Journal of the Operations Research
Society of Japan
2005, Vol. 48, No. 2, 135-147

LOCATING A SINGLE FACILITY IN THE PLANE IN THE PRESENCE OF A BOUNDED REGION AND DIFFERENT NORMS

Jack Brimberg
Royal Military College of Canada

Hossein Taghizadeh Kakhki
Ferdowsi University of Mashhad

George Orest Wesolowsky
McMaster University

(Received March 5, 2004; Revised January 27, 2005)

Abstract We consider the problem of locating a single new facility in the plane in the presence of a bounded region, where the distance measures are different in the interior and exterior of this region. This is, in fact, an extension of previous work for unbounded regions separated by a line (Brimberg et al. [2]). We explore the properties of the problem, propose exact and approximate solution procedures, and examine a special case.

Keywords: Facility planning, continuous location, single facility, convex programming, optimization

1. Introduction

Suppose $\Omega_1 \subset \Re^2$ is a closed bounded set (region) of polygonal shape, and Ω_2 is its exterior. Suppose also that in each region there are a given number of existing facilities (fixed points) with known weights, and that the distance measure in Ω_1 is induced by a norm k_1, and in Ω_2 a different norm k_2. We want to find the location of a single new facility so that the sum of distances from the existing facilities to this new facility is minimized.

Mathematically, the problem can be stated as:

$$\text{(P1)} \qquad \min \sum_{i=1}^{N} w_i d(X, P_i) = \sum_{P_i \in \Omega_1} w_i d(X, P_i) + \sum_{P_i \in \Omega_2} w_i d(X, P_i);$$

where:
N = total number of fixed points,
$P_i = (a_i, b_i)$, the coordinates of fixed point i, $i = 1, \ldots, N$,
w_i = the weight of fixed point i, $w_i > 0$, $i = 1, \ldots, N$,
$X = (x, y)$ the location of the new facility, unknown
$d(X, Y)$ = the shortest (geodesic) path distance between any two points X, $Y \in \Re^2$.

An application of this problem is the location of a facility within or outside an urban area where due to the layout of the streets the movement is *slow* within the city boundary, while outside this boundary the movement is *fast*. For mathematical convenience, the movement along a boundary is always assumed to be the faster of the two modes.

The closed region Ω_1 could also be a province, territory, or country with a different transportation grid. By increasing the number of sides, the polygon representation could be made as accurate as desired. Furthermore, Ω_1 would, in general, be nonconvex in shape.

A related problem with Euclidean distances on one side and rectilinear distances on the other side of a line has been studied by Parlar [10]. The author shows that the objective function is not convex in X, and formulates the problem as a mixed integer program. A modified *Weiszfeld* procedure is proposed to solve the problem, and the results are compared with those obtained using an *adaptive random search* procedure for three example problems. Batta and Palekar [1] examine a modeling framework for a mixed planar/network facility location problem. They analyze the p-median problem in a region with a network structure in some parts and a rectilinear structure in some other parts. Carrizosa, and Rodriguez-Chia [3] also address a p-facility minisum problem but with a metric induced by a *gauge* and a finite set of rapid transit lines. Mitchell and Papadimitriou [9] consider the problem of finding the shortest paths through a planar subdivision with weighted Euclidean metrics. Brimberg et al. [2] consider locating a facility in regions with varying norms, where the regions are half spaces, as opposed to the closed region we consider here. The unbounded half-planes allow some nice mathematical properties.

A related topic considers the location of facilities in the presence of regions that act as barriers to travel. For example, Dearing [4] examines the single facility location problem with rectilinear travel distances where specified regions act as barriers; that is travel through these regions is not permitted. Savas et al. [11] investigate a similar problem where the facility has a finite size. A global optimization approach is developed in McGarvey and Cavalier [8]. Other related references include Dearing et al. [5] and Wang et al. [12]. The model presented here may be viewed as a generalization of the location problem with barriers to travel. By assigning a high inflation factor to the norm k_1 in Ω_1 (see, e.g., Love et al. [7], ch.10, for a discussion of distance functions), we may penalize travel within Ω_1 to the extent that it becomes a barrier to travel.

The remainder of the paper is organized as follows. In the next section we examine the mathematical properties of our model. These general properties are applied in Section 3 to a simplified case. Section 4 builds a framework for finding exact and approximate solutions to the problem. We conclude the paper with a short summary and suggestions for future research.

2. Model Properties

The problem is depicted in Figure 1, which will be used to assist in developing some important properties. Here we have a nonconvex bounded polygonal region Ω_1 with distance norm k_1, and exterior region Ω_2 with norm k_2. Fixed points (or existing facilities) are located in both Ω_1 and Ω_2. The objective is to find the location X of a new facility that minimizes the cost function in (P1).

An important aspect of the problem is to determine the geodesic (or shortest) path as a function of X to each fixed point P_i. For example, consider the shortest distance from an interior point $X_1 \in \Omega_1$ to exterior point $P_1 \in \Omega_2$, shown in Figure 1. In this case, the geodesic path intersects the line segment $L = [V_1, V_2]$ of the boundary at some point Z. We present the following preliminary result.

Property 1 *Consider a fixed point $P \in \Omega_2$, an $X \in \Omega_1$, and a fixed (straight) line segment L of any orientation.*

Let

$$d(X,P) = \min_{Z\in L}\{k_1(X,Z) + k_2(Z,P)\}. \tag{1}$$

Then d is a convex function of X.

