Journal of the Operations Research
Society of Japan
Vol. 44, No. 3, September 2001

FUSION OF MULTI-DIMENSIONAL POSSIBILISTIC INFORMATION VIA POSSIBILISTIC LINEAR PROGRAMMING

Peijun GUO
Kagawa University

Tomoe Entani
Osaka Prefecture University

Hideo Tanaka
Toyohashi Sozo College

(Received January 31, 2000; Final February 8, 2001)

Abstract In this paper, multi-source possibilistic information is represented by a set of possibilistic constraints to characterize decision variables from different information aspects. Possibilistic linear programming is used to integrate multi-source possibilistic information into the upper and the lower possibility distributions of decision vector.

1. Introduction

The remarkable advance of computer techniques has brought about a present-day information age characterized by the acceleration, intellectualization and globalization of information, which has stimulated a more emergent requirement for dealing with huge and sophisticated information in the real world. Information fusion is one of newly-emerging information techniques which tries to offer an integrated information from multiple information sources which are conflicting, partially inconsistent or reflect different aspect of information, such as multi-sensor and multi-expert pool where signals from different sensors represent the different or partially common information and likewise the experts also have different or partially common background and interests. Generally speaking, information fusion models can be built based on probability and possibility theories, respectively, to represent the intrinsic uncertainty contained in multi-source information. The probability networks, such as Bayes networks and Markov networks are well-known probability methods for information fusion where information is represented by a conditional probability distribution and fusion procedure is based on Bayes formula[13]. Dempster-Shafer theory of evidence (DS) is an important tool of information fusion where the fusion procedure is based on the Dempster's rule of combination [16]. Dubois, Prade and Yager proposed several information fusion models based on possibility theory [1,2,19,20]. The approaches related to information fusion also have been researched in the papers [3,5,6,7,18] for decision analysis. In this paper, another method for information fusion is proposed where multi-source information is represented by a set of possibilistic constraints to characterize decision variables from different information aspects. Each possibilistic constraint leaves some feasible region for decision variable. The feasible set of decision variables from all constraints conflicting with each other is characterized by their upper and lower possibility distributions obtained by possibilistic linear programming.

It is the first time to deal with information fusion via possibilisitic programming problems. However, it is been well-known that fuzzy linear programming initiated by Zimmermann [21] has been widely used and got many achievement in both applications and theories [9,10,11,14,15]. Generally speaking, in fuzzy linear programming problems, the coefficients of decision variables are fuzzy numbers while decision variables are crisp ones. This means that in uncertain environment, a crisp decision is made to meet some decision criteria. On the other hand, Tanaka et al. [17] initially proposed a possibilistic linear programming formulation where the coefficients of decision variables are crisp while decision variables are obtained as fuzzy numbers, and LP technique is used to obtain the largest possibility distribution of the decision variables. As an extension of that idea, Guo et al. [4] have used LP and quadratic programming (QP) techniques to obtain different fuzzy solutions to enable a decision maker select a preferable one. Further, Guo et al. [8] dealt with the interactive case of decision variables in which exponential distribution functions are used so that the formulation is very complex and a little bit difficult to be solved. This paper reconsiders the meaning of getting possibility distributions of decision variables from the viewpoint of information fusion. For avoiding the difficult of understanding, the triangular fuzzy numbers are used to represent possibilistic information.

2. Information Fusion via Possibilistic Linear Programming

Suppose that multi-source information related to decision vector $\mathbf{x}=[x_1,...,x_n]'$ can be described by a set of possibilistic constraints as follows:

$$b_{i1}x_1+\cdots+b_{in}x_n \approx^- C_i,\quad i=1,...,m_1, \tag{1}$$
$$b_{i1}x_1+\cdots+b_{in}x_n \approx^+ C_i,\quad i=m_1+1,...,m,$$

where the symbols $\approx^-$ and $\approx^+$, defined later, represent two kinds of "approximately satisfy", C_i is a fuzzy number(i=1,...,m), x_j is a decision variable (j=1,...,n) and b_{ij} is an associated crisp coefficient. The ith constraint of (1) characterizes information on decision vector **x** from the ith information source. The left-hand side of the constraint is some technical conditions to attain such possibilistic information. The fuzzy number C_i in (1), for example, can be regarded as a fuzzy goal given by a head with only considering the benefit of his own department in some company where the center of fuzzy number is an ideal point and the spread of fuzzy number represents some tolerance. As a result, a reasonable plan should be feasible for constraints from all of such departments.

