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RETAILER'S INVENTORY POLICY UNDER SUPPLIER'S PARTIAL TRADE CREDIT POLICY

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Abstract This paper discusses the economic order quantity (EOQ) under partial trade credit. In 1985, Goyal assumed that:

(i) The unit selling price and the unit purchasing price were equal.

(ii) The supplier would offer the retailer full trade credit under condition of delay payments.

The main purpose of this paper wants to modify Goyal's model to presume that the unit selling price and the unit purchasing price are not necessarily equal. Furthermore, in this paper, we assume that the supplier would offer the retailer partial trade credit not full trade credit. That is, the retailer must make a partial payment to the supplier when the order is received. Then, the retailer must pay off the remaining balance at the end of the permissible delay period. Under these conditions, we model the retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory cycle time and optimal order quantity. One theorem is developed to efficiently determine the optimal inventory policy for the retailer. We deduce some previously published results of other researchers as special cases. Finally, numerical examples are given to illustrate the theorem obtained in this paper.

Keywords: Inventory, EOQ, partial trade credit, permissible delay in payments

1. Introduction

The classical economic order quantity (EOQ) model assumes that the retailer must be paid for the items as soon as the items were received. However, in real-life situations, the supplier hopes to stimulate his products, he will offer the retailer a delay period, which is the trade credit period, in paying for the amount of purchasing cost. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. An interest is charged if the payment is not settled by the end of the trade credit period. Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible period allowed by the supplier.

Many researchers discussed this topic that investigate inventory problems under varying conditions. Goyal [9] established a single-item inventory model under permissible delay in payments. Chung [6] developed an alternative approach to determine the economic order quantity under condition of permissible delay in payments. Aggarwal and Jaggi [1] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Chang et al. [3] extended this issue to the varying rate of deterioration. Liao et al. [14] and Sarker et al. [15] investigated this topic with inflation. Jamal et al. [12] and Chang and Dye [2] extended this issue with allowable shortage. Chang et al. [4] extended this issue with linear trend demand. Chen and Chuang [5] investigated light buyer's inventory policy under trade credit by the concept of discounted cash flow.

Hwang and Shinn [11] modeled an inventory system for retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payment. Jamal et al. [13] and Sarker et al. [16] addressed the optimal payment time under permissible delay in payment with deterioration. Teng [18] assumed that the selling price not equal to the purchasing price to modify the inventory model under permissible delay in payments. Chung et al. [8] discussed this issue under the selling price not equal to the purchasing price and different payment rule. Shinn and Hwang [17] determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailer's order size, and also the demand rate is a function of the selling price. Chung and Huang [7] extended this problem within the EPQ framework and developed an efficient procedure to determine the retailer's optimal ordering policy. Huang [10] extended this issue under two levels of trade credit and developed an efficient solution procedure to determine the optimal lot-sizing policy of the retailer.

Goyal [9] is frequently cited when the inventory systems under conditions of permissible delay in payments are discussed. Goyal [9] implicitly makes the following assumptions:

(1) The unit selling price and the unit purchasing price are assumed to be equal. However, the unit selling price is not lower than the unit purchasing price in general. Consequently, the view point of Goyal [9] is debatable sometimes.

(2) Goyal [9] and all above published models assumed that the supplier would offer the retailer fully permissible delay in payments.

According to the above arguments, this paper will adopt the following assumptions to modify Goyal's model [9].

(i) The unit selling price and the unit purchasing price are not necessarily equal.

(ii) The supplier would offer the retailer partial trade credit not full trade credit. That is, the retailer must make a partial payment to the supplier when the order is received. Then, the retailer must pay off the remaining balance at the end of the permissible delay period.

The main purpose of this paper wants to incorporate the above assumptions (i) and (ii) to develop the retailer's inventory system as a cost minimization problem to investigate the optimal retailer's inventory policy.

