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A DISTRIBUTION FREE NEWSBOY PROBLEM UNDER SHORTAGE-LEVEL CONSTRAINTS

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Abstract Since order quantity has only been decided at a spot selling time in the classical newsboy problem, both the purchase timing and the time-variant variance of the forecasted demand are neglected. When the buyer purchases seasonal commodity by changing his pre-planned schedule for more purchase discount, it is necessary for the buyer to forecast the demand of spot selling time, which may increase the variance of the demand. This paper deals simultaneously with optimal purchase timing and order quantity under a specified shortage-level limit in a distribution free newsboy problem. Besides, a numerical example is used to illustrate the largest amount called Expected Value of Additional Information, which decision-maker will be willing to pay for the knowledge of the normal distribution. Furthermore, the effects of parameters and economical meanings are also included. The resultant outcomes could apply to some cases in futures commodity contracts.

1. Introduction

We may consider the classical newsboy problem as the scenarios on the demand forecasted at time t ahead of spot selling time T, and therefore it only decides order quantity on the tradeoff between over-stocking and under-stocking to minimize cost (to maximize profit) in the classical newsboy problem.

Since the classical newsboy problem has only one decision variable in it, it is oversimplified. We improved it to be an extended one with two decision variables: (1) when to order (to decide the purchase timing); (2) how many to order (to decide the order quantity). The purchase timing was necessarily considered because it affects not only the purchase cost but also the accuracy of the forecasted demand. The later the purchase timing is, i.e., near the spot selling time, the more accurate it is, i.e., the less the variance of the forecasted demand is. The larger variance of the forecasted demand results from forecasting the demand earlier. The characteristic exists mostly in the futures commodity contracts. It makes much difference between this paper and the classical newsboy problem, which are summarized as follows:

(1) The classical newsboy problem was assumed that the purchase cost per unit is fixed. We assumed that the supplier gives more price discount to stimulate the buyer to purchase earlier for decreasing the inventory level. Based on the different purchase discount at different purchase timing, the model is completely different from the other model in the literature such as Anvari[1], Anvari & Kusy[2], Chung[5] and Pfeifer[13], contrarily, which are formulated on the different sales discount at different sales quantity. Although Eeckhout *et al.*[6] proposed that a buyer can reorder the inventory by expensive purchase price, this paper shows different opinions in which buyer can only order once.

The assumption in price discount is different from the applications in the literature of

the classical newsboy problem. For example, Khouja[9] formulated a model on the scenarios as sale price discount on the demand. Contrarily we formulated our model on the effects of purchase price discount at different purchase timing on the purchase cost.

(2) There is a difference in conducting under-stocking between the classical newsboy problem and this paper. Since the shortage cost includes penalty cost specified in the contract; discount asked by customers for shortage of goods and for inconvenience; imputation loss; and loss from losing potential customer, it is really difficult to measure shortage cost by experience. Therefore, some researchers like Aardal *et al.*[3], Moon & Choi[11] formulated their inventory model with service-level constraint. In order to widen the application of this model, we utilize the average shortage rate subject to a specified shortage rate limit to deal with the under-stocking.

(3) It was assumed that the variance of the forecasted demand was fixed in the literature of the newsboy problem such as Gerchak & Parlar[8], Lau[10], Walker[14, 15]. Namely, the variance is invariant with the time. When the buyer forecasts the demand, the nearer the spot selling time is, the less the variance of the forecasted demand is; i.e., the less the forecast bias of the buyer is. The effects of forecast bias on the expected inventory quantity and expected shortage quantity were simultaneously considered in the model of this paper. This innovation has never been mentioned in the related literatures of the newsboy problem.

(4) When the buyer purchases the seasonal products, he can evaluate the mean and variance of the demand from experience. However, he always can't identify the exact type of distribution function of the demand. Some scholars extended the newsboy problem with distribution free such as Gallego & Moon[7], Moon & Choi[11, 12], but all of them didn't include the purchase timing into the model. A distribution free newsboy problem with purchase timing was formulated to extend the application in the paper.