It is obvious that if "$Y_i \approx^- C_i$" and "$\approx^+$" (soft equal) become "=" (hard equal) and correspondingly C_i becomes a crisp value in (1), we can rarely obtain the feasible solutions for the case of $m>n$. However, each possibilisitc constraint leaves some feasible region for decision variables. Because the feasible regions left from possibilistic constraints are conflicting and partially inconsistent in nature, the problem considered now is to obtain an integrated feasible region of decision vector, which satisfies all of possibilistic constraints. Let S_i (i=1,...,m) be a feasible region from the ith possibilistic constraint, the fused feasible region denoted as S is as follows

$$S=\bigcap_{i=1,...,m} S_i. \tag{2}$$

It can be seen that fusion operator here is intersection which satisfies the following three conditions:

(I) Idempotency: if the feasible regions from all possibilistic constraints are the same, the fused region should be this same one.
(II) Commutativity: the indexing of constraints is irrelevant.
(III) Monotonicity: If $\forall i$, $S_i \supseteq \hat{S}_i$, then $\bigcap_{i=1,...,m} S_i \supseteq \bigcap_{i=1,...,m} \hat{S}_i$.

Let us consider how to obtain the feasible region from possibilistic constraints in (1). For the sake of simplicity, assume that C_i is a symmetrical triangular fuzzy number denoted as $(c_i,d_i)_T$ where c_i and d_i are its center and spread with the condition $c_i-d_i\geq 0$. The feasible region of $\mathbf{x}=[x_1,...,x_n]'$ left from the ith possibility constraint is characterized by a symmetrical triangular fuzzy decision vector $\mathbf{A}=[A_1,......,A_2]'$. The membership function of $\mathbf{A}$ is defined as follows:

$$\Pi_{\mathbf{A}}(\mathbf{x})=\Pi_{A_1}(x_1)\wedge\Pi_{A_2}(x_2)\wedge\cdots\wedge\Pi_{A_n}(x_n) \tag{3}$$

where

$$\Pi_{A_i}(x_i)=\begin{cases}1-|x_i-a_i|/r_i, a_i-r_i\le x_i\le a_i+r_i, r_i>0\\ 0 \quad , \quad otherwise\end{cases}, \tag{4}$$

a_i and r_i are the center and spread of fuzzy number A_i. $\mathbf{A}$ is simply denoted as $(\mathbf{a},\mathbf{r})_T$, with $\mathbf{a}=[a_1,\cdots,a_n]'$ ($a_i\ge 0, i=1,\ldots,n$) and $\mathbf{r}=[r_1,\cdots,r_n]'$ ($r_i\ge 0, i=1,\ldots,n$). Denote two typical possibilistic constraints in (1) as follows:

$$b_{i1}x_1+\cdots+b_{in}x_n \approx^- C_i, \quad i=1,\ldots,m_1, \tag{5}$$

$$b_{j1}x_1+\cdots+b_{jn}x_n \approx^+ C_j, \quad j=m_1+1,\ldots,mj=m_1+1,\ldots,m, \tag{6}$$

where the subscripts i and j correspond to symbols "$\approx^-$" and "$\approx^+$", respectively. The left-hand side of possibilistic constraint of (1) is written as

$$Y_i=b_{i1}x_1+\cdots+b_{in}x_n. \tag{7}$$

Since $\mathbf{x}=[x_1,x_2,\cdots,x_n]'$ becomes a possibilistic vector $\mathbf{A}=[A_1,\ldots\ldots,A_2]'$, the possibility distribution of fuzzy set Y_i can be obtained by the extension principle as

$$\Pi_{Y_i}(y)=(y_i,s_i)_T=(\mathbf{b}_i'\mathbf{a},\mathbf{b}_i'\mathbf{r})_T, \tag{8}$$

where center y_i and spread s_i are $\mathbf{b}_i'\mathbf{a}$ and $\mathbf{b}_i'\mathbf{r}$, respectively, and $\mathbf{b}_i=[b_{i1},\cdots,b_{in}]'\ge\mathbf{0}$.

Constraint $Y_i\approx^- C_i$ is explained as "Y_i is a little bit smaller than C_i", defined by the following inequalities.

$$y_i-(1-h)s_i\le c_i-(1-h)d_i, \quad i=1,\ldots,m_1, \tag{9}$$

$$y_i+(1-h)s_i\le c_i+(1-h)d_i, \quad i=1,\ldots,m_1, \tag{10}$$

$$y_i\ge c_i-(1-h)d_i, \qquad i=1,\ldots,m_1, \tag{11}$$

where h is a predetermined possibility level by decision-makers. It can be understood that the higher h is, the stricter constraints are. A graphical explanation of $Y_i\approx^- C_i$ is given by Figure 1 which shows that "Y_i is a little bit smaller than C_i", denoted as $Y_i\approx^- C_i$, means that the left and right endpoints of the h-level set of Y_i are smaller than the left and right ones of C_i, respectively described in (9) and (10), but the center of Y_i is confined to be larger than the left endpoint of C_i shown by the dotted line described in (11).

