mathbf{v}\mathbf{x}_j \leq 1, \quad j = 1, \dots, N, \\ & \mathbf{v} \geq \mathbf{0}, \quad \mathbf{u} \geq \mathbf{0}. \end{aligned} \quad (3.6)$$

We can also derive the same results by setting the denominator equal to a constant and maximizing the numerator. So we have the following linear program.

$$\max \theta = \mathbf{u}\mathbf{y}_o \quad (3.7)$$

$$\begin{aligned} \text{subject to } & \mathbf{v}\mathbf{x}_o = 1, \\ & \mathbf{u}\mathbf{y}_j \leq \mathbf{v}\mathbf{x}_j, \quad j = 1, \dots, N \\ & \mathbf{v} \geq \mathbf{0}, \quad \mathbf{u} \geq \mathbf{0}. \end{aligned} \quad (3.8)$$

Then, DEA applies the weight parameters derived for DMU $_o$ to all other DMUs and regards DMU $_o$ as efficient only when its efficiency score is maximum among all DMUs. The efficiency scores of efficient DMUs are set to unity, and the efficient DMUs form an efficient frontier represented by a polygon with those optimum DMUs located at the apexes. An example with two input variables (the output is always unity) is shown in Figure 1. DMUs C, E, and F are efficient among the seven DMUs. The efficiency score of inefficient DMUs, i.e., A, B, D, and G, is given by a/b , where b is the distance between the origin and the DMU and a is the distance between the origin and the point where the line passing through the origin and the DMU crosses the efficient frontier. For example, the efficiency score of DMU $_B$ is the length of segment OP divided by the length of segment OB.

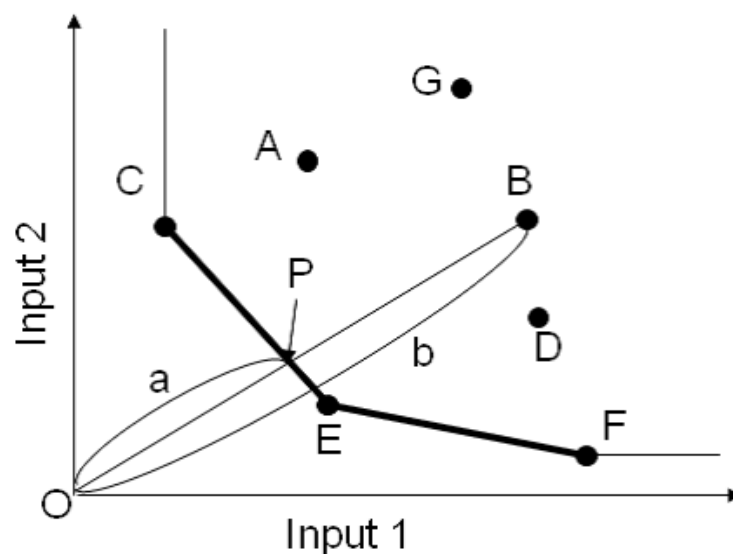


Figure 1: Example of DEA

3.2. Applying DEA to network topology design

Let us regard a physical topology candidate as a DMU and use DEA to select optimum topologies. When evaluating a business, for example, DEA uses the consumed factors, i.e., the invested capital or labor, as input variables and uses the outcomes, i.e., the sales or profit, as output variables. In the case of applying DEA to a network topology evaluation, we can regard the facilities, i.e., deployed links and nodes, as the input, and we can regard the resultant performance criteria achieved by these facilities, i.e., the average path length or the amount of traffic on the most congested link, as the output. Hence, we choose V_1 and V_2 described in Section 2.2 as input variables and V_3 and V_4 as output variables. Because the efficiency of a DMU with larger output variables is better, we subtract V_3 or V_4 from the maximum values among all DMUs to get the output variables. That is, we use $\max_j V_{3,j} - V_{3,i}$ and $\max_j V_{4,j} - V_{4,i}$ as output variables of DMU_i , where the third and fourth criteria of DMU_i are $V_{3,i}$ and $V_{4,i}$, respectively. However, this treatment is arbitrary, and we can also apply other DEA models to the network topology design. For example, we can also construct DEA model without outputs [13], so we also applied DEA to the candidate topology set when choosing all the four criteria, V_1 , V_2 , V_3 , and V_4 , as input variables and using a constant output.

4. Numerical Evaluation

4.1. Network model

The network model (the Japanese archipelago model [9]) used in the numerical evaluation is shown in Figure 2. Closed circles represent edge nodes and dotted circles and links represent positions where we can put core nodes or links. The link lengths between nodes i and j , d_{ij} (km) are summarized in Table 1 [9]. As mentioned in Section 2.2, we consider the sum of path lengths weighted by path traffic and the amount of traffic on maximally loaded link as the third and fourth criteria evaluating each topology candidate. To obtain these criteria, we need to give the traffic demand matrix between each pair of edge nodes. In the following evaluation, the amount of traffic from edge node s to d , T_{sd} , was set to be proportional to the product of U_s and U_d , where U_k was the population accommodated by edge node k

shown in Table 2. That is,

$$T_{sd} = \frac{T \cdot U_s U_d}{\sum_{\substack{m,n \\ m \neq n}} U_m U_n}, \tag{4.1}$$

where T is the total amount of traffic within the network, and we set $T = 10$ Tbps.

A loop topology is a typical and simple topology in which connectivity between all pairs of edge nodes is maintained in the case of a single link failure. It is often used in backbone networks. Here, we assumed a loop topology as the initial physical topology and put links on the locations shown by solid lines in Figure 2 in all the candidate topologies. The degree of all possible positions of core nodes on the loop was two, so no switching function was necessary at these positions. In the initial physical topology, we did not put any core nodes on the loop.

Besides the links used in the initial physical topology, 17 possible positions were left for links, so we were able to construct 2^{17} physical topologies in total. However, after removing all topologies with core nodes of degree 1 or unused links, as mentioned in Section 2.3, we had 83,868 candidate topologies. The time needed to obtain these candidates was about 75 seconds.

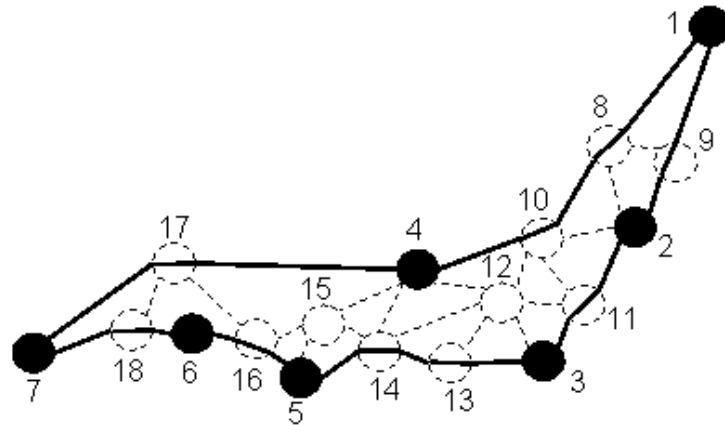


Figure 2: Network model

Table 1: Link lengths between node i and node j

i	j	d_{ij}	i	j	d_{ij}	i	j	d_{ij}
1	8	379	4	14	157	8	10	230
1	9	364	4	15	192	10	11	163
2	8	170	4	17	355	10	12	168
2	9	173	5	14	129	11	12	151
2	10	157	5	15	38	12	13	163
2	11	200	5	16	40	12	14	204
3	11	89	6	16	135	13	14	148
3	12	174	6	18	132	14	15	99
3	13	143	7	17	317	15	16	59
4	10	254	7	18	217	16	17	222
4	12	132	8	9	83	17	18	121

