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SETUP COST AND LEAD TIME REDUCTIONS ON STOCHASTIC INVENTORY MODELS WITH A SERVICE LEVEL CONSTRAINT

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Abstract The stochastic inventory models analyzed in this paper explore the problem of lead time associated with setup cost reductions for the continuous review and periodic review inventory models. For these two models with a mixture of backorders and lost sales, we respectively assume that their mean and variance of the lead time demand and protection interval (i.e., lead time plus review period) demand are known, but their probability distributions are unknown. We develop a minimax distribution free procedure to find the optimal solution for each case.

1. Introduction

As point out by Silver [16], if the quantitative models are to be more useful as aids for managerial decision making, they must permit some of the usual parameters to be treated as decision variables. Recently, motivated by the Japanese successful experience of using Just-In-Time (JIT) production, which evidenced that substantial advantages and benefits can be attained by controlling lead time, several authors have presented various inventory models with lead time reductions (see, e.g., Liao and Shyu [6], Ouyang *et al.* [10], Moon and Choi [7], Hariga and Ben-Daya [4] and Ouyang and Chuang [9]). We note that these papers are focusing on the benefits from lead time reductions in which setup cost is treated as a fixed constant.

In some practical situations, setup cost can be controlled and reduced through efforts such as worker training, procedural changes, and specialized equipment acquisition. For example, according to Silver *et al.* [17], the implementation of electronic data interchange (EDI) may reduce the fixed setup cost and result in new replenishment policy and the corresponding lower cost. Setup (setup cost and/or setup time) control has been a topic of interest for many researchers in the field of production/inventory management. Initially, Porteus [12] introduced the concept and developed a framework of investing in reducing setup cost on the classical economic order quantity (EOQ) model. Porteus [13] then extended [12] to consider the discounted effects on the EOQ model with setup cost reduction. Billington [1] considered the economic production quantity (EPQ) model without backorders and included the setup cost as a function of capital expenditure. Nasri *et al.* [8] investigated the effects of setup cost reduction on the EOQ model with stochastic lead time. Kim *et al.* [5] presented several classes of setup cost reduction functions and described a general solution procedure on the EPQ model. Paknejad *et al.* [11] presented a quality-adjusted lot-sizing model with stochastic demand and constant lead time, and studied the benefits of lower setup cost in the model. Sarker and Coates [14] extended EPQ model with setup cost reduction under stochastic lead time and finite number of investment possibilities to reduce setup cost. However, the underlying assumption in these setup cost reduction models is that lead time is a prescribed constant or a random variable, which therefore is not subject to control.

Though both the lead time and setup cost have been recognized as cruxes of elevating productivity, there has been little literature simultaneously examining the effects of these two factors on the inventory-control systems. And hence, here we would like to investigate such an issue and extend the recent study presented by Ouyang and Chuang [9], who applied the minimax distribution free procedure to deal with the lead time reduction for the continuous review and periodic review inventory models with stochastic partial backorders. We note that the minimax distribution free approach was originally proposed by Scarf [15]; it is based on the conservative attitude of making the best out of the worst possible conditions, specifically, for a situation where only the first two moments (or mean and variance) of the demand distribution are known but the exact distributional form is unknown. Furthermore, Gallego and Moon [2] have presented a new and very compact proof of the optimality of Scarf's [15] ordering rule. Besides, the previous works on distribution free approach, setup cost reduction and lead time reduction are well documented in [17].

In this paper, instead of the fixed setup cost assumption in [9], we consider setup cost as one of the decision variables, which can be varied through capital investment. We seek to minimize the sum of capital investment cost of reducing setup cost and inventory related cost by simultaneously optimizing the setup cost, order quantity, reorder point and lead time for continuous review model; and optimizing the setup cost, review period, target inventory level and lead time for periodic review model. For the stochastic lead time demand and protection interval demand, we consider the same case as in [9], that is, their probability distributions are unknown but only the first two moments are known. We solve the formulated inventory models by applying the minimax distribution free approach as described earlier. The rest of this paper is organized as follows. In Section 2, we first review Ouyang and Chuang's [9] models. Then investing capital in reducing setup cost is examined in Section 3. In Section 4, numerical examples are given to illustrate the results obtained in this study. Finally, some concluding remarks and some future research problems are given in Section 5.

