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OPTIMAL HOSTAGE RESCUE PROBLEM WHERE AN ACTION CAN ONLY BE TAKEN ONCE — CASE WHERE ITS EFFECTIVENESS VANISHES THEREAFTER —

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Abstract In this paper, we propose a mathematical model for an optimal hostage rescue problem. Suppose that a person is taken as a hostage and that a decision has to be made from among three alternatives: storm for rescue, wait up to the next point in time for an opportunity to present itself or take an action of negotiation which might save the situation. Here, it is assumed that the action of negotiation can only be taken once and its effectiveness completely vanishes thereafter. The objective is to find an optimal decision rule so as to maximize the probability of the hostage not being killed.

Keywords: Hostage rescue, dynamic programming, myopic property

1. Introduction

Hostage events frequently occur in many places all over the world for different reasons. Typical examples in recent years include:

1. A 44-year-old, knife-wielding man, took a female receptionist hostage at the Kyoto bureau of NHK in Japan on January 18, 2002. He barricaded himself in the building and demanded to speak to Prime Minister Junichiro Koizumi by telephone. Police officers stormed the site after four and a half hours, rescued the hostage and arrested the man [4].
2. Two Chechens hijacked a Russian airliner and enforced the flight with 167 passengers to land in the Saudi Arabian City of Medina on March 15, 2001. They demanded halting the genocide in Chechnya and sitting at the negotiating table to find a peaceful solution to the conflict. Saudi security forces stormed the hijacked Russian plane the next day, freeing remaining hostages. However, three people - a flight attendant, a hijacker and a passenger - were killed with several others injured [5].
3. A Myanmar man wielding a toy pistol stormed into the secure area of an airport in H.K. on July 31, 2000, and took a cleaning woman hostage, forced the woman to board a Cathay Pacific Boeing 747 scheduled to fly to Paris and England. He surrendered after two and a half hours [6].
4. A Spanish man hijacked a domestic Iberia Air Lives flight with 131 people on board on June 23, 1998, and demanded to be flown to Tel Aviv. However, a four-hour standoff ended when he surrendered peacefully to police after speaking to his psychiatrist [7].

In view of such a situation, a successful rescue of hostages has become an urgent issue to be tackled worldwide. In order to solve the problem, we think that the most important decision for the person in charge of a crisis settlement is the timing of hostage rescue operation, together with taking into account the safety of hostages, the demands of the perpetrators, and the repercussions of success or failure in a rescue attempt, and so on. The purpose of this paper is to propose a mathematical model for an optimal hostage rescue problem by using the concept of a sequential stochastic decision process and examine the properties of an optimal rescuing rule.

Up to the present, the author has examined a model for solving the problem in [2] where two decision alternatives, storming for rescue or waiting up to the next point in time, were available. However, as seen in many hostage events, negotiators can take various actions to deal with the perpetrator(s); for example, persuading the perpetrator(s) to surrender by subjecting him to a relative's voice, or submitting to his demands to be flown to another country, or providing a means of escape, paying the ransom, releasing his comrades in prison, and so on. Therefore, it is necessary to include such an action of negotiation in our rescue decision, i.e., we should make a rescue decision from among three alternatives: storm for rescue, wait up to the next point in time, or take an action of negotiation. The author has already proposed a basic model in [3] where such an action of negotiation can only be taken once and its effectiveness lasts up to the deadline. In this new paper we propose another model where such an action of negotiation can only be taken once and its effectiveness completely vanishes thereafter.

Unfortunately, concerning hostage rescue problems, with the exception of the author's two papers [2] [3], we have been unable to find any reference material utilizing a mathematical approach.

2. Model

The process in the sequential decision problem dealt with in this paper is defined as discrete-time model with a finite planning horizon. Let us number the points in time backward from the final point in time on the horizon, time 0, as 0, 1, $\cdots$, and so on. Further, let the time internal between two successive points, say times t and $t-1$, be called the period t. Here, we assume that storming for rescue is the only course of action at the deadline (time 0), prompted by some reason, say, the hostage's health condition, the degree of the perpetrator(s) desperation, and so on.

In this model, we suppose that one person is taken as a hostage at any given point in time t, and a decision has to be made from among three alternatives: storm for rescue, wait up to the next point in time for an opportunity to present itself or take an action of negotiation which might save the situation. For simplicity, by `S`, `W` and `A` let us denote the above three decisions, respectively. Further, let us assume that the action of negotiation can only be taken once and it is effective only at that time, i.e., the effectiveness of the action of negotiation completely vanishes thereafter.

