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DISSOLUTION OF DILEMMA BY NEWLY DEFINING CRITERIA MATRIX IN ANP

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Abstract In a decision-making process, one finds that there are alternatives which can be categorized into several groups, based on his or her perspectives, and/or on expected outcome, such as whether his or her choice brings about gains or losses, or that such a choice is right or wrong. Besides, there are many cases with a dilemma where it is hard to select the best choice through transitive relations. This paper presents a new method to dissolve this dilemma by using ANP (Analytic Network Process) with a newly defined criteria matrix. The case of the super-matrix of ANP is examined in an alternative matrix with a dilemma. The paper shows mathematically that the eigenvector of the super-matrix serves as a solution to this simple dilemma, describes the proposed method, and attempts to apply this method to other dilemmas. The proposed method is limited to up to three alternatives.

Keywords: AHP, decision analysis, ANP, dilemma, eigenvector, fallacy of composition

1. Introduction

There are many cases with a dilemma where it is hard to select the best choice through transitive relations. Enterprises often confront with such a dilemma in the process of corporate activities. In case a firm plans a new product by catering to customer's needs, based on the concept of reversing the value chain, for example, there often can be differences of opinion between manufacturers who are closely connected with customers, and those belong to the development and management department with a strategic viewpoint. In short, such disagreements can be regarded as an evaluation problem with a dilemma.

Arrow [1] proved that irrationality can be observed in a society where an evaluation is conducted by multiple types of people in the general possibility theorem, and when a social decision-making is carried out of more than three choices. Such irrationality is often brought about by the difference of viewpoint retained by respective evaluator. Thus, decision-making technique is extremely important in solving this evaluation problem with a dilemma. Just as Simon [23] rightly stated, "To manage is to decide to make."

Because AHP (Analytic Hierarchy Process) [12] is one of such decision-making techniques, which require transitive relations, the influence of a dilemma is incorporated into Consistency Index (C.I.), and has been treated as a decision-making stress problem. When one gets a larger view of the evaluation problem with a dilemma, it is found, as a result of the research of reversal problem of the order in AHP, that it is similar to an opportunity. Because the reversal of order in the decision-making method constitutes a fatal flaw of the technique, Choo et al. [3], Saaty [13–19], Schoner et al. [21, 22], Belton and Gear [2], Dyer [4], Kinoshita et al. [7–11], Harker et al. [5, 6], and Tamura et al. [26] have voiced objection and disagreed with one another. This prompted some other proposed methods for evading this reversal problem. In order to introduce the weights of criteria relative to alternatives, Kinoshita et al. developed Dominant AHP, Linking pin AHP [3, 21, 22])and a simple super-matrix with ANP [16] which leads to the same solution as that of dominant AHP, and dissolved the issue of the reversal of order. Kinoshita et al. developed Concurrent Convergence Method and Concurrent Convergence Method of Evaluation Value to put dominant AHP for practical use. Sugiura and Kinoshita [24, 25] proved that the evaluation problem with a dilemma can be solved by using Concurrent Convergence of Evaluation Value method. However, a method to dissolve this problem by ANP has not been proposed yet. This paper describes a method to dissolve this dilemma problem by using ANP, and explains some findings concerning ANP through several examples. The proposed method is limited to up to three alternatives.

The rest of this paper is organized as follows. In Section 2, a dissolution method of a dilemma problem is introduced. A new ANP method is proposed, and an interpretation of the proposed method is given in Section 3. Some examples of application and knowledge concerning ANP are presented in Section 4. In Section 5, the paper attempts to discuss further on some other matters, and Section 6, the conclusion.

2. Dissolution Method of Dilemma

In this section, a new dissolution method of a dilemma is presented. Firstly, Kinoshita's method for solving a dilemma is introduced. Secondly, a new dissolution method is proposed.

2.1. Previous method

A need for decision-making determines man's subsequent behavior. Therefore, any method should offer a decision-maker preferable information on alternatives, along with risk-related information for each of them. However, there is no decision-making method which offers significant information on the evaluation problem with a circulative nature, such as "Rock > Scissors", "Scissors > Paper", "Paper > Rock". Referring to the idea of Arrow [1], Sugiura and Kinoshita [25] stated that such a dilemma can be classified into two categories, a simple dilemma and a dilemma with a fallacy of composition, and presented a unified solution. When alternatives are evaluated based on a specifically selected choice, a simple dilemma can arise. Therefore, the rock-paper-scissors becomes an issue of this simple dilemma. When two or more decision-makers exist, and when each of them holds a different viewpoint, a dilemma with a fallacy of composition occurs.

Triantaphyllou [27] and Sugiura et al. [24, 25] treated the dilemma in AHP as a structural issue, and compared two alternatives concerning the cancellation of the order reversal problem. For instance, Sugiura and Kinoshita assumed three types of Japanese Sinkansen; "Kodama", "Hikari", and "Nozomi" as alternatives, and "Amenity (C1)" and "Economy (C2)" as criteria in the example of such simple dilemma. The comparison of "Kodama" and "Hikari" is focus on "Amenity (C1)", "Nozomi" and "Kodama" is equivalent to "Amenity (C1)" and "Economy (C2)". And the comparison of "Hikari" and "Nozomi" is focus on "Economy (C2)". In this example, it was impossible to set priorities among three alternatives (Table 1), i.e. Kodama (0.68) > Hikari (0.32), Hikari (0.55) > Nozomi (0.45). By the same token, it was not possible to assume transitive relations such as Kodama > Nozomi, because Nozomi (0.6) > Kodama (0.4) being the case.

They took the ratios of the evaluation values, and examined the evaluation by AHP (Table 2) [24]. The evaluation values in the left column were obtained as the evaluation

Table 1: Illustration of simple dilemma

	$C_1(0.8)$	$C_2(0.2)$	Value
Kodama	0.7	0.6	$0.68(a_1)$
Hikari	0.3	0.4	$0.32(a_2)$

	$C_1(0.5)$	$C_2(0.5)$	Value
Nozomi	0.9	0.3	$0.6(b_1)$
Kodama	0.1	0.7	$0.4(b_2)$

	$C_1(0.3)$	$C_2(0.7)$	Value
Hikari	0.2	0.7	$0.55(c_1)$
Nozomi	0.8	0.3	$0.45(c_2)$

Table 2: Pairwise comparison

	Kodama	Hikari	Nozomi	Eigenvalue
Kodama	1	2.1250	0.6667	0.3712 (1)
Hikari	0.4760	1	1.2222	0.2749 (3)
Nozomi	1.5000	0.8182	1	0.3539 (2)

value of "Kodama" is 1. It was calculated by a_1, a_2, b_1, and b_2. The values in the middle column were obtained as the evaluation value of "Hikari" is 1, and it was calculated by a_1, a_2, c_1 and c_2. The values in the right column were obtained as the evaluation value of "Nozomi" is 1, and it was calculated by b_1, b_2, c_1 and c_2. From Table 2, we can acquire the following priority weights, or Kodama (0.3712) > Nozomi (0.3539) > Hikari (0.2749).