Table 2: Edge node populations

k	$U_k(10^3)$	k	$U_k(10^3)$	k	$U_k(10^3)$	k	$U_k(10^3)$
1	5,683	3	41,059	5	26,313	7	16,292
2	11,822	4	9,847	6	15,910		

4.2. Properties of candidate topologies

Table 3 summarizes the minimum, maximum, average, and coefficient of variation (CV) of the four criteria: V_1 , the number of nodes, V_2 , the total link length (10^3 km), V_3 , the total path length weighted by path traffic (10^3 km \times Gbps), and V_4 , the amount of traffic on the maximally loaded link (Gbps), for 83,868 candidate topologies. Moreover, Figure 3 shows the cumulative distribution of each of the four criteria. Because the number of topologies increases as N grows, the average of V_1 was close to the maximum value. Hence, in many of the topologies among the 83,868 candidates, core nodes and links were put at a lot of possible locations. In these topologies, the flexibility of path allocation was high, and the averages of V_3 and V_4 were close to the minimum value.

Table 3: Properties of 83,868 topologies

	Minimum	Maximum	Average	CV
V_1	7	18	14.48	0.122
V_2	3.31	5.76	4.59	0.065
V_3	6.01	9.74	7.13	0.072
V_4	2.18	5.22	3.46	0.136

The correlation coefficients of the four criteria calculated for 83,868 candidate topologies are summarized in Table 4. Because the number of deployed core nodes increased as the deployed link count grew, V_1 showed the same tendency as V_2 . In topologies in which paths were allocated more flexibly, the average path length and the maximum link-load were decreased. Therefore, V_3 and V_4 showed the same tendency. As a result, positive correlation was observed between two criteria belonging to the same criterion group when V_1 and V_2 were assigned to criterion group A and V_3 and V_4 were assigned to criterion group B. In particular, a strong positive correlation was observed between V_1 and V_2 . On the other hand, a negative correlation was observed between two criteria belonging to different groups. This is because we need to increase the topological redundancy to improve the criteria belonging to group B, which means that we need to increase the number of links and core nodes deployed. In the physical topology set, the network costs (V_1 and V_2) and the path optimization (V_3 and V_4) are incompatible.

169 topology candidates formed a Pareto frontier when we evaluated 83,868 topologies using the concept of the Pareto optimum mentioned in [8]. In this approach, we cannot focus on a small optimum set.

Table 4: Correlation coefficients of four variables

	V_2	V_3	V_4
V_1	0.694	-0.318	-0.345
V_2	*	-0.530	-0.401
V_3	*	*	0.305

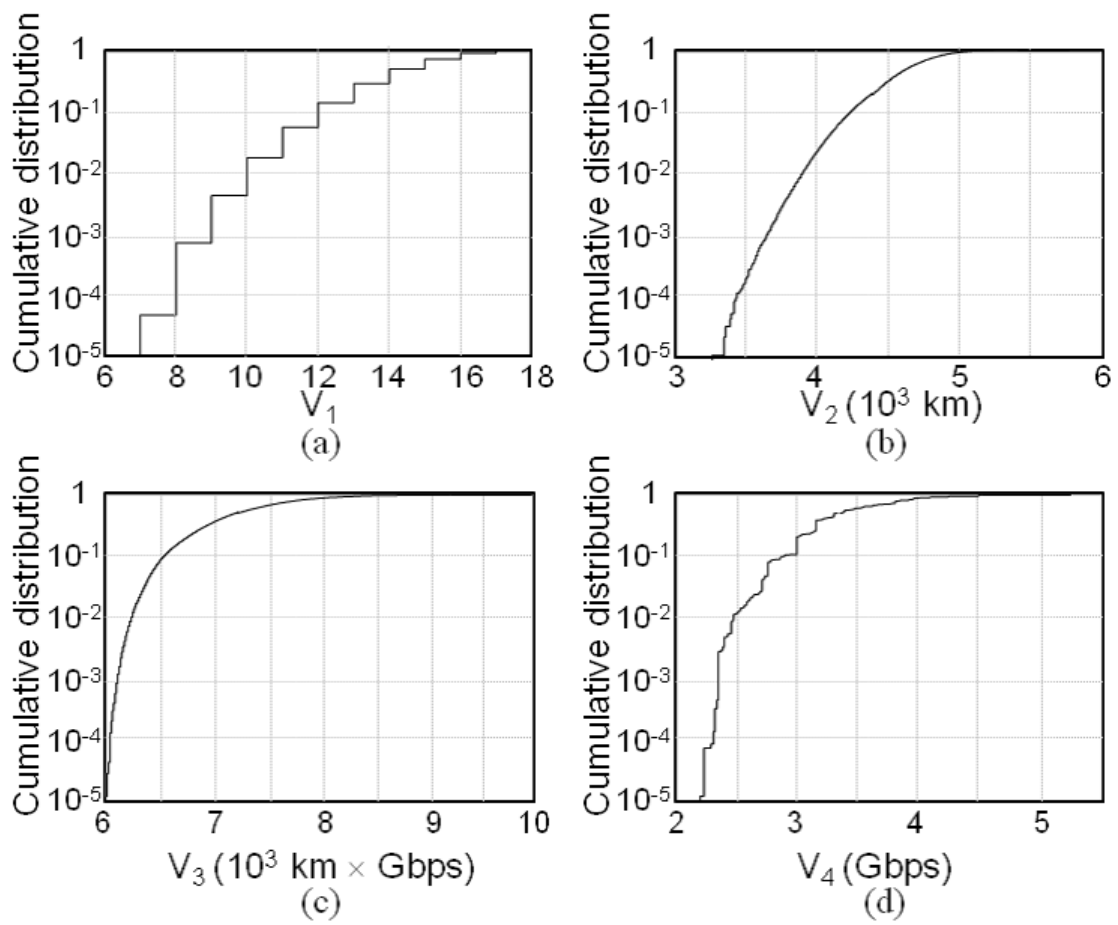


Figure 3: Cumulative distribution of each of four criteria

4.3. Results of applying DEA

Using the Benchmarking library of R [6], we applied DEA of two-input and two output model to all the 83,868 candidate topologies, and we had seven efficient topologies with $\theta_i = 1$. The values of the four criteria and their normalized score (NS) for these seven topologies are summarized in Table 5. NS is the relative value among all the candidates. It is defined as $(X_{ij} - \text{Min}X)/(\text{Max}X - \text{Min}X)$, where $X_{ij} \equiv 1/V_{ij}$, and $\text{Max}X$ and $\text{Min}X$ are the maximum and minimum values of X_{ij} . The NS of the topology with the minimum value of X_{ij} is zero, and that of the topology with the maximum value of X_{ij} is unity.