2. Review of Ouyang and Chuang's Models

In a recent study, Ouyang and Chuang [9] considered two stochastic inventory models (continuous review and periodic review) under the following circumstances. During the stockout period, only a fraction of excess demand is backordered and the remaining fraction is lost. The backorder rate, β, is assumed to be a random variable with mean M_β. Instead of having a stockout cost term in the objective function, a service level constraint is employed. Lead time, L, can be decomposed into n mutually independent components each having a different crashing cost for reducing lead time. By the assumptions outlined above, they proposed:

Model 1. Continuous review inventory model

$$Min\ \ EAC(Q,r,L) = A\frac{D}{Q} + h\Big[\frac{Q}{2} + r - DL + (1-M_\beta)E(X-r)^+\Big] + C(L)\frac{D}{Q}$$

$$Subject\ to\ \ \frac{E(X-r)^+}{Q} \le \alpha; \qquad (1)$$

Model 2. Periodic review inventory model

$$Min\ EAC(T,R,L) = \frac{A}{T} + h\Big[R - DL - \frac{DT}{2} + (1-M_\beta)E(X-R)^+\Big] + \frac{C(L)}{T}$$

$$Subject\ to\quad \frac{E(X-R)^+}{D(T+L)} \le \alpha, \tag{2}$$

where

A = setup cost per order
D = average demand per year
h = inventory holding cost per unit per year
Q = order quantity
r = reorder point for the continuous review case
R = target inventory level for the periodic review case
L = length of lead time
T = length of a review period, $T > L$
α = proportion of demands that are not met from stock; hence, $1-\alpha$ is the service level, $0 < \alpha < 1/2$
X = lead time/protection interval demand
$f(x)$ = *p.d.f.* of X with finite mean DL $(D(T+L))$ and standard deviation $\sigma\sqrt{L}(\sigma\sqrt{T+L})$ for the continuous (periodic) review case, where σ denotes the standard deviation of the demand per unit time
$E(\cdot)$ = mathematical expectation
x^+ = maximum value of x and 0, i.e., $x^+ = Max\{x,0\}$
$C(L)$ = lead time crashing cost per cycle; $C(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)$, for which i-th component of lead time L has a minimum duration a_i, normal duration b_i, and a crashing cost per unit time c_i, where $c_1 \le c_2 \le \cdots \le c_n$; L_i denotes the length of lead time with components $1, 2, \ldots, i$ crashed to their minimum duration, that is, $L_i = \sum_{j=1}^{n} b_j - \sum_{j=1}^{i}(b_j - a_j)$, $i = 1, 2, \cdots, n$. Also, let $L_0 = \sum_{j=1}^{n} b_j$ and $C(L_0) = 0$.

For a situation that the distributional form of the lead time/protection interval demand X is unknown, Ouyang and Chuang [9] utilized Lemma 1 in Gallego and Moon [2], and the assumptions $r = DL + k\sigma\sqrt{L}$ and $R = D(T+L) + \delta\sigma\sqrt{T+L}$ (k and δ are safety factors), to find the worst case for $E(X-r)^+$ and $E(X-R)^+$, i.e.,

$$E(X-r)^+ \le \frac{1}{2}\sigma\sqrt{L}(\sqrt{1+k^2} - k), \tag{3}$$

and

$$E(X-R)^+ \le \frac{1}{2}\sigma\sqrt{T+L}\Big(\sqrt{1+\delta^2} - \delta\Big). \tag{4}$$