The Concurrent Convergence Method of Evaluation Value was used to verify whether the solution in AHP is appropriate altogether [24]. Let M_i^{YX} (i=1, 2, 3) denote an evaluation ratio of X and Y in terms of i-th criterion. AHP was defined in the Concurrent Convergence Method as follows.

$$\begin{array}{c} \\ Kodama \\ Hikari \\ Nozomi \end{array}\!\!\begin{array}{c} \begin{array}{ccc} Kodama & Hikari & Nozomi \end{array} \\ \begin{pmatrix} 1 & 2.1250 & 0.6667 \\ 0.4706 & 1 & 1.2222 \\ 1.5000 & 0.8182 & 1 \end{pmatrix} \end{array} = \begin{array}{c} \\ X \\ Y \\ Z \end{array}\!\!\begin{array}{c} \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{pmatrix} 1 & M_2^{YX} & M_3^{ZX} \\ M_1^{XY} & 1 & M_3^{ZY} \\ M_1^{XZ} & M_2^{YZ} & 1 \end{pmatrix} \end{array},$$

$$\begin{array}{c} \\ X \\ Y \\ Z \end{array}\!\!\begin{array}{c} \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{pmatrix} 1 & \sqrt[3]{M_1^{YX}\cdot M_2^{YX}\cdot M_3^{YX}} & \sqrt[3]{M_1^{ZX}\cdot M_2^{ZX}\cdot M_3^{ZX}} \\ \sqrt[3]{M_1^{XY}\cdot M_2^{XY}\cdot M_3^{XY}} & 1 & \sqrt[3]{M_1^{ZY}\cdot M_2^{ZY}\cdot M_3^{ZY}} \\ \sqrt[3]{M_1^{XZ}\cdot M_2^{XZ}\cdot M_3^{XZ}} & \sqrt[3]{M_1^{YZ}\cdot M_2^{YZ}\cdot M_3^{YZ}} & 1 \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} 1 & 1.3505 & 1.0490 \\ 0.7405 & 1 & 0.7768 \\ 0.9533 & 1.2874 & 1 \end{pmatrix}$$

All the ratios of the evaluation of final alternatives turned out to be equal, and the priorities (0.3712, 0.2749, 0.3539) denote the normalized eigenvector. It shows that the priority level was decided specifically as a result of a problem like a simple dilemma, as is shown in the example.

2.2. Proposed method

AHP usually requires the information of all the alternatives, or the ratios of the evaluation items (criteria). However, it might be difficult to evaluate it in pairwise comparison with missing values. Thus, a dilemma, or a circulative matrix, shown in Table 1, needs to be examined again. $U_\square$ is a circulative matrix represented the evaluation value in Table 1. Supposing that X, Y, and Z were the alternatives, the evaluation value a_1 and a_2 were obtained as X and Y. The missing values have appeared with the sign of "$\square$".

$$U_\square = \begin{pmatrix} a_1 & b_1 & \square \\ a_2 & \square & c_2 \\ \square & b_3 & c_3 \end{pmatrix} \tag{2.1}$$

Let's examine each of the elements of pairwise comparison matrix, represented by A, B and C. A, B and C are: $A = a_1/a_2$, $B = b_1/b_3$ and $C = c_2/c_3$

$$U_{AHP} = \begin{pmatrix} a_1/a_1 & a_1/a_2 & b_1/b_3 \\ a_2/a_1 & a_2/a_2 & c_2/c_3 \\ b_3/b_1 & c_3/c_2 & c_3/c_3 \end{pmatrix} = \begin{pmatrix} 1 & A & B \\ 1/A & 1 & C \\ 1/B & 1/C & 1 \end{pmatrix} \tag{2.2}$$

We think about ANP that defines these two matrices, by assuming a matrix where missing evaluation values denoted by $W_\square$ are interpolated. So, the element of no evaluation is assumed to be zero, then it is rewritten as follows: $U_\square$ as U, and $W_\square$ as W. Since the evaluated element is assumed to be one, ANP will be made from the criteria matrix. The super-matrix of ANP is made from the alternatives matrix U, with its missing value being zero, and the criteria matrix W with its evaluation value being assumed to be one.

$$W = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \tag{2.3}$$

The definition of this criteria matrix W is insufficient. The eigenvector of UW turns out to be (0.3620, 0.2778, 0.3602), the order is maintained, and the error margin with each eigenvector remains within ± 0.001 compared with the vector of U_{ANP}. Therefore, we may obtain the same solution as Equation (2.2) by ANP, when adequate criteria matrix W is defined. Next, we examine the eigenvector of ANP based on the criteria matrix W, by taking the matrix inverse of U.

$$W = \begin{pmatrix} \frac{b_3c_2}{a_1b_3c_2+a_2b_1c_3} & \frac{b_1c_3}{a_1b_3c_2+a_2b_1c_3} & 0 \\ \frac{a_2c_3}{a_1b_3c_2+a_2b_1c_3} & 0 & \frac{a_1c_2}{a_1b_3c_2+a_2b_1c_3} \\ 0 & \frac{a_1b_3}{a_1b_3c_2+a_2b_1c_3} & \frac{a_2b_1}{a_1b_3c_2+a_2b_1c_3} \end{pmatrix} \tag{2.4}$$

The eigenvector to the maximum eigenvalue k of this ANP matrix is assumed to be x and z.

$$\begin{pmatrix} 0 & W \\ U & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = k \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.5}$$

The overall judgment of the alternatives is represented by eigenvector z to the maximum eigenvalue. Next, we compare this eigenvector z and the eigenvector y at the maximum eigenvalue α in Equation (2.2). By assuming $y(y_1, y_2, y_3)$ and $y_3 = 1$, each element of the eigenvector is obtained as follows: $y_1 = \frac{B\{AC(1-\alpha)-B\}}{B(1-\alpha)-AC}$ and $y_2 = \frac{C\{B(1-\alpha)-AC\}}{AC(1-\alpha)-B}$ (See **Appendix A**).

On the other hand, because the eigenvector $z(z_1, z_2, z_3)$ can be expressed also by $1 - 1/(1-k^2) = \alpha - 1$ as $z_3 = 1$, the following equations are obtained. (See **Appendix B**)

$$z_1 = \frac{B\{(B+AC)(1-k^2)-AC\}}{(B+AC)(1-k^2)-B} = \frac{B\{B-(1-\alpha)AC\}}{AC-(1-\alpha)B} = \frac{B\{(1-\alpha)AC-B\}}{(1-\alpha)B-AC} = y_1$$
$$z_2 = \frac{C\{(B+AC)(1-k^2)-B\}}{(B+AC)(1-k^2)-AC} = \frac{C\{AC-(1-\alpha)B\}}{B-(1-\alpha)AC} = \frac{C\{(1-\alpha)B-AC\}}{(1-\alpha)AC-B} = y_2$$

Thus, in spite of α and k, the eigenvector of UW is the same as the eigenvector of U_{AHP}. Therefore, the dilemma can be solved by this proposed method, by calculating the eigenvector in ANP to the maximum eigenvalue.