An extended newsboy problem with normal distribution under shortage-level constraints is postulated by Chen and Chuang[4]. Under the scenarios of the unidentified distribution function of the demand, a distribution free newsboy problem under shortage-level constraints is proposed in the paper. The model in this paper could be used in making decisions for some kinds of future commodity contracts. It is also suitable for the wholesaler to decide how many and when to buy the seasonal agriculture products. For example, when to order roses and how many roses to order on the eve of Valentine's Day is a typical case.

2. Assumptions and Notations

2.1. Assumptions

- (1) When price discount for early purchase is offered by the supplier within period [0, T], the buyer has to decide the purchase timing t at time 0 and then decide the order quantity q.
- (2) The buyer allows his customer to preorder products. The preordered must be sent to the customer ahead of time T in order to avoid loosing the opportunity of the sale.
- (3) The buyer needs to forecast the demand X_0 within [0, T] at time 0. X_0 is a random variable with mean μ and variance σ^2 . The preordered timing for the customer is evenly distributed within [0, T]. Hence, once the purchase timing for the buyer has been decided at time t, the mean of the demand within [t, T] forecasted at time t is $\mu\left(\frac{T-t}{T}\right)$ and

the variance is $\left[\sigma\left(\frac{T-t}{T}\right)\right]^2$. As to decision time point 0, the demand within [0, t] is a random variable (with mean $\mu \frac{t}{T}$ and variance is $\left[\sigma \frac{t}{T}\right]^2$); Yet, as to the decision time

point t, the demand within [0, t] is a deterministic value (an observable historical data).

2.2. Notations

2.2.1. Parameters

- T = spot selling time (= the length of price discount offered by the supplier)
- δ = unit purchase price discount ahead of one unit time
- c_t = unit purchase cost at time t in which $c_t = c \delta(T t)$, $0 \le t \le T$, satisfies $c_T = c$ and $c_0 = c - \delta T$
- $\nu\,=\,{\rm unit}$ salvage value unsold at time T
- h =unit holding cost per unit time

2.2.2. Function

 $\begin{aligned} X_t &= \text{the forecasted demand within } [t,T] \text{ which should be forecasted at decision time} \\ &= \text{point } t \text{ after the purchase timing } t \text{ has been decided, in which } E(X_t) = E(\text{demand within } [0,t] \text{ forecasted at time } 0) + E(\text{demand within } [t,T] \text{ forecasted at time } 0) = \\ &= \frac{t}{T}\mu + \frac{T-t}{T}\mu = \mu, \ Var(X_t) = Var(\text{demand within } [0,t] \text{ forecasted at time } t) = \\ &= Var(\text{demand within } [t,T] \text{ forecasted at time } t) = 0 + \left(\frac{T-t}{T}\sigma\right)^2 = \left(\frac{T-t}{T}\sigma\right)^2. \end{aligned}$

$$F_t$$
 = the distribution function of the random variable X_t

 $\Omega_t = \{F_t \mid F_t \text{ is a distribution function with mean } \mu \text{ and variance } [(1 - t/T)\sigma]^2\}$

2.2.3. Decision variable

- t =purchase timing that has been decided by the buyer at time 0, $0 \le t \le T$
- q = order quantity that has been decided by the buyer at time t after purchase timing t had been decided

3. The Proposed Model

To maintain the sales stable, the supplier always sells products at a price discount to stimulate the buyer to purchase products ahead of schedule. Since the purchase cost at (t,q) minus the one at $(t - \Delta t, q)$ equals to

$$[c_t q + h(T-t)q] - [c_{t-\Delta t}q + h(T-t+\Delta t)q] = (\delta - h)q\Delta t, \ \forall q.$$

If $\delta < h$, it does not make sense of the price discount offered by the supplier. Therefore, it is the same as the classical newsboy problem and the optimal purchase timing t^* must be T.

If the buyer decides to purchase products at time t ahead of time T, the order quantity q is based on the distribution function F_t of the forecasted demand X_t . The holding cost would be hq(T-t) for holding these products from time t to T. If q is greater than demand X_t , the expected salvage value of unsold products at time T would be $\nu E[q - X_t]^+$, which $[q - X_t]^+ = \max\{0, q - X_t\}$. If q is less than demand X_t , the expected shortage quantity for unsatisfied demand at time T would be $E[X_t - q]^+$.

