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HUB NETWORK DESIGN MODEL IN A COMPETITIVE ENVIRONMENT WITH FLOW THRESHOLD

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Abstract We consider a hub network design model based on the Stackelberg hub location model, where two firms compete with each other to maximize their own profit. The firm as a leader first locates p hubs and decides which OD pairs should be in services on the condition that the other firm as a follower locates q hubs and decides its strategies in a similar way after that. To avoid the possibility of unprofitable services, we incorporate flow threshold constraints into the model. We formulate the leader's problem as a bilevel programming problem with the follower's problem as a lower level problem. We solve the problem with the complete enumeration method and a greedy heuristic. The main objective is to make it clear how the network structure can be affected by the flow threshold constraints and the competitor's strategies.

Keywords: Facility planning, hub location, network design

1. Introduction

Since O'Kelly [4] formulated a discrete hub location problem as a quadratic integer programming problem, a variety of hub location models have been studied in the last two decades. However, studies on hub location problems in a competitive environment are scarce. Marianov, Serra and ReVelle [2] first addressed a competitive hub location model with the objective of maximizing the sum of captured flow and solved the problem using a tabu search heuristic. Sasaki et al. [5] developed the Stackelberg hub location model, where two firms compete to maximize their own profit. A similar Stackelberg location-allocation model was presented by Serra and ReVelle [6] with the objective of minimizing the maximum market share captured by the follower firm.

Most hub location models studied so far assume that the firms provide their services for all OD pairs in a market. As a result, they also have to operate some routes with extremely low flows. To avoid the possibility of such unprofitable services, we incorporate flow threshold constraints into the model, which prohibit providing services not expecting enough captured flows. Campbell [1] first introduced threshold scheme on arcs as well as arc capacities into hub location models. We consider a hub network design model in a competitive environment, where hub locations and operating routes (services) are both determined.

The firm as a leader first locates p hubs and decides which OD pairs should be in services on the condition that the other firm as a follower locates q hubs and decide its strategies in a similar way after that. We formulate the leader's problem as a bilevel programming problem with follower's problem as the lower level problem. By introducing flow threshold constraints in to a competitive hub location model, we can enrich the model so as to develop a comprehensive hub network design model for more practical use.

This paper is organized as follows. In Section 2, we briefly review the Stackelberg hub location model which forms the basis of the new hub network design model. In Section 3, we explain the presented model and formulate it as a bilevel programming problem. In Section 4, we explain how to solve the problem by using a brute force procedure and a greedy heuristic. In Section 5, we show computational results using real airlines' data, i.e. the CAB data. In Section 6, we give concluding remarks and mention some future work.

2. Brief Review of Stackelberg Hub Location Model

In this section, we briefly review the Stackelberg hub location model [5], which forms the basis of a new competitive hub network design model. In the Stackelberg hub location model, we assume the following conditions:

1. There is one big firm and several medium firms in the market. They provide services using one hub for each OD pair with the objective of maximizing their own profit.
2. The trip demands among all OD pairs are assumed to be known and symmetric.
3. The level of captured passengers is determined by the logit function [3]. Specifically, we assume that there are k services available for an OD pair and let $u_i (i = 1, \cdots, k)$ be the disutility of the i-th service. Then the level of captured passengers for the i-th service is determined by
$$L_i(u) = \frac{\exp[-\alpha u_i]}{\sum_{j=1}^{k} \exp[-\alpha u_j]}, \quad i = 1, \cdots, k, \tag{2.1}$$
where $\alpha > 0$ is a parameter.
4. The airfare for an OD pair is the same regardless of which firm provides the service, i.e., there is no price competition.
5. The set of OD pairs by which the leader provides services is predetermined. The sets of OD pairs by which the followers provide services are also predetermined and subsets of the leader's set. The followers' sets are mutually disjoint, i.e., there is no competition among the followers.
6. A big firm is the leader and the other firms are the followers. After the leader locates its hub, the followers locate their hubs simultaneously. The leader firm knows that the follower firms are going to locate their new hubs after knowing the leader's decision. So the leader firm has to locate its new hub, given that the follower firms make optimal decisions.
7. Each hub can be located anywhere on the plane (continuous location model) and there is no capacity limit on the passengers who use it. Hubs are only for the use of a facility for transfer and they have no trip demand of their own.
8. All services are provided via one hub (one-stop service). Services through more than one hub and nonstop services are not allowed.

Under these assumptions, each firm locates its new hub one by one. Sasaki and Fukushima [5] reported interesting computational results of Stackelberg hub location model. Specifically, they make it clear how the optimal location and the market share are affected by the rival firms. On the other hand, the results bring new issues for further improvements of the model. As in the case with many hub location models addressed so far, the firms often have to provide services even if they capture few demand. Since the network structure is necessarily fixed if the hub locations are given, the firms are forced to provide such unprofitable services. Moreover, the assumption that the service sets are predetermined seems to be unrealistic. To overcome these problems, it may be useful to consider a hub network design model, where the optimal location and the services to be provided are both determined. More precisely, we incorporate flow threshold constraints into the model to deal with the problems. We describe the threshold as a lower limit of the market share of each OD pair rather than the actual amount of captured demand. Namely, firms cannot provide any services whose captured market share does not reach to the predetermined level.