Suppose that the action of negotiation has not yet been taken up to time t. Let p $(0 < p < 1)$ be the probability of the hostage being killed if the decision `S` is made, and let q and r $(0 < q < 1,\ 0 \le r < 1$ and $0 < q + r < 1)$ be the probabilities of the hostage being, respectively, killed and released if the decision `W` is made; accordingly, $1-q-r$ is the probability of the hostage being neither killed nor released; let $\lambda = 1-q-r$ $(0 < \lambda < 1)$. Now, noting the fact that taking an action of negotiation will influence the probabilities q and r to a greater or lesser degree, in this model let us assume that if an action of negotiation is taken, then the q and r thus far change into q' and r', respectively, and that the q' and r' return again to q and r thereafter; let $\lambda' = 1 - q' - r'$ $(0 < q' < 1,\ 0 \le r' < 1$, and $0 < \lambda' < 1)$. Here, the cases of $p = 0$, $p = 1$, $q = 0$, $q = 1$, $r = 1$, $q' = 0$, $q' = 1$, $r' = 1$,

$q + r = 1$ and $q' + r' = 1$ make the problem trivial. Accordingly, all are excluded from the definition of the model. The flow chart of decision in this model can be depicted as in Figure 1.

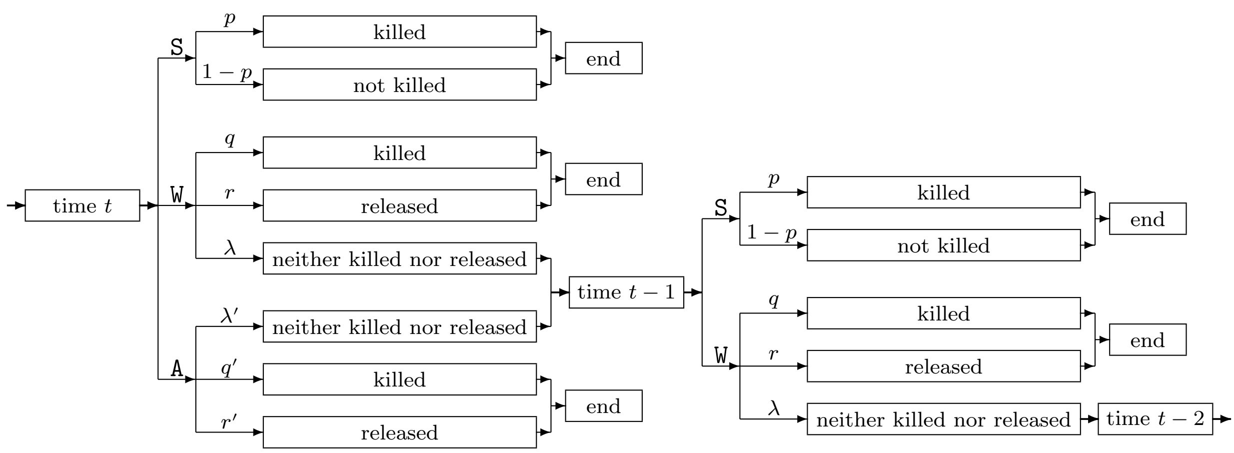

Figure 1: Flow chart of decision.

The objective here is to find the optimal decision rule so as to maximize the probability of the hostage not being killed.

3. Optimal Equation

Let S be the probability of the hostage not being killed at any time if the decision S is made. Then, we have

$$S = 1 - p. \tag{3.1}$$

Suppose that an action of negotiation *has not yet* been taken up to time t. In this case, let W_t (A_t) is the maximum probability of the hostage not being killed from times t to 0 (the deadline) if the decision W (A) is made, and further, let v_t be the maximum probability of the hostage not being killed, starting from time $t \geq 0$. Then, we get

$$v_0 = S, \tag{3.2}$$
$$v_t = \max\{S, W_t, A_t\}, \quad t \geq 1. \tag{3.3}$$

Suppose that an action of negotiation *has already* been taken up to time t. In this case, let W'_t is the maximum probability of the hostage not being killed from times t to 0 if the decision W is made, and further, let v'_t be the maximum probability of the hostage not being killed, starting from time $t \geq 0$. Then, we obtain

$$v'_0 = S, \tag{3.4}$$
$$v'_t = \max\{S, W'_t\}, \quad t \geq 1. \tag{3.5}$$

Therefore, the W_t, A_t and W'_t for $t \geq 1$ can be expressed as follows:

$$W_t = r + \lambda v_{t-1}, \tag{3.6}$$
$$A_t = r' + \lambda' v'_{t-1}, \tag{3.7}$$
$$W'_t = r + \lambda v'_{t-1}. \tag{3.8}$$

Note that the expression Eq.(3.6) should be written as $W_t = q \times 0 + r \times 1 + \lambda v_{t-1}$, we can see that the implication of Eq.(3.6). Similarly, Eqs.(3.7) and (3.8) can be also interpreted.