<u>Proof</u>: The proof follows analogously to the one in Lemma 3 of Brimberg et al. [2] where L is a line dividing $\Re^2$ into two half planes. We give it here for completeness.

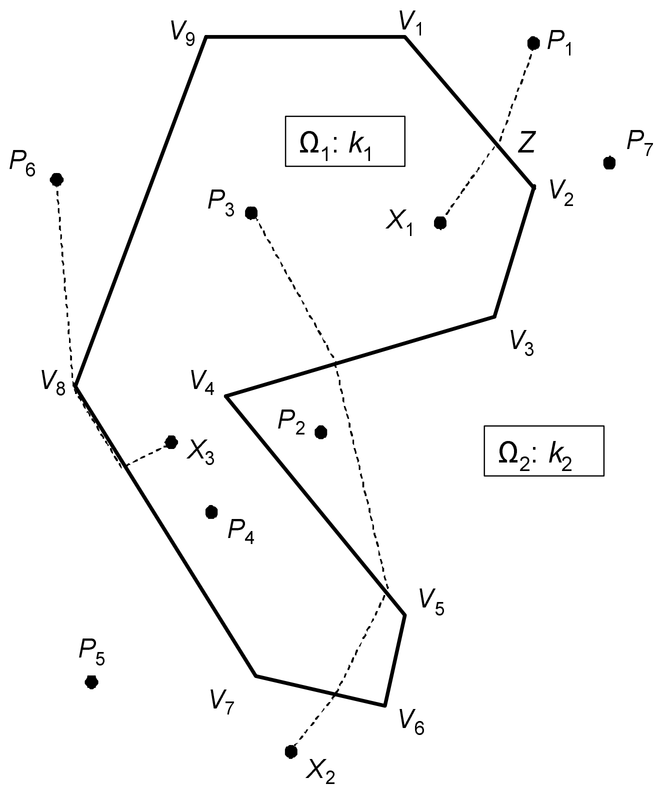

Figure 1: An illustration of the problem

Since k_1 and k_2 are norms, it follows that the composite distance function $D(X,Z) = k_1(X,Z) + k_2(Z,P)$ is a convex function in (X,Z). Therefore,

$$D(V,W) \leq \lambda D(X,Y) + (1-\lambda)D(U,Z)$$

where (X,Y), (U,Z) are any points in $\Re^4$ and

$$(V,W) = \lambda(X,Y) + (1-\lambda)(U,Z), \quad 0 \leq \lambda \leq 1.$$

Choose $Y, Z \in L$ such that $D(X,Y) = d(X,P)$, $D(U,Z) = d(U,P)$. It follows that:

$$W = \lambda Y + (1-\lambda)Z \in L$$

and

$$d(V,P) \leq D(V,W) \leq \lambda d(X,P) + (1-\lambda)d(U,P). \quad \square$$

Due to the nonconvex shape of Ω_1, the geodesic path may be more convoluted, passing through several boundary edges as shown going from $X_2 \in \Omega_2$ to $P_3 \in \Omega_1$ in Figure 1. We extend the preceding result to handle this case as follows.

Property 2 *Consider a fixed point P, fixed (straight) line segments $L_1, L_2, ..., L_n$ of arbitrary orientations and distances induced by arbitrary norms $k_0, k_1, ..., k_n$. Let*

$$d(X,P) = \min_{Z_i \in L_i, i=1,...,n} \{k_0(X,Z_1) + \sum_{i=1}^{n-1} k_i(Z_i, Z_{i+1}) + k_n(Z_n, P)\}. \tag{2}$$

Then d is a convex function of X.

Proof: The proof follows in a similar fashion as before. For example, let $n = 2$, and define

$$D(X, Z_1, Z_2) = k_0(X, Z_1) + k_1(Z_1, Z_2) + k_2(Z_2, P).$$

Then D is convex in (X, Z_1, Z_2). Let $(X_1, Y_1, Y_2), (X_2, Z_1, Z_2)$ be any two points in $\Re^6$, and $(V_1, V_2, V_3) = \lambda(X_1, Y_1, Y_2) + (1 - \lambda)(X_2, Z_1, Z_2), \quad 0 \le \lambda \le 1$. It follows that

$$D(V_1, V_2, V_3) \le \lambda D(X_1, Y_1, Y_2) + (1 - \lambda)D(X_2, Z_1, Z_2).$$

Choose $Y_1, Z_1 \in L_1,\ Y_2, Z_2 \in L_2$, and such that $D(X_1, Y_1, Y_2) = d(X_1, P),\ D(X_2, Z_1, Z_2) = d(X_2, P)$. We get: $V_2 = \lambda Y_1 + (1 - \lambda)Z_1 \in L_1,\ V_3 = \lambda Y_2 + (1 - \lambda)Z_2 \in L_2$, and

$$d(V_1, P) \le D(V_1, V_2, V_3) \le \lambda d(X_1, P) + (1 - \lambda)d(X_2, P). \quad \square$$

The geodesic path between two points may also contain segments of the boundary separating Ω_1 and Ω_2. This is shown in Figure 1 for the shortest distance between X_3 and P_6. Here,

$$d(X_3, P_6) = \min_{Z \in [V_7, V_8]} \{k_1(X_3, Z) + k_2(Z, V_8)\} + k_2(V_8, P_6), \tag{3}$$

where it is assumed that k_1 is the "slow" norm and k_2 is the "fast" norm. Such a case is equivalent to the case examined in Property 2 where one or more edges (L_i) degenerate to single points. It follows again that $d(X, P)$ is a convex function of X.

