Journal of the Operations Research
Society of Japan
2005, Vol. 48, No. 4, 308-317

ESTIMATION OF FIRMS EFFICIENCIES USING A KALMAN FILTER AND STOCHASTIC EFFICIENCY MODEL

Tohru Ueda Kenichi Hoshino
Seikei University

(Received March 1, 2005; Revised July 28, 2005)

Abstract The efficiencies of firms should be evaluated, based on their historical financial data. Here a process for evaluation is proposed. Each historical data set is reduced to a distribution function with a mean and a variance estimated by Kalman filter. Then, a stochastic efficiency model is applied to this reduced data. Then a new efficiency measure is proposed and compared with existing measures. An application is made to three kinds of firms.

Keywords: DEA, forecasting, Kalman filter

1. Introduction

The efficiency of firms should be evaluated from a variety of viewpoints, and thus Data Envelopment Analysis (DEA) is used as a method of evaluating firms with many kinds of inputs and outputs. Evaluation based on comparison among firms at a single time point reflects spatial variance, but firms have their history and have longitudinal data. Window analysis is well-known as an analytic method for longitudinal data. However, this method does not seem to be appropriate for analysis of future performance, but seems to be appropriate for analysis of past performance.

As usual DEA uses one value by each kind of inputs and outputs respectively, we must decide which value should be selected. In a stochastic efficiency model shown in [7] a variance of each kind of inputs and outputs should be estimated.

The Kalman filter gives the minimum mean squared error of forecasts, that is, this is appropriate for analysis of future performance and gives not only an estimate of future value, but also an estimate of its variance. We can use these values as parameters of a stochastic efficiency model. We propose a new measure of the efficiency score called the modified efficiency score and compare it with existing measures.

2. The Kalman Filter

The Kalman filter is presented by system equations and an observation equation as shown in [5]. In this paper system equations at time t are given by

$$\mathbf{x}(t+1) = F\mathbf{x}(t) + GU(t), \tag{1}$$

and an observation equation is given by

$$y(t) = H(t)^t\mathbf{x}(t) + w(t), \tag{2}$$

where the superscript, t, is not time t, but means **t**ransposition, and

$$\mathbf{x}(t)^t = (T(t), T(t-1), L(t)), \tag{3}$$

$T(t)$: an element presenting trend at time t,

$$\nabla^k T(t) = u(t) \sim N(0, \tau^2); \nabla T(t) = T(t) - T(t-1), \tag{4}$$

($k = 2$ is used in this paper.)

$L(t)$: an element presenting effect of an accidental event at time t,

$$F = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

$$G^t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ U(t)^t = (u(t), 0), \tag{6}$$

$$H(t)^t = (1, 0, d(t, N));\ w(t) \sim N(0, \omega^2), \tag{7}$$

$$d(t, N) = \begin{cases} 1 & : \ t = N, \\ 0 & : \ t \neq N. \end{cases} \tag{8}$$

The expectation and variance of forecasts are given by

$$y(t|t-1) = H(t)^t \mathbf{x}(t|t-1) \tag{9}$$

$$R(t|t-1) = H(t)^t P(t|t-1) H(t) + \omega^2 \tag{10}$$

where $A(t|t-1)$ is a forecast of $A(t)$ at time $(t-1)$, for example, $y(t|t-1)$ and $R(t|t-1)$ are the expectation and variance of forecasts of $y(t)$ at time $(t-1)$ and these values are renewed as follows and used in a stochastic efficiency model:

(Renewal process on time)

$$\begin{aligned} \mathbf{x}(t|t-1) &= F\mathbf{x}(t-1|t-1) \\ P(t|t-1) &= FP(t-1|t-1)F^t + GQG^t \end{aligned} \tag{11}$$

where Q is a variance-covariance matrix of U.