3. Formulation of Hub Network Design Model

Before we formulate the model, we provide a model description to make it clear the difference compared with the Stackelberg hub location model. Suppose that one leader firm and one follower firm exist in a market and they compete with each other to maximize their own profit as the same in the Stackelberg hub location model. The major difference is the network structure. Although the Stackelberg hub location model allows to locate hubs anywhere in a plane, we rather consider a discrete network model, where demand nodes and hub candidates are both given as a discrete node set. Let Firm A denote the leader firm and Firm B denote the follower firm. We employ the following notations:

- N: the set of demand nodes, $|N| = n$.
- H: the set of hub candidates, $|H| = h$.
- Π: the set of OD pairs, $\Pi \subseteq N \times N$.
- d_π: the direct distance between OD pair $\pi \in \Pi$.
- $c_{\pi k}$: the actual travel distance between OD pair $\pi \in \Pi$ via hub $k \in H$.
- t_π: the flow threshold of OD pair $\pi \in \Pi$, $0 \leq t_\pi \leq 0.5$.
- W_π: the trip demand (the number of passengers) for OD pair $\pi \in \Pi$.
- F_π: the airfare for OD pair $\pi \in \Pi$.
- M: a large number.

Note that the flow threshold t_π is given by the market share. We introduce the design variables to describe which OD pairs should be in services as well as the location variables. The decision variables of the firms are as follows:

- x_k: binary variable such that $x_k = 1$ if node $k \in H$ is selected as a Firm A's hub, and 0 otherwise.
- y_k: binary variable such that $y_k = 1$ if node $k \in H$ is selected as a Firm B's hub, and 0 otherwise.
- u_π: binary variable such that $u_\pi = 1$ if Firm A's service is provided on OD pair π, and $u_\pi = 0$ otherwise.
- v_π: binary variable such that $v_\pi = 1$ if Firm B's service is provided on OD pair π, and $v_\pi = 0$ otherwise.

As in the Stackelberg hub location model, we suppose that the captured demand level determined by the logit function given in (2.1), which is a function of service disutility. The disutility of Firm A's service between OD pair π using Firm A's hub $k \in H$ is defined as the ratio of the actual travel distance to the direct distance between the OD pair π, i.e., $c_{\pi k}/d_\pi$. If the firm does not locate hub $k \in H$, no services through hub $k \in H$ are available.

In such a case, the disutility of all service disutility through hub $k \in H$ is defined to be infinity. Therefore, the disutility of Firm A's service $\eta^A_\pi(x_k)$ between OD pair π using Firm B's hub $k \in H$ is given by

$$\eta^A_{\pi k}(x_k) = \begin{cases} c_{\pi k}/d_\pi, & \text{if } x_k = 1, \\ \infty, & \text{if } x_k = 0, \end{cases} \qquad \pi \in \Pi, k \in H.$$

In a similar manner, the disutility of Firm B's service $\eta^B_\pi(y_k)$ between OD pair π using Firm B's hub $k \in H$ is given by

$$\eta^B_{\pi k}(y_k) = \begin{cases} c_{\pi k}/d_\pi, & \text{if } y_k = 1, \\ \infty, & \text{if } y_k = 0. \end{cases} \qquad \pi \in \Pi, k \in H.$$

Suppose that both Firm A and Firm B provide their services on an OD pair $\pi \in \Pi$. Then the market share of OD pair π captured by Firm A and Firm B are given by

$$\phi_\pi(x, y) = \frac{\sum_{k \in H} \exp[-\alpha \eta^A_{\pi k}(x_k)]}{\sum_{k \in H} \exp[-\alpha \eta^A_{\pi k}(x_k)] + \sum_{k \in H} \exp[-\alpha \eta^B_{\pi k}(y_k)]}, \tag{3.1}$$

and

$$\psi_\pi(x, y) = \frac{\sum_{k \in H} \exp[-\alpha \eta^B_{\pi k}(y_k)]}{\sum_{k \in H} \exp[-\alpha \eta^A_{\pi k}(x_k)] + \sum_{k \in H} \exp[-\alpha \eta^B_{\pi k}(y_k)]} = 1 - \phi_\pi(x, y), \tag{3.2}$$

with a constant $\alpha > 0$, $x = (x_1, x_2, \cdots, x_h)^\top$, and $y = (y_1, y_2, \cdots, y_h)^\top$, respectively.

By taking design variables u_π and v_π into consideration, the actual market share captured by Firm A and Firm B are given by

$$\Phi_\pi(x, y, u, v) = \frac{\sum_{k \in H} \exp[-\alpha \eta^A_{\pi k}(x_k)] u_\pi}{\sum_{k \in H} \exp[-\alpha \eta^A_{\pi k}(x_k)] u_\pi + \sum_{k \in H} \exp[-\alpha \eta^B_{\pi k}(y_k)] v_\pi} \tag{3.3}$$

and

$$\Psi_\pi(x, y, u, v) = \frac{\sum_{k \in H} \exp[-\alpha \eta^B_{\pi k}(y_k)] v_\pi}{\sum_{k \in H} \exp[-\alpha \eta^A_{\pi k}(x_k)] u_\pi + \sum_{k \in H} \exp[-\alpha \eta^B_{\pi k}(y_k)] v_\pi}, \tag{3.4}$$

where $u = (u_1, u_2, \cdots, u_{|\Pi|})^\top$ and $v = (v_1, v_2, \cdots, v_{|\Pi|})^\top$, respectively. Consequently, the total revenues of Firm A and Firm B are given by

$$f(x, y, u, v) = \sum_{\pi \in \Pi} F_\pi W_\pi \Phi_\pi(x, y, u, v), \tag{3.5}$$

and

$$g(x, y, u, v) = \sum_{\pi \in \Pi} F_\pi W_\pi \Psi_\pi(x, y, u, v), \tag{3.6}$$

respectively. Now we formulate the problem. First we consider Firm B's problem. Given the Firm A's hub locations, Firm B will locate q hubs so as to maximize its total revenue. So Firm B's problem, which is called HNDP-B, is written as follows:

[HNDP-B]

$$\begin{aligned}
\text{maximize}_{y,v} \quad & g(x,y,u,v) && && \\
\text{subject to} \quad & t_\pi - \psi_\pi(x,y) \le M(1-v_\pi), && \pi \in \Pi, && (3.7)\\
& \sum_{k\in H} y_k = q, && && (3.8)\\
& y_k \le 1 - x_k, && k \in H, && (3.9)\\
& y_k \in \{0,1\}, && k \in H, && \\
& v_\pi \in \{0,1\}, && \pi \in \Pi. &&
\end{aligned}$$

Constraints (3.7) prohibit providing services whose captured market share is less than flow threshold t_π. Constraint (3.8) ensures that Firm B locates q hubs. Constraints (3.9) means that once Firm A locates hub $k \in H$, Firm B never locates hub $k \in H$. Firm A solves its own problem subject to the condition that Firm B finds the optimal solution of HNDP-B. More precisely, $[y,v] \in \arg\max\{g(x,y,u,v)|y \in Y, v \in V\}$ should be a constraint in Firm A's problem, where Y and V denote the feasible regions of y and v, respectively. Hence, Firm A's problem is stated as the following bilevel programming problem:

[HNDP]

$$\begin{aligned}
\text{maximize} \quad & f(x,y,u,v) && && \\
\text{subject to} \quad & t_\pi - \phi_\pi(x,y) \le M(1-u_\pi), && \pi \in \Pi, && (3.10)\\
& \sum_{k\in H} x_k = p, && && (3.11)\\
& x_k \in \{0,1\} && k \in H, && \\
& u_\pi \in \{0,1\}, && \pi \in \Pi, && \\
& [y,v] \in \arg\max\{g(x,y,u,v)|y \in Y, v \in V\}. && &&
\end{aligned}$$

Constraints (3.10) prohibit providing services whose captured market share is less than flow threshold t_π. Constraint (3.11) ensures that Firm A locates p hubs.

First, we establish that all demand is satisfied in HNDP. From (3.3) and (3.5), the value of function $f(x,y,u,v)$ increases as the value of u_π increases. Also from (3.4) and (3.6), the value of function $g(x,y,u,v)$ increases as the value of v_π increases. It follows that Firm A's service on OD pair π that satisfies the threshold constraint $\phi_\pi(x,y) \ge t_\pi$ should be provided, i.e., $u_\pi = 1$ at the optimal solution. In a similar way, Firm B's service on OD pair π that satisfies the threshold constraint $\psi_\pi(x,y) \ge t_\pi$ should be provided, i.e., $v_\pi = 1$ at the optimal solution. In addition, $\phi_\pi(x,y) + \psi_\pi(x,y) = 1$ is always satisfied for all π by (3.1) and (3.2). Moreover, we define the value of t_π ranges from 0 to 0.5 and hence at least $\phi_\pi(x,y) \ge t_\pi$ or $\psi_\pi(x,y) \ge t_\pi$ is always satisfied. Therefore, at least one of the two firms provides a service for each OD pair, implying that, all demand is satisfied, while passengers may not always take the most desired service.

4. Solution Method

4.1. Complete enumeration method

We can obtain an optimal solution by the complete enumeration method. Assuming that x and y are fixed, we specify the following two sets: $H_A^1 = \{k \in H | x_k = 1\}$ and $H_B^1 = \{k \in H | y_k = 1\}$. Then the market share of Firm A and Firm B of OD pair π are given by

$$\tilde{\phi}_\pi = \frac{\sum_{k \in H_A^1} \exp[-\alpha c_{\pi k}/d_\pi]}{\sum_{k \in H_A^1} \exp[-\alpha c_{\pi k}/d_\pi] + \sum_{k \in H_B^1} \exp[-\alpha c_{\pi k}/d_\pi]}$$

and

$$\tilde{\psi}_\pi = \frac{\sum_{k \in H_B^1} \exp[-\alpha c_{\pi k}/d_\pi]}{\sum_{k \in H_A^1} \exp[-\alpha c_{\pi k}/d_\pi] + \sum_{k \in H_B^1} \exp[-\alpha c_{\pi k}/d_\pi]} = 1 - \tilde{\phi}_\pi,$$

respectively. Moreover, we define $\Pi_A^0 = \{\pi \in \Pi | \tilde{\phi}_\pi < t_\pi\}$ and $\Pi_B^0 = \{\pi \in \Pi | \tilde{\psi}_\pi < t_\pi\}$. It is necessary to be $u_\pi = 0$ for all $\pi \in \Pi_A^0$ and $v_\pi = 0$ for all $\pi \in \Pi_B^0$ to satisfy the constraints (3.7) and (3.10). As in the previous section, $f(x, y, u, v)$ and $g(x, y, u, v)$ are increasing functions of u_π and v_π, respectively. Consequently, $u_\pi = 1$ for all $\pi \notin \Pi_A^0$ and $v_\pi = 1$ for all $\pi \notin \Pi_B^0$ to maximize the objective value under the condition that x and y are fixed. From the above observation, we see that to examine all possible combinations of x and y is sufficient to obtain the optimal solution of HNDP.

4.2. Greedy heuristic

Although we can develop a complete enumeration method by examining all possible combinations of possible hub sets of both firms. However, the number of such combinations is still ${}_nC_p \cdot {}_{n-p}C_q$. Hence, the CPU time required by the complete enumeration grows rapidly as the problem size increases. Another possibility is to develop some heuristic methods. Since the Firm A's problem has the constraint that Firm B always find an optimal solution, we have to solve the Firm B's problem exactly to ensure the feasibility of HNDP. The basic idea is that we first determine the Firm A's hubs in the following greedy manner, then we solve the Firm B's problem exactly by the complete enumeration to ensure the feasibility of HNDP. However, when $p < q$, this approach doesn't work well, so we determine Firm B's hubs in a greedy manner beforehand to determine Firm A's hubs in this case. Then we solve the Firm B's problem exactly.