Moreover, these seven topologies are shown in Figure 4, in which gray circles represent the deployed core nodes. For example, DEA judged topology 4 to be efficient because of the desirable values of V_1 and V_3 . Because DEA emphasizes the strong properties of DMU, the efficiency of DMU tends to be large when a specific criterion is excellent even if other criteria are not so good. For example, V_4 was emphasized in topologies 1, 2, and 3, V_3 was emphasized in topologies 5 and 6, and V_1 was emphasized in topology 7. Hence, topologies, such as topology 1, with an excellent value in one criterion and poor values in other criteria, as well as well-balanced topologies, such as topology 4, with desirable values in all the criteria, were selected as optimum ones when DEA was applied. The merit of using DEA is that we can narrow down the vast number of topology candidates to a small set of desirable candidates. The network topology designer can easily choose one topology from the limited set of desirable ones based on the importance of each criterion for him or her.

Next, we applied DEA of four-input model to all the 83,868 candidate topologies. We had 33 efficient topologies, and Table 6 summarizes the values of the four criteria and their NS for these 33 topologies. Figure 5 plots the efficiency score of each candidate topology in the four-inputs model against that in the two-inputs and two-outputs model. The efficiency scores of candidates widely spread between zero and unity in the two-inputs and two-outputs model, whereas the efficiencies of all the candidates were greater than about 0.8 in the four-inputs model. Therefore, in the four-inputs model, many more candidates were efficient, and it is more difficult for network designers to focus on a small number of desirable candidates although having a reasonable candidate set might be helpful.

Table 5: Properties of optimum topologies evaluated by DEA

	V_1		V_2		V_3		V_4	
	Value	NS	Value (10^3 km)	NS	Value (10^3 km \times Gbps)	NS	Value (Gbps)	NS
1	14	0.18	4.31	0.46	6.92	0.66	2.40	0.85
2	11	0.40	4.05	0.57	7.30	0.54	2.60	0.72
3	12	0.32	4.40	0.42	6.97	0.64	2.40	0.85
4	8	0.80	4.00	0.60	6.71	0.73	2.99	0.53
5	9	0.64	4.16	0.52	6.06	0.98	2.99	0.53
6	11	0.40	4.28	0.47	6.11	0.96	2.72	0.66
7	8	0.80	4.01	0.59	7.09	0.60	2.99	0.53

4.4. Comparison with metaheuristic design method

As mentioned in Section 1, various design methods of network topologies based on metaheuristic approach have been proposed. To clarify the effectiveness of applying DEA to