2. Model Formulation and the Convexity

The following notations and assumptions will be used throughout:

Notations:

D = demand rate per year
A = ordering cost per order
c = unit purchasing price
s = unit selling price
h = unit stock holding cost per year excluding interest charges
I_e = interest earned per \$ per year
I_k = interest charged per \$ in stocks per year
M = the trade credit period in years
α = the percentage of permissible delay in payments, $0 \leq \alpha \leq 1$
T = the cycle time in years
$TVC(T)$= the annual total relevant cost, which is a function of T

T^* = the optimal cycle time of $TVC(T)$
Q^* = the optimal order quantity = DT^*.

Assumptions:

(1) Demand rate, D, is known and constant.
(2) Shortages are not allowed.
(3) Time horizon is infinite.
(4) $s \geq c$.
(5) As the order is received, the retailer must make a partial payment $(1-\alpha)cDT$ to the supplier. Then, the retailer pays off the remaining balance αcDT and keeps profits at the end of the trade credit period.
(6) During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account.

The annual total relevant cost consists of the following elements:

(1) Annual ordering cost $= \frac{A}{T}$.
(2) Annual stock holding cost (excluding interest charges) $= \frac{DTh}{2}$.
(3) According to assumption (5), there are three cases to occur in interest charged per year.

Case 1: $\frac{M}{1-\alpha} \leq T$, shown in Figure 1.
Annual interest payable $= cI_k(\frac{DT^2}{2} - \alpha DTM)/T$.

Case 2: $M \leq T \leq \frac{M}{1-\alpha}$, shown in Figure 2.
Annual interest payable
$= cI_k[\frac{(1-\alpha)^2DT^2}{2} + \frac{D(T-M)^2}{2}]/T = \frac{cDI_k}{2}[(1-\alpha)^2T^2 + (T-M)^2]/T$.

Case 3: $T \leq M$, shown in Figure 3.
Annual interest payable $= cI_k[\frac{(1-\alpha)^2DT^2}{2}]/T$.

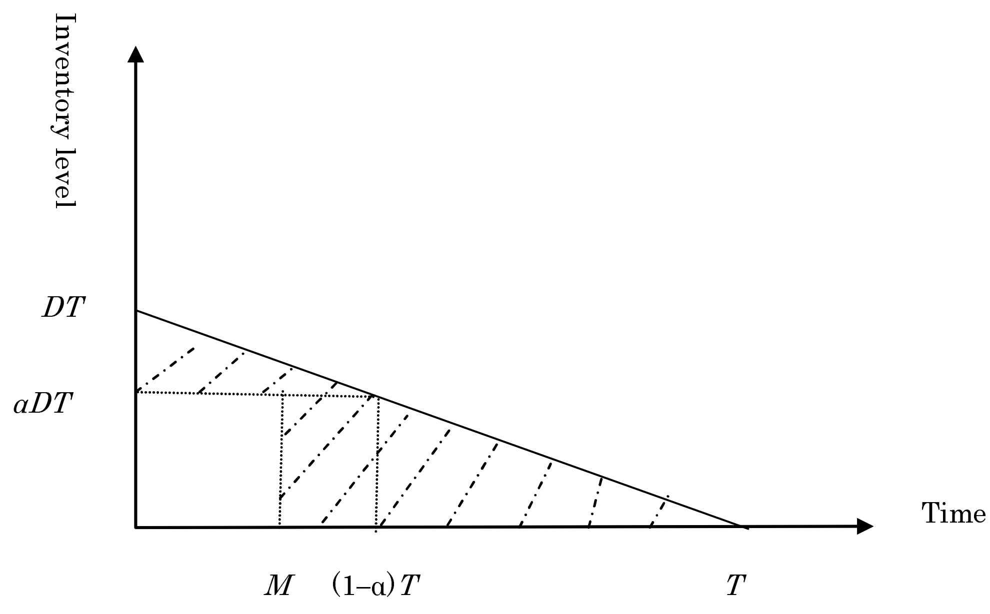

Figure 1: The inventory level and the total accumulation of interest payable when $M/(1-\alpha) \leq T$