Given a point X, we can determine the shortest path to each P_i, and then construct the objective function, $f(X) = \sum_{i=1}^{N} w_i d(X, P_i)$. From the preceding results, we may conclude that $f(X)$ is the sum of convex functions, and hence, is itself convex over a subset of $\Re^2$over which the shortest path to each P_i is thus defined (see(1) and (2)).

Aside from finding these shortest paths for a given X, the main difficulty in solving (P1) is that these paths (or combinations of edges used) change as a function of X. For example consider the fixed point P_7 in Figure 1, and let $X \in \Omega_1$. Depending on the X chosen, the shortest path to P_7 could conceivably intersect any one of the boundary edges or a combination thereof. Consider, for example, the trajectory of X satisfying the following equation:

$$k_1(X, Z_1^*(X)) + k_2(Z_1^*(X), P_7) = k_1(X, Z_2^*(X)) + k_2(Z_2^*(X), P_7) \tag{4}$$

where $Z_1^*(X), Z_2^*(X)$ are intersection points of the shortest paths through $L_1 = [V_1, V_2]$ and $L_2 = [V_2, V_3]$, respectively. Along this trajectory, the shortest path from X to P_7 is indifferent to edges L_1 and L_2; however, L_1 is the preferred edge on one side of the trajectory, and L_2 on the other. It follows in this way that Ω_1 (and similarly Ω_2) can be divided into a finite number of subsets such that the shortest path from X to P_i uses an identical sequence of edges, $\forall X$ in each subset.

We see that the functional form of $f(X)$ will change whenever a better sequence of edges becomes available for constructing the geodesic path to one of the fixed points. The functional form of $f(X)$ may also change for a second reason: a given sequence of edges (or path) becomes infeasible. This key property of the model is illustrated in Figure 2. The shortest path from X to P_i crosses the edge $[V_3, V_4]$. The similar path from X' to P_i, giving $d(X', P_i) = \min_{Z \in [V_3, V_4]} \{k_2(X', Z) + k_1(Z, P_i)\}$, is no longer feasible, since the line joining X' and the optimal intersection point, $Z^* \in [V_3, V_4]$, crosses the Ω_1 region (i.e., intersects another edge first).

Definition 1 *A sequence of edges, L_t, $t = 1, ..., n$, is said to be feasible between two points if the corresponding sequence of intersection points, Z_t^*, $t = 1, ..., n$, giving the shortest path for that sequence (see (2)), results in a path that does not intersect any edges other than the specified sequence. The path so constructed is termed a feasible path. Note that a feasible path in this context always refers to the shortest path for a given sequence of edges.*

Consider again the feasible paths from X to P_i in Figure 2. As shown in the figure, let Z_1^* denote the intersection point, also referred to as the "gate" point, of the feasible path with leading edge $L_1 = [V_3, V_4]$. Note that Z_1^* is a function of X and P_i. We obtain the following useful result.

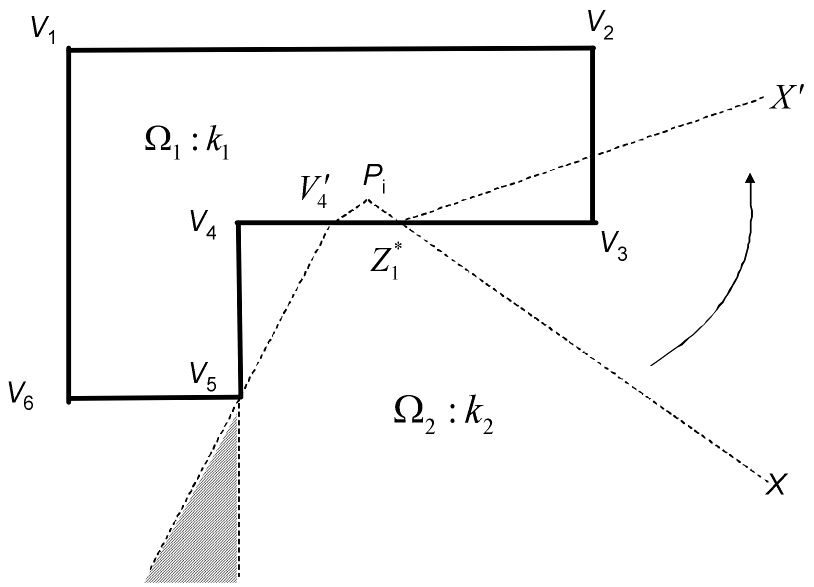

Figure 2: Infeasible paths and lines of vision

Property 3 *Consider the ray, starting at Z_1^* that joins Z_1^* to X and continues beyond X until another boundary edge is reached, and let Y be any point on this ray. The feasible path from Y to P_i using L_1 as the leading edge intersects L_1 at the same "gate" point Z_1^* as for X.*

<u>Proof:</u> Assume without loss of generality that k_1 and k_2 are differentiable round norms (see for example, Drezner [6], chapter 1, for a classification of round norms and block norms). The directional derivative of $D(Y, Z) = k_2(Y, Z) + k_1(Z, P_i)$ evaluated along the edge L_1 at $Z = Z_1^*$ is given by:

$$\frac{d}{ds} D(Y, Z_1^*) = \frac{d}{ds} k_2(Y, Z_1^*) + \frac{d}{ds} k_1(Z_1^*, P_i). \quad (5)$$