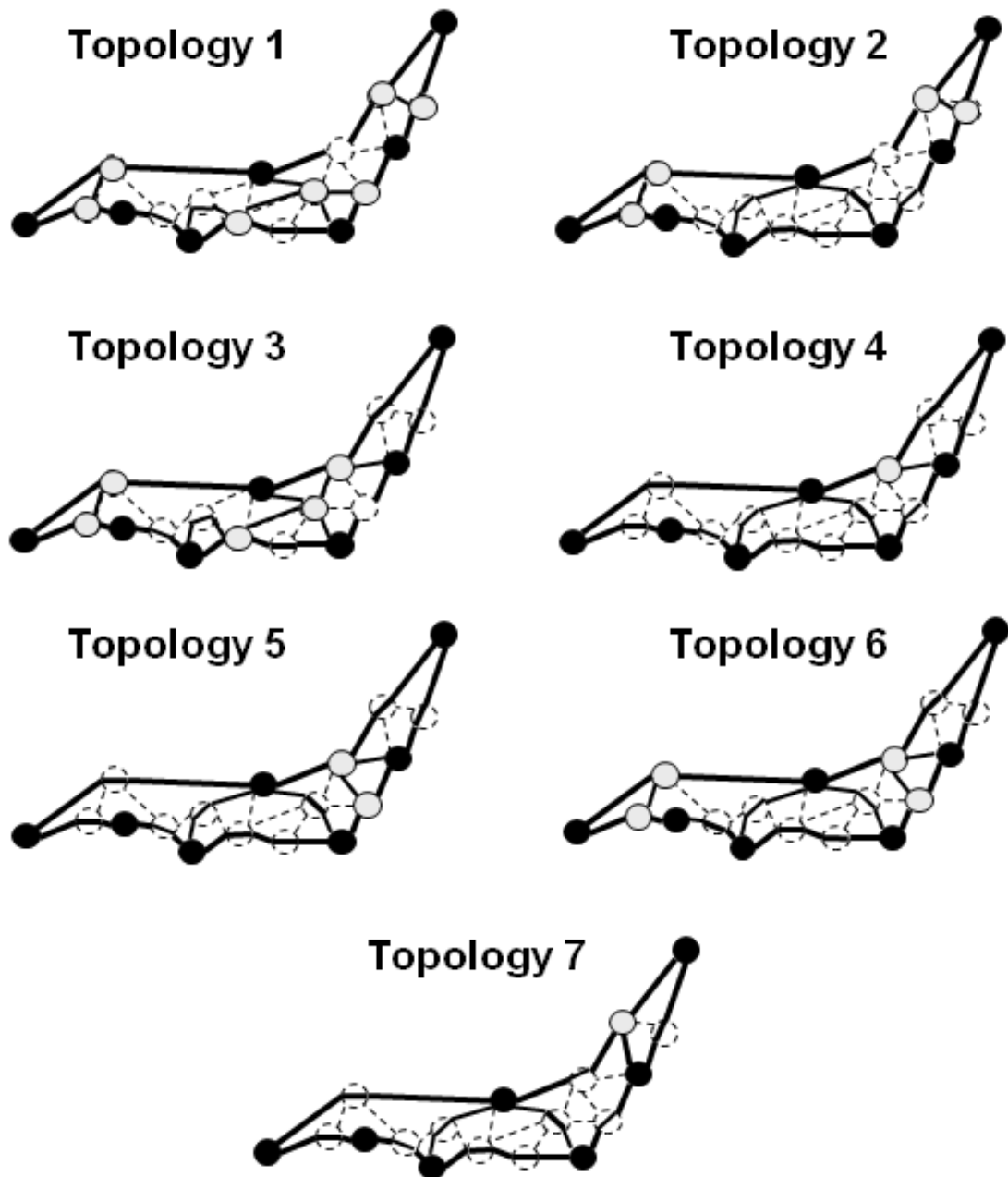


Figure 4: Optimum topologies evaluated by DEA

Table 6: Properties of optimum topologies evaluated by DEA in four-inputs model

	V_1		V_2		V_3		V_4	
	Value	NS	Value (10^3 km)	NS	Value (10^3 km \times Gbps)	NS	Value (Gbps)	NS
1	9	6.64	4.16	0.52	6.06	0.98	2.99	0.53
2	7	1.00	3.54	0.85	8.06	0.34	3.89	0.25
3	7	1.00	3.61	0.80	8.49	0.24	3.56	0.33
4	7	1.00	3.84	0.68	7.35	0.52	3.37	0.39
5	8	0.80	3.46	0.90	7.77	0.41	3.62	0.32
6	8	0.80	4.00	0.59	6.71	0.73	2.99	0.53
7	8	0.80	4.01	0.59	7.09	0.60	2.99	0.53
8	9	0.64	3.47	0.89	7.53	0.47	3.91	0.24
9	9	0.64	3.39	0.94	8.20	0.30	3.77	0.28
10	9	0.64	3.62	0.80	7.39	0.51	3.30	0.42
11	9	0.64	4.00	0.59	6.61	0.76	2.99	0.53
12	9	0.64	3.92	0.63	7.15	0.58	2.99	0.53
13	9	0.64	3.62	0.80	6.77	0.71	4.46	0.12
14	9	0.64	3.76	0.72	6.79	0.70	3.30	0.42
15	9	0.64	4.16	0.52	6.06	0.98	2.99	0.53
16	10	0.51	3.58	0.82	7.63	0.45	3.28	0.42
17	10	0.51	3.54	0.85	7.38	0.52	3.62	0.32
18	11	0.40	3.55	0.84	7.18	0.57	3.91	0.24
19	11	0.40	3.70	0.75	6.66	0.75	4.46	0.12
20	11	0.40	4.22	0.49	6.01	1.00	2.99	0.53
21	13	0.24	4.59	0.34	6.01	1.00	2.99	0.53
22	13	0.24	4.34	0.44	6.09	0.97	2.72	0.66
23	14	0.18	4.66	0.32	6.09	0.97	2.55	0.75
24	11	0.40	4.28	0.47	6.11	0.96	2.72	0.66
25	15	0.13	5.03	0.20	6.12	0.95	2.33	0.89
26	15	0.13	4.89	0.24	6.13	0.95	2.33	0.89
27	16	0.08	4.84	0.26	6.83	0.69	2.18	1.00
28	16	0.08	4.99	0.21	6.38	0.85	2.21	0.98
29	13	0.24	4.80	0.27	6.95	0.65	2.30	0.91
30	14	0.18	4.95	0.22	6.25	0.90	2.30	0.91
31	14	0.18	4.31	0.45	6.92	0.66	2.40	0.84
32	12	0.32	4.40	0.42	6.97	0.64	2.40	0.84
33	13	0.24	4.55	0.36	6.27	0.89	2.46	0.80

network topology design, we compare the results of DEA with those obtained by the topology design method using the generic algorithm (GA) proposed in [20]. As mentioned in Section 1, GA considers only a single criterion as the optimization target. Although the optimized criterion of obtained topologies has a desirable value, the other criteria of obtained topologies are seriously degraded. When designing network topologies, it is ideal to select topologies with reasonable values in all the criteria according to the relative importance of each criterion. By comparing the results of DEA with those of GA, we will clarify the effectiveness of applying DEA to network topology design.

As the optimization function f , we used V_1, V_2, V_3 , or V_4 . Although just a single criterion is optimized in GA, we can indirectly consider other criteria by giving the allowable boundaries on them. Therefore, in addition to the constraint described in Sections 2.2 and 2.3, we introduced the upper bound B_i on each of the criteria except the optimization target f . In other words, when f was V_1 for example, we try to find the optimum network topologies with the minimum value of V_1 among candidates which satisfy $V_2 \leq B_2, V_3 \leq B_3$, and $V_4 \leq B_4$. We set B_i to the 100 r percent point of V_i among all the 83,868 candidates. In other words, the value of the x-axis giving the cumulative distribution of r in Figure 3 was set to B_i .