(Modification based on observed values)

$$\begin{aligned} \mathbf{x}(t|t) &= \mathbf{x}(t|t-1) + K(t)\{y(t) - y(t|t-1)\} \\ P(t|t) &= P(t|t-1) - K(t)H(t)^t P(t|t-1) \\ K(t) &= P(t|t-1)H(t)/\{H(t)^t P(t|t-1)H(t) + \omega^2\} \end{aligned} \tag{12}$$

Here we suppose that an accidental event occurred at time N, but some accidental event may occur.

We insist that such Kalman filter as shown in (1)-(12) should be used for evaluation of future performance.

3. Stochastic Efficiency Model

In usual DEA each Decision Making Unit (DMU) O $(= 1, 2, \ldots, n)$ has deterministic inputs $\mathbf{X}_o = (x_{1o}, x_{2o}, \ldots, x_{mo})$ and deterministic outputs $\mathbf{Y}_o = (y_{1o}, y_{2o}, \ldots, y_{so})$ and its efficiency in Charnes, Cooper and Rhodes [4] (CCR) model is measured by

$$\max \sum_r u_r y_{ro} / \sum_i v_i x_{io}$$

where

$$\sum_r u_r y_{rj} / \sum_i v_i x_{ij} \le 1 \quad (j = 1, 2, ..., n)$$

However, in this paper the case where inputs and outputs are not deterministic is discussed. Assume that for each DMU O (=1,2,. . . ,n) its inputs have multivariate normal distributions with mean, $\mathbf{X}_o = (x_{1o}, x_{2o}, \ldots, x_{mo})^t$, and variations, $\Delta\mathbf{X}_o = (\Delta x_{1o}, \Delta x_{2o}, \ldots, \Delta x_{mo})^t$ and its outputs have multivariate normal distributions with mean, $\mathbf{Y}_o = (y_{1o}, y_{2o}, \ldots, y_{so})^t$, and variations, $\Delta\mathbf{Y}_o = (\Delta y_{1o}, \Delta y_{2o}, \ldots, \Delta y_{so})^t$, where their variance-covariance matrix is given by Σ_o and the confidence region of stochastic variations, $\delta \equiv (\Delta\mathbf{X}_o, \Delta\mathbf{Y}_o)$, at probability level α is given by

$$S_\alpha = \{\delta \mid \delta\Sigma_o^{-1}\delta^t \le \chi^2_{m+s}(\alpha)\} \tag{13}$$

and let the worst inputs and outputs at probability level αge

$$(\mathbf{X}_{o\alpha}, \mathbf{Y}_{o\alpha}) = (\mathbf{X}_o + \Delta\mathbf{X}_o, \mathbf{Y}_o - \Delta\mathbf{Y}_o \mid \delta\Sigma_o^{-1}\delta^t \le \chi^2_{m+s}(\alpha)) \tag{14}$$

where $\chi^2_{m+s}(\alpha)$ is an α percentile of the χ^2 distribution . Let the minimum efficiency score obtained within S_α be W_α.

When a number of DMUs, n, is small, compared with a sum, $(m + s)$, of numbers of inputs and outputs, relatively many DMUs may be evaluated as efficient in the usual DEA. On the lines of [7], we calculate the maximum α_{max} which gives $W_\alpha = 1$ within S_α. The more α_{max} is, the more robust the efficient state is. For efficient DMU in CCR model this problem can be formulated as follows:

[**Problem 1**]

$$\begin{array}{ll} \max & \alpha \\ \text{subject to} & W_\alpha = \mathbf{u}^t\mathbf{Y}_{o\alpha} = 1, \\ & \mathbf{v}^t\mathbf{X}_{o\alpha} = 1, \\ & \mathbf{v}^t\mathbf{X}_j \ge \mathbf{u}^t\mathbf{Y}_j;\ j \ne o, \\ & \delta\Sigma_o^{-1}\delta^t \le \chi^2_{m+s}(\alpha), \end{array}$$

where $\mathbf{X}_j$ and $\mathbf{Y}_j$ are inputs and outputs of DMU j.