Here, for convenience in the later discussions, let us define

$$W_0 = A_0 = W'_0 = S, \tag{3.9}$$
$$U = r + \lambda S, \tag{3.10}$$
$$U' = r' + \lambda' S. \tag{3.11}$$

Then, Eqs.(3.3) and (3.5) also hold for $t = 0$, and from Eqs.(3.6) to (3.8) we have

$$W_1 = W'_1 = U, \tag{3.12}$$
$$A_1 = U'. \tag{3.13}$$

From Eq.(3.12) we see that U is the maximum probability of the hostage not being killed from times 1 to 0, provided that the decision `W` is made at time 1 whether or not the action of negotiation has been taken up to time 1. From Eq.(3.13) we see that U' is the maximum probability of the hostage not being killed from times 1 to 0, provided that the decision `A` is made at time 1.

4. Preliminaries

Lemma 4.1. *Both v_t and v'_t are nondecreasing in t for $t \geq 0$, and all of W_t, A_t and W'_t are nondecreasing in t for $t \geq 1$. Further, they converge to finite numbers v, v', W, A and W', respectively, as $t \to \infty$.*

Proof. From Eqs.(3.2) to (3.5) we have $v_1 \geq S = v_0$ and $v'_1 \geq S = v'_0$. Suppose $v_{t-1} \geq v_{t-2}$ and $v'_{t-1} \geq v'_{t-2}$. Then, $W_t \geq W_{t-1}$, $A_t \geq A_{t-1}$ and $W'_t \geq W'_{t-1}$ from Eqs.(3.6) to (3.8). Thus, $v_t = \max\{S, W_t, A_t\} \geq \max\{S, W_{t-1}, A_{t-1}\} = v_{t-1}$ and $v'_t = \max\{S, W'_t\} \geq \max\{S, W'_{t-1}\} = v'_{t-1}$. Accordingly, the monotonicities of v_t and v'_t in t hold. Further, the monotonicities of W_t, A_t and W'_t in t also hold from Eqs.(3.6) to (3.8). Now, since v_t, v'_t, W_t, A_t and W'_t are all bounded because they are all probabilities, it follows that their limits as $t \to \infty$ exist. □

Lemma 4.2. *Let $U > S$. Then,*

(a) $v'_t = W'_t$ *for* $t \geq 0$;

(b) $v_t = \max\{W_t, A_t\}$ *for* $t \geq 1$;

(c) *for* $t \geq 1$ *we have* $A_t = r' + \lambda' W'_{t-1}$, $W'_t = r + \lambda W'_{t-1}$ *and*

$$W'_t = r(1 - \lambda^{t-1})/(1 - \lambda) + \lambda^{t-1} U; \tag{4.1}$$

(d) $W' = r/(1 - \lambda)$;

(e) W'_t *is strictly increasing in* t *for* $t \geq 1$;

(f) *suppose that a certain* t° ($t^\circ \geq 1$) *exists such that* $v_{t^\circ} = A_{t^\circ}$, *then*

1. *if* $r/q \geq r'/q'$, *then* $v_t = W_t$ *for* $t \geq t^\circ + 1$;
2. *if* $r/q < r'/q'$, *then* $v_t = A_t$ *for* $t \geq t^\circ$.

Proof. (a , b) Noting Eqs.(3.4) and (3.9), we have $v'_0 = W'_0$. From Lemma 4.1., Eq.(3.12) and the assumption we get $W'_t \geq W'_1 = U > S$ and $W_t \geq W_1 = U > S$ for $t \geq 1$. Then, $v'_t = W'_t$ for $t \geq 1$ due to Eq.(3.5), and $v_t = \max\{W_t, A_t\}$ for $t \geq 1$ due to Eq.(3.3).

(c , d) Using Eqs.(3.7), (3.8) and (a), we obtain $A_t = r' + \lambda' W'_{t-1}$ and $W'_t = r + \lambda W'_{t-1}$ for $t \geq 1$. Further, from Eq.(3.12) we get $W'_t = r(1 + \lambda + \cdots + \lambda^{t-2}) + \lambda^{t-1} W'_1 = r(1 - \lambda^{t-1})/(1 - \lambda) + \lambda^{t-1} U$ for all $t \geq 1$. Thus, from this we have $W' = r/(1 - \lambda)$ as $t \to \infty$.