3. Describing the Proposed Method

The missing value is assumed to be zero in the preceding section, bringing the evaluation matrix to be defined. In this section, a decision-making process is described by reviewing the evaluation of dissatisfaction (degree of dissatisfaction for the evaluated value of alternatives), and the criteria matrix is interpreted as the missing value.

It is mathematically proved that Equation (2.5) has the same eigenvector as Equation (2.2). Here, the meaning of W will be defined through the interpretation of a figure. Let's assume that the alternatives matrix with the missing value to be $U_\square$, and that the criteria matrix with the dissatisfaction for this evaluation value to be $W_\square$. When the alternatives, "Kodama", "Hikari", and "Nozomi", are assumed to be $A_i (i = 1, 2, 3)$, it is possible to consider a situation where virtually three different evaluators, or T_1, T_2, and T_3, evaluate alternatives.

$$U_\square = \begin{array}{r} Kodama : A_1 \\ Hikari : A_2 \\ Nozomi : A_3 \end{array} \begin{pmatrix} a_1 & b_1 & \square \\ a_2 & \square & c_2 \\ \square & b_3 & c_3 \end{pmatrix} \tag{3.1}$$

For instance, the evaluation value given to alternative "Kodama" is either a_1 or b_1. The evaluation value of other alternatives are equally shown in alternatives matrix $U_\square$. However, it is impossible to set priorities only by the alternatives matrix $U_\square$. Therefore, it is necessary to define the new matrix $W_\square$ as the criteria matrix W, based on the inverse matrix of U. We assume such a matrix where the matrix $W_\square$ is reflected as the evaluation value. It is necessary that the alternatives tell the evaluators their dissatisfaction accurately, which eventually help them define the priorities. Therefore, in order to measure the degree of dissatisfaction accurately, the combination of a_i, b_j, and c_k, from which the evaluation value is given to all alternatives, has to be examined. Thus, the evaluations of dissatisfaction in terms of the evaluation values of the alternatives are considered in this section. The relationship of evaluators and the evaluation values are shown from Figure 1 to Figure 9. When the assessment is a case with evaluator T_i, the arrow of T_i, is bold line.

3.1. The evaluation of dissatisfaction in terms of the evaluation value of the "Kodama"

(1) The assessment is as follows with the case with evaluator T_1 (Figure 1).
The conversion coefficient is assumed to be $1/R$, and the evaluation of the degree of dissatisfaction is shown by multiplying the evaluation value of the other two alternatives by the

evaluation value of “Kodama”. The element in the row of $W_{\square}$ corresponds with the element in the line of $U_{\square}$ in ANP. Then, as is shown in the figure on the left, the degree of dissatisfaction is denoted by b_3c_2/R times for evaluation value a_1 of “Kodama”. Therefore, the two alternatives need $a_1b_3c_2/R$ from evaluator T_1 so that they can be satisfied. Moreover, the dissatisfaction degree, or the figure on the right, is shown as a_2c_3/R times against the evaluation value b_1 of “Kodama”, and the value necessary for evaluator T_1 is denoted by $a_2b_1c_3/R$. Therefore, the satisfaction rating of “Kodama” that evaluator T_1 has to judge is expressed by $a_1b_3c_2/R + a_2b_1c_3/R$.
(2) The assessment is as follows for the case with evaluator T_2 (Figure 2).
As for the evaluator, applying the two alternatives at the same time is not relevant, and in this case, no evaluation is acquired for “Hikari”. The dissatisfaction degree becomes b_1c_3/R times for evaluation value a_1 of “Kodama”. Therefore, the two alternatives require $a_1b_1c_3/R$ from evaluator T_2 to be satisfied.
(3) The assessment is as follows for the case with evaluator T_3 (Figure 3).
In this case, the evaluation value is given to “Kodama,” excluding evaluator T_1. The dissatisfaction degree turns out to be a_1c_2/R times for the evaluation value b_1 of “Kodama”, and $a_1b_1c_2/R$ will be needed from evaluator T_3. Here, the following matrix $W_{\square}$ shows the dissatisfaction degree in terms of the evaluation value of “Kodama”.

$$W_{\square} = \begin{pmatrix} b_3c_2/R & b_1c_3/R & \square \\ a_2c_3/R & \square & a_1c_2/R \\ \square & \square & \square \end{pmatrix} \tag{3.2}$$

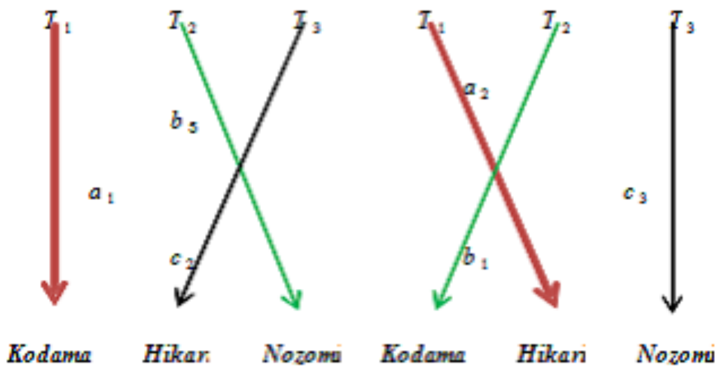

Figure 1: The assessment is a case with evaluator T_1

3.2. The evaluation of dissatisfaction in terms of the evaluation value of “Hikari”

(1) The assessment is as follows with the case with evaluator T_1 (Figure 4).
The value of a_2 and c_2 on “Hikari” are given from evaluator T_1 and evaluator T_3, respectively. The dissatisfaction degree must be b_3c_2/R for “Hikari” of evaluator T_1 concerning value a_2. Moreover, the value necessary for evaluator T_1 is $a_2b_3c_2/R$. Because the element in the second row of the two lines of $U_{\square}$ is left blank, the dissatisfaction degree concerning the evaluation value of “Hikari” cannot be demonstrated in this combination.
(2) The assessment is as follows with the case with evaluator T_2 (Figure 5).
There are two sets of combination in this case. The standard of dissatisfaction turns out to be c_2, according to the evaluation of T_2 on “Nozomi” in the figure on the left. Then, the dissatisfaction degree turns out to be a_1b_3/R times for the evaluation value c_2 of “Hikari”.