Let H_A denotes the set of Firm A's hubs and H_B denotes the set of Firm B's hubs. If Firm A selects a new hub $l \notin H_A \cup H_B$ under the condition that H_A and H_B are given. Then Firm A's market share of OD pair π is given by

$$R_{\pi l}^A = \begin{cases} 1 & \text{if } \bar{R}_{\pi l}^A > 1 - t_\pi \\ \bar{R}_{\pi l}^A & \text{if } t_\pi \le \bar{R}_{\pi l}^A \le 1 - t_\pi \\ 0 & \text{if } \bar{R}_{\pi l}^A < t_\pi \end{cases},$$

where

$$\bar{R}_{\pi l}^A = \frac{\sum_{k \in H_A} \exp[-\alpha c_{\pi k}/d_\pi] + \exp[-\alpha c_{\pi l}/d_\pi]}{\sum_{k \in H_A} \exp[-\alpha c_{\pi k}/d_\pi] + \sum_{k \in H_B} \exp[-\alpha c_{\pi k}/d_\pi] + \exp[-\alpha c_{\pi l}/d_\pi]}.$$

Thus, the total revenue of Firm A is written by

$$S_l^A = \sum_{\pi \in \Pi} F_\pi W_\pi R_{\pi l}^A.$$

From these observation, we can regard a candidate

$$l' = \arg \max_{l \notin H_A \cup H_B} S_l^A$$

as the best choice for Firm A at this stage. In a similar manner, we define $R_{\pi l}^B, \bar{R}_{\pi l}^B$ and s_l^B for Firm B. Then $l' = \arg\max_{l \notin H_A \cup H_B} S_l^B$ is the best choice for Firm B at this stage. Note that this greedy heuristic brings a feasible solution in some cases. More precisely, if $q = 1$ and $|H_B| = \emptyset$, then the greedy solution of Firm B, $H_B = \{l'\}$ brings a feasible solution of HNDP under the condition that Firm A's hub set H_A satisfies $|H_A| = p$. We can simply describe a greedy heuristic as follows.

[Greedy Heuristic]

Step 0: Set $H_A := \emptyset$ and $\bar{H}_B := \emptyset$. If $p < q$, then go to Step 3.

Step 1: If $|H_A| = q$, then go to Step 2. Compute $S_l^A(l \notin H_A \cup \bar{H}_B)$ and select $l_A \in \arg\max_{l \notin H_A \cup \bar{H}_B} S_l^A$. Set $H_A := H_A \cup \{l_A\}$. Compute $S_l^B(l \notin H_A \cup \bar{H}_B)$ and select $l_B \in \arg\max_{l \notin H_A \cup \bar{H}_B} S_l^B$. Set $\bar{H}_B := \bar{H}_B \cup \{l_B\}$. Go to Step 1.

Step 2: If $|H_A| = p$, then go to Step 4. Otherwise, compute $S_l^A(l \notin H_A \cup \bar{H}_B)$ and select $l_A \in \arg\max_{l \notin H_A \cup \bar{H}_B} S_l^A$. Set $H_A := H_A \cup \{l_A\}$. Goto Step 2.

Step 3: Compute $S_l^B(l \in H)$ and arrange them in the increasing order: $S_{l_1}^B, S_{l_2}^B, \cdots, S_{l_{|H|}}^B$. Set $\bar{H}_B := \{l_1, l_2, \cdots, l_q\}$. Compute $S_l^A(l \notin \bar{H}_B)$ and arrange them in the increasing order: $S_{l_1}^A, S_{l_2}^A, \cdots, S_{l_{|H \setminus \bar{H}_B|}}^A$. Set $H_A := \{l_1, l_2, \cdots, l_p\}$.

Step 4: If $p = q = 1$, then $\{H_A, \bar{H}_B\}$ is an approximate solution. Otherwise, compute an optimal H_B under the condition that Firm A's hub set is H_A. The obtained $\{H_A, H_B\}$ is an approximate solution.

5. Computational Results

In this section, we report some computational results for the proposed model HNDP and examine how the optimal location and the total revenue affected by the flow threshold constraints and the passengers' preference (i.e., parameter α). Computer programs were coded in MATLAB R14 (version 7.0.1). All programs were carried out on a DELL DIMENSION 8300 computer with Intel Pentium 4 processor available in speeds of 3.0GHz operated under Windows XP professional with 2.0 GB DDR-SDRAM memory. We prepared the demand data based on the well-known U.S. 25 cities data evaluated in 1970 by CAB (Civil Aeronautics Board). For airfare data, we used the data supplied by http://www.airfare.com/. All figures presented in this section are prepared by using MATLAB and Mapping Toolbox (version 2.0.1).

For simplicity, we assume that the threshold is the same in all OD pairs and denote t as the common flow threshold. We also assume that all demand nodes are hub candidates, that is, $H = N$. We solved 168 problems with $n = 25$ and various values of parameter p, q, t and α. More precisely, we solved the problem with $(p, q) = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)\}$ varying α from 1 to 4 by 1 and t from 0.0 to 0.5 by 0.1. In the results with $t = 0.5$, one of the firms captures all demand and the other captures nothing except a few cases in which passengers of an OD pair π are evenly

Table 1: Computational results for complete enumeration and greedy heuristic

			CE	Greedy heuristic			
					%error		
n	p	q	CPU (sec.)	CPU (sec.)	ave.	min	max
25	1	1	0.33	0.30	1.42	0.0	7.39
25	1	2	0.82	0.31	8.69	0.0	72.93
25	2	1	0.80	0.32	5.10	0.05	13.60
25	2	2	6.08	0.34	10.95	0.0	20.59
25	2	3	41.41	0.49	22.98	6.01	70.52
25	3	2	40.74	0.35	6.47	0.93	13.87
25	3	3	273.84	0.49	8.17	0.0	30.09

