# REBALANCE SCHEDULE OPTIMIZATION OF
# A LARGE SCALE PORTFOLIO UNDER TRANSACTION COST

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*Abstract*    This paper is concerned with an optimization problem associated with a rebalancing schedule of a large scale fund subject to nonconvex transaction cost. We will formulate this problem as a 0-1 mixed integer programming problem under linear constraints using absolute deviation as the measure of risk. This problem can be solved by an integer programming software if the size of the universe is small. However, it is still beyond the reach of the state-of-the-art technology to solve a large scale rebalancing problem. We will show that we can now solve these problems almost exactly within a practical amount of time by using an elaborate heuristic approach.

## 1.  Introduction

According to the standard single-period portfolio optimization framework [15], one constructs a portfolio on the efficient frontier. However, this portfolio may not be efficient after, say three months, since the efficient frontier will shift as the elapse of time. Therefore, we will have to check the portfolio periodically and rebalance the portfolio if the deviation of the current portfolio from the new efficient frontier is not acceptable. This rebalance will incur a certain amount of transaction costs [4, 5, 19]. However, we had to wait more than a decade before this problem was solved to optimality since transaction cost functions are usually nonconvex.

This problem has been studied by some researchers. For example, in Konno and Wijayanayake [9], they considered a rebalancing portfolio optimization problem with concave transaction cost functions and they proposed a branch and bound algorithm to solve this problem. In Konno and Yamamoto [11], they compared transaction costs between some rebalancing strategies in the Japanese stock market. They showed that the rebalancing strategy considered transaction costs exactly was the best strategy than the others which is usually used by practitioners. Also, in Guastaroba et al. [6], they considered a rebalancing portfolio optimization problem with fixed and proportional transaction costs. They formulated this problem as a mixed integer linear programming problem using the conditional value at risk as a measure of risk and compared between some rebalancing strategies in the Germany stock market.

If the amount of fund and the number of assets under consideration is relatively small, these approaches work well. If on the other hand the amount of fund is large, the market impact cost significantly affects the performance of the portfolio. This problem was first studied in a path-breaking paper by Perold [18]. He considered transaction cost functions

consisting of concave transaction fee functions and convex market impact cost functions. Market impact cost is associated with a large amount of transaction of assets with smaller circulation in the market. When one buys (sells) a large amount of assets with smaller circulation, the unit price will increase (decrease), thereby decreases the rate of return of the portfolio.

Recently, Konno and Wijayanayake [10] and Konno et al. [7] formulated this problem as a minimization of a piecewise linear nonconvex objective function under linear constraints, using absolute deviation of the rate of return as a measure of risk [13]. They successfully solved small to medium size problems using a branch and bound algorithm developed in the field of global optimization.

Pension fund managers who manage a very large portfolio such as more than 100 billion yen employ "gradual rebalancing" strategy, which they conduct rebalance over a period of a few days to alleviate the market impact costs. In fact, Chan and Lakonishok [3] showed that the typical institutional investors used the gradual rebalancing strategy because their trades were so large. They showed that the institutional investors traded only the amount of 20% of their trading plan within a day and that the amount of 53% or more was bought and sold over four days or more. However, there are few researches about practical methods of the gradual rebalancing strategy.

Recently, Kritzman et al. [14] proposed a multi-period stochastic programming approach for calculating a minimal transaction cost rebalance schedule to a target portfolio. However, there are no guarantees to rebalance to the optimal portfolio since they solve an optimal portfolio construction problem and an optimal rebalancing problem separately. Also, it is difficult to solve a large scale problem since they use a time consuming dynamic programming approach.

In this paper, we will consider a problem which decides an optimal rebalance schedule and an optimal target portfolio simultaneously with nonconvex transaction costs. We formulate this problem as a 0-1 mixed integer linear programming problem which can be solved by using state-of-the-art mathematical programming software if the size of the problem is small enough. To solve a large scale problem, we will employ a "problem reduction" strategy which worked remarkably well for portfolio optimization problems with nonconvex transaction costs [2, 20]. Also, we will report the results of computational experiments using real stock data in the Tokyo Stock Exchange, which demonstrate that our approach leads to a remarkably good result.