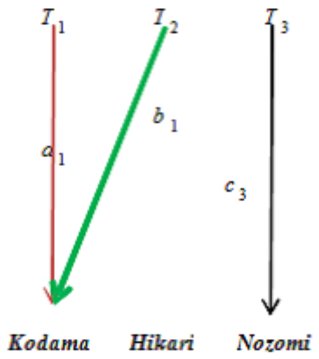

Figure 2: The assessment is a case with evaluator T_2

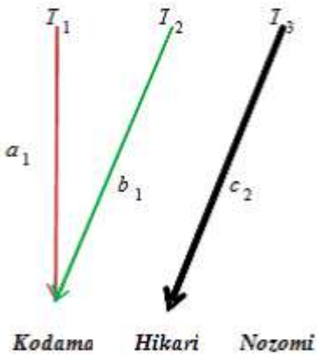

Figure 3: The assessment is a case with evaluator T_3

Therefore, two alternatives require $a_1b_3c_2/R$ to satisfy evaluator T_2. The dissatisfaction degree in the figure on the right turns out to be b_1c_3/R times for the evaluation value a_2 for "Hikari", the value necessary for evaluator T_2 is $a_2b_1c_3/R$. Therefore, the satisfaction ratio of "Hikari" by evaluator T2 has to be $a_1b_3c_2/R + a_2b_1c_3/R$.

(3) The assessment is as follows with the case with evaluator T_3 (Figure 6).

The value of a_2 and c_2 of "Hikari" are given from evaluator T_1 and evaluator T_3, respectively. The degree of dissatisfaction must be a_2b_1/R for "Hikari" by evaluator T_3 concerning value c_2. Moreover, the value required for evaluator T_3 is $a_2b_1c_2/R$. However, because the element of $U_\square$ in the second row of the two lines is left blank, the dissatisfaction degree cannot be shown in the ratio as well as in the paragraph (1) in Section 3.2.

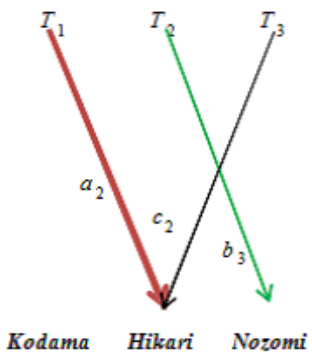

Figure 4: The assessment is a case with evaluator T_1

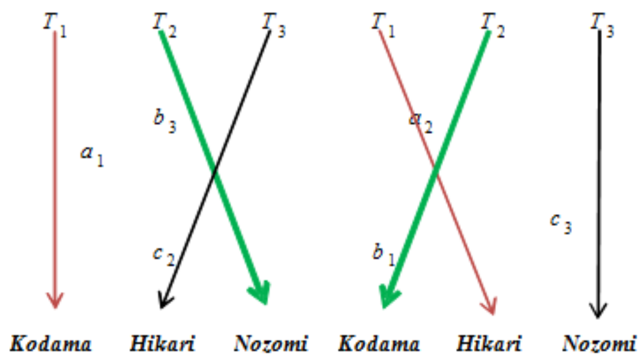

Figure 5: The assessment is a case with evaluator T_2

The dissatisfaction degree concerning the evaluation value of "Hikari" is shown as follows.

$$W_\square = \begin{pmatrix} \square & b_1c_3/R & \square \\ \square & \square & \square \\ \square & a_1b_3/R & \square \end{pmatrix} \tag{3.3}$$

3.3. The evaluation of dissatisfaction in terms of the evaluation value of "Nozomi"

(1) The assessment is as follows with the case with evaluator T_1 (Figure 7).

The value of b_3 and c_3 for "Nozomi" are given from evaluator T_2 and evaluator T_3, respectively. The level of dissatisfaction turns out to be b_3 according to the evaluation of T_2 on

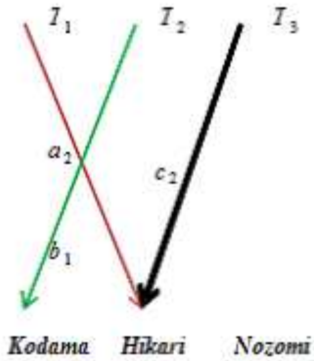

Figure 6: The assessment is a case with evaluator T_3

"Nozomi". Then, the dissatisfaction degree turns out to be a_2c_3/R times for the evaluation value b_3 of "Nozomi". The two alternatives require $a_2b_3c_3/R$ from evaluator T_1 so that they can be satisfied.
(2) The assessment is as follows with the case with evaluator T_2 (Figure 8).
The value of b_3 and c_3 for "Nozomi" are given from evaluator T_2 and T_3, respectively. The degree of dissatisfaction turns out to be c_3 according to the evaluation of T_2 on "Nozomi". Then, the dissatisfaction degree turns out to be a_1b_3/R times for the evaluation value b_3 of "Nozomi". The two alternatives require the value $a_1b_3c_3/R$ from evaluator T_2 so that they can be satisfied.

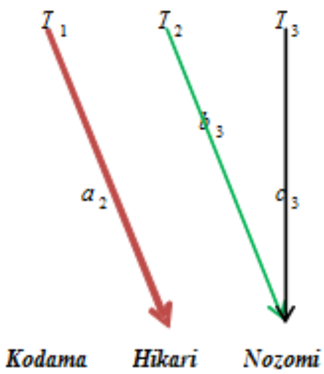

Figure 7: The assessment is a case with evaluator T_1

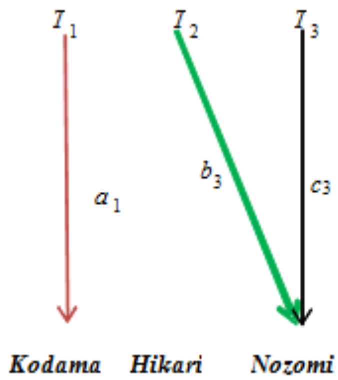

Figure 8: The assessment is a case with evaluator T_2

(3) The assessment is as follows with the case with evaluator T_3 (Figure 9).
There are two sets of combination in this case. The degree of dissatisfaction is denoted by b_3 according to the evaluation of T_3 on "Hikari," as is shown in the figure on the left. Then, the dissatisfaction degree turns out to be a_1c_2/R times for the evaluation value b_3 of "Nozomi". The two alternatives require $a_1b_3c_2/R$ from evaluator T_3 to be satisfied. The dissatisfaction degree demonstrated by the figure on the right is a_2b_1/R times to the evaluation value c_3 of "Nozomi", the value required from evaluator T_3 is $a_2b_1c_3/R$. Therefore, the satisfaction ratio of "Nozomi" to be judged by evaluator T_3 turns out to be $a_1b_3c_2/R + a_2b_1c_3/R$.