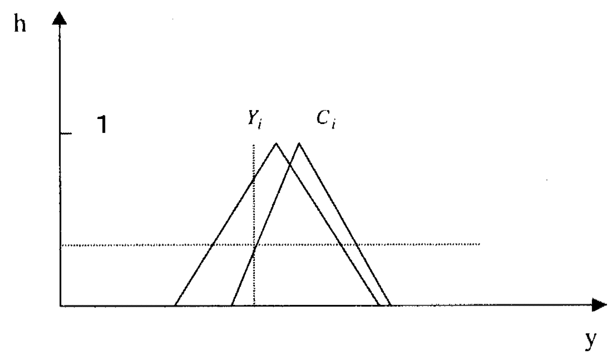

Figure 1. Explanation of $Y_i\approx^- C_i$ with degree h

Similarly, the constraint $Y_j\approx^+ C_j$ is explained as "Y_j is a little bit larger than C_j", defined by the following inequalities:

$$y_i-(1-h)s_i\ge c_i-(1-h)d_i, i=1,\ldots,m_1, \tag{12}$$

$$y_i+(1-h)s_i\ge c_i+(1-h)d_i, i=1,\ldots,m_1, \tag{13}$$

$$y_i\le c_i+(1-h)d_i, \qquad i=1,\ldots,m_1. \tag{14}$$

It is obvious from the above definitions that possibilistic constraint conditions leave some feasible region for decision variables described by inequalities (9)-(14). Considering (8), possibilistic constraint (5) can be rewritten as

$$\mathbf{b}_i'\mathbf{a}-(1-h)\mathbf{b}_i'\mathbf{r}\le c_i-(1-h)d_i,\quad i=1,\ldots,m_1, \tag{15}$$

$$\mathbf{b}_i'\mathbf{a}+(1-h)\mathbf{b}_i'\mathbf{r}\le c_i+(1-h)d_i,\quad i=1,\ldots,m_1, \tag{16}$$

$$\mathbf{b}_i'\mathbf{a}\ge c_i-(1-h)d_i,\qquad i=1,\ldots,m_1, \tag{17}$$

and possibilistic constraint (6) can be rewritten as

$$\mathbf{b}_j'\mathbf{a}-(1-h)\mathbf{b}_j'\mathbf{r}\ge c_j-(1-h)d_j,\quad j=m_1+1,\ldots,m, \tag{18}$$

$$\mathbf{b}_j'\mathbf{a}+(1-h)\mathbf{b}_j'\mathbf{r}\ge c_j+(1-h)d_j,\quad j=m_1+1,\ldots,m, \tag{19}$$

$$\mathbf{b}_j'\mathbf{a}\le c_j+(1-h)d_j,\qquad j=m_1+1,\ldots,m. \tag{20}$$

From constraints (15) -(20), it is easy to understand that the center vector $\mathbf{a}$ should comply with the following constraints,

$$c_i-(1-h)d_i\le\mathbf{b}_i'\mathbf{a}\le c_i,\quad i=1,\ldots,m_1, \tag{21}$$

$$c_j\le\mathbf{b}_j'\mathbf{a}\le c_j+(1-h)d_i,\ j=m_1+1,\ldots,m. \tag{22}$$

It implies that the center vector must exist in the region formed by (21) and (22). The following LP problem is used to find out a candidate of center vector.

$$\max_{\mathbf{a}}\quad g(\mathbf{a}) \tag{23}$$

$$\begin{aligned}
\text{s. t.}\quad & \mathbf{b}_i'\mathbf{a}\ge(c_i-(1-h)d_i)(1+\beta),\quad i=1,\ldots,m_1,\\
& \mathbf{b}_i'\mathbf{a}\le c_i(1-\beta),\quad i=1,\ldots,m_1,\\
& \mathbf{b}_j'\mathbf{a}\le(c_j+(1-h)d_j)(1-\beta),\quad j=m_1+1,\ldots,m,\\
& \mathbf{b}_j'\mathbf{a}\ge c_j(1+\beta),\quad j=m_1+1,\ldots,m,\\
& \beta\ge0,\\
& \mathbf{a}\ge\mathbf{0},
\end{aligned}$$

where objective function $g(\mathbf{a})$ is given by decision-makers to characterize his preference for selecting center vector, which is regarded as a reference point in the feasible region of decision variables. Parameter $\beta\ge0$ is used to reduce the original feasible region to guarantee the obtained center vector inside the region formed by (21) and (22). It should be noted that the selected parameter β by decision-makers should satisfy the following theorem.