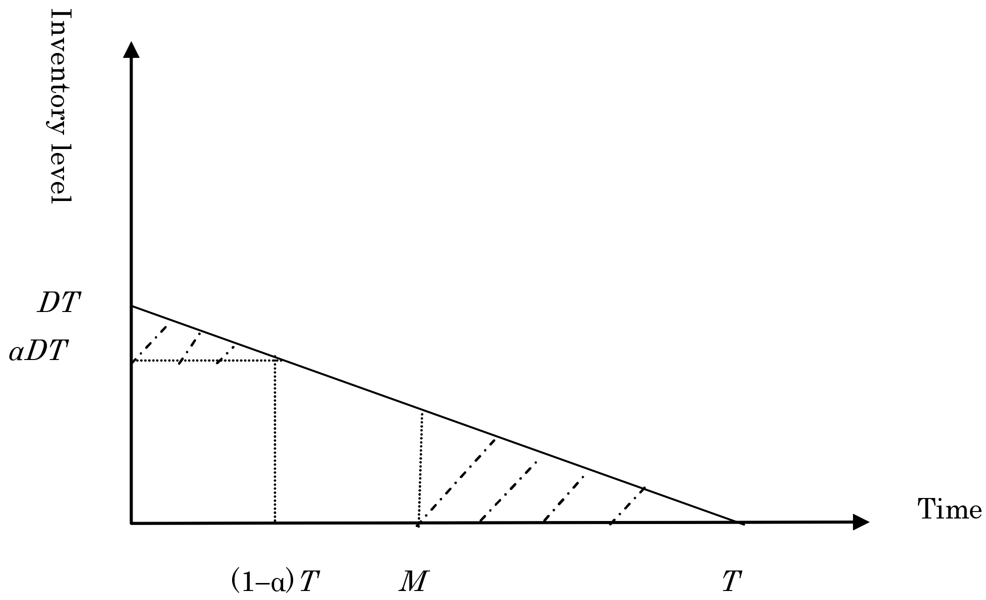

Figure 2: The inventory level and the total accumulation of interest payable when $M \leq T \leq M/(1-\alpha)$

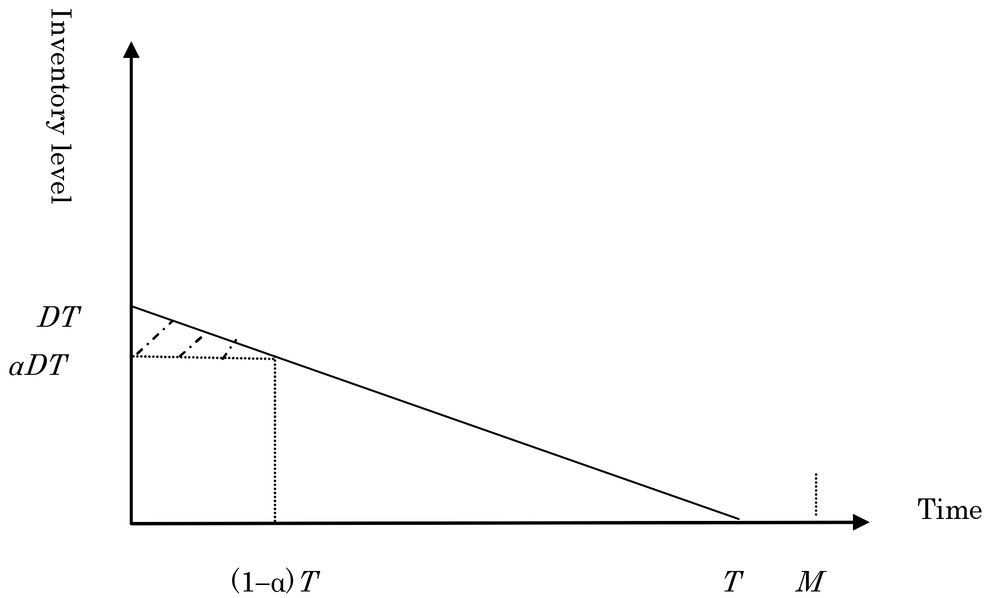

Figure 3: The inventory level and the total accumulation of interest payable when $0 < T \leq M$

(4) According to assumption (6), there are two cases to occur in interest earned per year.