The dissatisfaction degree concerning the evaluation value of "Nozomi" is shown as

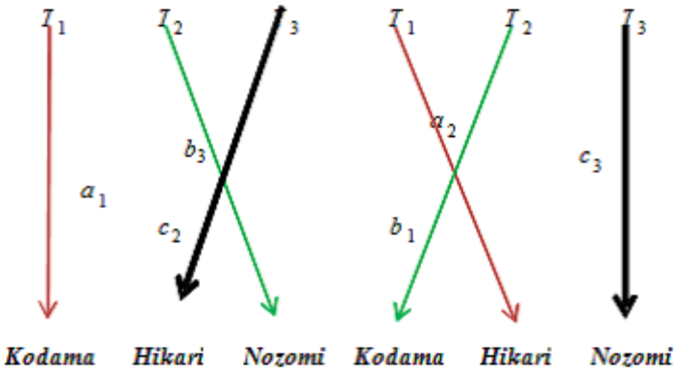

Figure 9: The assessment is a case with evaluator T_3

follows.

$$W_\square = \begin{pmatrix} \square & \square & \square \\ a_2c_3/R & \square & \square \\ \square & a_1b_3/R & a_2b_1/R \end{pmatrix} \tag{3.4}$$

Finally, the three undefined parts left in the matrix $W_\square$ can be obtained from Equation (3.2) to Equation (3.4). Therefore, the value necessary for the satisfaction ratio is brought about by the matrix $U_\square$ with the missing value, and is shown as follows.

$$\begin{aligned} U_\square W_\square &= \begin{pmatrix} a_1 & b_1 & \square \\ a_2 & \square & c_2 \\ \square & b_3 & c_3 \end{pmatrix} \begin{pmatrix} b_3c_2/R & b_1c_3/R & \square \\ a_2c_3/R & \square & a_1c_2/R \\ \square & a_1b_3/R & a_2b_1/R \end{pmatrix} \\ &= \begin{pmatrix} (a_1b_3c_2 + a_2b_1c_3)/R & a_1b_1c_3/R & a_1b_1c_2/R \\ a_2b_3c_2/R & (a_1b_3c_2 + a_2b_1c_3)/R & a_2b_1c_2/R \\ a_2b_3c_3/R & a_1b_3c_3/R & (a_1b_3c_2 + a_2b_1c_3)/R \end{pmatrix} \end{aligned} \tag{3.5}$$

Equation (3.5) holds that even if the missing value is assumed to be zero, and that the evaluators in (1) and (3) of the paragraph in Section 3.2 can be satisfied. Therefore, $U_\square U_\square$ of the equation turns out to be UW, when assuming that blank parts to be zero, taking into the consideration the fact that the missing value signifies “there is no evaluation”.

Whereas, the elements other than the missing value in the matrix $W_\square$ are the same as the inverse matrix of U. As a result, it can be said that W shows the ratio of dissatisfaction concerning the element of U. Because the position of zero remains unchanged as long as it is symmetrical in the matrix of U, W is obtained by taking the inverse matrix of U and by replacing the missing value position with zero, as is shown by Equation (2.4), when assuming that $R = a_1b_3c_2 + a_2b_1c_3$.

$$U_\square W_\square = UW = \begin{pmatrix} 1 & \frac{a_1b_1c_3}{a_1b_3c_2+a_2b_1c_3} & \frac{a_1b_1c_2}{a_1b_3c_2+a_2b_1c_3} \\ \frac{a_2b_3c_2}{a_1b_3c_2+a_2b_1c_3} & 1 & \frac{a_2b_1c_2}{a_1b_3c_2+a_2b_1c_3} \\ \frac{a_2b_3c_3}{a_1b_3c_2+a_2b_1c_3} & \frac{a_1b_3c_3}{a_1b_3c_2+a_2b_1c_3} & 1 \end{pmatrix} \tag{3.6}$$

4. Example of Application and Knowledge of ANP

The paper has discussed the issue of a simple dilemma so far. Next, we need to go on to discuss the dilemma pointed out by Triantaphyllou, and a dilemma with a fallacy of composition. In this section, the effectiveness of the proposed method will be demonstrated through some examples.

4.1. Dilemma of Triantaphyllou [27]

Table 3 and Table 4 show the cases of order reversal which arises in AHP, as was pointed out by Triantaphyllou. Such a dilemma can emerge frequently when there are many criteria and alternatives. In this section, we will apply the proposed method to Example 1 and Example 2 used by Triantaphyllou.

Table 3: Example 1 of Triantaphyllou

	C1(2/7)	C2(2/7)	C3(3/7)	Evaluation
A1	9/19	2/12	2/7	0.3054
A2	5/19	1/12	4/7	0.3439
A3	5/19	9/12	1/7	0.3507
	A3 >	A2 >	A1	

	C1(2/7)	C2(2/7)	C3(3/7)	Evaluation
A1	9/14	2/3	2/6	0.5170
A2	5/14	1/3	4/6	0.4830

	C1(2/7)	C2(2/7)	C3(3/7)	Evaluation
A1	5/10	1/10	4/5	0.5143
A2	5/10	9/10	1/5	0.4857

	C1(2/7)	C2(2/7)	C3(3/7)	Evaluation
A1	9/14	2/11	2/3	0.5213
A2	5/14	9/11	1/3	0.4787
	A1 >	A2 >	A3	

The priority level of alternative A1, A2, and A3 was expressed as A3 > A2 > A1 in Example 1, and A2 > A3 > A1 in Example 2. However, the eigenvector by the pairwise comparison of three alternatives turned out to be (0.3505, 0.3319, 0.3176) in Example 1, and (0.2913, 0.3696, 0.3391) in Example 2 by Triantaphyllou.

When the proposed method is applied to these examples, U_1W_1 in Example 1, and U_2W_2 in Example 2, respectively, are expressed as follows.

$$U_1W_1 = \begin{pmatrix} 1 & 0.5232 & 0.5540 \\ 0.4777 & 1 & 0.5202 \\ 0.4512 & 0.4804 & 1 \end{pmatrix} \text{ and } U_2W_2 = \begin{pmatrix} 1 & 0.4090 & 0.4056 \\ 0.5992 & 1 & 0.5656 \\ 0.6042 & 0.4332 & 1 \end{pmatrix}$$

Each eigenvector turns out to be (0.3499, 0.3325, 0.3176) in Example 1, i.e. A1 > A2 > A3 and (0.2913, 0.3696, 0.3391) in Example 2, i.e. A2 > A3 > A1. The result is almost the same as the value Triantaphyllou [27] obtained from the calculation. Thus, from the above two examples, it is possible to point out that a dilemma may arise when two or more alternatives are evaluated simultaneously, or when the difference between the evaluation values is marginal. The method of Triantaphyllou is the same as the one proposed by Kinoshita et al. [24] shown in Equation (2.2) concerning the use of AHP. Regrettably, however, Triantaphyllou examined only the reversal of order, and did not specify the method for dissolving issues concerning dilemma evaluation. It can be said that this proposed method is more descriptive and superior to that of Triantaphyllou because the dissolution