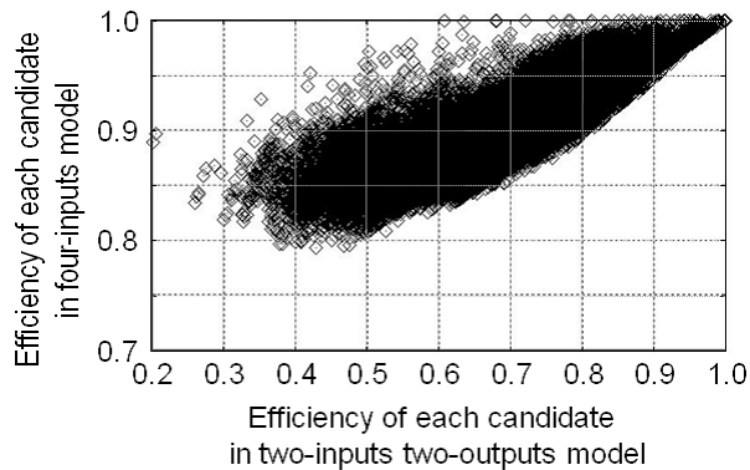


Figure 5: Scattergram of efficiency score of each candidate in two DMU models

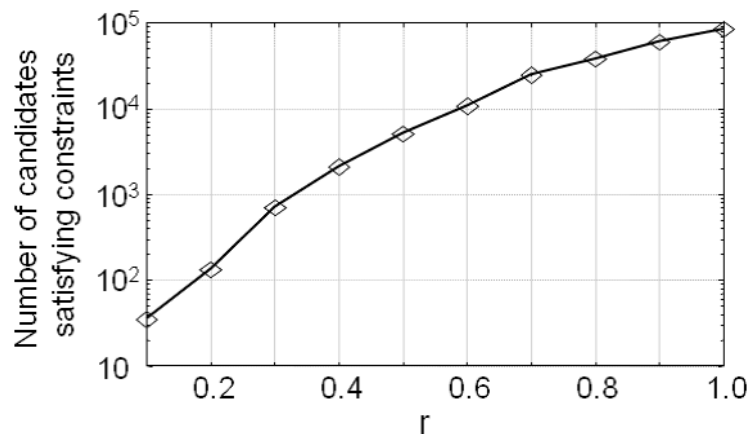


Figure 6: Number of candidates satisfying constraints against r

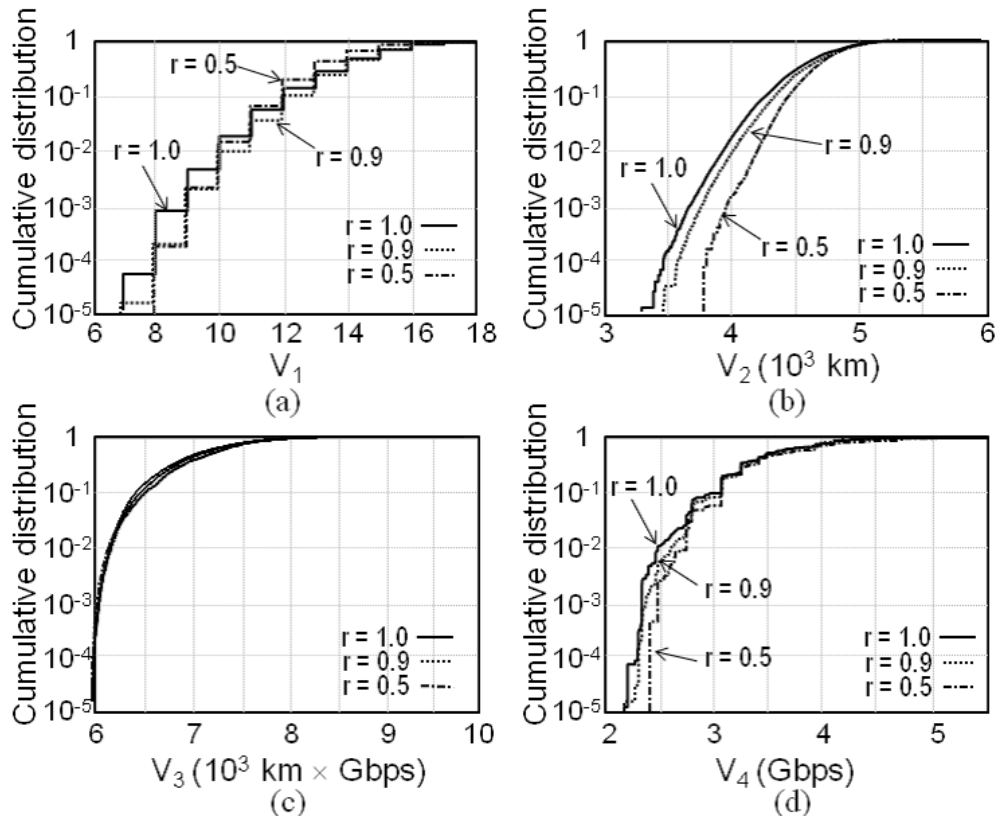


Figure 7: Cumulative distribution of each of four criteria for three values of r

Although we can expect to avoid selecting topologies with terrible values in criteria except f by introducing the upper bound on each of the criteria except the optimization target f , it is anticipated that the optimality of the obtained topologies for f is degraded. Figure 6 plots the number of candidate topologies satisfying the constraint described in Sections 2.2 and 2.3 as well as the constraint of B_i against r . As r decreased, fewer candidate topologies can satisfy the constraints, so the number of candidates satisfying the constraints dramatically decreased.

Figure 7(a) shows the cumulative distribution of V_1 of the candidate topologies satisfying the upper bound of criteria when f was V_1 for the three values of r . When setting $r = 1$, the upper bound of criteria were not considered, and all the 83,868 topologies were candidates. Figure 7(b), (c), and (d) also shows the cumulative distribution of V_2 , V_3 , and V_4 when f was V_2 , V_3 , and V_4 , respectively. When f was V_3 , the cumulative distribution of V_3 of the candidate set was almost identical for the three values of r . However, when f was V_1 , V_2 , or V_4 , more topologies with f close to the minimum value were excluded from the candidate set as r decreased. Therefore, by introducing the upper bound on each of the criteria except the optimization target f , the optimality of the obtained topologies for f will be degraded.

4.4.1. Designing topologies using GA

The GA is a stochastic optimization heuristic in which explorations in solution space are carried out by limiting the population genetics stated in Darwin's theory of evolution. We need to represent a solution to the problem as a genome consisting of binary strings. At each generation, the population comprises a group of N_p individuals (chromosomes) generated by the parent selection, genetic operations (crossover and mutation), and replacement.

Each candidate topology can be modeled as a chromosome consisting of $L = 17$ bit

binary strings. We set the unique number from 1 to 17 at each of the $L = 17$ candidate positions for setting links, and there was a link at candidate position x if the binary string at x was unity; otherwise, there was no link at position x . Each chromosome was evaluated based on the objective function f , i.e., V_1 , V_2 , V_3 , or V_4 . Initially, N_p candidates satisfying the constraint were randomly selected. To produce N_p chromosomes at generation t , the following procedure was repeated [20]. First, two chromosomes were randomly selected from the population of generation $t - 1$, and the chromosome with smaller f was selected. This was repeated twice, and we obtained two parent chromosomes. Next, with probability p_c , the selected parent chromosomes were recombined. This crossover operation was performed by randomly selecting the crossover point between unity and L and exchanging the portions of the two parent chromosomes beyond this point. The generated chromosome was inserted to the new generation chromosome set if it satisfied the constraint.

Mutation was used to change the value of a gene to prevent the convergence of the solutions to bad local optima. With probability p_m , one bit of the gene at the randomly selected position was inverted. If the obtained chromosome satisfied the constraint, it was inserted to the new generation set. Finally, the N_p chromosomes with the smallest f were selected from N_p chromosomes of generation $t - 1$ and N_p chromosomes of the new generation set as the population of generation t . The procedure of making each generation was repeated T times, and some candidates with the smallest f were output as the best candidate set.

4.4.2. Numerical comparison

We set T to 100, and we set $p_c = 0.95$ and $p_m = 0.01$ because it is desirable to set p_c close to unity and p_m to a small value [20]. We set r to 0.9. Figure 8(a) shows the average rank of V_1 of the ten candidates with the smallest V_1 in the population at each generation for three values of N_p when the objective function f was V_1 . Figure 8(b), 8(c), and 8(d) also depicts the average rank of V_2 , V_3 , or V_4 of the ten candidates with the smallest V_2 , V_3 , or V_4 in the population at each generation when f was V_2 , V_3 , or V_4 , respectively. When the top 10 topologies obtained by GA completely agreed with those when selecting the 10 topologies with the smallest V_i among all the candidates satisfying the constraint, the average rank was $\sum_{i=1}^{10} i/10 = 5.5$, so we also show the ideal result 5.5 in the figures.

When $N_p = 10$, the average rank of the top ten candidates at each generation did not monotonically decrease, and the obtained candidates at the T -th generation were not desirable. However, even when we set N_p to 100, we can obtain the desirable candidates close to the ideal ones with the average rank of 5.5 when f was V_2 or V_3 . When N_p was 1,000, the obtained candidate set was close to the ideal one in all the four cases. Hereafter, we set $N_p = 1,000$.

Table 7 summarizes the values of the four criteria and their normalized scores for the top six candidates with the smallest V_1 in the population at the T -th generation when f was V_1 . Figure 9 shows the topologies of these top six candidates. When V_1 was minimized, topologies with a small number of links tended to be selected. As shown in Figure 9, the obtained topologies were similar to some topologies selected by DEA shown in Figure 4. Table 8 and Figure 10 also show the properties and topologies of the top six candidates with the smallest V_2 when f was V_2 . The obtained topologies were also similar to these topologies selected by DEA.

Table 9 and Figure 11 show the properties and topologies of the top six candidates with the smallest V_3 when f was V_3 . When V_3 was minimized, topologies with a large number of links tended to be selected. V_1 and V_2 , which have a negative correlation with V_3 , of the obtained topologies were degraded. This clarified that if we design the network topology

optimizing just a single criterion, the other criteria are degraded in the obtained topologies when quality was regarded as important.

This tendency was more remarkable when V_4 was regarded as most important. Table 10 and Figure 12 show the properties and topologies of the top six candidates with the smallest V_4 when f was V_4 . Topologies with smallest values of V_4 were selected, and V_1 and V_2 of the obtained topologies were seriously degraded. By setting a smaller value to r , we can avoid selecting topologies with terrible values in any criteria. Tables 11 and 12 show the properties of the top six candidates with the smallest V_4 in the setting of $f = V_4$ when $r = 0.1$ and $r = 0.5$, respectively. We confirmed that V_4 , i.e., the optimized criterion, was degraded although the other criteria were improved in the selected topologies by decreasing r . This is because that fewer candidates can satisfy the constraints as r decreased as shown in Figure 6.