In the next section we will formulate the optimal rebalancing problem as a mixed 0-1 integer linear programming problem. Also we will develop a heuristic approach for large scale problems.

Section 3 will be devoted to the result of computational experiments using market data. It will be shown that a heuristic approach presented in Section 2 works very well for problems of practical size. Finally in Section 4, we will present concluding remarks.

## 2. Formulation

### 2.1. Rebalance schedule optimization problem

We consider a single period portfolio rebalancing problem that a fund manager refreshes his/her portfolio constructed before a few months. However, it is difficult to rebalance for a day because the market capitalization of this portfolio is very large, say 100 billion yen and the rebalance need the high market impact cost. Therefore we consider the following gradual rebalancing optimization problem.

Let us assume that there exists $N$ stocks $S_i (i = 1, 2, \ldots, N)$ in the market and let $R_i$ be the random variable representing the rate of return of $S_i$. We consider a problem of rebalancing the current portfolio $\boldsymbol{x_0} = (x_{10}, x_{20}, \ldots, x_{N0})$ to an optimal portfolio during the period of $T$ days. Let $\boldsymbol{x_t} = (x_{1t}, x_{2t}, \ldots, x_{Nt})$, $t = 1, 2, \ldots, T$ be a portfolio at day $t$.

We assume that $(R_1, R_2, \ldots, R_N)$ is distributed over a finite set of points $(r_{1s}, r_{2s}, \ldots, r_{Ns})$, $s = 1, 2, \ldots, S$ and that

$$\Pr\{(R_1, R_2, \ldots, R_N) = (r_{1s}, r_{2s}, \ldots, r_{Ns})\} = \frac{1}{S}, \ s = 1, 2, \ldots, S.$$

These data can be collected from the historical data where $(r_{1s}, r_{2s}, \ldots, r_{Ns})$ is the achieved rate of return during period $s$, or generated by a scenario model which $(r_{1s}, r_{2s}, \ldots, r_{Ns})$ is the rate of return under scenario $s$. Also we assume that the distribution does not change over the period of $T$ days.

Then the expected return $r_i$ of $R_i$ and the net expected rate of the return at day $T$ can be represented as follows:

$$r_i = \frac{1}{S} \sum_{s=1}^{S} r_{is}, \ i = 1, 2, \ldots, N, \tag{2.1}$$

$$\mathrm{E}[R(\boldsymbol{x_T})] = \sum_{i=1}^{N} r_i x_{iT} - \frac{1}{M} \sum_{t=1}^{T} \sum_{i=1}^{N} c(|x_{it} - x_{it-1}|M), \tag{2.2}$$

where $M$ is the total amount of investment and $c(\cdot)$ is a nonconvex transaction cost function depending on the amount of transaction $|x_{it} - x_{it-1}|M$, which will be introduced in detail in next subsection.

We do not consider the intermediate expected returns of the portfolios during the period of $T$ days but consider only an expected return of a finished portfolio at day $T$. We suppose that we rebalance the portfolio to take high expected rate of return during several days using the expected rate of returns of each asset which estimated at day 0, such as the end of months. Also, we assume that $M$ is constant during the period of $T$ days. Strictly, we have to use $\mathrm{E}[M_t]$, $t = 1, 2, \ldots, T$, which are variables, instead of $M$. However, this formulation can not be solved because the rank of nonconvexity is large. Moreover, we consider that this assumption do not affect the objective value because the rate of return of each asset at a day is relatively small.

The standard measure of risk associated with a portfolio is variance or standard deviation [15]. However, we employ the absolute deviation [13] which has a remarkable computational advantage when we try to solve problems under nonconvex transaction cost, minimal transaction unit constraint and cardinality constraint, etc. (See Konno and Yamamoto [12], Angelelli et al. [1] for details).

Also, some theoretical properties of the absolute deviation were proved. Ogryczak and Ruszczynski [17] demonstrated that the absolute deviation is an authentic measure of risk from the viewpoint of second order stochastic dominance.