Table 4: Example 2 of Triantaphyllou

	C1(4/22)	C2(9/22)	C3(9/22)	Evaluation	Normalize
A1	9/9	5/8	2/8	0.5398	0.2844
A2	1/9	8/8	5/8	0.6850	0.3609
A3	8/9	2/8	8/8	0.6730	0.3546
	A2 >	A3 >	A1		

	C1(4/22)	C2(9/22)	C3(9/22)	Evaluation	Normalize
A1	1/8	8/8	5/8	0.6875	0.4979
A2	8/8	2/8	8/8	0.6932	0.5021

	C1(4/22)	C2(9/22)	C3(9/22)	Evaluation	Normalize
A1	9/9	5/8	2/5	0.6011	0.4176
A2	1/9	8/8	5/5	0.8384	0.5824

	C1(4/22)	C2(9/22)	C3(9/22)	Evaluation	Normalize
A1	9/9	5/5	2/8	0.6932	0.4856
A2	8/9	2/5	8/8	0.7343	0.5144
	A3 >	A2 >	A1		

of the dilemma is explained by ANP, which clarifies the interaction among alternatives and criteria.

4.2. Discussion of a dilemma with a fallacy of composition

A fallacy of composition is a state where each choice of some part of the whole is appropriate, however, the overall effects can be negative. In recent years, a life cycle of commodities tends to be short, bringing risks to grow. This is because the value-chain can reverse in the process, while commodities are produced in close connection with customers' needs. Therefore, decision-making becomes a vital issue for enterprises.

For instance, let's assume a case where the evaluation of a planning for the development of a certain commodity is given separately according to categories shown in Table 5. The table shows the ratios of satisfaction for farm of enterprise, development section and production department. The farm of enterprise evaluates the three commodities according to the

Table 5: Dilemma with a fallacy of composition [25]

	Farm	Development	Production	Total
Commodity 1	83	72	65	220
Commodity 2	77	56	85	218
Commodity 3	64	85	70	219

following order, or Commodity 1 (83) > Commodity 2 (77) > Commodity 3 (64), while the order of evaluation given by the development section is Commodity 3 (85) > Commodity 1 (72) > Commodity 2 (56). In the evaluation of the production department, the order turns out to be Commodity 3(70) > Commodity 2(85) > Commodity 1 (65).

Here, we can witness a fallacy of composition. It is impossible to set overall priorities in this case, because the differences among the total value for each section are marginal. Let's apply the proposed method to Table 5, which results in the following evaluation matrices.

$$U_1 = \begin{array}{c} \begin{array}{ccc} F & D & P \end{array} \\ \begin{pmatrix} 83 & 72 & 0 \\ 77 & 0 & 85 \\ 0 & 85 & 70 \end{pmatrix} \end{array},\ U_2 = \begin{array}{c} \begin{array}{ccc} D & P & F \end{array} \\ \begin{pmatrix} 72 & 65 & 0 \\ 56 & 0 & 77 \\ 0 & 70 & 64 \end{pmatrix} \end{array},\ U_3 = \begin{array}{c} \begin{array}{ccc} P & F & D \end{array} \\ \begin{pmatrix} 65 & 83 & 0 \\ 85 & 0 & 56 \\ 0 & 64 & 85 \end{pmatrix} \end{array} \tag{4.1}$$

The following equations are obtained from Equation (3.6).

$$\begin{aligned} U_1W_1 &= \begin{array}{c} \begin{array}{ccc} F & D & P \end{array} \\ \begin{pmatrix} 1 & 0.4235 & 0.5143 \\ 0.5632 & 1 & 0.4771 \\ 0.4638 & 0.5 & 1 \end{pmatrix} \end{array}, \\ U_2W_2 &= \begin{array}{c} \begin{array}{ccc} D & P & F \end{array} \\ \begin{pmatrix} 1 & 0.4823 & 0.5803 \\ 0.486 & 1 & 0.4513 \\ 0.4040 & 0.5194 & 1 \end{pmatrix} \end{array}, \\ U_3W_3 &= \begin{array}{c} \begin{array}{ccc} P & F & D \end{array} \\ \begin{pmatrix} 1 & 0.5508 & 0.3628 \\ 0.3659 & 1 & 0.4745 \\ 0.5553 & 0.4247 & 1 \end{pmatrix} \end{array} \end{aligned}$$

When these evaluations are synthesized by taking the geometric mean method, the evaluation matrix UW is obtained.

$$UW = \begin{array}{c} \begin{array}{ccc} F & D & P \end{array} \\ \begin{pmatrix} 1 & 0.8734 & 0.9509 \\ 0.9265 & 1 & 0.8458 \\ 0.8510 & 0.9568 & 1 \end{pmatrix} \end{array} \tag{4.2}$$

The matrix UW of Equation (4.2) is decomposed as Equation (4.1) and the UW is calculated again. By repeating this step, the final converged matrix is obtained.

$$UW = \begin{pmatrix} 1 & 1.0196 & 1.0066 \\ 0.9808 & 1 & 0.9873 \\ 0.9934 & 1.0128 & 1 \end{pmatrix} \tag{4.3}$$

Because the eigenvector of Equation (4.3) (0.3362, 0.3298, 0.3340) denotes the normalized eigenvector, the priority becomes Commodity 1(0.3362) $>$ Commodity 3(0.3340) $>$ Commodity 2(0.3298). Sugiura et al. [25] have acquired the same evaluating value by using the Concurrent Convergence Method of Evaluation Value. Therefore, the legitimacy of this method is verified.

5. Discussion and Implications

(1) Meaning of the criteria matrix W

The essence of this paper lies in the fact that it has found the criteria matrix W based on the matrix inverse to the alternatives matrix U where a dilemma arises. In other words, it demonstrates that a dilemma can be dissolved by using ANP, which consists of the alternatives matrix U and the criteria matrix W. In Section 2, it proves that "All the alternatives are dissatisfied with the evaluation." This is because the basis of W is the matrix inverse of U. However, in Section 3, it is uncertain whether the assumption is applicable also to the criteria matrix W. For instance, the dissatisfaction degree is defined by the evaluation values of products of other alternatives relative to the element of the evaluation value, but it is not specified whether dissatisfaction was exhibited. Thus, we have to consider how an evaluator judges the needs for alternatives, because there can always be dissatisfaction when there are needs. It is assumed that evaluators T_1, T_2, and T_3 give the evaluation value of z_1, z_2, and z_3 for the need of alternatives, respectively, and that $z_1 + z_2 + z_3 = 1$. For example, evaluator T_1 gives the evaluation value z_1 of one or less to the need value of $(a_1b_3c_2 + a_2b_1c_3)/R$ of "Kodama". Moreover, evaluator T_2 gives z_2 to the need value of $a_1b_1c_3/R$ and z_3 to the need value of $a_1b_1c_2/R$. The evaluation values of z_1, z_2, and z_3 have reduced the need value.