3.1. State representation

Supposed that the buyer is facing the following situations:

(i) The sum of unit purchase cost $c - \delta T$ at time 0 and unit holding cost hT in the time period [0, T] is greater than unit salvage value ν ; i.e., inequality

$$c - \delta T + hT > \nu \tag{1}$$

holds. The reason for satisfying Eq. (1) is: if $\nu > c - \delta T + hT$, the buyer will certainly earn profit $\nu - [c - (\delta - h)T]$ for purchasing one unit at time 0, i.e., the more products the

buyer purchases, the more profit he earns. That is to say, in the absence of Eq. (1) at time 0, it will lead to an infinite order quantity and so negative infinite expected total cost for the buyer. It is impractical!

(ii) Although the buyer can estimate the mean μ and the variance $[(1 - t/T)\sigma]^2$ of the demand, he doesn't know what type of the distribution function F_t is, i.e., the buyer only knows $F_t \in \Omega_t$.

(iii) If order quantity is less than the demand, the expected shortage of quantity would be $E[X_t - q]^+$. The buyer specified that the average shortage level, the ratio of expected shortage of quantity to the expected demand quantity, isn't greater than the shortage rate limit β , $0 \leq \beta \leq 1$ so as to maintain the service level, and it can be defined as

$$\frac{\max_{F_t \in \Omega_t} E[X_t - q]^+}{E[X_t]} \le \beta.$$
(2)

(iv) The buyer minimizes the upper bound of the expected total cost L(t,q) so as to deal with the effects of inexact type F_t on the inventory cost, i.e.,

$$\min_{(t,q)} \max_{F_t \in \Omega_t} L(t,q) = (c - \delta(T-t))q + h(T-t)q - \nu E[q - X_t]^+$$
(3)

where the first item of L(t,q) is purchase cost, the second is holding cost, and the third is expected salvage value for the unsold product. The expected total cost doesn't include the setup cost, because it doesn't influence the optimal solution of the model.

3.2. Model derivation

Follow Gallego & Moon [7] and extend it, we get

$$\max_{F_t \in \Omega_t} E[q - X_t]^+ \le \frac{\left[\left(1 - \frac{t}{T}\right)^2 \sigma^2 + (q - \mu)^2\right]^{\frac{1}{2}} + (q - \mu)}{2} \tag{4}$$

and

$$\max_{F_t \in \Omega_t} E[X_t - q]^+ \le \frac{\left[\left(1 - \frac{t}{T}\right)^2 \sigma^2 + (q - \mu)^2\right]^{\frac{1}{2}} - (q - \mu)}{2}.$$
(5)

Substitute Eq. (4), Eq. (5) into Eq. (2), Eq. (3) and rearrange it, the optimal inventory policy (t^*, q^*) of a distribution free newsboy problem under a specified shortage rate limit is the optimal solution of the following model.

Model(I):

$$\min_{(t,q)} L(t,q) = \left(c - (\delta - h)(T - t) - \frac{\nu}{2}\right)q - \frac{\nu \left[\left(\frac{T - t}{T}\right)^2 \sigma^2 + (q - \mu)^2\right]^{\frac{1}{2}}}{2} + \frac{\nu \mu}{2} \quad (6)$$

s.t.
$$\left[\left(\frac{T-t}{T} \right)^2 \sigma^2 + (q-\mu)^2 \right]^{\frac{1}{2}} - (q-\mu) \middle/ 2\mu \le \beta$$
(7)
$$0 \le q$$
$$0 \le t \le T$$

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The above-analyzed problem of the model (I) is not the exact problem to be solved but an approximate one.