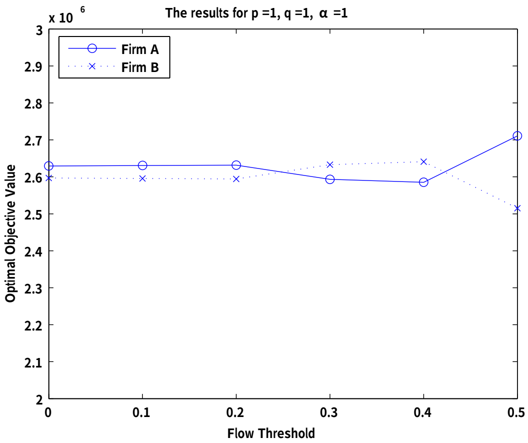
Table 2: Deviation of heuristic solutions

Threshold	ave.	min	max
0.0	4.71	0.0	14.54
0.1	5.02	0.0	14.74
0.2	5.01	0.0	19.18
0.3	6.54	0.0	31.08
0.4	13.41	0.0	42.79
0.5	19.98	0.0	72.93

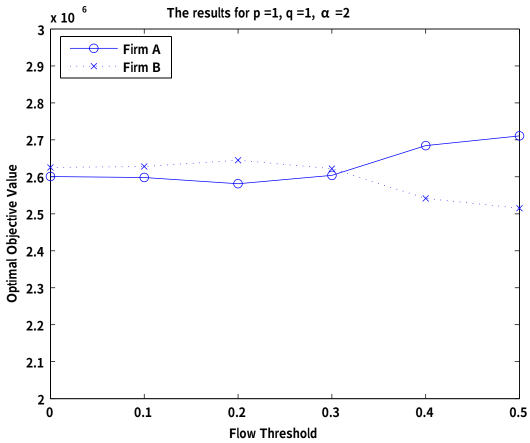
shared by the two firms, i.e., $\Phi_\pi(x, y) = \Psi_\pi(x, y)$. On the other hand, the results with $t = 0.0$ are exactly the same as those of problems with no flow threshold constraints. As we mention previously, we used a logit function so as to reflect passengers' various preferences. Note that the value of α becomes large, passenger preferences approach to all-or-nothing assignment.

Table 1 shows the computational results for the complete enumeration and the greedy heuristic. We got optimal solutions for 33 out of 168 examples. Although the greedy heuristic provides solutions in a short time, the maximum deviation from the optimal solutions is quite large especially when $p < q$. This is presumably due to lack of consideration of the leader's advantage, since the heuristic selects Firm B's q hubs first in a greedy manner and Firm A's p hubs after that in the same manner. Based on our preliminary test for some greedy approaches, this seems to provide better solutions compare with that which has been applied to the problem with $p \geq q$. Table 2 shows that the deviation of greedy heuristic solutions depends on the value of thresholds. When the threshold constraints are tight, the deviation becomes large.

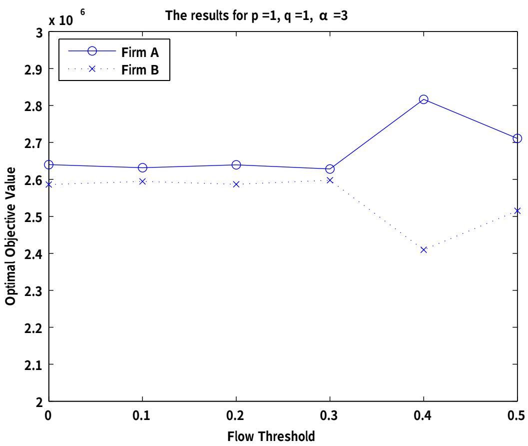
Next we examine how the flow threshold constraints and the value of α affect the optimal objective values. Figure 1 shows the optimal objective values for the problems with $p = q = 1$, Figure 2 shows the optimal objective values for the problems with $p = q = 2$ and Figure 3 shows the optimal objective values for the problems with $p = 3$ and $q = 2$. In each figure, the solid line denotes the result for Firm A and the dotted line denotes the result for Firm B. The optimal objective values of Firm A are always larger than those of Firm B regardless of the value of α and t when (p, q)=(2,2) and (3,2) (See Figure 2 and Figure