4. New Efficiency Measure

Solutions of Problem 1 give the efficiency score, $W_\alpha = 1$. Therefore in order to distinguish many DMU with the efficiency score, 1, we propose a new efficiency measure, $\tilde{W}_o$, using multipliers $\mathbf{v}^*$ and $\mathbf{u}^*$ obtained by solving Problem 1:

$$\tilde{W}_o = (\mathbf{u}^{*t}\mathbf{Y}_o)/(\mathbf{v}^{*t}\mathbf{X}_o).$$

If $W_\alpha = 1$, $\mathbf{v}^{*t}\mathbf{X}_{o\alpha} = \mathbf{v}^{*t}(\mathbf{X}_o + \Delta\mathbf{X}_o) = 1$ and $\mathbf{u}^{*t}\mathbf{Y}_{o\alpha} = \mathbf{u}^{*t}(\mathbf{Y}_o - \Delta\mathbf{Y}_o) = 1$. Then $\mathbf{v}^{*t}\mathbf{X}_o \le 1$ and $\mathbf{u}^{*t}\mathbf{Y}_o \ge 1$. Therefore, this measure, $\tilde{W}_o$, gives a larger efficiency score than 1 for semi-positive δ (all are not zero), where for inefficient DMU the same efficiency scores as CCR model are given.

5. Application

The following three data groups were studied.

(1) 31 electric device manufacturers
(2) 19 department stores

(3) 38 supermarkets

We use two kinds of indices as the management indices. The first indices were used in [8] as an altered discriminant function of Z value which was proposed by E. I. Altman [1]. The alternative indices were selected for altering of the first indices.

5.1. First indices

The following indices are used for evaluation.

y_1: (finance expense)/(sales)
y_2: (current income)/(current expense)
y_3: (cash flow)/(number of Employees and directors)
y_4: $(a_4^{(t)}/a_4^{(t-1)} - 1)$
  $a_4^{(t)}$: (gross margin of year t)/(mean number of Employees and directors during year t)
y_5: $(a_5^{(t)}/a_5^{(t-1)} - 1)$
  $a_5^{(t)}$: (fixed assets of year t)/(mean number of Employees and directors during year t)
y_6: (surplus)/(net capital)
y_7: {(liquid assets increase)−(non-liquid assets increase)}
  /(liquid assets on the beginning of the fiscal year)

In DEA these indices are treated as outputs, where all inputs are assumed to be equal to one and y_j (j=1,7) are replaced by ([maximum of them over all DMUs]– [y_j of the targeted DMU]).

We forecasted each index value of 2001, using indices values from 1986 to 2000 based on the Kalman filter represented by (1)-(12).

Coefficients of variation, that is, $CV = \sqrt{R(2001|2000)}/y(2001|2000)$ are shown in Table 1 and 2, where $R(n|n-1)$ and $y(n|n-1)$ are given by (9) and (10). Figure 1 shows variation of index y_1 at Mabuchi-Motor, where $R(2001|2000)$=3.06, $y(2001|2000)$=2.39 and CV=0.732 for y_1. As shown in these tables and figure, many forecasts have large variations, because corresponding observed values have large variations. However, we must be ready for such large variation and we use $R(2001|2000)$ and $y(2001|2000)$ for estimation of α_{max}.

Evaluation results for CCR efficient DMUs of each data groups are shown in Table 3, 4 and 5, where super-efficiency scores proposed by [2] are also shown in these tables for the purpose of comparison.

We think that α_{max} is not appropriate as a measure for efficiency comparison, because several DMUs have a value zero. Super-efficiency scores can take such very large values as Toshibatec in Table 3. However, our measure, $\tilde{W}_o$, take more moderate values than super-efficiency scores. This may be an advantage of our measure. However, though Sogo and Mycal went down, they were evaluated as efficient DMUs. Therefore the alternative indices are used in the next session.