Since
$$\frac{\partial L(t,q)}{\partial t} = (\delta - h)q + \frac{\nu(T-t)\sigma^2}{2T^2 \left[\left(\frac{T-t}{T}\right)^2 \sigma^2 + (q-\nu)^2 \right]^{\frac{1}{2}}} > 0, \quad \forall t \in [0,t]$$

from Eq. (6) and the left-hand side of Eq. (7) is decreasing with t, the optimal solution (t^*, q^*) must make Eq. (7) binding. Therefore the feasible solution of the model (I) could be constrained to the (t, q) such that the left-hand side of Eq. (7) is equal to the right-hand. Let Eq. (7) be equality and rearrange it, t relates to q as follows:

$$\frac{T-t}{T} = Q, \quad \text{in which} \quad Q = \sqrt{\frac{4\beta\mu}{\sigma^2}[q-\mu(1-\beta)]} \quad \text{and} \quad 0 \le Q \le 1.$$
(8)

Use Eq. (8) and substitute it into Eq. (6), Model (I) could be transformed to Model (II). Model (II):

$$\min_{0 \le Q \le 1} J(Q) = -(\delta - h)T \frac{\sigma^2 Q^3}{4\beta\mu} + (c - \nu)\frac{\sigma^2 Q^2}{4\beta\mu} - (\delta - h)T(1 - \beta)\mu Q + c(1 - \beta)\mu$$
(9)

where the relation between the feasible solution Q of Model (II) and feasible solution (t, q) of Model (I) is shown as Eq. (8).

4. Solution

Let

$$D = \frac{c - \nu}{(\delta - h)T} \tag{10}$$

and

$$G = \frac{\mu \sqrt{(1-\beta)\beta}}{\sigma}.$$
(11)

From Eq. (1) and Eq. (10), we obtain

$$D > 1 \tag{12}$$

Eq. (9) derivative with respect to Q and use Eq. (10) and Eq. (11), we obtain

$$\frac{dJ(Q)}{dQ} = -\frac{3(\delta-h)T\sigma^2}{4\beta\mu} \left[Q^2 - \frac{2(c-\nu)}{3(\delta-h)T}Q + \frac{4\beta\mu^2(1-\beta)}{3\sigma^2} \right]$$
(13)

$$= \begin{cases} = -\frac{3(\delta - h)T\sigma^2}{4\beta\mu}(Q - Q_l)(Q - Q_s), & \text{if } D > 2\sqrt{3}G\\ \leq 0, \forall Q & \text{if } D \leq 2\sqrt{3}G \end{cases}$$
(14)

where

$$Q_{l} = \frac{D}{3} + \sqrt{\left(\frac{D}{3}\right)^{2} - \frac{4G^{2}}{3}},$$
(15)

$$Q_s = \frac{D}{3} - \sqrt{\left(\frac{D}{3}\right)^2 - \frac{4G^2}{3}}.$$
 (16)

From Eq. (13), if $D > 2\sqrt{3}G$, the graph of J(Q), $Q \in [0, \infty)$, is shown as Figure 1; in which $\overline{Q_s}$ could be obtained as follows:

Using Eq. (9) and Eq. (16),

$$J(Q) - J(Q_s) = (Q - Q_s) \left(\frac{-(\delta - h)T\sigma^2}{4\beta\mu}\right) \left\{ Q^2 + QQ_s + Q_s^2 - \frac{c - \nu}{T(\delta - h)}(Q + Q_s) + \frac{4\beta\mu^2(1 - \beta)}{\sigma^2} \right\}$$

Using Eq. (13): $0 = 3Q_s^2 - \frac{2(c-\nu)}{(\delta-h)T}Q_s + \frac{4\beta\mu^2(1-\beta)}{\sigma^2}$.

$$= (Q - Q_s)^2 \left(\frac{-(\delta - h)T\sigma^2}{4\beta\mu}\right) \left(Q - \left(\frac{D}{3} + 2\sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}\right)\right)$$

therefore,

$$\overline{Q_s} = \frac{D}{3} + 2\sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}.$$
(17)

By Eq. (15) and Figure 1, we conclude:

If
$$D > 2\sqrt{3}G$$
 and $Q_s < 1 < \overline{Q_s}$, then $Q^* = Q_s$; otherwise, $Q^* = 1$ (18)



Figure 1: The graph of J(Q) (if $D > 2\sqrt{3}G$)

Using Eq. (16) and Eq. (17), the following process is satisfied:

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$$\iff \begin{cases} D > \frac{3}{2} + 2G^2, & \text{if } G \ge \frac{\sqrt{3}}{2} \\ D > 2\sqrt{4G^2 + 1} - 1, & \text{if } G \le \frac{\sqrt{3}}{2} \\ \Leftrightarrow D > \frac{3}{2} + 2G^2, \quad \forall G. \end{cases}$$
(19)

Using Eq. (16), Eq. (18) and Eq. (19), the optimal solution of the model (II) is summarized as Proposition 1.