(e) From Lemma 4.1. and Eq.(3.12) we have $U = W_1' \le W_t' \le W'$ for $t \ge 1$, i.e., $W_1' \le W'$. If $W_1' = W'$, then $W_t' = U$ for $t \ge 1$. Thus, from this, Eqs.(3.8) and (3.10) we get $v_{t-1}' = S$. Accordingly, $S \ge W_{t-1}' \ge U$ from Eq.(3.5) and the result above, which contradicts the assumption $U > S$. Consequently, it must be $W_1' < W'$. Further, from $W_1' < W'$ and (d) we have $r-(1-\lambda)W_1' > 0$. Thus, noting (c), we get $W_2' - W_1' = r-(1-\lambda)W_1' > 0$. Suppose $W_{t-1}' - W_{t-2}' > 0$. Then, $r-(1-\lambda)W_{t-2}' > 0$. Therefore, $W_t' - W_{t-1}' = r-(1-\lambda)W_{t-1}' = \lambda(r-(1-\lambda)W_{t-2}') > 0$. Accordingly, W_t' is strictly increasing in t.

(f) Suppose $v_{t^\circ} = A_{t^\circ}$ with $t^\circ \ge 1$. Then, $W_{t^\circ+1} = r + \lambda A_{t^\circ}$ from Eq.(3.6).

(f1) Suppose $r/q \ge r'/q'$. Then, from (c) we have

$$W_{t^\circ+1} = r + \lambda A_{t^\circ} = r + \lambda(r' + \lambda' W_{t^\circ-1}') = r + r'\lambda + \lambda\lambda' W_{t^\circ-1}', \tag{4.2}$$

$$\begin{aligned} A_{t^\circ+1} - W_{t^\circ+1} &= (r' + \lambda' W_{t^\circ}') - (r + \lambda A_{t^\circ}) \\ &= r' + \lambda'(r + \lambda W_{t^\circ-1}') - r - \lambda(r' + \lambda' W_{t^\circ-1}') = r'q - rq' \le 0, \end{aligned} \tag{4.3}$$

hence $A_{t^\circ+1} \le W_{t^\circ+1}$. Further, from (b) we have $v_{t^\circ+1} = W_{t^\circ+1}$. Suppose $v_{t-1} = W_{t-1}$ for $t > t^\circ + 1$. Then, noting Eqs.(3.6) and (4.2), we have

$$\begin{aligned} W_t &= r + \lambda W_{t-1} = r(1 + \lambda + \cdots + \lambda^{t-t^\circ-2}) + \lambda^{t-t^\circ-1} W_{t^\circ+1} \\ &= r(1 + \lambda + \cdots + \lambda^{t-t^\circ-2}) + \lambda^{t-t^\circ-1}(r + r'\lambda + \lambda\lambda' W_{t^\circ-1}') \\ &= r(1-\lambda^{t-t^\circ})/(1-\lambda) + r'\lambda^{t-t^\circ} + \lambda'\lambda^{t-t^\circ} W_{t^\circ-1}', \qquad t > t^\circ + 1. \end{aligned} \tag{4.4}$$

Now, from (c) we get

$$W_{t-1}' = r + \lambda W_{t-2}' = r(1-\lambda^{t-t^\circ})/(1-\lambda) + \lambda^{t-t^\circ} W_{t^\circ-1}', \quad t > t^\circ + 1,$$

and further, using this and (c), we immediately obtain

$$A_t = r' + \lambda' W_{t-1}' = r' + r\lambda'(1-\lambda^{t-t^\circ})/(1-\lambda) + \lambda'\lambda^{t-t^\circ} W_{t^\circ-1}', \quad t > t^\circ + 1. \tag{4.5}$$

Therefore, it follows from Eqs.(4.4) and (4.5) that

$$A_t - W_t = (r'q - rq')(1-\lambda^{t-t^\circ})/(1-\lambda) \le 0, \quad t > t^\circ + 1,$$

i.e., $A_t \le W_t$. Thus, $v_t = W_t$ for $t > t^\circ + 1$ due to (b). Accordingly, $v_t = W_t$ for $t \ge t^\circ + 1$.

(f2) Suppose $r/q < r'/q'$. Then, it follows from $v_{t^\circ} = A_{t^\circ}$ that the assertion holds for $t = t^\circ$. Suppose $v_{t-1} = A_{t-1}$ for $t > t^\circ$. Then, noting (c) and Eq.(3.6), we obtain

$$\begin{aligned} A_t - W_t &= (r' + \lambda' W_{t-1}') - (r + \lambda A_{t-1}) \\ &= r' + \lambda'(r + \lambda W_{t-2}') - r - \lambda(r' + \lambda' W_{t-2}') = r'q - rq' > 0, \end{aligned}$$

i.e., $A_t > W_t$. Thus, $v_t = A_t$ for $t > t^\circ$ from (b). Accordingly, $v_t = A_t$ for $t \ge t^\circ$. □

5. Analysis

In this section, we examine the properties of the optimal decision rule for the model, classifying combinations of the parameters, p, q, r, q' and r' into the following three cases:

$$\text{Case 1}: \begin{cases} S \ge U, \\ S > U', \end{cases} \qquad \text{Case 2}: \begin{cases} U' \ge S, \\ U' \ge U, \end{cases} \qquad \text{Case 3}: \begin{cases} U > S, \\ U > U', \end{cases}$$

which exclusively and exhaustively include all the combinations of the parameters. The three cases mean the following: Suppose that the process starts from time 1. Then, the cases 1, 2 and 3 imply that, storming for rescue, taking an action of negotiation and waiting up to the time 0 are optimal, respectively.