Table 1: Forecasted coefficient of variation of each index for data group (1)

	y_1	y_2	y_3	y_4	y_5	y_6	y_7
msc.panasonic	0.467	0.049	0.014	0.432	0.875	0.021	1.691
Toshibatec	0.108	0.049	0.012	0.292	0.080	0.047	0.218
Mabuchi-Motor	0.732	0.046	0.005	0.777	0.223	0.021	0.257
Matsushita	0.222	0.048	0.015	0.181	0.312	0.016	0.142
Sharp	0.216	0.046	0.008	0.573	0.502	0.030	0.853
Sony	0.334	0.051	0.012	0.308	0.275	0.028	8.477
Aiwa	1.665	0.056	0.010	0.188	0.664	0.046	0.246
Sanyo	0.192	0.047	0.012	0.178	1.208	0.029	0.251
kme.panasonic	0.192	0.048	0.015	5.951	0.396	0.026	1.225
Pioneer	0.466	0.048	0.022	0.253	0.264	0.107	1.455
Columbia	0.462	0.058	0.039	0.665	0.907	0.066	0.150
JvcVictor	0.438	0.049	0.017	0.828	0.568	0.025	0.200
Foster	0.363	0.050	0.012	0.173	0.319	0.035	0.876
Clarion	0.120	0.053	0.008	0.352	0.471	0.049	0.243
Marantz	0.094	0.051	0.012	0.344	0.271	0.027	0.150
Yokowo	0.136	0.050	0.022	19.03	0.988	0.035	0.310
Zojirushi	0.248	0.055	0.019	0.222	1.951	0.021	0.285
Teak	0.153	0.055	0.008	0.220	0.323	0.041	0.391
mke.panasonic	0.218	0.048	0.007	0.456	5.999	0.032	0.177
TOA	0.104	0.058	0.017	0.159	0.185	0.036	0.990
Nakamichi	0.542	0.053	0.009	0.147	0.636	0.085	0.511
mrc.panasonic	0.179	0.047	0.037	0.106	3.194	0.022	0.163
Rion	0.131	0.056	0.016	35.16	0.132	0.024	0.153
Enplas	0.200	0.045	0.007	0.231	0.256	0.029	0.26
Hitachi	2.243	0.047	0.017	0.245	0.513	0.023	0.295
Toshiba	1.944	0.046	0.011	1.410	0.326	0.030	0.142
Melco	1.491	0.045	0.068	7.726	0.467	0.031	0.147
Fujielectric	0.614	0.054	0.012	0.086	0.195	0.047	1.285
NEC	0.883	0.051	0.011	0.123	0.724	0.038	0.492
Fujitsu*	-	0.050	0.059	0.250	0.235	0.031	1.422
Oki	0.799	0.054	0.010	0.078	0.517	0.087	0.203
mean of CV	0.532	0.05	0.018	2.489	0.773	0.038	0.763

*Because $E(y_1) = 0$, $CV(y_1)$ cannot be calculated.

Table 2: Mean forecasted coefficient of variation of each index by each group

	y_1	y_2	y_3	y_4	y_5	y_6	y_7
group (1)	0.532	0.050	0.018	2.489	0.773	0.038	0.763
group (2)	0.560	0.049	0.039	1.430	2.357	0.051	0.967
group (3)	0.515	0.056	0.054	3.273	1.466	0.066	0.889