Then by above inequalities and allowing k and δ as decision variables instead of r and R, respectively, they transformed model (1) to

$$\begin{aligned} &Min\ \ EAC(Q,k,L) = \frac{D[A+C(L)]}{Q} + \frac{hQ}{2} + h\sigma\sqrt{L}\Big[k + \frac{1}{2}(1-M_\beta)\Big(\sqrt{1+k^2}-k\Big)\Big] \\ &Subject\ to\quad \sigma\sqrt{L}\Big(\sqrt{1+k^2}-k\Big) \le 2\alpha Q, \end{aligned} \tag{5}$$

and model (2) to

$$\begin{aligned} &Min\ \ EAC(T,\delta,L) = \frac{A+C(L)}{T} + \frac{hDT}{2} + h\sigma\sqrt{T+L}\Big[\delta + \frac{1}{2}(1-M_\beta)\Big(\sqrt{1+\delta^2}-\delta\Big)\Big] \\ &Subject\ to\ \sigma\sqrt{T+L}\Big(\sqrt{1+\delta^2}-\delta\Big) \le 2\alpha D(T+L). \end{aligned} \tag{6}$$

Note that in models (5) and (6), the setup cost A is treated as a fixed constant.

3. Extending Models with Setup Cost Reduction

In contrast to Ouyang and Chuang [9], this study considers the setup cost A as one of the decision variables. To avoid any possible confusion, we follow the same notations and assumptions in [9] as described above, except that an investing option of setup cost reduction is included. We seek to minimizing the sum of the capital investment cost of reducing setup cost A and inventory related costs expressed in the objective functions of models (5) and (6) for the continuous review case and periodic review case, respectively.

Thus, for setup cost reduction, the continuous review model can be formulated as

$$\begin{aligned} &Min\ \ EAC(A,Q,k,L) = \eta M(A) + EAC(Q,k,L) \\ &Subject\ to\quad \sigma\sqrt{L}\Big(\sqrt{1+k^2}-k\Big) \le 2\alpha Q, \end{aligned} \tag{7}$$

and the periodic review model can be formulated as

$$\begin{aligned} &Min\ \ EAC(A,T,\delta,L) = \eta M(A) + EAC(T,\delta,L) \\ &Subject\ to\quad \sigma\sqrt{T+L}\Big(\sqrt{1+\delta^2}-\delta\Big) \le 2\alpha D(T+L), \end{aligned} \tag{8}$$

over $A \in (0, A_0]$, where η is the cost of capital per year, and $M(A)$ is given by a logarithmic investment function such as

$$M(A) = \frac{1}{b}\ln(\frac{A_0}{A}) \text{ for } A \in (0, A_0], \tag{9}$$

where b is percentage decrease in A per dollar increase in $M(A)$, and A_0 is the original setup cost. The logarithmic investment cost function is consistent with the Japanese experience as reported in Hall [3], and it has been used by Porteus [12, 13] and others.

From equation (9), we note that the setup cost level $0 < A \le A_0$. It implies that if the optimal setup cost obtained does not satisfy the restriction on A, then no setup cost reduction investment should be made. For this special case, the optimal setup cost is the original setup cost, i.e., $A = A_0$, and our models reduce to [9].

3.1. Solution of continuous review case

For the continuous review case, using model (5) and equation (9), model (7) becomes

$$\begin{aligned} Min\ EAC(A,Q,k,L) &= \frac{\eta}{b}\ln\left(\frac{A_0}{A}\right) + \frac{D[A+C(L)]}{Q} + \frac{hQ}{2} \\ &\quad + h\sigma\sqrt{L}\Big[k + \frac{1}{2}(1-M_\beta)\Big(\sqrt{1+k^2}-k\Big)\Big] \\ Subject\ to\quad & \sigma\sqrt{L}\Big(\sqrt{1+k^2}-k\Big) \le 2\alpha Q. \end{aligned} \tag{10}$$