Case 1: $M \leq T$.

Annual interest earned $= sI_e(\frac{DM^2}{2})/T$.

Case 2: $T \leq M$.

Annual interest earned $= sI_e[\frac{DT^2}{2} + DT(M-T)]/T = sI_eDT(M-\frac{T}{2})/T$.

From the above arguments, the annual total relevant cost for the retailer can be expressed as

TVC(T) = ordering cost + stock-holding cost + interest payable − interest earned.

$$TVC(T) = \begin{cases} TVC_1(T) & \text{if } \ T \geq \frac{M}{1-\alpha} & (1a) \\ TVC_2(T) & \text{if } \ M \leq T \leq \frac{M}{1-\alpha} & (1b) \\ TVC_3(T) & \text{if } \ 0 < T \leq M & (1c) \end{cases}$$

where

$$TVC_1(T) = \frac{A}{T} + \frac{DTh}{2} + cI_kDT(\frac{T}{2} - \alpha M)/T - sI_eDM^2/2T, \quad (2)$$

$$TVC_2(T) = \frac{A}{T} + \frac{DTh}{2} + cI_kD[(1-\alpha)^2T^2 + (T-M)^2]/2T - sI_eDM^2/2T, \quad (3)$$

and

$$TVC_3(T) = \frac{A}{T} + \frac{DTh}{2} + (1-\alpha)^2 cI_kDT^2/2T - sI_eDT(M - \frac{T}{2})/T. \quad (4)$$

Since $TVC_1(\frac{M}{1-\alpha}) = TVC_2(\frac{M}{1-\alpha})$ and $TVC_2(M) = TVC_3(M)$, *TVC(T)* is continuous and well-defined. All $TVC_1(T)$, $TVC_2(T)$, $TVC_3(T)$ and *TVC(T)* are defined on $T > 0$. Equations (2), (3), and (4) yield

$$TVC_1{'}(T) = -(\frac{2A - sDM^2I_e}{2T^2}) + D(\frac{h + cI_k}{2}), \quad (5)$$

$$TVC_1{''}(T) = \frac{2A - sDM^2I_e}{T^3}, \quad (6)$$

$$TVC_2{'}(T) = -\left[\frac{2A + DM^2(cI_k - sI_e)}{2T^2}\right] + \frac{D\{h + cI_k[1 + (1-\alpha)^2]\}}{2}, \quad (7)$$

$$TVC_2{''}(T) = \frac{2A + DM^2(cI_k - sI_e)}{T^3}, \quad (8)$$

$$TVC_3{'}(T) = -\frac{A}{T^2} + \frac{D\{h + [(1-\alpha)^2cI_k + sI_e]\}}{2} \quad (9)$$

and

$$TVC_3{''}(T) = \frac{2A}{T^3} > 0. \quad (10)$$

Let $\beta_1 = 2A - sDM^2I_e$ and $\beta_2 = 2A + DM^2(cI_k - sI_e)$, then we can find $\beta_1 \leq \beta_2$. Equation (10) implies that $TVC_3(T)$ is convex on $T > 0$. However, $TVC_1(T)$ is convex on $T > 0$ if $\beta_1 > 0$ and $TVC_2(T)$ is convex on $T > 0$ if $\beta_2 > 0$. Furthermore, we have $TVC_1{}'(\frac{M}{1-\alpha}) = TVC_2{}'(\frac{M}{1-\alpha})$ and $TVC_2{}'(M) = TVC_3{}'(M)$. Therefore, equations 1(a, b, c) imply that *TVC(T)* is convex on $T > 0$ if $\beta_1 > 0$. Then we can obtain following results.