5.1. Case of $S \geq U$ and $S > U'$

Theorem 5.1. *Let $S \geq U$ and $S > U'$. Then, $v_t = v'_t = S$ for $t \geq 1$.*

Proof. Suppose $S \geq U$ and $S > U'$. Then, $v_1 = v'_1 = S$ from Eqs.(3.3), (3.5), (3.12) and (3.13). Suppose $v_{t-1} = v'_{t-1} = S$. Then, from Eqs.(3.6) to (3.8), (3.10) and (3.11) we get $W_t = W'_t = r + \lambda S = U$ and $A_t = r' + \lambda' S = U'$. Thus, $v_t = v'_t = S$ for $t \geq 1$ due to Eqs.(3.3) and (3.5). □

5.2. Case of $U' \geq S$ and $U' \geq U$

Theorem 5.2. *Let $U' \geq S$ and $U' \geq U$.*

(a) *Suppose that $U \leq S$. Then, $v'_t = S$ and $v_t = A_t$ for all $t \geq 1$.*

(b) *Suppose that $U > S$. Then,*

1. *if $r/q < r'/q'$, then $v_t = A_t$ for $t \geq 1$;*
2. *if $r/q \geq r'/q'$, then $v_1 = A_1$ and $v_t = W_t$ for $t \geq 2$.*

Proof. Let $U' \geq S$ and $U' \geq U$. Then, $v_1 = A_1$ from Eqs.(3.3), (3.12) and (3.13).

(a) Suppose $U \leq S$. (i) From Eqs.(3.5) and (3.12) we have $v'_1 = S$. By induction starting with this, it can be proven that $v'_t = S$ for $t \geq 1$ holds from Eqs.(3.5), (3.8) and (3.10). (ii) Using $v'_t = S$ for $t \geq 1$, Eqs.(3.7), (3.11) and the assumption $U' \geq S$, we get

$$A_t = r' + \lambda' v'_{t-1} = r' + \lambda' S = U' \geq S, \quad t \geq 1. \tag{5.1}$$

Further, since $v_1 = A_1$, it follows that the assertion $v_t = A_t$ holds for $t = 1$. Suppose $v_{t-1} = A_{t-1}$. Then, $v_{t-1} = U'$ due to Eq.(5.1), and hence $W_t = r + \lambda U'$ from Eq.(3.6). Thus, noting Eqs.(5.1) and (3.10), we have

$$A_t - W_t = U' - (r + \lambda U') = (1-\lambda)U' - r \geq (1-\lambda)S - r = S - U \geq 0, \quad t \geq 1.$$

Accordingly, $v_t = A_t$ for $t \geq 1$ due to Eqs.(3.5) and (5.1).

(b1) Suppose $U > S$ and $r/q < r'/q'$. Then, from $v_1 = A_1$ and Lemma 4.2. (f2) with $t^\circ = 1$ we have $v_t = A_t$ for $t \geq 1$.

(b2) Suppose $U > S$ and $r/q \geq r'/q'$. Then, from $v_1 = A_1$ and Lemma 4.2. (f1) with $t^\circ = 1$ we have $v_t = W_t$ for $t \geq 2$. □

5.3. Case of $U > S$ and $U > U'$

In this case, we first define the symbol δ and parameter t_δ, and further, give a lemma. They will be used in Theorem 5.3. stated later.

$$\delta = -(r' - r)/(\lambda' - \lambda), \qquad \lambda' \neq \lambda, \tag{5.2}$$

$$t_\delta = \{t \mid W'_{t-1} < \delta \leq W'_t\}, \quad t_\delta \geq 2 \tag{5.3}$$

if it exists. It is clear from Lemma 4.2. (e) that the t_δ is unique if it exists.

Lemma 5.1.

(a) *If $U > S$ and $U > U'$, then $W_t = W'_t$ for $t \geq 1$.*

(b) *If $U > U'$ and $\lambda' > \lambda$, then $\delta > 0$.*

(c) *If $U > S$, $\lambda' > \lambda$ and $\delta < W'$, then $r/q < r'/q'$.*

Proof. (a) Suppose $U > S$ and $U > U'$. Then, from Eqs.(3.3), (3.12) and (3.13) we have $v_1 = W_1 = W'_1$. Suppose $v_{t-1} = W_{t-1} = W'_{t-1}$. Then, $W_t = r + \lambda W'_{t-1} = W'_t$ for $t \geq 1$ due to Eq.(3.6) and Lemma 4.2. (c).