The inequality constraint in model (10) can be converted into equality by adding a nonnegative slack variable, S^2. Thus, the Lagrangean function is given by

$$\begin{aligned} & EAC(A,Q,k,L,\lambda,S) \\ &= EAC(A,Q,k,L) + \lambda\Big[\sigma\sqrt{L}\Big(\sqrt{1+k^2}-k\Big) + S^2 - 2\alpha Q\Big] \\ &= \frac{\eta}{b}\ln\left(\frac{A_0}{A}\right) + \frac{D[A+C(L)]}{Q} + \frac{hQ}{2} + h\sigma\sqrt{L}\Big[k+\frac{1}{2}(1-M_\beta)\Big(\sqrt{1+k^2}-k\Big)\Big] \\ &\quad + \lambda\Big[\sigma\sqrt{L}\Big(\sqrt{1+k^2}-k\Big) + S^2 - 2\alpha Q\Big], \end{aligned} \tag{11}$$

where λ is a Lagrange multiplier.

For any given (A,Q,k,λ,S), $EAC(A,Q,k,L,\lambda,S)$ is concave in $L \in [L_i, L_{i-1}]$, because

$$\frac{\partial^2 EAC(A,Q,k,L,\lambda,S)}{\partial L^2} = -\frac{1}{4}hk\sigma L^{-3/2} - \frac{1}{8}\sigma L^{-3/2}\Big(\sqrt{1+k^2}-k\Big)\Big[h(1-M_\beta)+2\lambda\Big] < 0.$$

Hence, for fixed (A,Q,k,λ,S), the minimum expected total annual cost will occur at the end points of the interval $[L_i, L_{i-1}]$. On the other hand, for any given $L \in [L_i, L_{i-1}]$, the optimal solution (A,Q,k,λ,S) to minimize $EAC(A,Q,k,L,\lambda,S)$ needs to satisfy the following equations:

$$0 = \frac{\partial EAC(A,Q,k,L,\lambda,S)}{\partial A} = -\frac{\eta}{Ab} + \frac{D}{Q}, \tag{12}$$

$$0 = \frac{\partial EAC(A,Q,k,L,\lambda,S)}{\partial Q} = -\frac{D[A+C(L)]}{Q^2} + \frac{h}{2} - 2\lambda\alpha, \tag{13}$$

$$0 = \frac{\partial EAC(A,Q,k,L,\lambda,S)}{\partial k} = h\sigma\sqrt{L}\Big[1 - \frac{1}{2}(1-M_\beta)\frac{\sqrt{1+k^2}-k}{\sqrt{1+k^2}} - \frac{\lambda}{h}\frac{\sqrt{1+k^2}-k}{\sqrt{1+k^2}}\Big], \tag{14}$$

$$0 = \frac{\partial EAC(A,Q,k,L,\lambda,S)}{\partial \lambda} = \sigma\sqrt{L}\Big(\sqrt{1+k^2}-k\Big) + S^2 - 2\alpha Q, \tag{15}$$

and

$$0 = \frac{\partial EAC(A,Q,k,L,\lambda,S)}{\partial S} = 2\lambda S. \tag{16}$$

From equation (16), we first note that $\lambda = 0$ or $S = 0$. However, if $\lambda = 0$, then equation (14) will result in $\frac{k}{\sqrt{1+k^2}} = -\frac{1+M_\beta}{1-M_\beta} < 0$, which is infeasible since k is safety factor and the value of k should be nonnegative. Thus, it is clear that the slack variable $S = 0$, that is, the inequality constraint in model (10) becomes equality. Then, solving equations (12)-(15)

for A, Q, λ and k, respectively, leads to

$$A = \frac{\eta Q}{bD}, \tag{17}$$

$$Q = \left\{\frac{2D[A + C(L)]}{h - 4\lambda\alpha}\right\}^{1/2}, \tag{18}$$

$$\lambda = h\Big[\frac{\sqrt{1+k^2}}{\sqrt{1+k^2} - k} - \frac{1}{2}(1 - M_\beta)\Big], \tag{19}$$

and

$$\sqrt{1+k^2} - k = \frac{2\alpha Q}{\sigma\sqrt{L}}. \tag{20}$$

Furthermore, combining equations (18)-(20), we obtain the order quantity

$$Q = \left\{\frac{4\alpha D[A + C(L)] + h\sigma^2 L}{2\alpha h(1 - 2\alpha M_\beta)}\right\}^{1/2}. \tag{21}$$

Consequently, we can develop the following algorithm to find the optimal value of (A, Q, k, L).