Proposition 1

(i) If
$$D > \frac{3}{2} + 2G^2$$
, then $Q^* = \frac{D}{3} - \sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}$. (20)
(ii) If $D \le \frac{3}{2} + 2G^2$, then $Q^* = 1$.

From Proposition 1, the optimal solution Q^* was decided by the relative size between parameter G and parameter D. Parameter $G = \frac{\mu\sqrt{(1-\beta)\beta}}{\sigma}$, called demand-distribution-related parameter, is relative to the demand distribution parameters μ, σ, β and not to the inventory cost parameters. Parameter $D = \frac{c-\nu}{(\delta-h)T}$, called inventory-cost-related parameter, is relative to the inventory cost parameters c, δ, T, h, ν and not to the demand distribution parameters.

Using Eq. (8) and Proposition 1, the optimal solution of the model (I) could be concluded as Proposition 2.

Proposition 2

(i) If
$$D > \frac{3}{2} + 2G^2$$
, then $t^* = T\left(1 - \frac{D}{3} + \sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}\right)$ and

$$q^* = \frac{\left(\frac{D}{3} - \sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}\right)^2 \sigma^2}{4\beta\mu} + \mu(1 - \beta).$$
(21)

(ii) If
$$D \le \frac{3}{2} + 2G^2$$
, then $t^* = 0$ and $q^* = \frac{\sigma^2}{4\beta\mu} + \mu(1-\beta)$. (22)

Expected Value of Additional Information and Sensitivity Analysis 5.1. Expected value of additional information

Let the optimal solution under a certain distribution (e.g. the normal distribution, we assume it to be a normal distribution for comparing with Chen and Chuang [4]) be (t_N^*, q_N^*) and the optimal solution under the distribution free be (t_F^*, q_F^*) . Here F represents the worst distribution. If we use the quantity (t_F^*, q_F^*) instead of (t_N^*, q_N^*) when the real distribution is certain, the largest amount that we would be willing to pay for the knowledge of these certain distribution is called Expected Value of Additional Information (EVAI), which is defined as $EVAI = L^N(t_F^*, q_F^*) - L^N(t_N^*, q_N^*)$. What the expected total cost $L^N(T_F^*, q_F^*)$ under a normal distribution is to (t_F^*, q_F^*) , and $L^N(t_N^*, q_N^*)$ to (t_N^*, q_N^*) .

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Example 1. Supposed $\mu = 10000$ units, $\sigma = 2000$ units, T = 60 days, h = 1.2 dollars/per day, $\delta = 1.5$ dollars/per day, c = 100 dollars/per unit, $\nu = 20$ dollars/per unit, s = 120 dollars/per unit, $\beta = 0.05$. Using (10), (11), (21) and Maple V software, the optimal purchase timing t_F^* is 18.039 and the optimal order quantity q_F^* is 10478.136 units under the shortage-level constraints. From Chen and Chuang [4], the optimal solution (t_N^*, q_N^*) with normal distribution is (5.15, 11828.15) under the shortage-level constraints. Using Maple V software, $L^N(18.039, 10478.136) = 982112.03$ and $L^N(5.15, 11828.15) = 907803.7$, therefore EVAI = 74308.33.