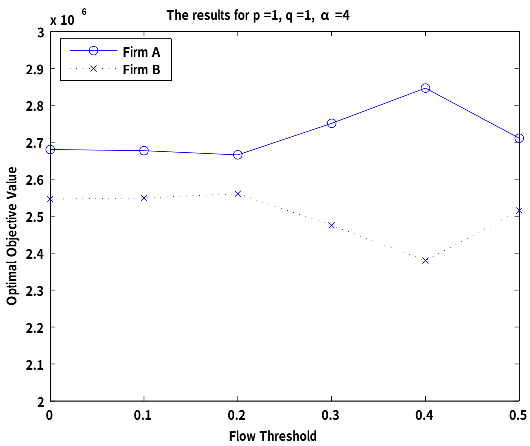
(a) $\alpha = 1$



(b) $\alpha = 2$



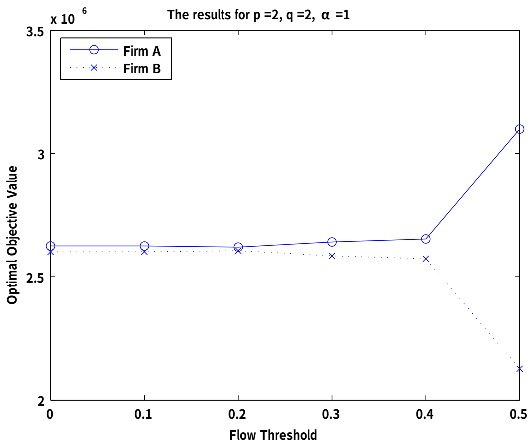
(c) $\alpha = 3$



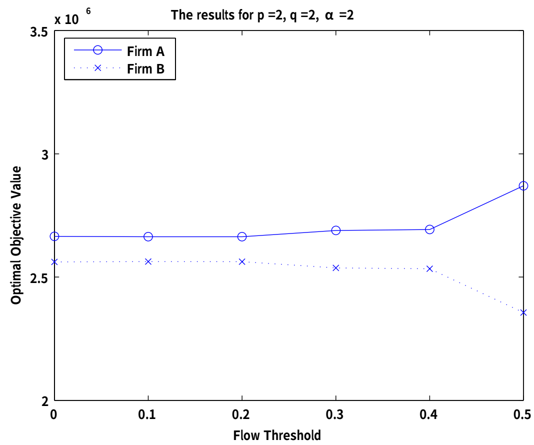
(d) $\alpha = 4$

Figure 1: Optimal objective value for $n = 25, p = 1, q = 1$

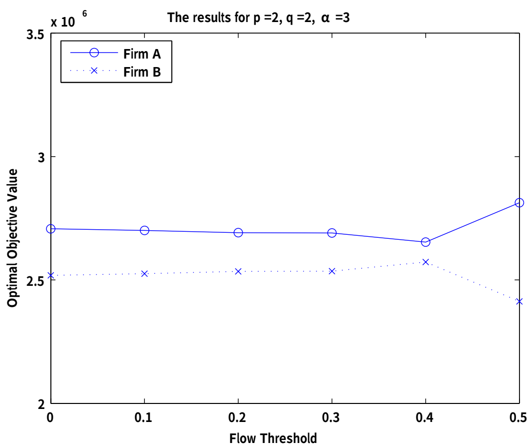
3). However, it is not necessarily the case, for example, Figure 1(a) and 1(b) indicate that Firm B's optimal objective values are larger. It follows that the leader does not always take advantage even on the condition that the follower is not allowed to select hubs which are selected by the leader. Moreover, in the result with (p, q)=(1,2) and (2,3), Firm B's optimal value is always larger. The reason is simply that the market share also depends on the number of located hubs. There is no clear relationship between the value of thresholds and the leader's optimal values, however, the results indicate that it is advantageous to the leader in the case $t = 0.5$. Figure 4 displays optimal hub locations with $p = q = 2$ and $\alpha = 1$. Figure 5 displays optimal hub locations with $p = 3, q = 2$ and $\alpha = 3$. "A" and "B" in the figures denote the optimal locations of Firm A and Firm B, respectively. These figures show that optimal hub locations are very sensitive to the flow thresholds, hence the flow threshold is one of the important factors of designing hub location network. In the firms' point of view, the results may rather negative in the sense that stable hub locations are hard to find.



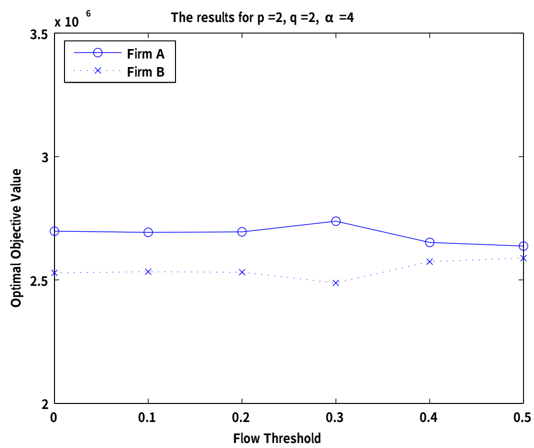
(a) $\alpha = 1$



(b) $\alpha = 2$

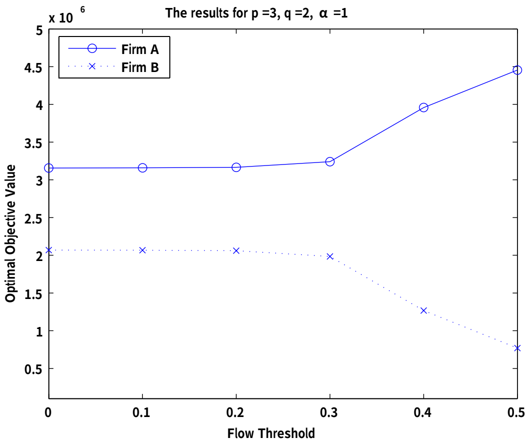


(c) $\alpha = 3$

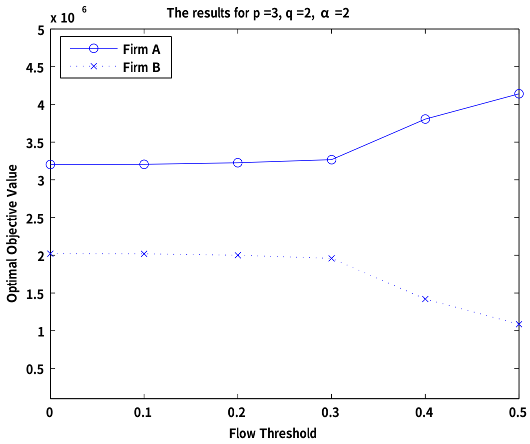


(d) $\alpha = 4$

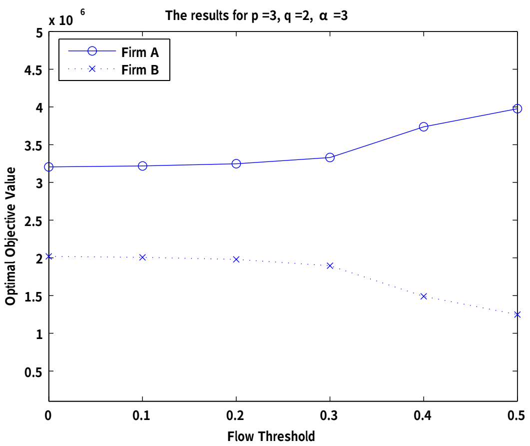
Figure 2: Optimal objective value for $n = 25, p = 2, q = 2$