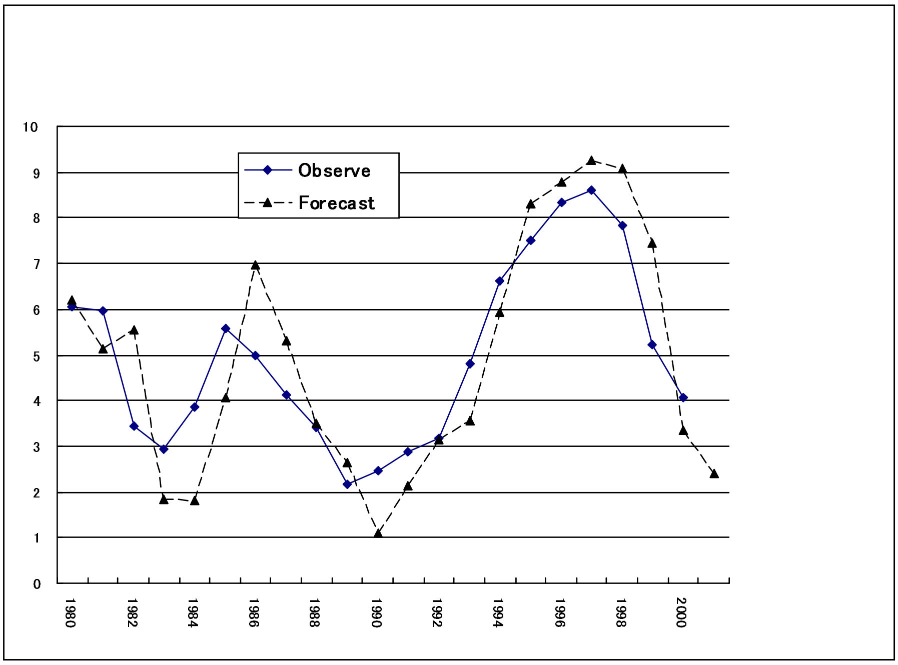

Figure 1: Variation of y_1 at Mabuchi-Motor

Table 3: Comparison among efficiency measures for data group (1)

Rank	α_{max}		Super-efficiency		Proposed measure	
1	Toshibatec	84.82%	Toshibatec	3.846	Mabuchi-motor	1.860
2	Mabuchi-motor	51.55%	Mabuchi-motor	2.546	Nakamichi	1.813
3	Matsushita	9.54%	Nakamichi	2.116	Toshibatec	1.758
4	Nakamichi	3.83%	Matsushita	1.550	Matsushita	1.694
5	Toshiba	1.30%	Toshiba	1.454	Toshiba	1.586
6	Marantz	0.00%	Marantz	1.074	Marantz	1.076
7	Fujitsu	0.00%	Melco	1.050	Melco	1.056
8	Hitachi	0.00%	Fujitsu	1.032	Fujitsu	1.033
9	Melco	0.00%	Hitachi	1.024	Hitachi	1.025
10	Msc.Panasonic	0.00%	Msc.panasonic	1.021	Msc.panasonic	1.020
11	Enplas	0.00%	Enplas	1.008	Enplas	1.008

Table 4: Comparison among efficiency measures for data group (2)

Rank	α_{max}		Super-efficiency		Proposed measure	
1	Izutsuya	0.80%	Izutsuya	1.906	Izutsuya	1.535
2	Hankyu-Dept	0.39%	Hankyu-Dept	1.395	Hankyu-Dept	1.452
3	Matsuzakaya	0.12%	Isetan	1.331	Matsuya	1.336
4	Matsuya	0.03%	Matsuya	1.318	Isetan	1.306
5	Sogo	0.01%	Daimaru	1.233	Nagano-Tokyu	1.292
6	Isetan	0.01%	Nagano-Tokyu	1.212	Daimaru	1.267
7	Nagano-Tokyu	0.00%	Matsuzakaya	1.198	Matsuzakaya	1.236
8	Daimaru	0.00%	Platz-Kintetsu	1.154	Sogo	1.153
9	Platz-Kintetsu	0.00%	Sogo	1.141	Iwataya	1.147
10	Iwataya	0.00%	Iwataya	1.099	Platz-Kintetsu	1.145
11	Tokyu-Depart	0.00%	Tokyu-Depart	1.078	Tokyu-Depart	1.078
12	Daiwa-Dp	0.00%	Daiwa-Dp	1.044	Takashimaya	1.052
13	Takashimaya	0.00%	Takashimaya	1.034	Daiwa-Dp	1.043
14	Saikaya	0.00%	Saikaya	1.029	Saikaya	1.034