That is to say, an excessive need has been generated because all alternatives are dissatisfied with the evaluation value. The evaluator should attempt to enhance evaluation to an adequate level, by reducing the value of dissatisfaction for each alternative. Thus, the sum of the evaluation value concerning the need of "Kodama" can reach the proper evaluation value. As a result, the evaluation values of z_1, z_2, and z_3 turn out to be the eigenvectors of UW because evaluators T_1, T_2, and T_3 equally do their evaluation properly, concerning the need value of "Hikari" and "Nozomi". Thus, the criteria matrix W, shown as the degree of dissatisfaction in the ratio, on the assumption that "All the alternatives are dissatisfied," plays an important role in composing UW and in obtaining a proper evaluation, when supposing that the alternative matrix U and the criteria matrix W are independent and that they are forming a state of feedback.

(2) Normalization of ANP

Though the eigenvector of ANP, or the alternative matrix U in Example 2, is normalized, and remains unchanged, the eigenvector of the super-procession, where criteria matrix W is normalized, turns out to be (0.2818, 0.3799, 0.3382) without reaching the same value. This can be explained as follows. ANP, consisting of the alternative matrix U and criteria matrix W, is shown by using A, B and C as follows.

$$\begin{pmatrix} Aa_2 & Bb_3 & 0 \\ a_2 & 0 & Cc_3 \\ 0 & b_3 & c_3 \end{pmatrix} \cdot \begin{pmatrix} \frac{C}{a_2(AC+B)} & \frac{B}{a_2(AC+B)} & 0 \\ \frac{1}{b_3(AC+B)} & 0 & \frac{AC}{b_3(AC+B)} \\ 0 & \frac{A}{c_3(AC+B)} & \frac{B}{c_3(AC+B)} \end{pmatrix} = \begin{pmatrix} 1 & \frac{AB}{B+AC} & \frac{ABC}{B+AC} \\ \frac{C}{B+AC} & 1 & \frac{BC}{B+AC} \\ \frac{1}{B+AC} & \frac{A}{B+AC} & 1 \end{pmatrix} \tag{5.1}$$

Though the alternative matrix U is not normalized, Equation (3.5) shows that the values of A, B and C do not change. In other words, the eigenvector of UW does not change even though A, B and C take positive numbers. However, when the criteria matrix W is normalized, neither a_2, b_3 nor c_3 are independent, and the element of UW is also different from Equation (3.5). That is to say, the criteria matrix W cannot be normalized. In AHP and in ANP, it is quite natural that the criteria matrix W and the alternative matrix U are normalized. Moreover, the result indicates the fact that it is not easy to normalize the criteria matrix although normalization is necessary in the super-matrix of Saaty.

(3) The proposed method concerning a fallacy of composition problem.

Dilemma as a whole has been handled in Section 4.2, though there is consistency in the evaluation in each section. However, in actual circumstances, it is not easy to accomplish consistency because the differences among the elements of the eigenvector are marginal.

The evaluation matrix E is produced based on Commodity 1 in the first row, Commodity 2 in the second row and Commodity 3 in the third row.

$$E = \begin{pmatrix} 1 & 1.2857 & 0.9286 \\ 0.9277 & 1 & 1.2143 \\ 0.7711 & 1.5179 & 1 \end{pmatrix} \tag{5.2}$$

When the evaluation matrix E is defined as the AHP, the eigenvector is (0.3329, 0.3267, 0.3404) and differs from the values (0.3362, 0.3298, 0.3340) obtained through the geometric mean method. However, in this example, the evaluation value by the geometric mean method and the evaluation value by the eigenvector are different, and the order is also reversed. Now, let us discuss the proposed method. Equation (5.2) is rewritten in the general form as follows.

$$U = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \tag{5.3}$$

The evaluation value of this evaluation matrix U is assumed to have synthesized simple dilemma matrix $D_i (i = 1, 2, 3)$. The geometric mean method is taken to consolidate the simple dilemma matrices.

$$DW = \begin{pmatrix} 1 & \sqrt[3]{\frac{a_1b_1c_1(a_2b_1c_3+a_3b_2c_1)}{a_2b_2c_2(a_1b_3c_2+a_2b_1c_3)}} & \sqrt[3]{\frac{a_1b_1c_1(a_1b_3c_2+a_3b_2c_1)}{a_3b_3c_3(a_1b_3c_2+a_2b_1c_3)}} \\ \sqrt[3]{\frac{a_2b_2c_2(a_1b_3c_2+a_3b_2c_1)}{a_1b_1c_1(a_1b_3c_2+a_2b_1c_3)}} & 1 & \sqrt[3]{\frac{a_2b_2c_2(a_2b_1c_3+a_3b_2c_1)}{a_3b_3c_3(a_1b_3c_2+a_2b_1c_3)}} \\ \sqrt[3]{\frac{a_3b_3c_3(a_2b_1c_3+a_3b_2c_1)}{a_1b_1c_1(a_1b_3c_2+a_2b_3c_1)}} & \sqrt[3]{\frac{a_3b_3c_3(a_1b_3c_2+a_3b_2c_1)}{a_2b_2c_2(a_1b_3c_2+a_2b_3c_1)}} & 1 \end{pmatrix} \tag{5.4}$$

Here, the evaluation matrix which is assumed to be U is taken up again, the final matrix is obtained.

$$ConvergedDW = \begin{pmatrix} 1 & \sqrt[3]{\frac{a_1b_1c_1}{a_2b_2c_2}} & \sqrt[3]{\frac{a_1b_1c_1}{a_3b_3c_3}} \\ \sqrt[3]{\frac{a_2b_2c_2}{a_1b_1c_1}} & 1 & \sqrt[3]{\frac{a_2b_2c_2}{a_3b_3c_3}} \\ \sqrt[3]{\frac{a_3b_3c_3}{a_1b_1c_1}} & \sqrt[3]{\frac{a_3b_3c_3}{a_2b_2c_2}} & 1 \end{pmatrix} \tag{5.5}$$

The estimate of Equation (5.3) is given by the geometric mean method. When the pairwise comparisons are presented in Equation (5.3), the geometric mean and the eigenvector yield the same values. The evaluation value of Equation (5.2) becomes the same as the one obtained by the Concurrent Convergence Method, and also that of Equation (4.3). Therefore, AHP cannot be applied to this example because all the elements do not belong to pair comparisons.

6. Conclusion

This paper presents a new method for dissolving a dilemma by using ANP. The following conclusion can be drawn from the study.

(1)The legitimacy of this method is proven by the solution in ANP.

This method is more descriptive and superior to the one proposed by Triantaphyllou, because the dissolution of a dilemma is described by ANP, which clarifies the interaction among alternatives and criteria.