Theorem 1.

(A) *If* $\beta_2 \leq 0$, *then TVC(T) is convex on* $(0, M]$ *and concave on* $[M, \infty)$.
(B) *If* $\beta_1 \leq 0$ *and* $\beta_2 > 0$, *then TVC(T) is convex on* $(0, M/(1-\alpha)]$ *and concave on* $[M/(1-\alpha), \infty)$.
(C) *If* $\beta_1 > 0$, *then TVC(T) is convex on* $(0, \infty)$.

3. Determination of the Optimal Cycle Time T^*

Let $TVC_i{}'(T_i^*) = 0$ for all $i = 1, 2, 3$. We can obtain

$$T_1^* = \sqrt{\frac{2A-sDM^2I_e}{D(h+cI_k)}} \quad \text{if } \beta_1 > 0, \tag{11}$$

$$T_2^* = \sqrt{\frac{2A+DM^2(cI_k-sI_e)}{D\{h+cI_k[1+(1-\alpha)^2]\}}} \quad \text{if } \beta_2 > 0, \tag{12}$$

and

$$T_3^* = \sqrt{\frac{2A}{D\{h+[(1-\alpha)^2cI_k+sI_e]\}}}. \tag{13}$$

T_1^* given by Equation (11) becomes the optimal cycle Time T^* if and only if $T_1^* \geq M/(1-\alpha)$. We substitute Equation (11) into $T_1^* \geq M/(1-\alpha)$, then $T^* = T_1^*$

$$\text{if and only if } -(2A - sDM^2I_e) + D(\frac{M}{1-\alpha})^2(h + cI_k) \leq 0.$$

Likewise, T_2^* given by Equation (12) becomes the optimal cycle Time T^* if and only if $M \leq T_2^* \leq M/(1-\alpha)$. We substitute Equation (12) into $M \leq T_2^* \leq M/(1-\alpha)$, then $T^* = T_2^*$

$$\begin{aligned} \text{if and only if} \quad & -(2A - sDM^2I_e) + D(\tfrac{M}{1-\alpha})^2(h + cI_k) \geq 0 \quad \text{and} \\ & -2A + DM^2\{h + [(1-\alpha)^2cI_k + sI_e]\} \leq 0. \end{aligned}$$

And last, T_3^* given by Equation (13) becomes the optimal cycle Time T^* if and only if $T_3^* \leq M$. We substitute Equation (13) into $T_3^* \leq M$, then $T^* = T_3^*$

$$\text{if and only if } -2A + DM^2\{h + [(1-\alpha)^2cI_k + sI_e]\} \geq 0.$$

Furthermore, we let

$$\Delta_1 = -(2A - sDM^2I_e) + D(\frac{M}{1-\alpha})^2(h + cI_k). \tag{14}$$

and

$$\Delta_2 = -2A + DM^2\{h + [(1-\alpha)^2 cI_k + sI_e]\}. \tag{15}$$

Then, we have $\Delta_1 > \Delta_2$ from Equations (14) and (15). Summarized above arguments, the optimal cycle time T^* can be obtained as follows.

Theorem 2.

(A) If $\Delta_2 \geq 0$, then $TVC(T^*) = TVC(T_3^*)$ and $T^* = T_3^*$.
(B) If $\Delta_1 > 0$ and $\Delta_2 < 0$, then $TVC(T^*) = TVC(T_2^*)$ and $T^* = T_2^*$.
(C) If $\Delta_1 \leq 0$, then $TVC(T^*) = TVC(T_1^*)$ and $T^* = T_1^*$.