But

$$(Y - Z_1^*) = r(X - Z_1^*), \quad (6)$$

where $r > 0$ since X and Y are on the same ray from Z_1^*. Thus

$$\begin{aligned} &\frac{d}{ds}k_2(Y, Z_1^*) = \lim_{\|\delta Z\| \to 0} \left\{ \frac{k_2(Y, Z_1^* + \delta Z) - k_2(Y, Z_1^*)}{\|\delta Z\|} \right\} \\ &= \lim_{\|\delta Z\| \to 0} \left\{ \frac{r(k_2(X, Z_1^* + \delta Z/r) - k_2(X, Z_1^*))}{\|\delta Z\|} \right\} \\ &= \lim_{\|\delta Z\| \to 0} \left\{ \frac{(k_2(X, Z_1^* + \delta Z/r) - k_2(X, Z_1^*))}{\|\delta Z/r\|} \right\} \\ &= \frac{d}{ds}k_2(X, Z_1^*) \end{aligned} \tag{7}$$

We conclude that

$$\frac{d}{ds}D(Y, Z_1^*) = \frac{d}{ds}k_2(X, Z_1^*) + \frac{d}{ds}k_1(Z_1^*, P_i) = 0 \tag{8}$$

and hence Z_1^* is also on the shortest path from Y to P_i that uses L_1. □

Property 3 extends, in a straightforward fashion, to feasible paths that traverse more than one boundary edge. Furthermore, we may use this property to divide the plane in polygonal regions for each P_i such that given paths to P_i are feasible only for those X in the corresponding polygonal regions. For example, referring to Figure 2, we find the gate point V_4' on edge $[V_3, V_4]$ for the corresponding feasible path from vertex V_5 to demand point P_i; that is,

$$V_4' = \arg \min_{Z \in [V_3, V_4]} \{k_2(V_5, Z) + k_1(Z, P_i)\}.$$

It follows that the path from X to P_i that uses edge $L_1 = [V_3, V_4]$, and only this edge, is feasible only for those points X belonging to the lower right-hand quadrant formed by $[V_4, V_5]$ and $[V_4, V_3]$ plus the wedge (shaded area) extending below V_5 formed by the extensions of $[V_4, V_5]$ and the ray from V_4' to V_5.

We are now ready to state the main result.

Theorem 1 *Problem (P1) is equivalent to solving a finite number of convex programs.*

Proof: From Property 3 (and the subsequent discussion), it follows that the plane may be subdivided into a finite number of polygonal regions, $S_k,\ k = 1, ..., K$, such that a finite number of candidate shortest paths exists to each P_i, where each path is feasible, for all points $X \in S_k$. These regions may be further subdivided into a finite number of convex polygonal regions, $S_k',\ k = 1, ..., K'$, as required.

Furthermore, each alternate form of the objective function obtained by all possible combinations of feasible paths is a convex function by Properties 1 and 2. Therefore, we conclude that the optimal solution of (P1) may be found by solving a finite number of convex programs and retaining the best solution of all of them. □

The shape of the objective function may now be characterized as follows:

1) In any polygonal region, S_k', $f(X)$ is the minimum of a set of convex functions; i.e.,

$$f(X) = \min\{f_{1k}(X), ..., f_{N_k k}(X)\},\ \forall X \in S_k',$$

where each $f_{rk},\ r = 1, ..., N_k$, is a convex function of X. If the unrestricted minimum of f_{rk} given by X_{rk}^* is an interior point of S_k', and $f(X_{rk}^*) = f_{rk}(X_{rk}^*)$, with no ties then X_{rk}^* is a local minimum of $f(X)$ and a candidate solution.

2) Different sets of functions $\{f_{1k}, ..., f_{N_k k}\}$ apply to different S_k'. Thus, a local minimum (and candidate solution) may also occur on the boundary separating adjacent S_k'.

The characterization of the objective function given above is studied in the next section for a special case.

3. A Special Case: Formulation and Analysis

We now consider the case of a rectangular area where distances are rectilinear on the inside and Euclidean on the outside. This gives $\Omega_1 : \ell_1$ and $\Omega_2 : \ell_2$, as shown in the Figure 3.

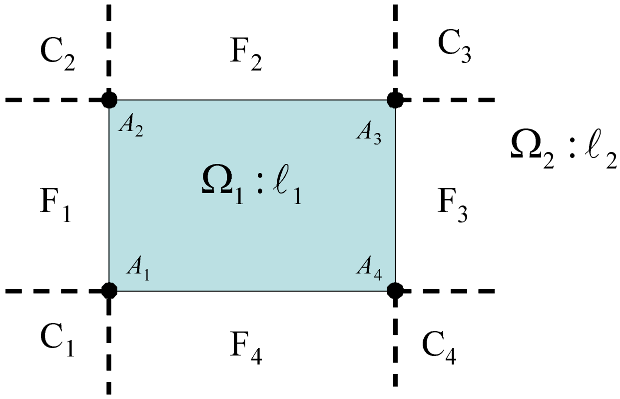

Figure 3: Labeling regions and vertices

Figure 3 divides the plane into regions. C_1, C_2, C_3, and C_4 will be referred to as "corner regions, and will be of particular interest. As shown in Figure 4, even this simplified case produces a wide variety of possible distances. Note the path $P_5 - P_6$, which was derived in Brimberg et al. [2].