Absolute deviation of a portfolio at day $T$ is defined as follows:

$$W[R(\boldsymbol{x_T})] = \mathrm{E}[|R(\boldsymbol{x_T}) - \mathrm{E}[R(\boldsymbol{x_T})]|] \tag{2.3}$$

$$= \frac{1}{S} \sum_{s=1}^{S} \left| \sum_{i=1}^{N} (r_{is} - r_i) x_{iT} \right|. \tag{2.4}$$

Let us define the investable set $X$ as follows:

$$X = \left\{ (\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_T}) \in R^N \;\middle|\; \begin{array}{l} \sum_{i=1}^{N} x_{it} = 1; \; \sum_{t=1}^{T}\sum_{i=1}^{N} |x_{it} - x_{it-1}| \leq \kappa; \\ 0 \leq x_{it} \leq u_i, \; i = 1, 2, \ldots, N, \; t = 1, 2, \ldots, T \end{array} \right\}, \quad (2.5)$$

where $u_i, i = 1, 2, \ldots, N$ are upper bounds of investment into $S_i$ and $\kappa$ is constant representing the acceptable level of turnover.

To obtain an optimal rebalance schedule, we have to solve the following mathematical programming problem:

$$\left|\begin{array}{ll} \text{maximize} & \sum_{i=1}^{N} r_i x_{iT} - \dfrac{1}{M} \sum_{t=1}^{T}\sum_{i=1}^{N} c(|x_{it} - x_{it-1}|M) \\ \text{subject to} & \dfrac{1}{S} \sum_{s=1}^{S} \left| \sum_{i=1}^{N} (r_{is} - r_i) x_{iT} \right| \leq w \\ & (\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_T}) \in X, \end{array}\right. \quad (2.6)$$

where $w$ is a constant representing the acceptable level of risk.

It is well known that the constraint of the problem (2.6) can be represented as a system of linear inequalities by introducing a set of nonnegative variables $v_s, s = 1, 2, \ldots, S$ as follows [16]:

$$\left|\begin{array}{ll} \text{maximize} & \sum_{i=1}^{N} r_i x_{iT} - \dfrac{1}{M} \sum_{t=1}^{T}\sum_{i=1}^{N} c(|x_{it} - x_{it-1}|M) \\ \text{subject to} & \dfrac{2}{S} \sum_{s=1}^{S} v_s \leq w \\ & v_s \geq \sum_{i=1}^{N} (r_{is} - r_i) x_{iT}, \; s = 1, 2, \ldots, S \\ & v_s \geq 0, \; s = 1, 2, \ldots, S \\ & (\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_T}) \in X. \end{array}\right. \quad (2.7)$$

## 2.2. Transaction cost function

We assume that the transaction cost function is a piecewise linear D.C. function as depicted in Figure 1(a). Let us decompose the total transaction cost function $c(\cdot)$ into a piecewise linear concave transaction fee function $c_1(\cdot)$ and a piecewise linear convex market impact cost function $c_2(\cdot)$ (Figure 1(b)).

It is well known that the market impact cost depends on circulation of each asset in the market. Then an asset which has smaller circulation in the market has the higher/steeper market impact cost function. We assume that the market impact cost depends on the characteristics of each asset and denote the market impact cost $c_{2i}(\cdot)$ for each $i$. Also, we will define the market impact cost $c_{2i}(\cdot)$ using the market capitalization of each asset instead of the circulation in the market in Section 3. This is a natural assumption because it is high correlation between the market capitalization and the market circulation.