(b) Since $U > U'$, we have $-(\lambda' - \lambda)S > r' - r$ due to Eqs.(3.10) and (3.11). Then, the assumption $\lambda' > \lambda$ yields $r' < r$ due to $S > 0$. Thus, $\delta > 0$ from Eq.(5.2).

(c) Suppose $U > S$. Then, from Lemma 4.2. (d) and Eq.(5.2) we get $\delta - W' = (rq' - r'q)/(1-\lambda)(\lambda'-\lambda)$. Thus, the assertion is true due to the assumption $\lambda' > \lambda$ and $\delta < W'$. □

Theorem 5.3. *Let $U > S$ and $U > U'$.*

(a) *Suppose that $\lambda' \leq \lambda$. Then, $v_t = W_t$ for $t \geq 1$.*

(b) *Suppose that $\lambda' > \lambda$. Then, $\delta > 0$. Further,*

1. *if $W' \leq \delta$, then $v_t = W_t$ for $t \geq 1$;*
2. *if $U < \delta < W'$, then there exists a unique t_δ ($t_\delta \geq 2$) such that $v_t = W_t$ for $1 \leq t \leq t_\delta$ and $v_t = A_t$ for $t > t_\delta$;*
3. *if $\delta \leq U$, then $v_1 = W_1$ and $v_t = A_t$ for $t \geq 2$.*

Proof. Let $U > S$ and $U > U'$. Then, $v_1 = W_1$ from Eqs.(3.3), (3.12) and (3.13).

(a) Suppose $\lambda' \leq \lambda$. Then, from Lemmas 5.1. (a), 4.2. (c), 4.1., Eqs.(3.12) and (3.10) we have

$$\begin{aligned} A_t - W_t = A_t - W'_t &= (r' + \lambda' W'_{t-1}) - (r + \lambda W'_{t-1}) \\ &= (\lambda' - \lambda) W'_{t-1} + r' - r \leq (\lambda' - \lambda) U + r' - r \\ &< (\lambda' - \lambda) S + r' - r = U' - U < 0, \quad t \geq 2. \end{aligned}$$

Hence, $v_t = W_t$ for $t \geq 2$ due to Lemma 4.2. (b). Accordingly, $v_t = W_t$ for $t \geq 1$.

(b) Suppose $\lambda' > \lambda$. Then, from Lemma 5.1. (b) we get $\delta > 0$.

(b1) Suppose $W' \leq \delta$. Then, $W'_t \leq \delta$ for all $t \geq 1$ due to Lemma 4.1.. Hence, from Eq.(5.2) we get $(\lambda' - \lambda) W'_{t-1} + r' - r \leq 0$ for $t \geq 2$. From this, Lemmas 5.1. (a) and 4.2. (c) we can obtain

$$A_t - W_t = A_t - W'_t = (\lambda' - \lambda) W'_{t-1} + r' - r \leq 0, \quad t \geq 2.$$

Hence, $v_t = W_t$ for $t \geq 2$ due to Lemma 4.2. (b). Accordingly, $v_t = W_t$ for $t \geq 1$.

(b2) Suppose $U < \delta < W'$. Then, from Lemma 5.1. (c) we have $r/q < r'/q'$, and from Eq.(3.12) we have $W'_1 < \delta < W'$. Therefore, it is from Lemma 4.2. (e) and Eq.(5.3) that there must exist a unique t_δ ($t_\delta \geq 2$). Hence, $W'_t < \delta$ for $1 \leq t < t_\delta$ and $W'_t \geq \delta$ for $t \geq t_\delta$, i.e., $W'_{t-1} < \delta$ for $2 \leq t \leq t_\delta$ and $W'_t \geq \delta$ for $t \geq t_\delta$. From these and Eq.(5.2) we get

$$(\lambda' - \lambda) W'_{t-1} + r' - r < 0, \quad 2 \leq t \leq t_\delta, \tag{5.4}$$

$$(\lambda' - \lambda) W'_t + r' - r \geq 0, \quad t \geq t_\delta. \tag{5.5}$$

Using Lemmas 5.1. (a), 4.2. (c) and Eq.(5.4), we obtain

$$A_t - W_t = A_t - W'_t = (\lambda' - \lambda) W'_{t-1} + r' - r < 0, \quad 2 \leq t \leq t_\delta.$$

Hence, $v_t = W_t = W'_t$ for $2 \leq t \leq t_\delta$ due to Lemma 4.2. (b). Accordingly, $v_t = W_t$ for $1 \leq t \leq t_\delta$. Further, from Eq.(3.6) we have $W_{t_\delta+1} = r + \lambda W'_{t_\delta}$. Noting Lemma 4.2. (c) and Eq.(5.5), we get $A_{t_\delta+1} - W_{t_\delta+1} = (\lambda' - \lambda) W'_{t_\delta} + r' - r \geq 0$. Hence, $v_{t_\delta+1} = A_{t_\delta+1}$ due to Lemma 4.2. (b). Therefore, it is from $r/q < r'/q'$ and Lemma 4.2. (f2) with $t^\circ = t_\delta + 1$ that $v_t = A_t$ for $t \geq t_\delta + 1$, i.e., $t > t_\delta$.