Table 5: Comparison among efficiency measures for data group (3)

Rank	α_{max}		Super-efficiency		Proposed measure	
1	Itoyokado	11.17%	Itoyokado	2.070	Yamazawa	1.873
2	Yorkbeni	0.67%	Yamazawa	1.822	Jujiya	1.837
3	Jujiya	0.19%	Jujiya	1.817	Itoyokado	1.547
4	Seiyu	0.01%	Yorkbeni	1.509	Yorkbeni	1.529
5	Marukyo	0.00%	Seiyu	1.242	Seiyu	1.228
6	Yamazawa	0.00%	Marukyo	1.222	Marukyo	1.205
7	SuperDaiei	0.00%	SuperDaiei	1.191	SuperDaiei	1.181
8	Taiyo	0.00%	Taiyo	1.181	U-Store	1.179
9	U-Store	0.00%	U-Store	1.152	Taiyo	1.165
10	Mycal	0.00%	Mycal	1.123	Mycal	1.127

5.2. The alternative indices

The following indices are used for evaluation on the line of [6].

z_1=ROA: (return)/(assets)
z_2: (operating profits)/(sales)
z_3: (sales)/(assets)
z_4: (current assets)/(current liability)
z_5=(liquidity on hand) : (cash and deposits on hand)/(sales)
z_6: (sales)/(fixed assets)

Evaluation results are shown in Tables 6-11.

Table 6: The second Comparison among efficiency measures for efficient DMUs in data group (1)

	α_{max}	Super-efficiency	Proposed measure
Mabuchi-Motor	96.82%	2.43	1.59
Teak	66.73%	1.75	1.48
msc.panasonic	0.47%	1.25	1.24
Sony	1.10%	1.24	1.23
mke.panasonic	0.00%	1.06	1.06

Table 7: The second comparison among efficiency measures for efficient DMUs in data group (2)

	α_{max}	Super-efficiency	Proposed measure
Hankyu-Dept	51.85%	1.923	1.932
Hanshin	33.05%	1.587	1.445
Isetan	2.94%	1.359	1.33
Meitetsu-Dept	0.45%	1.241	1.258
Maruei	0.01%	1.156	1.157
Daiwa	0.01%	1.114	1.128
Nagano-Tokyu	0.01%	1.132	1.121
Iwataya	0.00%	1.108	1.106
Matsuya	0.00%	1.064	1.078

Table 8: The second comparison among efficiency measures for efficient DMUs in data group (3)

	α_{max}	Super-efficiency	Proposed measure
Itoyokado	7.41%	1.482	1.396
Taiyo	0.00%	1.133	1.131
Aoki Super	0.00%	1.104	1.094
Yorkbenimaru	0.00%	1.079	1.073
MatsumotoKiyoshi	0.00%	1.06	1.06
Harashin	0.00%	1.008	1.008

In these tables Sogo and Mycal which went down are evaluated very lowly. This means that selection of indices in this section is better than Sec. 5.1, but we cannot recommend this selection easily, because [9] in year 2003 uses different indices from [8] in year 1993 as Z-values (See [1]) which predict corporate bankruptcy.

Table 9: Efficiency scoress for inefficient DMUs in data group (1)

Marantz	0.98	Yokowo	0.60
kme.panasonic	0.93	Clarion	0.56
Foster	0.87	Matsushita	0.55
Aiwa	0.85	Sharp	0.53
JvcVictor	0.78	Melco	0.53
Enplas	0.77	NEC	0.52
Toshibatec	0.77	Sanyo	0.52
TOA	0.77	Oki	0.51
mrc.panasonic	0.74	Toshiba	0.50
Rion	0.66	Fujielectric	0.50
Columbia	0.65	Hitachi	0.48
Zojirushi	0.62	Fujitsu	0.47
Pioneer	0.62	Nakamichi	0.18