Theorem 1 *The necessary condition for the existence of an optimal solution in (23) is that parameter β satisfies the following condition:*

$$\beta\le\Big(\min_{i=1,\ldots,m_1}\frac{(1-h)d_i}{2c_i-(1-h)d_i}\Big)\wedge\Big(\min_{j=m_1+1,\ldots,m}\frac{(1-h)d_j}{2c_j+(1-h)d_j}\Big) \tag{24}$$

Proof: Suppose that there is an optimal solution in (23). Considering the first two constraints of (23), we have

$$(c_i-(1-h)d_i)(1+\beta)\le c_i(1-\beta),\quad i=1,\ldots,m_1. \tag{25}$$

(25) leads to

$$(2c_i-(1-h)d_i)\beta\le(1-h)d_i,\quad i=1,\ldots,m_1. \tag{26}$$

The support of C_i is assumed to be non-negative. Therefore, the inequality $c_i-d_i\ge0$ is satisfied and the following inequality holds.

$$\beta\le\frac{(1-h)d_i}{2c_i-(1-h)d_i},\quad i=1,\ldots,m_1. \tag{27}$$

Considering the third and fourth constraints of (23), likewise we can have

$$\beta \le \frac{(1-h)d_j}{2c_j+(1-h)d_j}, \quad j=m_1+1,\ldots,m. \tag{28}$$

Inequalities (27) and (28) lead to (24).□

It should be noted that constraints (21) and (22) are the necessary conditions for $\mathbf{a}$ being a center vector rather than the sufficient conditions. As a reasonable center vector it is also needed to satisfy constraint conditions (15)-(20) with spread vector $\mathbf{r}$. We can find out a reasonable center vector via changing the value of β. The larger the value of β, the smaller the feasible set of center vector. If we set the different β for each constraint, by changing the value of β_i and $g(\mathbf{a})$, we can search all of points in the original feasible set of center vector. After determining one center vector, denoted as $\mathbf{a}_0$, we investigate two kinds of possibility distributions of fuzzy vector A, i.e., upper and lower possibility distributions denoted as $\Pi_u(\mathbf{x})$ and $\Pi_l(\mathbf{x})$, respectively, with the condition $\Pi_u \ge \Pi_l$, which have some similarities with rough sets concept [12]. The upper possibility distribution of A corresponds to the upper possibility distribution of Y and the lower possibility distribution of A corresponds to the lower possibility distribution of Y. Figure 2 gives a graphic explanation of upper and lower possibility distributions of Y restricted by the possibility constraint $Y \approx^- C$. Figure 2 shows that the region covered by the lower possibility distribution of Y is completely included by the h-level set of the given possibility information C while the region covered by the upper possibility distribution of Y completely includes the h-level set of the given possibility information C under the definition of $Y \approx^- C$. It can be understood that the upper and lower possibility distributions of Y are the upper and lower bounds of Y restricted by $Y \approx^- C$ for a fixed center of Y.

With considering a center vector $\mathbf{a}_0$, the problem for finding out the lower spread vector of A is formalized as the following LP problem.

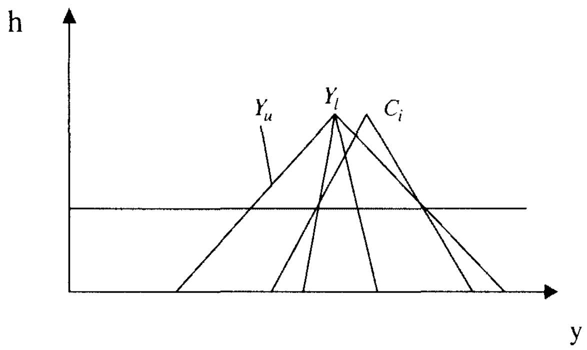

Figure 2. Illustration of upper and lower possibility distributions in $Y \approx^- C$