4. Special Case

When $\alpha = 1$ and $s = c$, let

$$TVC_4(T) = \frac{A}{T} + \frac{DTh}{2} + cI_k D(T-M)^2/2T - cI_e DM^2/2T, \tag{16}$$

and

$$TVC_5(T) = \frac{A}{T} + \frac{DTh}{2} - cI_e DT(M - \frac{T}{2})/T. \tag{17}$$

$$T_4^* = \sqrt{\frac{2A + cDM^2(I_k - I_e)}{D(h + cI_k)}} \tag{18}$$

and

$$T_5^* = \sqrt{\frac{2A}{D(h + cI_e)}}. \tag{19}$$

Then $TVC_i^{'}(T_i^*) = 0$ for $i = 4, 5$. Equations 1(a, b, c) will be reduced as follows:

$$TVC(T) = \begin{cases} TVC_4(T) & \text{if} \quad M \leq T & \text{(20a)} \\ TVC_5(T) & \text{if} \quad 0 < T \leq M & \text{(20b)} \end{cases}$$

Equations 20(a, b) will be consistent with equations (1) and (4) in Goyal [9], respectively. Hence, Goyal [9] will be a special case of this paper. Equation (15) can be modified as $\Delta_2 = -2A + DM^2(h + cI_e)$. If we let $\Delta = -2A + DM^2(h + cI_e)$, Theorem 2 can be modified as follows:

Theorem 3.

(A) If $\Delta > 0$, then $T^* = T_5^*$.
(B) If $\Delta < 0$, then $T^* = T_4^*$.
(C) If $\Delta = 0$, then $T^* = T_4^* = T_5^* = M$.

Theorem 3 has been discussed in Chung [6]. Hence, Theorem 1 in Chung [6] is a special case of Theorem 2 of this paper.

Table 1: Optimal solutions with various value of α

Example 1: Let $A = \$250$/order, $D = 2000$units/year, $c = \$100$/unit, $s = \$120$/unit, $h = \$5$/unit/year, $I_k = \$0.15/\$$/year, $I_e = \$0.1/\$$/year and $M = 0.1$year.						
α	Δ_1	Δ_2	Theorem	T^*	Q^*	$TVC(T^*)$
0	>0	>0	2-(A)	T_3^*=0.08839	177	3256.84
0.1	>0	>0	2-(A)	T_3^*=0.09261	185	2999.07
0.2	>0	>0	2-(A)	T_3^*=0.09695	193	2757.52
0.3	>0	<0	2-(B)	T_2^*=0.10118	202	2534.62
0.4	>0	<0	2-(B)	T_2^*=0.10499	209	2333.67
0.5	>0	<0	2-(B)	T_2^*=0.10858	217	2157.52
0.6	>0	<0	2-(B)	T_2^*=0.1118	223	2008.79
0.7	>0	<0	2-(B)	T_2^*=0.11452	229	1889.99
0.8	>0	<0	2-(B)	T_2^*=0.11659	233	1803.33
0.9	>0	<0	2-(B)	T_2^*=0.11788	236	1750.58