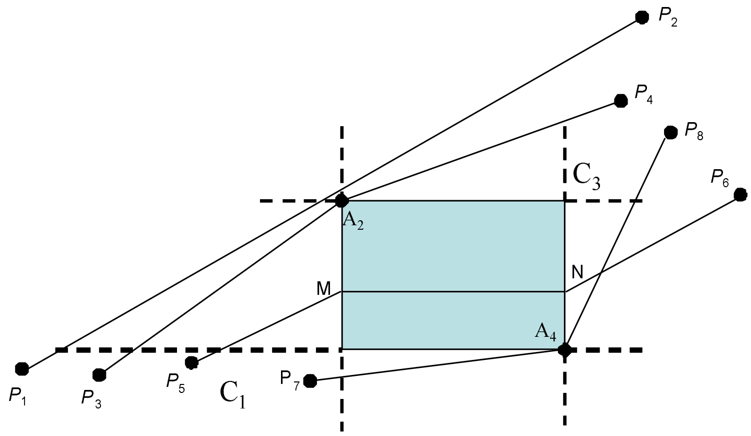

Figure 4: Sample paths

For illustrative and conceptual purposes we will simplify the situation further by restricting the facility location to the interior of the rectangle.

Without loss of generality, A_1 is set to (0,0) in Figure 5b. This figure illustrates that, through some algebraic manipulations, applied to the type of relation in (4), a corner region can be divided into two regions with a curve:

$$b = \frac{\sqrt{y}((x-y)\sqrt{x(y-2a)+2a^2} - \sqrt{x}\sqrt{y}(x-y-a))}{\sqrt{x}(2y-x)} \text{ for x} \neq 2y \tag{9}$$

such that in one the shortest path will be through some Z_1^* on the left side of the rectangle, and in the other through some point Z_2^* on the bottom side of the rectangle. This analysis

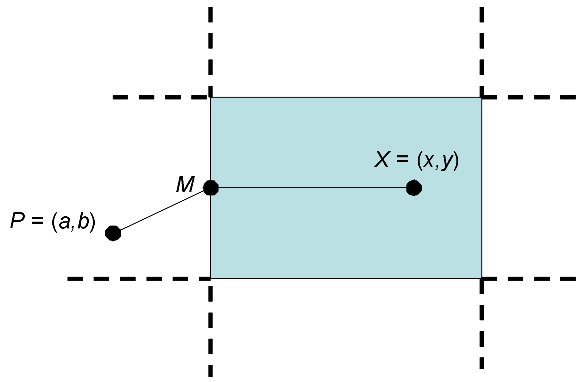

Figure 5a: Non-corner region to interior

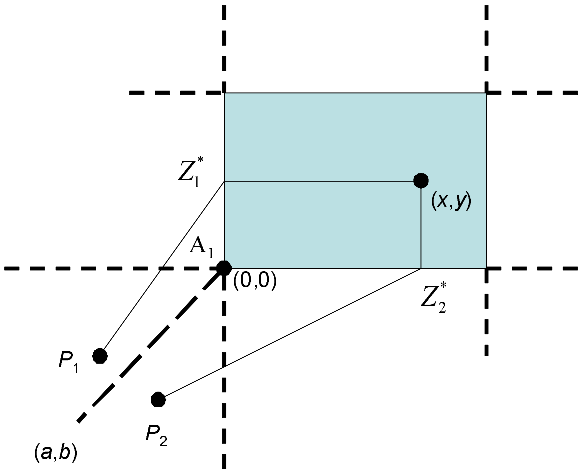

Figure 5b: Corner region to interior

can be transferred to the other corner regions by axis translation and rotation. We will call the curve in (9) *the corner demarcation curve.*

Each point in the corner region similarly creates an *interior demarcation curve*,

$$y = \frac{x((a-b)\sqrt{x^2 - 2ax + a^2 + b^2} + x(b-a) + a^2 - ab + b^2)}{a^2 - 2ax(a-b)} \tag{10}$$

which is illustrated for point (-4,-3) in Figure 6.

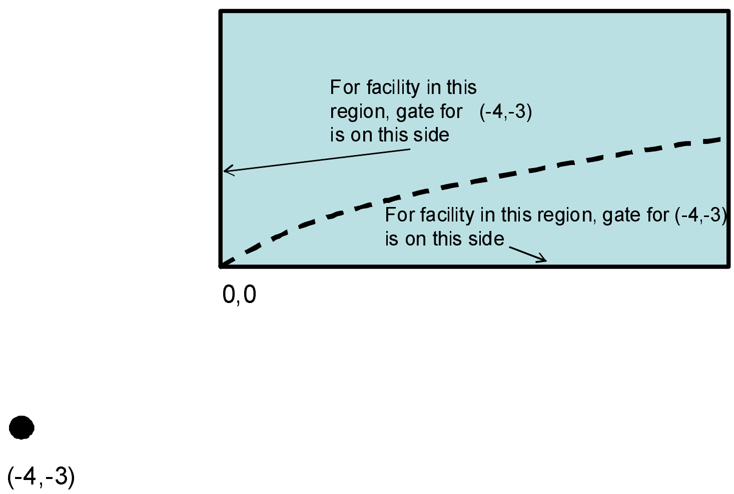

Figure 6: Example of interior demarcation curve

A set of curves for different corner points is shown in Figure 7.