Algorithm 1.

Step 1. For each L_i, $i = 0, 1, 2, \cdots, n$, perform (i)-(iv).
 (i) Start with $A_{i1} = A_0$.
 (ii) Substituting A_{i1} into equation (21) evaluates Q_{i1}.
 (iii) Using Q_{i1} determines A_{i2} from equation (17).
 (iv) Repeat (ii) and (iii) until no change occurs in the values of Q_i and A_i.

Step 2. Substitute Q_i into equation (20) to solve for k_i.

Step 3. Compare A_i and A_0.
 (i) If $A_i \le A_0$, A_i is feasible, then go to Step 4.
 (ii) If $A_i > A_0$, A_i is not feasible. Set $A_i = A_0$ and evaluate the corresponding values of Q_i from equation (21) and k_i from equation (20), then go to Step 4.

Step 4. For each (A_i, Q_i, k_i, L_i), compute the corresponding expected total annual cost $EAC(A_i, Q_i, k_i, L_i)$, $i = 0, 1, 2, \cdots\cdots, n$.

Step 5. Find $\underset{i=0,1,2,\cdots,n}{Min} EAC(A_i, Q_i, k_i, L_i)$.
 If $EAC(A^*, Q^*, k^*, L^*) = \underset{i=0,1,2,\ldots,n}{Min} EAC(A_i, Q_i, k_i, L_i)$, then (A^*, Q^*, k^*, L^*) is the optimal solution. And hence, the optimal reorder point $r^* = DL^* + k^*\sigma\sqrt{L^*}$.

3.2. Solution of periodic review case

For the periodic review case, by using model (6) and equation (9), model (8) becomes

$$Min\ EAC(A, T, \delta, L) = \frac{\eta}{b}\ln\left(\frac{A_0}{A}\right) + \frac{A + C(L)}{T} + \frac{hDT}{2} + h\sigma\sqrt{T+L}\Big[\delta + \frac{1}{2}(1 - M_\beta)\Big(\sqrt{1+\delta^2} - \delta\Big)\Big]$$

$$Subject\ to\quad \sigma\sqrt{T+L}\Big(\sqrt{1+\delta^2} - \delta\Big) \le 2\alpha D(T + L). \tag{22}$$

Hence, the Lagrangean function is given by

$$\begin{aligned} EAC(A,T,\delta,L,\lambda,S) &= \frac{\eta}{b}\ln\left(\frac{A_0}{A}\right)+\frac{A+C(L)}{T}+\frac{hDT}{2} \\ &+h\sigma\sqrt{T+L}\Big[\delta+\frac{1}{2}(1-M_\beta)\Big(\sqrt{1+\delta^2}-\delta\Big)\Big] \\ &+\lambda\Big[\sigma\sqrt{T+L}\Big(\sqrt{1+\delta^2}-\delta\Big)+S^2-2\alpha D(T+L)\Big], \quad (23) \end{aligned}$$

where λ is a Lagrange multiplier and S^2 is a nonnegative slack variable.