5.2. Sensitivity analysis

By Eq. (8), Eq. (20) and Eq. (11), we obtain

$$\frac{\partial q^*}{\partial G} = \frac{\partial q^*}{\partial Q^*} \frac{\partial Q^*}{\partial G} = \frac{2\sigma(T-t)\sqrt{\beta(1-\beta)}}{3T\beta\sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}} > 0$$
(23)

$$\frac{\partial t^*}{\partial G} = \frac{\partial t^*}{\partial Q^*} \frac{\partial Q^*}{\partial G} = -\frac{4T\mu\sqrt{\beta(1-\beta)}}{3\sigma\sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}} < 0$$
(24)

i.e., the optimal order quantity increases with demand-distribution-related parameter G, and the optimal purchase timing decreases with demand-distribution-related parameter G.

By Eq. (8), Eq. (20) and Eq. (10), we obtain

$$\frac{\partial q^{*}}{\partial D} = \frac{\partial q^{*}}{\partial Q^{*}} \frac{\partial Q^{*}}{\partial D} = \frac{\sigma^{2}(T-t)}{6T\beta\mu} \left(1 - \left(\sqrt{1-12\left(\frac{G}{D}\right)^{2}}\right)^{-1} \right); \text{ using } \left(\frac{D}{3}\right)^{2} > \left(\frac{2G}{\sqrt{3}}\right)^{2}$$

$$< 0 \qquad (25)$$

$$\frac{\partial t^{*}}{\partial D} = \frac{\partial t^{*}}{\partial Q^{*}} \frac{\partial Q^{*}}{\partial D} = -\frac{T}{3} \left(1 - \left(\sqrt{1-12\left(\frac{G}{D}\right)^{2}}\right)^{-1} \right); \text{ using } \left(\frac{D}{3}\right)^{2} > \left(\frac{2G}{\sqrt{3}}\right)^{2}$$

$$> 0 \qquad (26)$$

i.e., the optimal order quantity decreases with inventory-cost-related parameter D, and the optimal purchase timing increases with inventory-cost-related parameter D.

5.2.1. The effects of changing the standard deviation σ of the demand Use Eq. (11), Eq. (23) and rearrange it, we obtain

$$\frac{\partial q^*}{\partial \sigma} = \frac{\partial q^*}{\partial G^*} \frac{\partial G^*}{\partial \sigma} = -\frac{2\mu(1-\beta)(T-t)}{3T\sigma\sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}} < 0$$
(27)

i.e., the greater the standard deviation of the demand is, the less the optimal order quantity is.

Use Eq. (11), Eq. (24) and rearrange it, we obtain

$$\frac{\partial t^*}{\partial \sigma} = \frac{\partial t^*}{\partial G^*} \frac{\partial G^*}{\partial \sigma} = \frac{4T\mu^2(1-\beta)\beta}{3\sigma^3 \sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}} > 0$$
(28)

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i.e., the greater the standard deviation of the demand is, the later the optimal purchase timing is.

5.2.2. The effects of changing the mean μ of the demand Use Eq. (11), Eq. (23) and rearrange it, we obtain

$$\frac{\partial q^*}{\partial \mu} = \frac{\partial q^*}{\partial G^*} \frac{\partial G^*}{\partial \mu} = \frac{2(1-\beta)(T-t)}{3T\sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}} > 0$$
(29)

i.e., the greater the mean of the demand is, the greater the optimal order quantity is.

Use Eq. (11), Eq. (24) and rearrange it, we obtain

$$\frac{\partial t^*}{\partial \mu} = \frac{\partial t^*}{\partial G^*} \frac{\partial G^*}{\partial \mu} = -\frac{4T\mu(1-\beta)\beta}{3\sigma^2 \sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}} < 0$$
(30)

i.e., the greater the mean of the demand is, the earlier the optimal purchase timing is.

5.2.3. The effects of changing the shortage rate limit β

Use Eq. (11), Eq. (23) and rearrange it, we obtain

$$\frac{\partial q^*}{\partial \beta} = \frac{\partial q^*}{\partial G^*} \frac{\partial G^*}{\partial \beta} = \frac{\mu (1 - 2\beta)(T - t)}{3\beta T \sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}} = \begin{cases} > 0, & \text{if } \beta < \frac{1}{2} \\ < 0, & \text{if } \beta > \frac{1}{2} \end{cases}$$
(31)

i.e., when the shortage rate limit is less than 0.5 (in practice), the greater the shortage rate limit demand is, the greater the optimal order quantity is.