Table 10: Efficiency scoress for inefficient DMUs in data group (2)

Tokyu-Dept	0.985	Platz-Kintetsu	0.744
Daimaru	0.899	Saikaya	0.687
Matsuzakaya	0.820	Sanyo-Dept	0.677
Takashimaya	0.768	Izutsuya	0.554
Mitsukoshi	0.768	Sogo	0.257

Table 11: Efficiency scoress for inefficient DMUs in data group (3)

Wellmart	0.988	Fuji	0.566
Marukyo	0.922	Daiei	0.565
HacKimisawa	0.895	DOMY	0.551
Maruya	0.856	TobuStore	0.547
Inageya	0.809	Seiyu	0.533
heiwado	0.788	Uny	0.524
Yamazawa	0.767	Okuwa	0.503
Lifecorp	0.743	Jusco	0.501
SuperDaiei	0.700	Marukyu	0.477
U-Store	0.689	Maruetsu	0.472
Maruwa	0.670	Izumi	0.462
Jujiya	0.668	Flex	0.442
Tokyu Store	0.658	Kotobukiya	0.435
Yamanaka	0.624	Izumiya	0.428
Valor	0.590	Mycal	0.424
Kansai Super	0.588	Tenmaya-Store	0.382

6. Discussion

Each method used in this paper is well known, but their combinational usage is original.

We propose use of the Kalman filter for estimation of variances which is needed in the stochastic efficiency model but is difficult on a single time point evaluation. When there are longitudinal data, estimation of variance is naturally included in the Kalman filter, for example, in Equation (10). Of course we can apply formulations different from Equation (1) to Equation (7).

In a case of single input, efficient scores in DEA are given by a linear function of outputs/input and the function can be used as a discriminatory function if a cut-off point is determined. However, DEA is a method which stresses merits of each DMU and takes weakness of them lightly. Thus, there may be no problem of evaluation of DMUs with good performance, but for DMUs with worse performance we may be unable to predict their insolvency. In fact Sogo and Mycal went down, but they were evaluated as efficient DMUs in Sec.5.1. We also have a problem which indices should be selected because Sec.5.2. obtained a different efficiency score from Sec.5.1. for the same firm.

We think various methods should be used for evaluation of firms, according to purposes of analysis.

References

[1] E.I. Altman: Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *The Journal of Finance*, **23** (1968), 589-609.

[2] P. Andersen and N.C. Petersen: A procedure for ranking efficient units in Data Envelopment Analysis. *Management Science*, **39** (1993), 1261-1264.

[3] P.J. Brockwell and R.A. Davis: *Introduction to Time Series and Forecasting, Second ed.* (Springer-Verlag, 2002), Chap.8.

[4] A. Charnes, W.W. Cooper, and E. Rhodes: Measuring efficiency of decision making units. *European Journal of Operational Research*, **2** (1978), 429-444.

[5] G. Kitagawa and W. Gersch: A smoothness priors-state space modeling of time series with trend and seasonality. *Journal of the American Statistical Association*, **79** (1984), 378-389.

[6] A. Koyama: *Finance and Decision Making* (Asakura, 2001) (in Japanese).

[7] H. Morita and L.M. Seiford: Characteristics on stochastic DEA efficiency -reliability and probability being efficient-. *Journal of the Operations Research Society of Japan*, **42** (1999), 389-404.

[8] Toyo Keizai Inc.: *The Weekly Toyo keizai*, **5175** (23, Aug., 1993) (in Japanese).

[9] Toyo Keizai Inc.: *The Weekly Toyo keizai*, (27, Sept., 2003) (in Japanese).

Tohru Ueda
Department of Computer and Information Science
Faculty of Science and Technology
Seikei University
3-3-1, Kichijoji-Kitamachi, Musasino-shi
Tokyo, 180-8633 Japan
E-mail: `ueda@st.seikei.ac.jp`