By analogous arguments in the continuous review case, it can be verified that $EAC(A,T,\delta,L,\lambda,S)$ is a concave function of $L \in [L_i, L_{i-1}]$ for fixed (A,T,δ,λ,S). Thus, for fixed (A,T,δ,λ,S), the minimum value of $EAC(A,T,\delta,L,\lambda,S)$ will occur at the end points of the interval $[L_i, L_{i-1}]$. On the other hand, for fixed $L \in [L_i, L_{i-1}]$, the optimal solution (A,T,δ,λ,S) which minimizes $EAC(A,T,\delta,L,\lambda,S)$ can be obtained by solving

$$\frac{\partial EAC(A,T,\delta,L,\lambda,S)}{\partial A} = 0, \quad \frac{\partial EAC(A,T,\delta,L,\lambda,S)}{\partial T}=0, \quad \frac{\partial EAC(A,T,\delta,L,\lambda,S)}{\partial \delta}=0,$$
$$\frac{\partial EAC(A,T,\delta,L,\lambda,S)}{\partial \lambda} = 0 \text{ and } \frac{\partial EAC(A,T,\delta,L,\lambda,S)}{\partial S}=0,$$

simultaneously. The resulting solutions are

$$A = \frac{T\eta}{b}, \quad (24)$$

$$\Big[\frac{A+C(L)}{T^2}-\frac{hD}{2}+2\lambda\alpha D\Big](T+L)^{1/2} = \frac{h\sigma\delta}{2}+\frac{\sigma}{4}\Big(\sqrt{1+\delta^2}-\delta\Big)\Big[h(1-M_\beta)+2\lambda\Big], \quad (25)$$

$$\lambda = h\Big[\frac{\sqrt{1+\delta^2}}{\sqrt{1+\delta^2}-\delta}-\frac{1}{2}(1-M_\beta)\Big], \quad (26)$$

$$\sqrt{1+\delta^2}-\delta = \frac{2\alpha D}{\sigma}\sqrt{T+L}, \quad (27)$$

and

$$S = 0. \quad (28)$$

Furthermore, by substituting equations (26) and (27) into equation (25), we get the review period

$$T=\left\{\frac{2[A+C(L)]}{hD(1-2\alpha M_\beta)}\right\}^{1/2}. \quad (29)$$

The following algorithm can be utilized to find the optimal (A,T,δ,L).

Algorithm 2.

Step 1. For each L_i, $i=0,1,2,\cdots,n$, perform (i)-(iv).
 (i) Start with $A_{i1}=A_0$.
 (ii) Substituting A_{i1} into equation (29) evaluates T_{i1}.
 (iii) Using T_{i1} determines A_{i2} from equation (24).
 (iv) Repeat (ii) and (iii) until no change occurs in the values of T_i and A_i.

Step 2. Substitute T_i into equation (27) to solve for δ_i.

Step 3. Compare A_i and A_0.
 (i) If $A_i \le A_0$, then A_i is feasible, go to Step 4.

(ii) If $A_i > A_0$, then A_i is not feasible. Set $A_i = A_0$ and evaluate the corresponding values of T_i from equation (29) and δ_i from equation (27), then go to Step 4.

Step 4. For each $(A_i, T_i, \delta_i, L_i)$, compute the corresponding expected total annual cost $EAC(A_i, T_i, \delta_i, L_i)$, $i = 0, 1, 2, \cdots, n$.

Step 5. Find $\underset{i=0,1,2,\cdots,n}{Min} EAC(A_i, T_i, \delta_i, L_i)$.

If $EAC(A_*, T_*, \delta_*, L_*) = \underset{i=0,1,2,\cdots,n}{Min} EAC(A_i, T_i, \delta_i, L_i)$, then $(A_*, T_*, \delta_*, L_*)$ is the optimal solution. And hence, the optimal target inventory is $R_* = D(T_* + L_*) + \delta_* \sigma \sqrt{T_* + L_*}$.

4. Numerical Examples

The numerical examples given below are for illustrating the above solution procedure. Since the major work in this paper is to extend Ouyang and Chuang's [9] models, we will also compare and contrast the optimal policies between our models and [9] by utilizing the same data.

Example 1. Continuous review case

We consider an inventory system with the following data used in [9]: $D = 600$ units per year, $A_0 = \$200$ per order, $h = \$20$ per unit per year, $\sigma = 7$ units per week, the service level $1 - \alpha = 0.985$. The lead time has three components with data as shown in Table 1. The backorder rate β during the stockout period has a uniform distribution, i.e., the *p.d.f.* of β is $g(\beta) = 1$, $0 \leq \beta \leq 1$; $= 0$, otherwise. Hence, the mean of β is $M_\beta = 0.5$. Besides, for setup cost reduction, we take $\eta = 0.1$ per dollar per year and $b = 0.01538\%$.