Needless to say, market impact cost depends not only on the amount of his/her trade but also on the amount of other investors' trade in the market. However, it is difficult to handle this information in this model. For the sake of tractability, we use this formulation
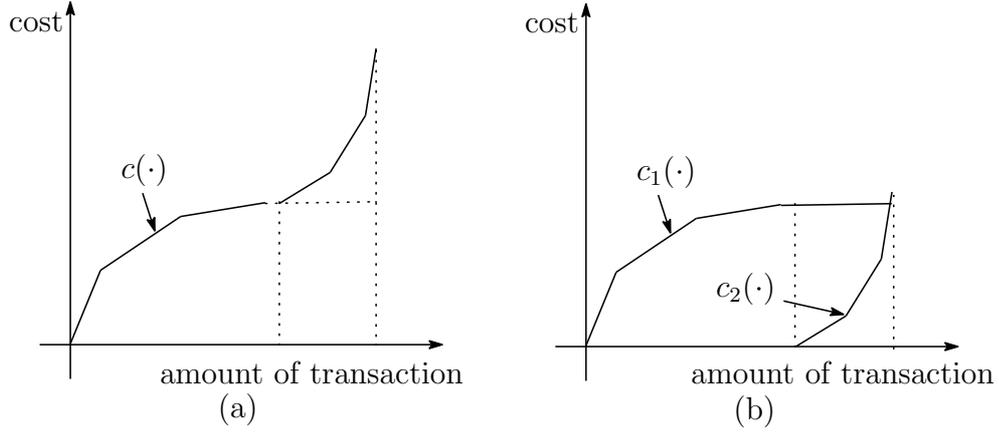
Figure 1: Transaction cost function

in this paper. Let us note that many other literatures about the market impact use this formulation.

Let us introduce a set of variables $\phi_{it}, \psi_{it}$ satisfying the condition:

$$x_{it} - x_{it-1} = \phi_{it} - \psi_{it}, \ \phi_{it}\psi_{it} = 0, \ \phi_{it} \geq 0, \ \psi_{it} \geq 0,$$
$$i = 1, 2, \ldots, N; \ t = 1, 2, \ldots, T. \tag{2.8}$$

Then we have

$$|x_{it} - x_{it-1}| = \phi_{it} + \psi_{it}, \ i = 1, 2, \ldots, N; \ t = 1, 2, \ldots, T.$$

Thus the problem (2.7) can be represented follows:

$$
\begin{aligned}
\text{maximize} \quad & \sum_{i=1}^{N} r_i x_{iT} - \frac{1}{M} \sum_{t=1}^{T} \sum_{i=1}^{N} \{c_1((\phi_{it} + \psi_{it})M) + c_{2i}((\phi_{it} + \psi_{it})M)\} \\
\text{subject to} \quad & \frac{2}{S} \sum_{s=1}^{S} v_s \leq w \\
& v_s \geq \sum_{i=1}^{N} (r_{is} - r_i) x_{iT}, \ s = 1, 2, \ldots, S \\
& v_s \geq 0, \ s = 1, 2, \ldots, S \\
& x_{it} - x_{it-1} = \phi_{it} - \psi_{it}, \ i = 1, 2, \ldots, N; \ t = 1, 2, \ldots, T \\
& \phi_{it} \geq 0, \ \psi_{it} \geq 0, \ i = 1, 2, \ldots, N; \ t = 1, 2, \ldots, T \\
& \phi_{it}\psi_{it} = 0, \ i = 1, 2, \ldots, N; \ t = 1, 2, \ldots, T \\
& (\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_T}) \in X.
\end{aligned}
\tag{2.9}
$$

**Proposition 2.1.** Complementarity conditions $\phi_{it}\psi_{it} = 0$ can be eliminated from the problem (2.9).

*Proof.* See Konno and Wijayanayake [10]. □

To solve the problem (2.9), we apply an integer programming approach used in Konno and Yamamoto [12].
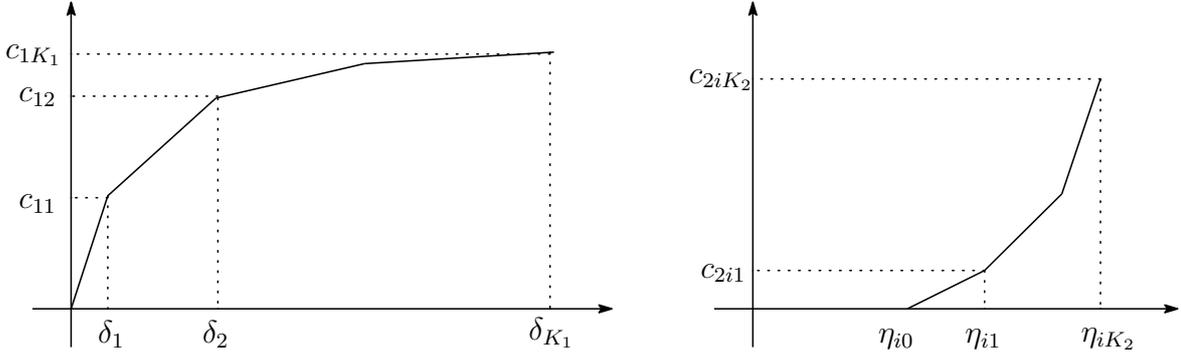
Figure 2: Piecewise linear commission fee and market impact functions