(2) It is necessary to note the following, concerning the normalization in ANP.

Normalization is often feasible in ANP. However, we found that it is not easy to normalize the criteria matrix when following the examples presented by Triantaphyllou.
(3) A fallacy of composition can arise from three simple dilemmas.
It is shown that a fallacy of composition can arise from three simple dilemmas, and that a dilemma with a fallacy of composition can be dissolved by this method. However, the proposed method is limited to up to three alternatives. The remaining issue is to expand and generalize the proposed method for n alternatives.

This method seems to be useful in setting priorities for the alternative matrix with missing values, which concerns a decision-making process. It resembles closely the way people make decisions. Finally, we wish that the idea presented in this paper could provide a fresh perspective for future research on decision-making.

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Appendices

A. Calculating the Eigenvector in AHP

Let's consider the following matrix of a dilemma.

$$U_{\square} = \begin{array}{r} \mathit{Kodama} \\ \mathit{Hikari} \\ \mathit{Nozomi} \end{array} \begin{pmatrix} a_1 & b_1 & 0 \\ a_2 & 0 & c_2 \\ 0 & b_3 & c_3 \end{pmatrix} \tag{A.1}$$

The Equation (A.1) can be altered to the form of the pairwise comparison, and the matrix U_{AHP} is represented by A, B, and C

$$U_{AHP} = \begin{pmatrix} a_1/a_1 & a_1/a_2 & b_1/b_3 \\ a_2/a_1 & a_2/a_2 & c_2/c_3 \\ b_3/b_3 & c_3/c_2 & c_3/c_3 \end{pmatrix} = \begin{pmatrix} 1 & A & B \\ 1/A & 1 & C \\ 1/B & 1/C & 1 \end{pmatrix}$$

Let's assume the maximum eigenvalue and the eigenvector to be α and $y(y_1, y_2, y_3)$.

$$y_1 + Ay_2 + By_3 = \alpha y_1 \tag{A.2}$$
$$y_1/A + y_2 + Cy_3 = \alpha y_2 \tag{A.3}$$
$$y_1/B + y_2/C + y_3 = \alpha y_3 \tag{A.4}$$

Equation (A.5) is obtained from Equation (A.2) and Equation (A.4).

$$y_1 = \frac{B\{AC(1-\alpha) - B\}}{B(1-\alpha) - AC} y_3 \tag{A.5}$$

Equation (A.6) is obtained from Equation (A.3) and Equation (A.4).

$$y_2 = \frac{C\{B(1-\alpha) - AC\}}{AC(1-\alpha) - B} y_3 \tag{A.6}$$

By assuming $y_3 = 1$,

$$y_1 = \frac{B\{AC(1-\alpha) - B\}}{B(1-\alpha) - AC}, y_2 = \frac{C\{B(1-\alpha) - AC\}}{AC(1-\alpha) - B} \tag{A.7}$$

are obtained as the eigenvector of U_{AHP}.

B. Calculating the Eigenvector in ANP

The matrix inverse of Equation (A.1) is shown by the following equation.

$$U^{-1} = \begin{pmatrix} \frac{b_3c_2}{a_1b_3c_2+a_2b_1c_3} & \frac{b_1c_3}{a_1b_3c_2+a_2b_1c_3} & \frac{-b_1c_2}{a_1b_3c_2+a_2b_1c_3} \\ \frac{a_2c_3}{a_1b_3c_2+a_2b_1c_3} & \frac{-a_2c_3}{a_1b_3c_2+a_2b_1c_3} & \frac{a_1c_2}{a_1b_3c_2+a_2b_1c_3} \\ \frac{-a_2b_3}{a_1b_3c_2+a_2b_1c_3} & \frac{a_1b_3}{a_1b_3c_2+a_2b_1c_3} & \frac{a_2b_1}{a_1b_3c_2+a_2b_1c_3} \end{pmatrix} \tag{B.1}$$

Let's think about the evaluation matrix W from whom element of (1,3), (2,2), and (3,1) of the matrix inverse U^{-1} is replaced by zero. Calculate the maximum eigenvalue k and eigenvector x and z in ANP. From $Wz = kx$ and $Ux = kz$, then UW is as follows.

$$UW = \begin{pmatrix} 1 & \frac{a_1b_1c_3}{a_1b_3c_2+a_2b_1c_3} & \frac{a_1b_1c_2}{a_1b_3c_2+a_2b_1c_3} \\ \frac{a_2b_3c_2}{a_1b_3c_2+a_2b_1c_3} & 1 & \frac{a_2b_1c_3}{a_1b_3c_2+a_2b_1c_3} \\ \frac{a_2b_3c_3}{a_1b_3c_2+a_2b_1c_3} & \frac{a_1b_3c_3}{a_1b_3c_2+a_2b_1c_3} & 1 \end{pmatrix} \tag{B.2}$$

Equation (B.2) is rewritten as follows:

$$(1-k^2)z_1 + \frac{AB}{AC+B}z_2 + \frac{ABC}{AC+B}z_3 = 0 \tag{B.3}$$
$$\frac{C}{AC+B}z_1 + (1-k^2)z_2 + \frac{BC}{AC+B}z_3 = 0 \tag{B.4}$$
$$\frac{1}{AC+B}z_1 + \frac{A}{AC+B}z_2 + (1-k^2)z_3 = 0 \tag{B.5}$$

From Equation (B.3) and Equation (B.5),

$$z_1 = \frac{B\{(1-k^2) - \frac{AC}{AC+B}\}}{(1-k^2) - \frac{B}{AC+C}} z_3 \tag{B.6}$$

is obtained.

From Equation (B.3) and Equation (B.4),

$$z_2 = \frac{C\{(1-k^2) - \frac{B}{AC+B}\}}{(1-k^2) - \frac{AC}{AC+B}} z_3 \tag{B.7}$$

is obtained.

When Equation (B.6) and Equation (B.7) are rewritten by $1 - 1/(1-k^2) = \alpha - 1$ under $z_3 = 1$,

$$z_1 = \frac{B\{(B+AC)(1-k^2) - AC\}}{(B+AC)(1-k^2) - B} = \frac{B\{B-(1-\alpha)AC\}}{AC-(1-\alpha)B} = \frac{B\{(1-\alpha)AC - B\}}{(1-\alpha)B - AC} = y_1$$
$$z_2 = \frac{C\{(B+AC)(1-k^2) - B\}}{(B+AC)(1-k^2) - AC} = \frac{C\{AC-(1-\alpha)B\}}{B-(1-\alpha)AC} = \frac{C\{(1-\alpha)B - AC\}}{(1-\alpha)AC - B} = y_2$$

are obtained. Therefore, the eigenvector of Equation (B.2) becomes equivalent with the eigenvector of U_{AHP}.

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