Use Eq. (11), Eq. (24) and rearrange it, we obtain

$$\frac{\partial t^*}{\partial \beta} = \frac{\partial t^*}{\partial G^*} \frac{\partial G^*}{\partial \beta} = -\frac{2T\mu^2(1-2\beta)}{3\sigma^2 \sqrt{\left(\frac{D}{3}\right)^2 - \left(\frac{2G}{\sqrt{3}}\right)^2}} = \begin{cases} <0, & \text{if } \beta < \frac{1}{2} \\ >0, & \text{if } \beta > \frac{1}{2} \end{cases}$$
(32)

i.e., the shortage rate limit is less than 0.5 (in practice), the greater the shortage rate limit is, the later the optimal purchase timing is.

5.2.4. The effects of changing the supplier's unit list price c Use Eq. (10), Eq. (25) and rearrange it, we obtain

$$\frac{\partial q^*}{\partial c} = \frac{\partial q^*}{\partial D^*} \frac{\partial D^*}{\partial c} = \frac{\sigma^2 (T-t)}{6(\delta-h)T^2 \beta \mu} \left(1 - \left(\sqrt{1 - 12\left(\frac{G}{D}\right)^2}\right)^{-1} \right) < 0$$
(33)

i.e., the greater the supplier's unit list price is, the less the optimal order quantity is.

Use Eq. (10), Eq. (26) and rearrange it, we obtain

$$\frac{\partial t^*}{\partial c} = \frac{\partial t^*}{\partial D^*} \frac{\partial D^*}{\partial c} = -\frac{1}{3(\delta - h)} \left(1 - \left(\sqrt{1 - 12\left(\frac{G}{D}\right)^2}\right)^{-1} \right) > 0$$
(34)

i.e., the greater the supplier's unit list price is, the greater the optimal purchase timing is.

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5.2.5. The effects of changing the unit purchase discount ahead of one unit time δ

Use Eq. (10), Eq. (25) and rearrange it, we obtain

$$\frac{\partial q^*}{\partial \delta} = \frac{\partial q^*}{\partial D^*} \frac{\partial D^*}{\partial \delta} = -\frac{\sigma^2 (c-\nu)(T-t)}{6(\delta-h)^2 T^2 \beta \mu} \left(1 - \left(\sqrt{1 - 12\left(\frac{G}{D}\right)^2}\right)^{-1} \right) > 0$$
(35)

i.e., the greater the unit purchase discount ahead of one unit time is, the greater the optimal order quantity is.

Use Eq. (10), Eq. (26) and rearrange it, we obtain

$$\frac{\partial t^*}{\partial \delta} = \frac{\partial t^*}{\partial D^*} \frac{\partial D^*}{\partial \delta} = \frac{(c-\nu)}{3(\delta-h)^2} \left(1 - \left(\sqrt{1-12\left(\frac{G}{D}\right)^2}\right)^{-1} \right) < 0$$
(36)

i.e., the greater the unit purchase discount ahead of one unit time is, the earlier the optimal purchase timing.

5.2.6. The effects of changing the unit holding cost per unit time h Use Eq. (10), Eq. (25) and rearrange it, we obtain

$$\frac{\partial q^*}{\partial h} = \frac{\partial q^*}{\partial D^*} \frac{\partial D^*}{\partial h} = \frac{\sigma^2 (c - \nu) (T - t)}{6(\delta - h)^2 T^2 \beta \mu} \left(1 - \sqrt{1 - 12 \left(\frac{G}{D}\right)^2} \right)^{-1} < 0$$
(37)

i.e., the greater the unit holding cost per unit time is, the less the optimal order quantity is.

Use Eq. (10), Eq. (26) and rearrange it, we obtain

$$\frac{\partial t^*}{\partial h} = \frac{\partial t^*}{\partial D^*} \frac{\partial D^*}{\partial h} = -\frac{(c-\nu)}{3(\delta-h)^2} \left(1 - \left(\sqrt{1-12\left(\frac{G}{D}\right)^2}\right)^{-1} \right) > 0$$
(38)

i.e., the greater the unit holding cost per unit time is, the later the optimal purchase timing is.