Let us introduce another set of auxiliary variables $\lambda_{itk}, k = 0, 1, \ldots, K_1$ and 0-1 variables $z_{itk}, k = 1, 2, \ldots, K_1$. Then the function $c_1(\cdot)$ can be represented as follows:

$$
\left|
\begin{aligned}
& c_1((\phi_{it} + \psi_{it})M) = \sum_{k=0}^{K_1} c_{1k}\lambda_{itk} \\
& (\phi_{it} + \psi_{it})M = \sum_{k=0}^{K_1} \delta_k \lambda_{itk} \\
& \sum_{k=0}^{K_1} \lambda_{itk} = 1 \\
& \sum_{k=1}^{K_1} z_{itk} = 1 \\
& \lambda_{it0} \leq z_{it1} \\
& \lambda_{itk} \leq z_{itk} + z_{itk+1}, \ k = 1, 2, \ldots, K_1 - 1 \\
& \lambda_{itK_1} \leq z_{itK_1}.
\end{aligned}
\right.
\tag{2.10}
$$

Similarly, the function $c_{2i}(\cdot)$ can be represented as linear equations without integer variables as follows:

$$
\left|
\begin{aligned}
& c_{2i}((\phi_{it} + \psi_{it})M) = \sum_{k=0}^{K_2} c_{2ik}\gamma_{itk} \\
& (\phi_{it} + \psi_{it})M = \sum_{k=0}^{K_2} \eta_{ik}\gamma_{itk} \\
& \sum_{k=0}^{K_2} \gamma_{itk} = 1.
\end{aligned}
\right.
\tag{2.11}
$$

This problem can be solved without difficulty if the problem is small, say when $N \leq 200$. When, however, $K_1 = 8$, $N = 1700$ and $T = 3$, then the number of 0-1 variables contained in (2.10) is $8 \times 1700 \times 3 = 40,800$, which is beyond the reach of the state-of-the art integer programming software. Thus we are forced to employ some sort of heuristic algorithm.

### 2.3. Heuristic algorithm

First we solve an approximate optimization problem of the problem (2.9) by using linear underestimator $\hat{c}_1(\cdot)$ of the commission fee function $c_1(\cdot)$ (Figure 3). Let $(x_{1t}^*, x_{2t}^*, \ldots, x_{Nt}^*), t =$

$1, 2, \ldots, T$ be an optimal solution, then we eliminate all assets which satisfy $\sum_{t=1}^{T} |x_{it}^* - x_{it-1}^*| = 0$, $i = 1, 2, \ldots, N$ from the original problem (2.9).

The reasons behind this heuristic are

**(i)** The weight of assets $i$ such that $\sum_{t=1}^{T} |x_{it}^* - x_{it-1}^*| = 0$ are not likely to have a large value in an optimal solution.

**(ii)** It is not profitable to purchase and/or sell smaller amount of $|x_{it} - x_{it-1}|$ since transaction fee per unit is relatively larger for smaller $|x_{it} - x_{it-1}|$.