Figure 13 plots a scattergram of the NS of each criterion against that of other criteria. In the figures, the results for criteria with a positive correlation, e.g., V_2 for V_1 , are omitted. We confirmed that some topologies with a desirable value for V_1 , V_2 , or V_3 also had desirable values for other criteria. On the other hand, no topologies with a desirable value for V_4 had desirable values in other criteria. This means that when we selected topologies with a desirable value in V_4 , the other criteria of the selected topologies were terrible. Therefore, it was desirable to select topologies which were well-balanced when V_4 was regarded as most important. Using DEA, we can obtain a small set of desirable topology candidates with diverse properties reflecting the relative importance of each evaluation criterion, and the obtained topologies were well-balanced ones as shown in Table 5 and Figure 4.

Table 7: Properties of top six topologies obtained by GA when $f = V_1$

	V_1		V_2		V_3		V_4	
	Value	NS	Value (10^3 km)	NS	Value (10^3 km \times Gbps)	NS	Value (Gbps)	NS
1	7	1.00	3.84	0.67	7.35	0.52	3.37	0.39
2	8	0.80	3.59	0.82	7.58	0.46	3.62	0.32
3	8	0.80	4.00	0.60	6.71	0.73	2.99	0.53
4	8	0.80	3.78	0.71	7.62	0.45	3.45	0.37
5	8	0.80	3.80	0.69	7.49	0.48	3.82	0.26
6	8	0.80	4.01	0.59	7.09	0.60	2.99	0.53

5. Conclusion

Network topology seriously affects network cost, reliability, throughput, and traffic pattern, etc, so we need to simultaneously consider these multiple criteria, which have different units, when evaluating network topologies. We applied DEA (data envelopment analysis), which has been widely used when evaluating the efficiency of a business or a project, to the network topology design. DEA derives the optimality of each candidate by emphasizing its strong criteria, and we can evaluate each topology candidate based on multiple criteria with different units. Using a Japanese archipelago backbone network model, we showed that just seven topology candidates were selected as the optimum ones. We also compared the results of DEA with those obtained by the topology design method using GA, and we clarified that