(b3) Suppose $\delta \leq U$. Then, $\delta \leq U = W'_1 \leq W'_t$ for $t \geq 1$ due to Lemma 4.1.. Hence, $W'_{t-1} \geq \delta$ for $t \geq 2$. Further, from Eq.(5.2) we have $(\lambda' - \lambda) W'_{t-1} + r' - r \geq 0$ for $t \geq 2$. Thus, from this, Lemmas 5.1. (a) and 4.2. (c) we get

$$A_t - W_t = A_t - W'_t = (\lambda' - \lambda) W'_{t-1} + r' - r \geq 0, \quad t \geq 2.$$

Accordingly, $v_t = A_t$ for $t \geq 2$ Lemma 4.2. (b). □

6. Conclusions

Note that an action of negotiation has not yet been taken at time t when the hostage taking occurs. Accordingly, by the definition of model, at that time t, it is sufficient to consider only v_t. Once the action of negotiation is taken at a certain time t' ($t' \leq t$), it is sufficient to consider only $v'_{t''}$ for $t'' \leq t'$ thereafter so long as the hostage is not released. Therefore, from Theorems 5.1., 5.2., 5.3. and Lemma 4.2. (a), it is seen that this model reveal that any one of the following five decisions is optimal.

DE-A Storm for rescue immediately.

DE-B Take an action of negotiation, and if the hostage is not released up to the next time, i.e., time $t-1$, then storm for rescue at time $t-1$.

DE-C Take an action of negotiation, and if the hostage is not released up to the next time, i.e., time $t-1$, then wait from time $t-1$ to the deadline.

DE-D Wait up to the deadline.

DE-E Wait up to time 1 and take an action of negotiation at time 1, and if the hostage is not released up to the deadline, then storm for rescue at the deadline.

Further, after analyzing Theorems 5.1., 5.2., 5.3. and Lemma 4.2. (a), we can exhaustively prescribe the optimal decision rules of this model by using the above five decisions as follows.

Optimal Decision Rule 6.1.

(a) Suppose that $S \geq U$ and $S > U'$. Then, DE-A is optimal for $t \geq 1$ (see Theorem 5.1.).

(b) Suppose that $U' \geq S$ and $U' \geq U$.
  1. Suppose that $U \leq S$. Then, DE-B is optimal for $t \geq 1$ (see Theorem 5.2. (a)).
  2. Suppose that $U > S$. Then,
     i. if $r/q < r'/q'$, then DE-C is optimal for $t \geq 1$ (see Theorem 5.2. (b1) and Lemma 4.2. (a));
     ii. if $r/q \geq r'/q'$, then DE-B is optimal for $t = 1$ and DE-E is optimal for $t \geq 2$ (see Theorem 5.2. (b2)).

(c) Suppose that $U > S$ and $U > U'$.
  1. Suppose that $\lambda' \leq \lambda$. Then, DE-D is optimal for $t \geq 1$ (see Theorem 5.3. (a)).
  2. Suppose that $\lambda' > \lambda$. Then,
     i. if $W' \leq \delta$, then DE-D is optimal for $t \geq 1$ (see Theorem 5.3. (b1));
     ii. if $U < \delta < W'$, then there exists a unique t_δ ($t_\delta \geq 2$), and hence DE-D is optimal for $1 \leq t \leq t_\delta$ and DE-C is optimal for $t > t_\delta$ (see Theorem 5.3. (b2) and Lemma 4.2. (a));
     iii. if $\delta \leq U$, then DE-D is optimal for $t = 1$ and DE-C is optimal for $t \geq 2$ (see Theorem 5.3. (b3) and Lemma 4.2. (a)).

Here, we will introduce the concept of *myopic property* used in later discussion.

In general, an optimal decision rule of a sequential decision process depends on time t. However, for some cases, the optimal decision becomes independent of time t although such cases are rare. In these cases, the optimal decision rule for any time t is the same as that for time 1. This implies that it is optimal to behave always *as if* only a single period of planning horizon remains; in other words, it is optimal to behave always *as if* the next point in time is the deadline. This property is usually called as the *myopic property* [1]. We may note that it is quite a singular property.