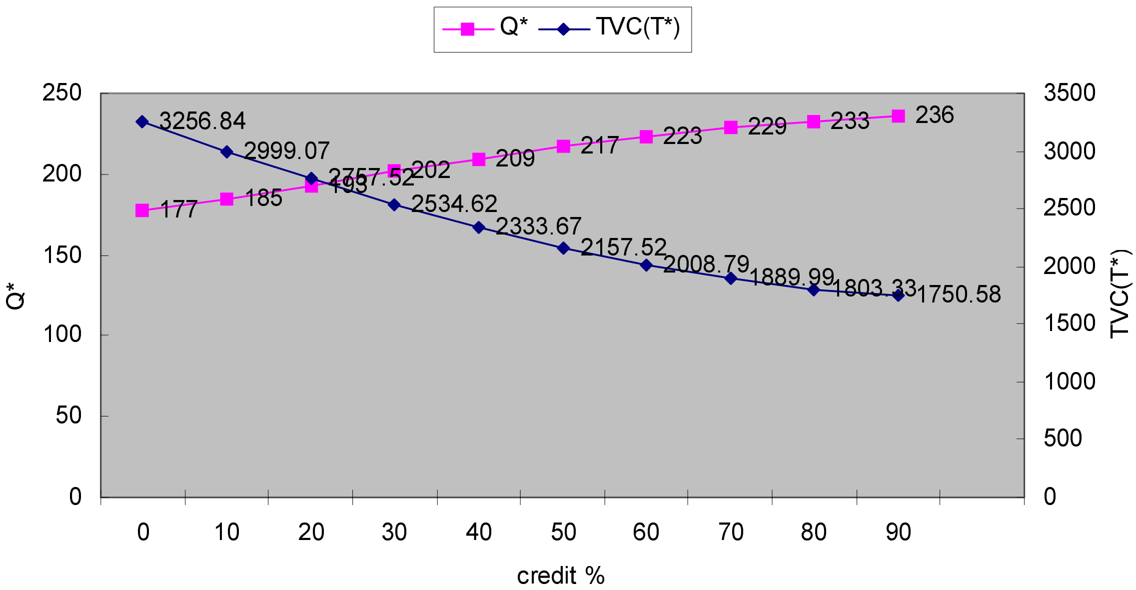

Figure 4: The behavior of the optimal order quantity and optimal annual total relevant cost in Example 1

Table 2: The optimal cycle time with various values of α and s

Example 2: Let $A = \$250$/order, $D = 2000$units/year, $c = \$50$/unit, $h = \$5$/unit/year, $I_k = \$0.15/\$$/year, $I_e = \$0.12/\$$/year and $M = 0.1$year.									
α	$s = \$100$/unit			$s = \$150$/unit			$s = \$200$/unit		
	Δ_1	Δ_2	T^*	Δ_1	Δ_2	T^*	Δ_1	Δ_2	T^*
0.3	>0	<0	T_2^*=0.11258	>0	>0	T_3^*=0.09681	>0	>0	T_3^*=0.08747
0.6	>0	<0	T_2^*=0.12233	>0	<0	T_2^*=0.10288	>0	>0	T_3^*=0.09098
0.9	>0	<0	T_2^*=0.12768	>0	<0	T_2^*=0.10738	>0	>0	T_3^*=0.09273

5. Numerical Examples

To illustrate all results obtained in this paper, let us apply the proposed method to efficiently solve the following numerical examples. The optimal cycle time and optimal order quantity are summarized in Table 1, Table 2 and Figure 4, respectively.

To study the effects of the percentage of permissible delay in payments, α, and unit selling price per item, s, on the optimal ordering policy for the retailer derived by the proposed method, we solve the examples 1 and 2 on Tables 1 and 2 with various values of α and s, respectively. The following inferences can be made based on Tables 1 and 2 and Figure 4. When α is increasing, the optimal cycle time and optimal order quantity for the retailer are increasing but the optimal annual total relevant cost is decreasing. So, the percentage of permissible delay in payments increasing, the effect of stimulating the retailer's demand will increase. When s is increasing, the optimal cycle time and optimal order quantity for the retailer are decreasing. This result implies that the retailer will order less quantity to take the benefits of the partial trade credit more frequently. This conclusion also had supported by Teng [18].

6. Conclusions

This paper presents an inventory model to modify Goyal's model [9] and provides a very efficient solution procedure to determine the optimal cycle time T^*. Theorem 2 helps the retailer accurately and quickly determining the optimal lot-sizing policy under minimizing the annual total relevant cost. If the percentage of permissible delay in payments equals to one and the unit selling price per item equals to the unit purchasing price per item, the inventory model discussed in this paper is reduced to Goyal [9]. From the final numerical examples, we find that the retailer will order less quantity to take the benefits of the partial trade credit more frequently when the unit selling price per item is larger than the unit purchasing price per item more and more. This conclusion also had supported by Teng [18].

A future study will further incorporate the proposed model into more realistic assumptions, such as finite replenishment rate, probabilistic demand, allowable shortages and I_e and I_k are not constant when T^* is a long period.

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