The exterior of the rectangle can now be divided into four regions R_1, R_2, R_3 and R_4, which determine the paths through the sides of the rectangle for each exterior point as shown in Figure 8. Unfortunately, the R_j's are functions of (x,y).

The objective function for minimizing the sum of weighted distances (for $X \in \Omega_1$) is

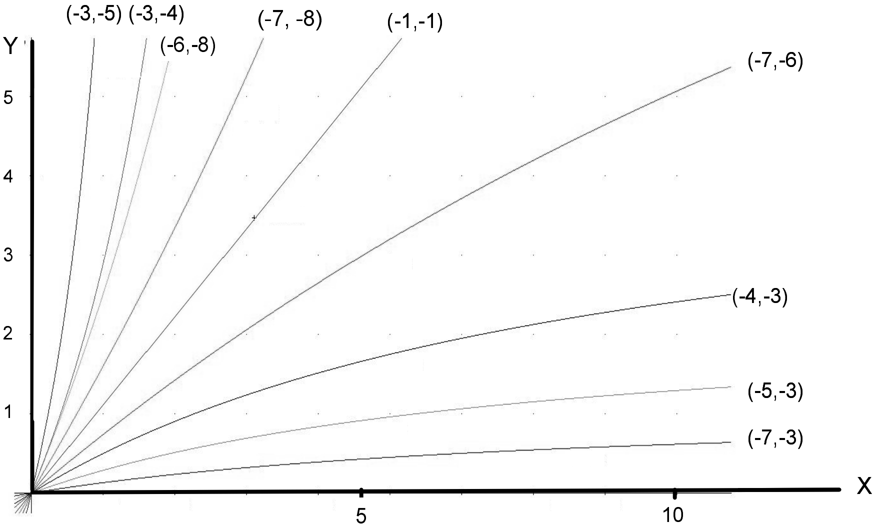

Figure 7: Interior demarcation curves for various corner points

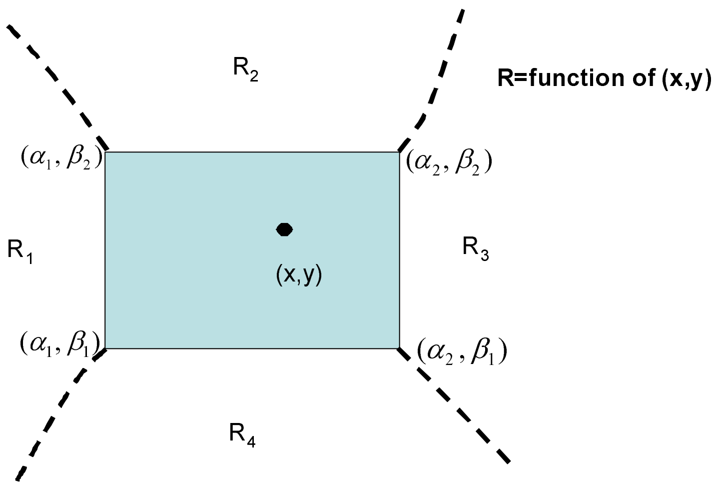

Figure 8: Notation

given by:

$$\begin{aligned} f(x,y) = & \sum_{P_i\in R_1} w_i\left((x-\alpha_1)+\sqrt{(a_i-\alpha_1)^2+(b_i-y)^2}\right) \\ & + \sum_{P_i\in R_2} w_i\left(\sqrt{(a_i-x)^2+(b_i-\beta_2)^2}+(\beta_2-y)\right) \\ & + \sum_{P_i\in R_3} w_i\left((\alpha_2-x)+\sqrt{(a_i-\alpha_2)^2+(b_i-y)^2}\right) \\ & + \sum_{P_i\in R_4} w_i\left(\sqrt{(a_i-x)^2+(b_i-\beta_1)^2}+(y-\beta_1)\right) \\ & + \sum_{P_i\in \Omega_1} w_i\left(|x-a_i|+|y-b_i|\right) \end{aligned} \tag{11}$$

If we ignore that the R_j's are functions of (x,y), we can obtain a 'false separability':

$$\begin{aligned} f_1(x) = & \sum_{P_i\in R_1} w_i(x-\alpha_1)+\sum_{P_i\in R_2} w_i\sqrt{(a_i-x)^2+(b_i-\beta_2)^2}+\sum_{P_i\in R_3} w_i(\alpha_2-x) \\ & +\sum_{P_i\in R_4} w_i\sqrt{(a_i-x)^2+(b_i-\beta_1)^2}+\sum_{P_i\in \Omega_1} w_i|x-a_i|, \end{aligned} \tag{12a}$$

$$\begin{aligned} f_2(y) = & \sum_{P_i\in R_1} w_i\sqrt{(a_i-\alpha_1)^2+(b_i-y)^2}+\sum_{P_i\in R_2} w_i(\beta_2-y) \\ & +\sum_{P_i\in R_3} w_i\sqrt{(a_i-\alpha_2)^2+(b_i-y)^2}+\sum_{P_i\in R_4} w_i(y-\beta_1)+\sum_{P_i\in \Omega_1} w_i|y-b_i|. \end{aligned} \tag{12b}$$

This leads us to the idea that the objective function is both 'neighborhood convex' and separable within regions created by the interior demarcation curves. This is illustrated in Figure 9. Note that the convexity derives directly from Properties 1 and 2 from the previous section.