In this paper we adopt a definition of myopic property that differs from the conventional one mentioned above. By $\mathcal{T}$ let us denote a given set of time t for a specified optimal decision rule, if it is optimal to behave for all $t \in \mathcal{T}$ as if only τ periods remain up to the deadline, then let the optimal decision rule be said to be a *τ-myopic property on $\mathcal{T}$*. Accordingly, it follows that the conventional definition of the myopic property is '1-myopic property on $\mathcal{T} = \{1, 2, \cdots\}$'.

Now, in order to make the Optimal Decision Rule 6.1. more understandable, let us summarize it as in Table 1. In the table, let DE-$\mathrm{X}_{\mathbb{T}}$ imply that DE-X is optimal for all t for which the statement $\mathbb{T}$ (referring to a time t when the hostage event occurs) is true.

Table 1: Summary of optimal decision rules.

$S \geq U,\ S > U'$			DE-$\mathrm{A}_{t\geq 1}$	: 1-myopic on $[1, 2, \cdots)$
$U' \geq S,\ U' \geq U$	$U \leq S$		DE-$\mathrm{B}_{t\geq 1}$	: 1-myopic on $[1, 2, \cdots)$
	$U > S$	$r/q < r'/q'$	DE-$\mathrm{C}_{t\geq 1}$	: 1-myopic on $[1, 2, \cdots)$
		$r/q > r'/q'$	DE-$\mathrm{B}_{t=1}$ DE-$\mathrm{E}_{t\geq 2}$	: 2-myopic on $[2, 3, \cdots)$
$U > S,\ U > U'$	$\lambda' \leq \lambda$		DE-$\mathrm{D}_{t\geq 1}$	: 1-myopic on $[1, 2, \cdots)$
	$\lambda' > \lambda$	$W' \leq \delta$	DE-$\mathrm{D}_{t\geq 1}$	: 1-myopic on $[1, 2, \cdots)$
		$U < \delta < W'$	DE-$\mathrm{D}_{1\leq t\leq t_\delta}$ DE-$\mathrm{C}_{t>t_\delta}$	: 1-myopic on $[1, t_\delta]$: $(t_\delta + 1)$-myopic on $[t_\delta + 1, t_\delta + 2, \cdots)$
		$\delta < U$	DE-$\mathrm{D}_{t=1}$ DE-$\mathrm{C}_{t\geq 2}$	: 2-myopic on $[2, 3, \cdots)$

Note : t_δ can be calculated by using Eqs.(4.1) and (5.3).

It should be noted in Table 1 that

A. For a certain space of parameters, there exists at most one *critical point* in time at which the optimal decision changes from one to another. It is either 1 or t_δ.

B. In order to demonstrate how to interpret the contents of the table, let us take one of the cells as example.

If $U > S$, $U > U'$, $\lambda' > \lambda$ and $U < \delta < W'$, then DE-$\mathrm{D}_{1\leq t\leq t_\delta}$, DE-$\mathrm{C}_{t>t_\delta}$ *is optimal.*

This implies that: when a hostage event occurs at time t, and the combination of p, q, r, q' and r' satisfies such the condition that $U > S$, $U > U'$, $\lambda' > \lambda$ and $U < \delta < W'$, we first calculate t_δ by using Eqs.(4.1) and (5.3). If $1 \leq t \leq t_\delta$, then DE-D is optimal and 1-myopic on $[1,\ t_\delta]$; if $t > t_\delta$, then DE-C is optimal and $(t_\delta + 1)$-myopic on $[t_\delta + 1,\ t_\delta + 2, \cdots)$.

C. As seen in Table 1, the optimal decision rule of this model has the myopic property that we defined.

7. Future Studies

Taking different real hostage situations into account, we feel that there is a need to modify the model from the following viewpoints:

1. We should consider the case where the effectiveness of an action of negotiation decreases gradually after it was taken.
2. In real hostage events, several acts of negotiation are available, in which the problem therefore arises as to when and what action of negotiation should be taken.
3. The author examined the case with more than one hostage in [2], which should be generalized by introducing simple or multiple actions of negotiation.
4. In many real cases, the perpetrator(s) operate with confused motives. This causes the probabilities p, q, and r to change randomly from one minute to the next. This consideration leads us to the model in which p, q, r, q', and r' are random variables with a

known or unknown distribution function. When it is unknown, we can and must update its unknown parameters by using Bayes' theorem.

5. In many real cases, the deadline is not always definite; in other words, it should be regarded as a random variable. A model with this assumption should be examined in the future.
6. In order for our models to be more realistic and effective, the probabilities p, q, r, q', and r' must be measured and known in advance for each hostage crisis. Although such a measurement would be a very difficult task, it should be tackled through the united efforts of researchers in different fields, say, statisticians, psychologists, sociologists, political scientists, engineers, and so on.

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