Hence most $x_{it}$ which satisfy these conditions are expected to be $x_{i0}$ in an optimal solution [20]. Moreover, we add some assets which have high expected rate of return to improve the objective value of this heuristic within the practical amount of computational time.
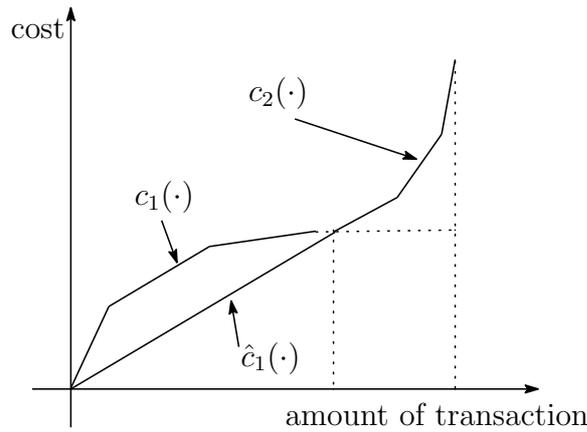


Figure 3: Underestimator of commission fee function

### Algorithm. Problem reduction

**Step 1.** Set Choose a positive integer $q$. $Q = \phi$.

**Step 2.** Solve the approximation problem of the problem (2.9). Let $(x_{1t}^*, x_{2t}^*, \ldots, x_{Nt}^*)$, $t = 1, 2, \ldots, T$ be an optimal solution and let $Q = \{i \mid \sum_{t=1}^{T} |x_{it}^* - x_{it-1}^*| > 0, \ i = 1, 2, \ldots, N\}$.

**Step 3.** If $|Q| > q$, then go to Step 4. Otherwise add $q - |Q|$ assets with the largest expected rate of return to $Q$.

**Step 4.** Solve the problem (2.9) under following conditions

$$\phi_{it} = 0, \ \psi_{it} = 0, \ t = 1, 2, \ldots, T; \ i \notin Q.$$

## 3. Computational Experiments

In this section, we will present the results of numerical simulations using the historical data of about 1700 stocks collected in the Tokyo Stock Exchange.

### 3.1. Problem setting

We prepare 12 data sets every March from 2000 to 2011 and calculate the current portfolio $\boldsymbol{x_0}$ by solving the mean-absolute deviation model 1 year ago. We choose $S = 36$, $u_i = 10\%$, $\kappa = 10\%$, $w$ is equal to a value of the risk of TOPIX Index and the amount of investment $M$ is 100 billion yen.

Also, we use the transaction fee function $c_1(\cdot)$ provided by a leading Japanese security company and the market impact cost function $c_{2i}(\cdot)$ depending on the amount of market capitalization of each asset (Table 1). Thus the transaction fee function is identical ($K_1 = 8$)

Table 1: Cost functions

| Commission Fee | | Market Impact | |
|---|---|---|---|
| transaction ($10^4$yen) | Cost($10^4$yen) | transaction (to size) | Cost($10^4$yen) |
| 0–50 | 1.47% | 0–0.33% | 1.50% |
| 50–70 | 1.1155%+0.16 | 0.33%–1.00% | 3.00% |
| 70–100 | 0.945%+0.30 | 1.00%–2.00% | 5.00% |
| 100–300 | 0.8925%+0.36 | 2.00%– | 10.00% |
| 300–500 | 0.84%+0.51 | | |
| 500–1000 | 0.714%+1.14 | | |
| 1000–3000 | 0.5775%+2.51 | | |
| 3000–5000 | 0.2625%+11.96 | | |
| 5000– | 0.105%+19.83 | | |

for all assets. Therefore the market impact cost of a small scale asset is larger than one of a large scale asset.

We choose $T$ from 1 to 5 because Chan and Lakonishok [3] showed that an institutional investor usually trade about 70% of his/her trading plan within 6 days. We assume that $M$ is constant and that the distribution of the rate of return does not change during $T$ days. This assumption is not valid if $T$ is longer than 1 week.

### 3.2. Computation time

Table 2 shows the statistics of the computation time of the data sets that can be solved within 6000 seconds using CPLEX12.2 (personal computer Intel Xeon(32GB, 3.33GHz, 64bit)). In Table 2, "count" means the number of data sets that was solved within 6000 seconds. The column corresponding to "ALL" represents the result for the problem (2.9) without using the heuristic procedure.