Table 1: Lead time data

Lead time component i	Normal duration b_i(days)	Minimum duration a_i(days)	Unit crashing cost c_i(\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Utilizing the above data and applying the Algorithm 1 yields the results as shown in Table 2.

Table 2: Solution procedure of Algorithm 1 (L_i in weeks)

i	L_i	$C(L_i)$	A_i	Q_i	$r_i(k_i)$	$EAC(A_i, Q_i, r_i, L_i)$
0	8	0	\$166	153	128(2.04)	\$3,133
1	6	5.6	151	140	99(1.92)	2,933
2	4	22.4	139	128	68(1.68)	2,764*
3	3	57.4	142	131	50(1.38)	2,800

* denotes the optimal solution.

From Table 2, the optimal inventory policy with the service level $1 - \alpha = 0.985$ can be easily found by comparing $EAC(A_i, Q_i, r_i, L_i)$, $i = 0, 1, 2, 3$. Therefore, we obtain the optimal setup cost $A^* = \$139$, the optimal order quantity $Q^* = 128$ units, the optimal reorder point $r^* = 68$ units, and the optimal lead time $L^* = 4$ weeks. The minimum expected total annual cost $EAC(A^*, Q^*, r^*, L^*) = \$2,764$. Comparing the optimal solutions with that of fixed setup cost case in [9] (i.e., $EAC(Q^*, r^*, L^*) = EAC(142, 65, 4) = \$2,798$), it can be seen that the smaller order quantity with the savings of expected total annual cost are realized through the reduction of setup cost.

Example 2. Periodic review case
We use the same data as in Example 1 and apply the Algorithm 2 to find the optimal solutions; the results are tabulated in Table 3.

Table 3: Solution procedure of Algorithm 2 (T_i, L_i in weeks)

i	L_i	$C(L_i)$	A_i	T_i	$R_i(\delta_i)$	$EAC(A_i,T_i,R_i,L_i)$
0	8	0	\$140	8.02	244(2.29)	\$3,399
1	6	5.6	80	6.42	205(2.78)	3,389*
2	4	22.4	93	7.45	195(2.91)	3,518
3	3	57.4	113	9.05	201(2.83)	3,740

* denotes the optimal solution.

From Table 3, the optimal value $(A_*, T_*, R_*, L_*) = (80, 6.42, 205, 6)$, and the minimum expected total annual cost $EAC(A_*, T_*, R_*, L_*) = \$3,389$. Comparing the optimal expected total annual cost with that of fixed setup cost case in [9] (i.e., $EAC(T_*, R_*, L_*) = EAC(9.80, 263, 8) = \$3,523$), the cost savings from the efforts of setup cost reduction for the period review case is equal to $EAC(T_*, R_*, L_*) - EAC(A_*, T_*, R_*, L_*) = \$3,523 - \$3,389 = \$134$.

Here we would like to point out that the savings of the expected total annual cost obtained in these two examples are only possible when the setup cost reduction can be achieved through investment.

5. Concluding Remarks

This paper extends the Ouyang and Chuang's [9] models by considering that the option of investing in reducing setup cost is available. The presented new models are based on the theme, *"changing the givens"* approach, which can be viewed as an application of JIT philosophy as indicated in Silver [16]. We study the joint effects of setup cost and lead time reductions for the continuous review and periodic review inventory systems. The results of the numerical examples show that the savings of expected total annual cost are realized when the setup cost reduction could be achieved through extra investments (by comparing the results with fixed setup cost models in [9]).

This paper considers that the quality of arrival order lots are perfect and acceptable, in future research on this problem, it would be interesting to deal with an arrival order lot including some defective items.

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