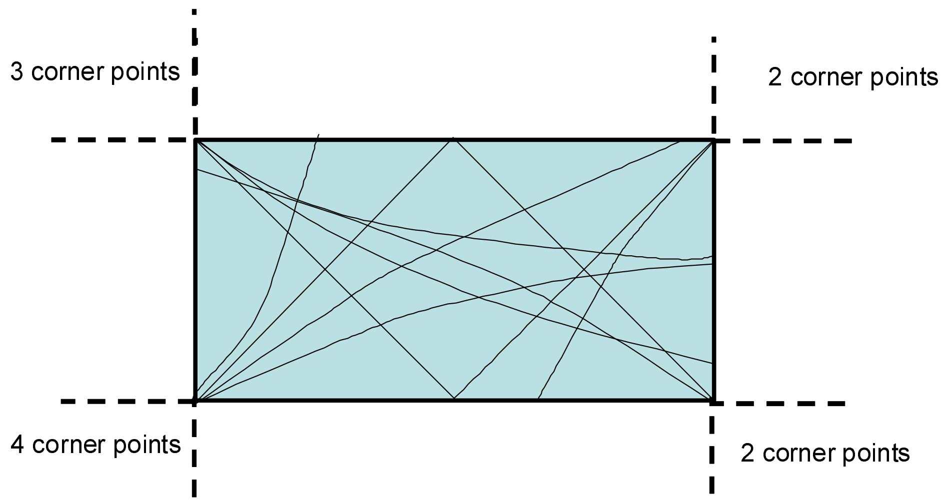

Figure 9: Interior area segments created by the interior demarcation curves

Referring to Figure 5b, we see that there are two feasible paths to be considered between any $X \in \Omega_1$ and corner point P_i. This implies, in compliance with Theorem 1, that up to $O(2^N)$ convex programs of the form

$$\min f_t(x), \quad s.t.\ X \in \Omega_1, \tag{13}$$

must be solved to find the optimal solution in the rectangle. However, from Figure 9 it is seen that each interior demarcation curve intersects no more than N other ones, so that

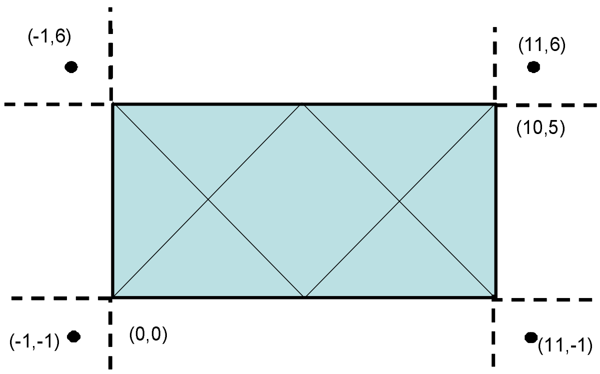

Figure 10a: Interior area segments

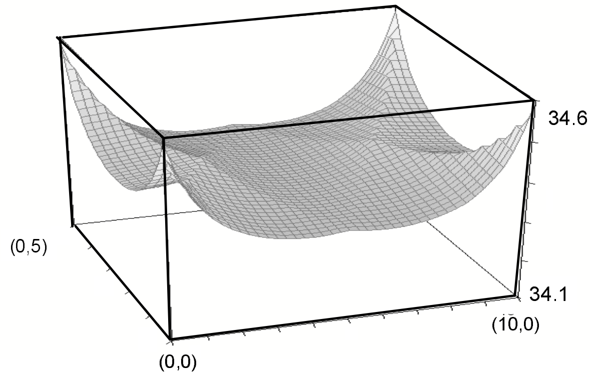

Figure 10b: Surface plot of objective function

the number of convex programs to be solved may be reduced to O(N^2). From a practical point of view, obtaining the complete set of (convex) objective functions corresponding to the interior area segments (see Figure 9), and minimizing each of these functions over the rectangle, may be an onerous task.

The shape of the objective function along any slice (or cross section) is seen to be piecewise convex with 'ridges' defined by the demarcation lines. The optimal solution within the rectangle is thus either an unrestricted minimum of one of the convex functions if it occurs in the interior of the rectangle, or it is a minimum on the boundary of the rectangle. This follows the characterization of the objective function discussed in Section 2. Also, note that the demarcation lines (as shown in Figure 9) are not needed explicitly in the solution process, but serve the purpose of identifying a smaller number of candidate sub-problems (convex programs) to solve.

A simple example with four corner points is given in Figure 10.

The optimal location in the rectangle is: (3.949,0), (6.051,0), (3.949,5), or (6.051,5), which may also be shown to be globally optimal.

4. Optimization Strategies

A general framework for solving (P1) may be summarized as follows:

Algorithm 1

Step 1: Divide the plane into the minimum number of convex polygonal regions ($S'_k,\ k = 1, ..., K'$) such that all candidate shortest paths, defined as sequences of edges, are identified for each S'_k, and are feasible, $\forall X \in S'_k$.

Step 2: For each S'_k, construct a representative number of objective functions ($f_{rk}(X)$) using the alternate feasible paths for S'_k identified in Step 1. Solve separately each of the corresponding convex programs ($\min f_{rk}(X),\ s.t.\ X \in S'_k$).