5.2.7. The effects of changing the unsold unit salvage value ν

Use Eq. (10), Eq. (25) and rearrange it, we obtain

$$\frac{\partial q^*}{\partial \nu} = \frac{\partial q^*}{\partial D^*} \frac{\partial D^*}{\partial \nu} = -\frac{\sigma^2 (T-t)}{6(\delta-h)T^2 \beta \mu} \left(1 - \left(\sqrt{1-12\left(\frac{G}{D}\right)^2}\right)^{-1} \right) > 0$$
(39)

i.e., The greater the unsold unit salvage value is, the greater the optimal order quantity is. Use Eq. (10), Eq. (26) and rearrange it, we obtain

$$\frac{\partial t^*}{\partial \nu} = \frac{\partial t^*}{\partial D^*} \frac{\partial D^*}{\partial \nu} = \frac{1}{3(\delta - h)} \left(1 - \left(\sqrt{1 - 12\left(\frac{G}{D}\right)^2}\right)^{-1} \right) < 0$$
(40)

i.e., the greater the unsold unit salvage value is, the earlier the optimal purchase timing is.

5.2.8. The effect of changing the length of price discount offered by the supplier T

Use Eq. (10), Eq. (25) and rearrange it, we obtain

$$\frac{\partial q^*}{\partial T} = \frac{\partial q^*}{\partial D^*} \frac{\partial D^*}{\partial T} = -\frac{\sigma^2 (c-\nu)(T-t)}{6(\delta-h)T^3\beta\mu} \left(1 - \left(\sqrt{1-12\left(\frac{G}{D}\right)^2}\right)^{-1}\right) > 0$$
(41)

i.e., the greater the length of price discount offered by the supplier is, the greater the optimal order quantity is.

Use Eq. (10), Eq. (26) and rearrange it, we obtain

$$\frac{\partial t^*}{\partial T} = \frac{\partial t^*}{\partial D^*} \frac{\partial D^*}{\partial T} = \frac{(c-\nu)}{3(\delta-h)T} \left(1 - \left(\sqrt{1-12\left(\frac{G}{D}\right)^2}\right)^{-1} \right) < 0$$
(42)

i.e., the greater the length of price discount offered by the supplier is, the earlier the optimal purchase timing is.

6. Conclusions

When average shortage rate is subject to a specified shortage rate limit, a mathematical model of a distribution free newsboy problem is formulated to decide simultaneously the optimal purchase timing and optimal order quantity.

From the research outcomes, the parameters of the distribution free newsboy problem could be divided into two categories: (i) demand-distribution-related parameter; (ii) inventory-cost-related parameter. Once the sizes of demand-distribution-related parameter and inventory-cost-related parameter are compared, the optimal purchase timing and order quantity of the mathematical model were decided. The optimal purchase timing decreases (increases) with demand-distribution-related parameter (inventory-cost-related parameter) while the optimal order quantity increases (decreases) with demand-distribution-related parameter (inventory-cost-related parameter). The effects of the mean of the demand, the standard deviation of the demand, the shortage rate limit (the supplier's unit list price, the unit purchase discount ahead of one unit time, the length of price discount offered by the supplier, the unit holding cost per unit time, the unsold unit salvage value) on the optimal purchase timing and optimal order quantity were obtained through the effects of the mean of the demand, the standard deviation of the demand, the shortage rate limit (the supplier's unit list price, the unit purchase discount ahead of one unit time, the length of price discount offered by the supplier, the unit holding cost per unit time, the unsold unit salvage value) on the demand-distribution-related parameter (the inventory-cost-related parameter) in the paper.

Additionally, Expected Value of Additional Information is defined as the largest amount that we would be willing to pay for the knowledge of a certain distribution, and then an example is illustrated under the optimal solution with the normal distribution and the optimal solution with the distribution free newsboy problem. All of these characteristics could be used to help the buyer to set the optimal inventory policy.

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