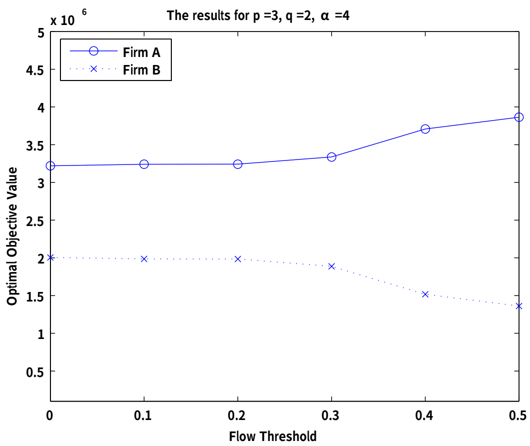
(a) $\alpha = 1$



(b) $\alpha = 2$

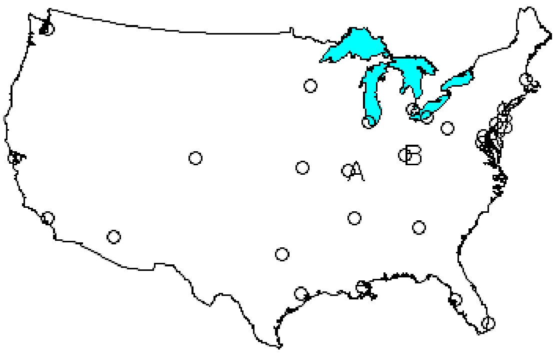


(c) $\alpha = 3$

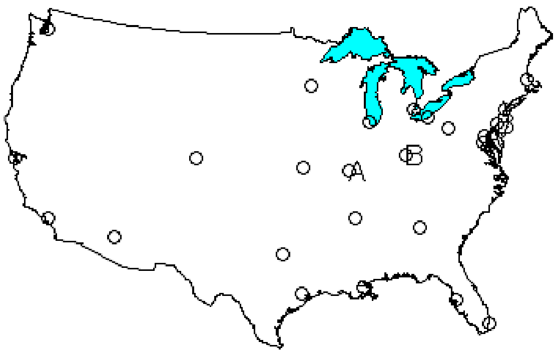


(d) $\alpha = 4$

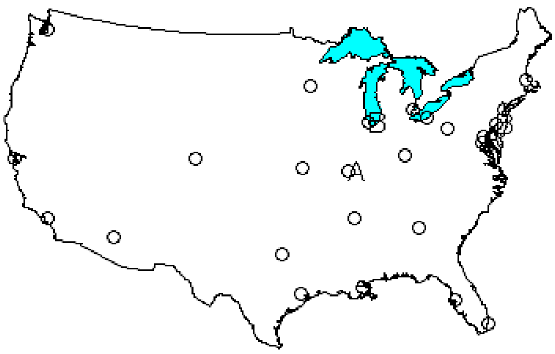
Figure 3: Optimal objective value for $n = 25, p = 3, q = 2$