The final solution is given by the best one obtained above. □

The representative objective functions may be selected in different ways in Step 2 of the algorithm. For example, a local search procedure may be implemented as follows.

1) Choose a random starting point, $X_0 \in S'_k$; set $t = 0$.

2) Construct the objective function $f_t(X)$ using shortest paths from X_t to all P_i.

3) Solve the convex program $\min\{f_t(X); X \in S'_k\}$, to obtain solution X^*_t.

4) If the shortest paths to X^*_t are unchanged from X_t (i.e., $f(X^*_t) = f_t(X^*_t)$), stop; else, $t \leftarrow t + 1,\ X_t \leftarrow X^*_t$, and return to Step 2.

The local search descends to a local minimum in S'_k and stops. The procedure would be repeated for each S'_k or a representative number of them. Also note that in the rare event X^*_t falls on a demarcation curve in Step 3, the tied shortest paths should be investigated as possible means of further descent.

To guarantee a global solution, an exhaustive search needs to be carried out in Step 2 of Algorithm 1. One possible strategy is use the Big Square-Small Square approach which is also frequently used in barrier and forbidden region problems (McGarvey et al. [8]). For example, in the special case examined in Section 3, we can divide the rectangle into 'bricks'. A lower bound on each brick is easily found by using the distance to the nearest corner or edge for each demand point, P_i, outside the rectangle. The optimal point inside the brick is easily obtained for those P_i within the rectangle.

Also, for bricks small enough, the exact solution can be found by solving a small number of associated sub-problems (see Figure 11). In this way, a branch-and-bound process may be implemented in an efficient manner.

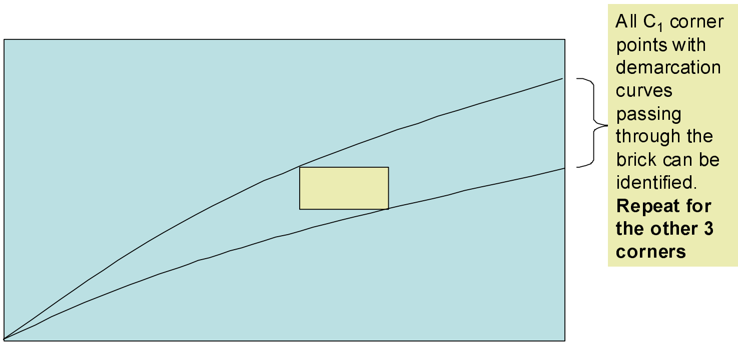

Figure 11: Finding the exact minimum inside a brick

5. Conclusions

In this paper we present a model for locating a single facility in the continuous plane where a closed bounded polygonal region exists with distance induced by one norm applying inside the region and distance induced by a different norm applying outside the region. The bounded region could represent, for example, a populated area where travel is slow compared to outside the area. By making travel very slow within the bounded region, this model may be viewed as a generalization of existing models in the literature where barriers to travel are examined. The analytical results and general solution approach presented are readily extended to problems with several bounded regions and several norms. We also illustrate the concepts by examining a special case.

Future research venues may include design, implementation, and testing of different solution approaches, and extension to the multi-facility problem.

Acknowledgement

This research was supported, in part, by the Natural Sciences and Engineering Research Council of Canada.

References

[1] R. Batta and U.S. Palekar: Mixed planar/network facility location problems. *Computers and Operations Research*, **15** (1988), 61-67.

[2] J. Brimberg, H. Kakhki, and G.O. Wesolowsky: Location among regions with varying norms. *Annals of Operations Research*, **122** (2003), 87-102.

[3] E. Carrizosa and A.M. Rodriguez-Chia: Weber problems with alternative transportation systems. *European Journal of Operational Research*, **97** (1997), 87-93.

[4] P.M. Dearing: An equivalence result for single facility planar location problems with rectilinear distance and barriers. *Annals of Operations Research*, **111** (2002), 89-111.

[5] P.M. Dearing, H.W. Hamacher, and K. Klamroth: Dominating sets for rectilinear center location problems with polyhedral barriers. *Naval Research Logistics*, **49** (2002), 647-665.

[6] Z. Drezner: *Facility Location: A Survey of Applications and Methods* (Springer Series in Operations Research, 1995).

[7] R.F. Love, J.G. Morris, and G.O. Wesolowsky: *Facilities Location, Models and Methods* (North-Holland, 1988).

[8] R.G. McGarvey, and T.M. Cavalier: A global optimal approach to facility location in the presence of forbidden regions. *Computers and Industrial Engineering*, **45** (2003), 1-15.

[9] J.S.B. Mitchell and C.H. Papadimitriou: The weighted region problem: Finding shortest paths through a weighted planar subdivision. *Journal of the Association for Computing Machinery*, **38** (1991), 18-73.

[10] M. Parlar: Single facility location problem with region-dependent distance metrics. *International Journal of Systems Science*, **25** (1994), 513-525.

[11] S. Savas, R. Batta, and R. Nagi: Finite-size facility placement in the presence of barriers to rectilinear travel. *Operations Research*, **50** (2002), 1018-1031.

[12] S.J. Wang, J. Bhadury and R. Nagi: Supply facility and input/output point locations in the presence of barriers. *Computers and Operations Research*, **29** (2002), 685-699.

George O. Wesolowsky
Michael G. DeGroote School of Business,
McMaster University,
Hamilton, ON, L8S 4M4, Canada
E-mail: wesolows@mcmaster.ca