Table 2: Computation time (sec)

| $T$ | $q$ | 0 | 50 | 100 | 200 | ALL |
|---|---|---|---|---|---|---|
| 1 | mean | 0.16 | 0.16 | 0.19 | 0.32 | 4.73 |
| | st. dev | 0.17 | 0.14 | 0.09 | 0.17 | 5.68 |
| | count | 12 | 12 | 12 | 12 | 12 |
| 2 | mean | 7.42 | 4.96 | 1.63 | 17.95 | 37.20 |
| | st. dev | 24.69 | 15.87 | 2.79 | 56.55 | 72.82 |
| | count | 12 | 12 | 12 | 12 | 11 |
| 3 | mean | 9.31 | 10.80 | 33.57 | 35.23 | 335.06 |
| | st. dev | 29.13 | 26.08 | 94.30 | 62.81 | 615.32 |
| | count | 12 | 12 | 12 | 12 | 9 |
| 4 | mean | 21.07 | 5.54 | 153.54 | 139.58 | 19.97 |
| | st. dev | 50.67 | 8.18 | 268.90 | 372.70 | 14.98 |
| | count | 11 | 11 | 10 | 8 | 6 |
| 5 | mean | 5.29 | 21.74 | 179.83 | 342.09 | 280.62 |
| | st. dev | 8.42 | 35.58 | 358.84 | 768.80 | 574.33 |
| | count | 10 | 10 | 9 | 7 | 6 |

We see from this table that we could solve all problems within 10 seconds without using

the heuristic when $T = 1$. However, the number of solvable case decreases when $T$ is larger. This is due to the fact that the number of integer variables increases as we increase $T$. Almost all cases could be solved within a practical amount of computation time using the heuristic algorithm.

### 3.3. Effect of heuristic

Table 3 shows the average of the relative error between an exact optimal solution (without heuristic) and an approximate solution with the heuristic.

$$r.e. = f^q/f^{ALL},$$

where $f^{ALL}$, $f^q$ are the objective values without the heuristic and with the heuristic, respectively.

Table 3: Relative error of the heuristic

| $T/q$ | 0 | 50 | 100 | 200 |
|---|---|---|---|---|
| 2 | 92% | 97% | 99% | 99% |
| 3 | 93% | 98% | 99% | 100% |
| 4 | 94% | 98% | 99% | 100% |
| 5 | 95% | 98% | 100% | 100% |

We see from this table that the objective value is almost the same as the exact objective value when $q \geq 50$.

### 3.4. Effect of gradual rebalance

Next, we will show the effect of gradual rebalance. Table 4 shows the details of the objective values. These values are the averages of the optimal objective values of 12 test problems when $q = 50$.

Table 4: Average of the objective values (%)

| $T$ | Objective Value | Expected Return | Commission Fee | Impact Cost |
|---|---|---|---|---|
| 1 | 2.675 | 3.099 | 0.013 | 0.411 |
| 2 | 2.822 | 3.130 | 0.017 | 0.291 |
| 3 | 3.005 | 3.260 | 0.019 | 0.236 |
| 4 | 3.052 | 3.265 | 0.021 | 0.192 |
| 5 | 3.326 | 3.504 | 0.024 | 0.154 |

We see from this table that the objective values increase as $T$ increases. Also, the commission fee increases when $T$ is larger because the number of assets subject to small amount of transaction increases. However, the increase is relatively small. Gradual rebalance can decrease the market impact cost and significantly increase the expected rate of return of the portfolio. This result shows that we can construct better portfolio using gradual rebalance, as expected.

Figure 4 shows the expected rate of return of the portfolio using gradual rebalance on March, 2006. We see from this figure that gradual rebalance results in a portfolio with higher expected return. This is due to the fact that small capitalization stocks with larger market impact cost can be incorporated in a portfolio when we use gradual rebalance.