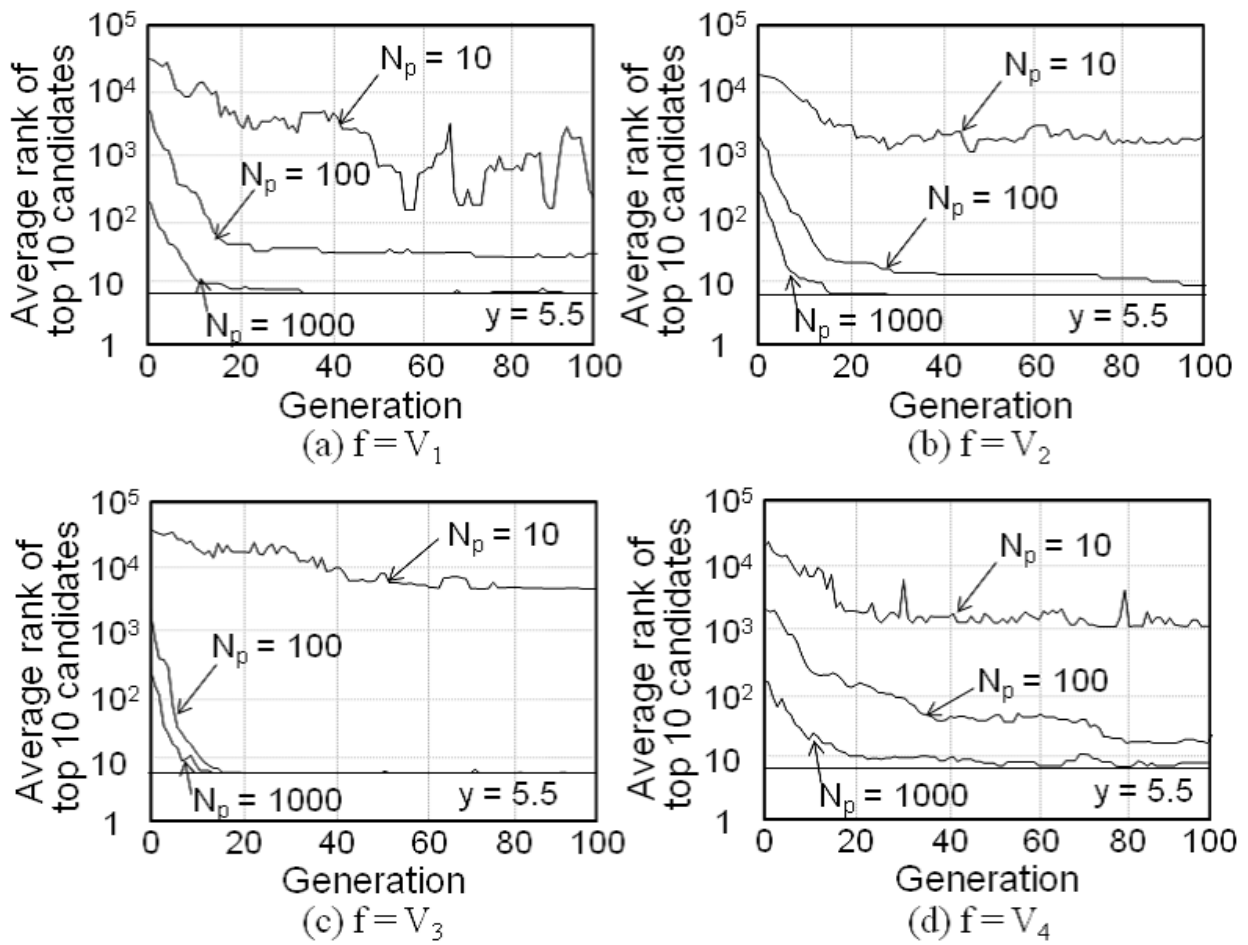


Figure 8: Average rank of the top 10 topologies at each generation of GA

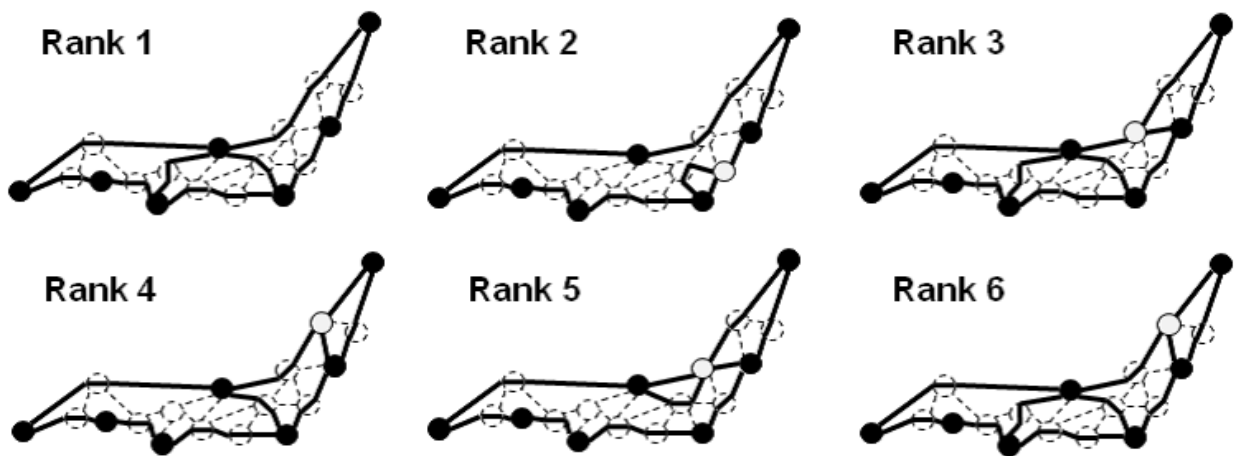
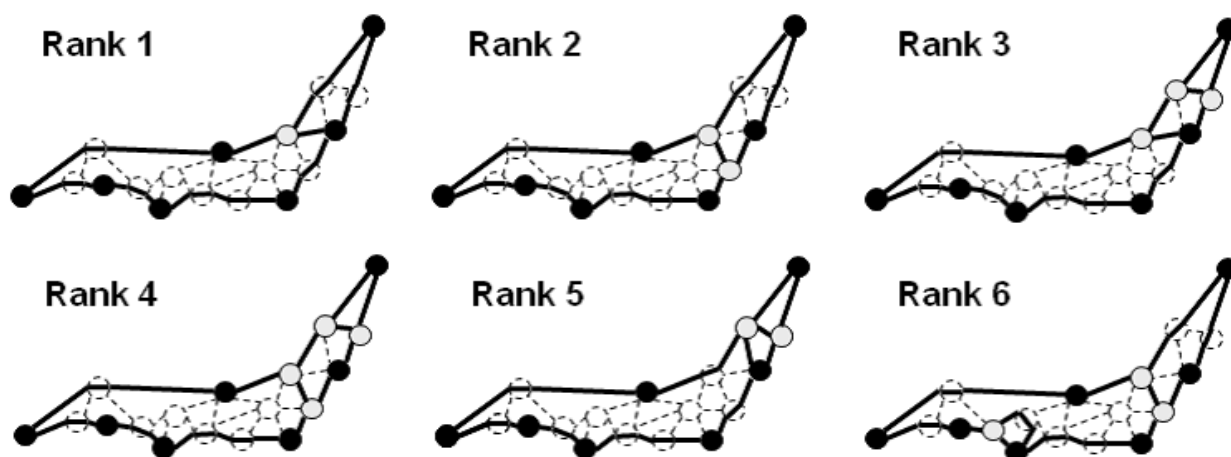


Figure 9: Topologies of the top six candidates with the smallest V_1 at the T -th generation when using GA with $f = V_1$

Table 8: Properties of top six topologies obtained by GA when $f = V_2$

	V_1		V_2		V_3		V_4	
	Value	NS	Value (10^3 km)	NS	Value (10^3 km \times Gbps)	NS	Value (Gbps)	NS
1	8	0.80	3.46	0.90	7.77	0.41	3.62	0.32
2	9	0.64	3.47	0.89	7.53	0.47	3.91	0.24
3	10	0.51	3.55	0.84	7.38	0.51	3.62	0.32
4	11	0.40	3.55	0.84	7.18	0.57	3.91	0.24
5	9	0.64	3.56	0.84	7.66	0.44	3.77	0.28
6	10	0.51	3.57	0.83	7.59	0.46	3.91	0.24

Figure 10: Topologies of the top six candidates with the smallest V_2 at the T -th generation when using GA with $f = V_2$ Table 9: Properties of top six topologies obtained by GA when $f = V_3$

	V_1		V_2		V_3		V_4	
	Value	NS	Value (10^3 km)	NS	Value (10^3 km \times Gbps)	NS	Value (Gbps)	NS
1	11	0.40	4.22	0.49	6.01	1.00	2.99	0.53
2	13	0.24	4.59	0.34	6.01	1.00	2.99	0.53
3	15	0.13	4.71	0.30	6.02	0.99	2.99	0.53
4	15	0.13	4.81	0.27	6.03	0.99	2.99	0.53
5	12	0.32	4.39	0.42	6.03	0.99	2.99	0.53
6	13	0.24	4.69	0.31	6.03	0.99	3.13	0.48

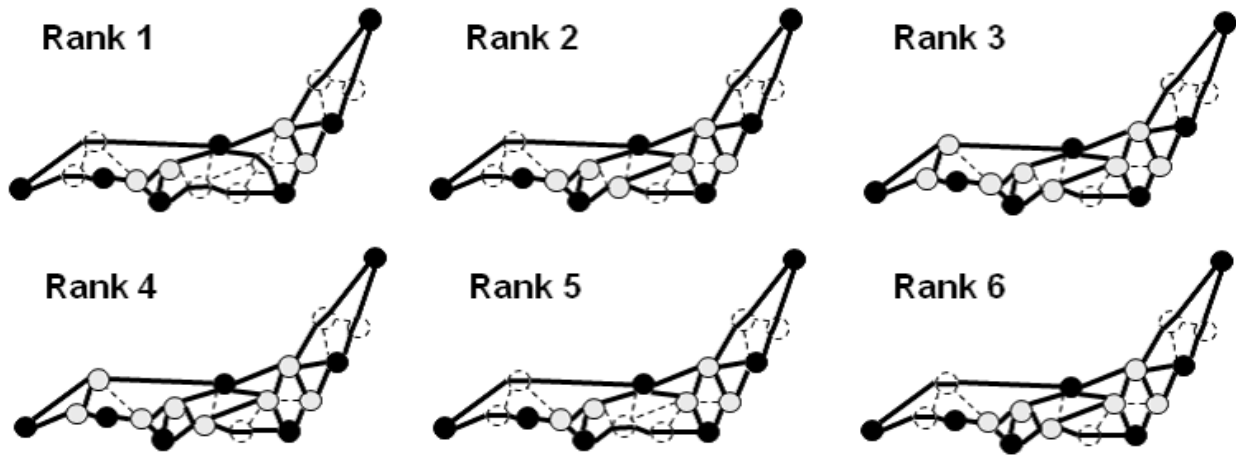


Figure 11: Topologies of the top six candidates with the smallest V_3 at the T -th generation when using GA with $f = V_3$

Table 10: Properties of top six topologies obtained by GA when $f = V_4$

	V_1		V_2		V_3		V_4	
	Value	NS	Value (10^3 km)	NS	Value (10^3 km \times Gbps)	NS	Value (Gbps)	NS
1	15	0.13	4.79	0.27	7.24	0.56	2.27	0.93
2	14	0.18	4.95	0.22	6.25	0.90	2.30	0.91
3	15	0.13	4.88	0.24	6.64	0.75	2.30	0.91
4	13	0.24	4.80	0.27	6.95	0.65	2.30	0.91
5	16	0.08	4.88	0.25	6.49	0.81	2.30	0.91
6	14	0.18	4.96	0.22	7.00	0.63	2.30	0.91