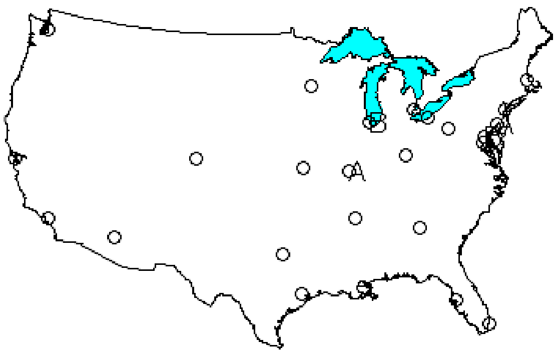
(a) Flow threshold=0.0



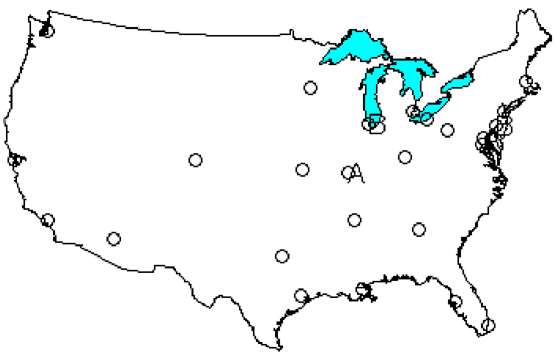
(b) Flow threshold=0.1



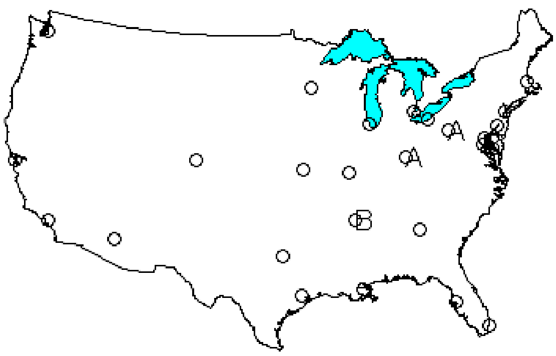
(c) Flow threshold=0.2



(d) Flow threshold=0.3

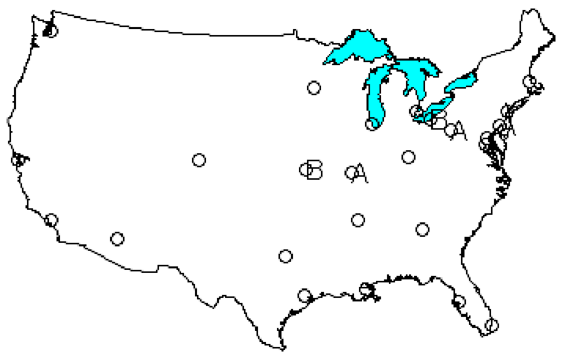


(e) Flow threshold=0.4

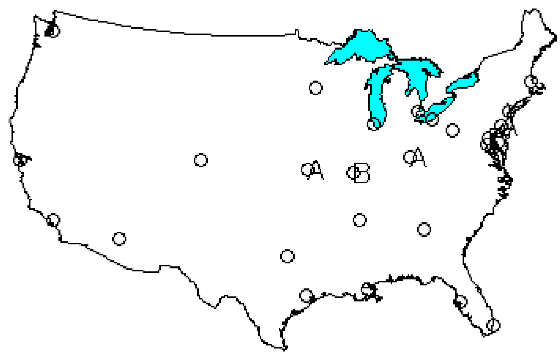


(f) Flow threshold=0.5

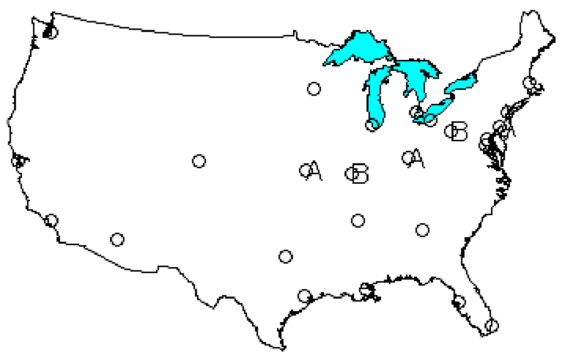
Figure 4: Results for $n = 25, p = 2, q = 2, \alpha = 1$



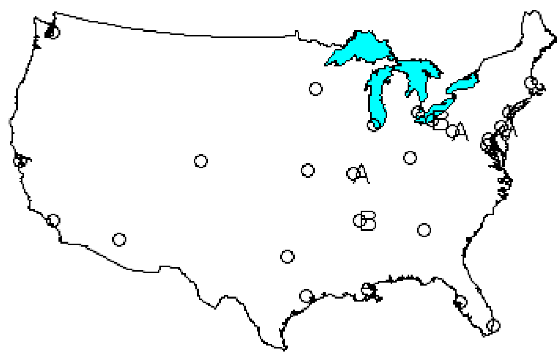
(a) Flow threshold=0.0



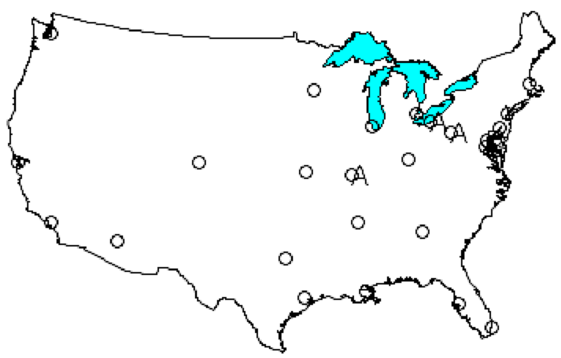
(b) Flow threshold=0.1



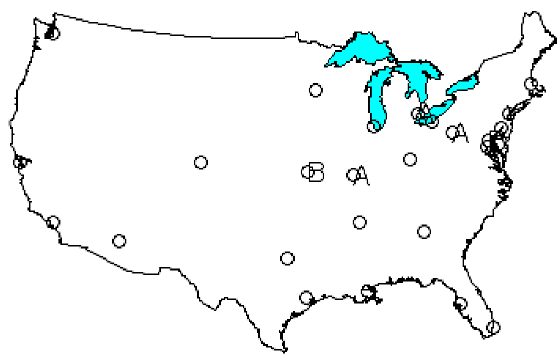
(c) Flow threshold=0.2



(d) Flow threshold=0.3



(e) Flow threshold=0.4



(f) Flow threshold=0.5

Figure 5: Results for $n = 25, p = 3, q = 2, \alpha = 3$

6. Conclusion and Future Work

We proposed a new hub network design problem in a competitive environment based on the Stackelberg hub location model. Specifically, we incorporated flow threshold constraints into the model to determine which services should be provided. We formulated the problem as a bilevel programming problem, where the upper and lower problems are both 0-1 integer programming problems. We solved 168 instances using the brute force procedure and a greedy heuristic. Computational results showed that optimal locations and the objective values are sensitive to the value of thresholds. This result pointed out that the flow threshold is one of the important factors in the hub network design. We also observed that the leader cannot always take advantage even in the follower is prohibited to select hubs which has been selected by the leader.

Although the greedy heuristic brings approximate solutions in a short time, the deviation from the optimal value significantly depends on the number of hubs and the threshold constraints. We should improve the greedy heuristic to solve large problems more efficiently. We also need to develop a branch-and-bound procedure to obtain optimal solutions. In the presented model, we incorporated the flow threshold constraint on each OD pair. Another possibility is considering arc flow oriented thresholds. Future work should concentrate on developing efficient solution methods as well as considering more realistic hub network design model.

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