Figure 5 shows the total cost which is the sum of the commission fee and the market impact cost. We see from this figure that the rebalance associated with $T = 1$ is subject to a
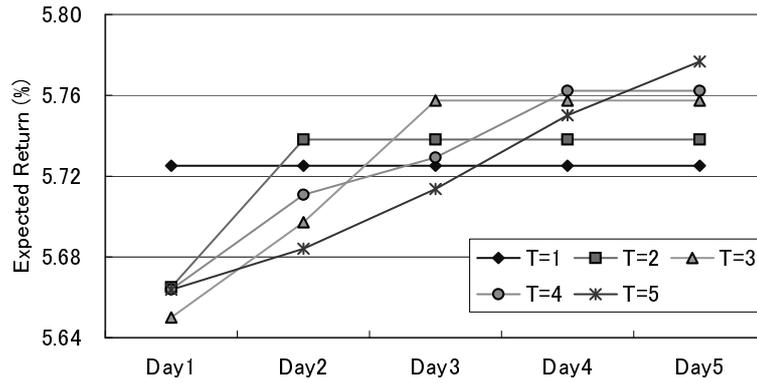
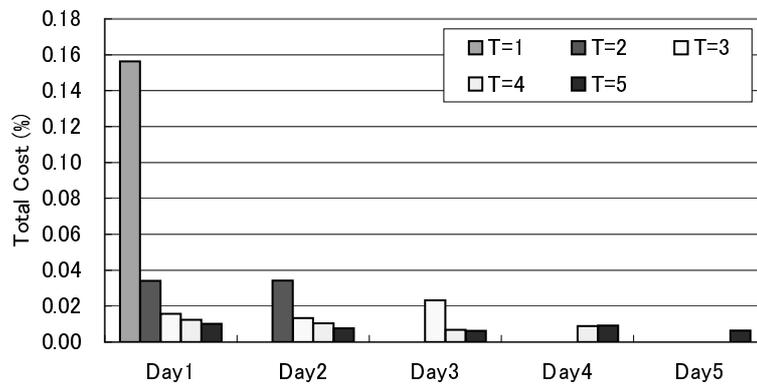Figure 4: Expected return using gradual rebalance



Figure 5: Total cost using gradual rebalance

large transaction cost. Total cost of the gradual rebalances decreases to one half. Therefore, gradual rebalance is an effective method for decreasing transaction cost.

Finally, we compare the efficient frontier in the case of $T = 1$ and $T = 5$. Figure 6 shows that the portfolio using gradual rebalance approaches to the efficient frontier without transaction cost. The portfolio using gradual rebalance is located closer to the efficient frontier than the one without gradual rebalance.
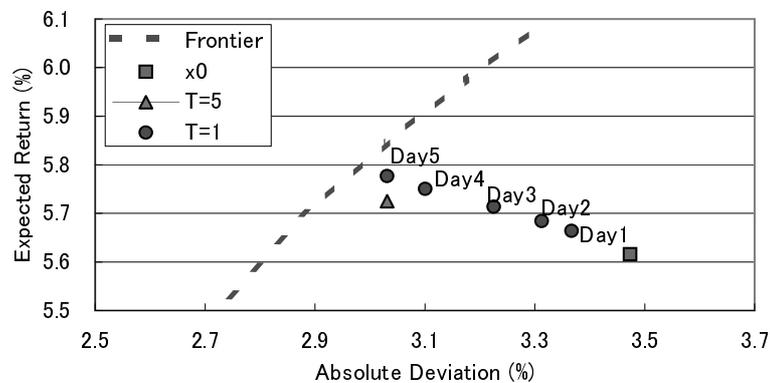


Figure 6: Efficient frontier

## 4.   Conclusions

In this paper we formulated the rebalance schedule optimization problem under market impact cost and showed the effectiveness of this approach using real market data. Let us note that the use of absolute deviation played a crucial role for solving a class of difficult problems within a practical amount of computation time. If, instead we employ standard deviation, problem could not be solved even if we use a heuristic approach used here. Also, let us note that absolute deviation is an authentic measure of risk as probed by Ogryczak and Ruszczynski [17] (See Konno and Koshizuka [8]). Also, let us emphasize that a heuristic "problem reduction" could significantly decrease the amount of computation time.

Further, we showed that the gradual rebalance portfolio constructed by this model can achieve larger expected rate of return with smaller transaction cost.

Transaction cost is very important problem for pension fund managers handling a large amount of fund. We showed that our approach can decrease the transaction cost effectively. We believe that this result would be of interest to researchers and practitioners in the field of asset management.

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