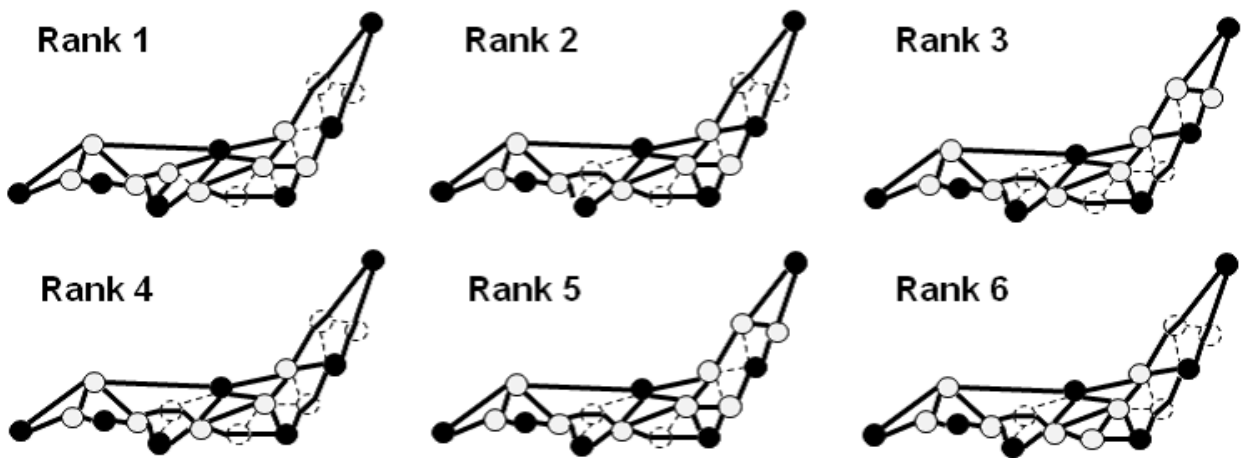


Figure 12: Topologies of the top six candidates with the smallest V_4 at the T -th generation when using GA with $f = V_4$

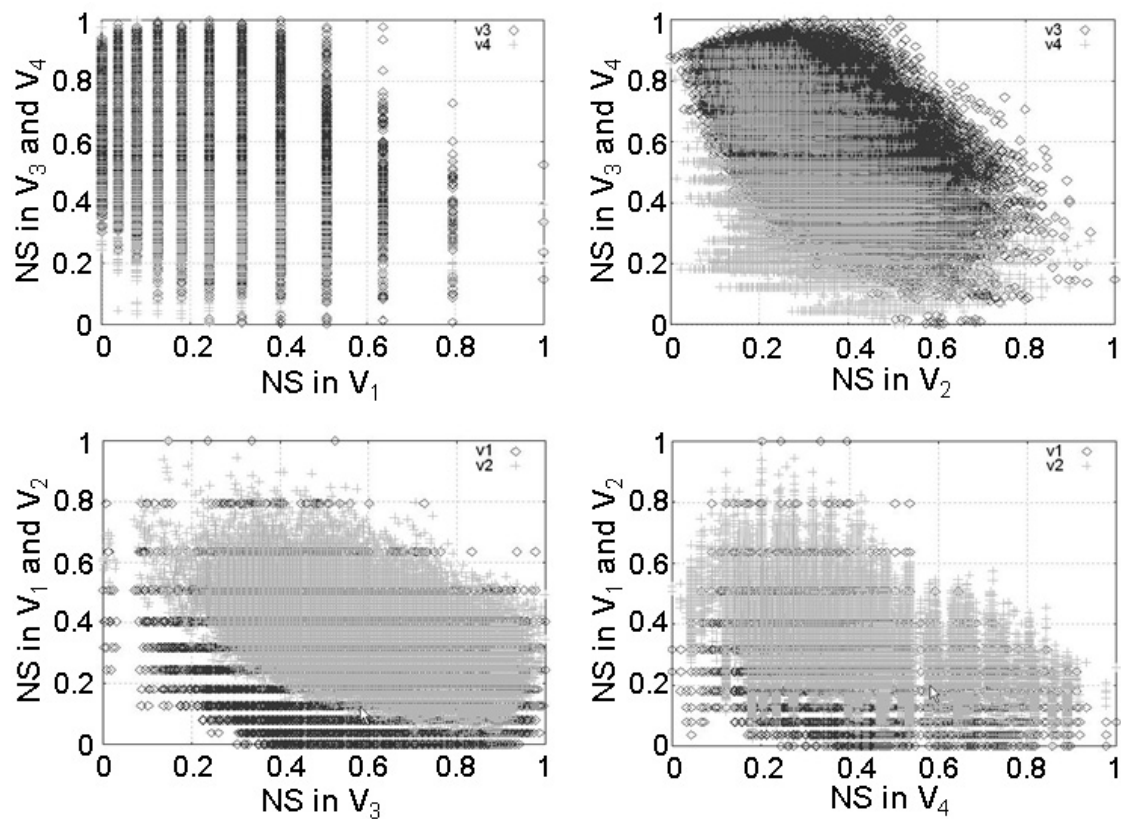


Figure 13: Scattergram of normalized scores

Table 11: Properties of top six topologies obtained by GA when $f = V_4$ and $r = 0.1$

	V_1		V_2		V_3		V_4	
	Value	NS	Value (10^3 km)	NS	Value (10^3 km \times Gbps)	NS	Value (Gbps)	NS
1	9	0.64	4.16	0.52	6.06	0.98	2.99	0.53
2	10	0.51	4.17	0.51	6.29	0.88	2.99	0.53
3	11	0.40	4.06	0.56	6.42	0.83	3.78	0.27
4	11	0.40	4.20	0.50	6.11	0.96	3.78	0.27
5	9	0.64	4.14	0.53	6.16	0.93	3.92	0.24
6	10	0.51	4.15	0.52	6.37	0.85	3.92	0.24

Table 12: Properties of top six topologies obtained by GA when $f = V_4$ and $r = 0.5$

	V_1		V_2		V_3		V_4	
	Value	NS	Value (10^3 km)	NS	Value (10^3 km \times Gbps)	NS	Value (Gbps)	NS
1	10	0.51	4.31	0.45	6.12	0.95	2.99	0.53
2	11	0.40	4.24	0.48	6.16	0.94	2.99	0.53
3	13	0.24	4.54	0.36	6.77	0.71	2.70	0.67
4	12	0.32	4.37	0.43	6.52	0.80	2.72	0.66
5	12	0.32	4.42	0.41	6.92	0.66	2.99	0.53
6	14	0.18	4.30	0.46	6.47	0.81	2.72	0.66

if we designed the network topology optimizing just a single criterion, the other criteria were degraded in the obtained topologies. On the other hand, DEA enables us to focus on a small number of desirable topologies among a huge number of possible candidates.

Acknowledgment

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