$$\min_{\mathbf{r}_l} \quad \mathbf{w}'\mathbf{r}_l \tag{29}$$

$$\begin{aligned}
\text{s. t.} \quad & -(1-h)\mathbf{b}_i'\mathbf{r}_l \le c_i-(1-h)d_i-\mathbf{b}_i'\mathbf{a}_0, \quad i=1,\ldots,m_1,\\
& (1-h)\mathbf{b}_i'\mathbf{r}_l \le c_i+(1-h)d_i-\mathbf{b}_i'\mathbf{a}_0, \quad i=1,\ldots,m_1,\\
& -(1-h)\mathbf{b}_j'\mathbf{r}_l \ge c_j-(1-h)d_j-\mathbf{b}_j'\mathbf{a}_0, \quad j=m_1+1,\ldots,m,\\
& (1-h)\mathbf{b}_j'\mathbf{r}_l \ge c_j+(1-h)d_j-\mathbf{b}_j'\mathbf{a}_0, \quad j=m_1+1,\ldots,m,\\
& \mathbf{r}_l \ge \mathbf{0},
\end{aligned}$$

where $\mathbf{w}$ is the weight vector of the lower spread vector $\mathbf{r}_l$.

Similarly, the problem for finding out the upper spread vector of A is formalized as the following LP problem.

$$\max_{\mathbf{r}_u} \quad \mathbf{w}'\mathbf{r}_u \tag{30}$$

$$\begin{aligned}
\text{s. t.} \quad & -(1-h)\mathbf{b}_i'\mathbf{r}_u \le c_i-(1-h)d_i-\mathbf{b}_i'\mathbf{a}_0, \quad i=1,\dots,m_1, \\
& (1-h)\mathbf{b}_i'\mathbf{r}_u \le c_i+(1-h)d_i-\mathbf{b}_i'\mathbf{a}_0, \quad i=1,\dots,m_1, \\
& -(1-h)\mathbf{b}_j'\mathbf{r}_u \ge c_j-(1-h)d_j-\mathbf{b}_j'\mathbf{a}_0, \; j=m_1+1,\dots,m, \\
& (1-h)\mathbf{b}_j'\mathbf{r}_u \ge c_j+(1-h)d_j-\mathbf{b}_j'\mathbf{a}_0, \quad j=m_1+1,\dots,m, \\
& \mathbf{r}_u \ge \mathbf{0}.
\end{aligned}$$

It can be seen that intersection operator of feasible sets in (2) is transformed into the above LP problems. It is obvious that constraints in (29) and (30) satisfy idempotency, commutativity and monotonicity. It means that if the feasible set formed by each possibilistic constraint in (1) is the same, the fused feasible set of decision variables obtained from (29) or (30) is the same as that one, and changing the indices of constraints in (1) can not make any difference of the solutions obtained from (29) and (30), and any reduction of the feasible set formed by constraints of (1) leads to the reduction of fused feasible set of decision variables obtained from (29) or (30). By considering the inclusion relation between the upper and lower possibility distributions, the problem for obtaining the upper and lower spreads of A simultaneously is introduced as follows:

$$\max_{\mathbf{r}_u,\mathbf{r}_l} \quad \mathbf{w}'(\mathbf{r}_u-\mathbf{r}_l) \tag{31}$$

$$\begin{aligned}
\text{s. t.} \quad & -(1-h)\mathbf{b}_i'\mathbf{r}_l \le c_i-(1-h)d_i-\mathbf{b}_i'\mathbf{a}_0, \quad i=1,\dots,m_1, \\
& (1-h)\mathbf{b}_i'\mathbf{r}_l \le c_i+(1-h)d_i-\mathbf{b}_i'\mathbf{a}_0, \quad i=1,\dots,m_1, \\
& -(1-h)\mathbf{b}_j'\mathbf{r}_l \ge c_j-(1-h)d_j-\mathbf{b}_j'\mathbf{a}_0, \quad j=m_1+1,\dots,m, \\
& (1-h)\mathbf{b}_j'\mathbf{r}_l \ge c_j+(1-h)d_j-\mathbf{b}_j'\mathbf{a}_0, \quad j=m_1+1,\dots,m, \\
& -(1-h)\mathbf{b}_i'\mathbf{r}_u \le c_i-(1-h)d_i-\mathbf{b}_i'\mathbf{a}_0, \quad i=1,\dots,m_1, \\
& (1-h)\mathbf{b}_i'\mathbf{r}_u \le c_i+(1-h)d_i-\mathbf{b}_i'\mathbf{a}_0, \quad i=1,\dots,m_1, \\
& -(1-h)\mathbf{b}_j'\mathbf{r}_u \ge c_j-(1-h)d_j-\mathbf{b}_j'\mathbf{a}_0, \quad j=m_1+1,\dots,m, \\
& (1-h)\mathbf{b}_j'\mathbf{r}_u \ge c_j+(1-h)d_j-\mathbf{b}_j'\mathbf{a}_0, \quad j=m_1+1,\dots,m, \\
& \mathbf{r}_u-\mathbf{r}_l \ge \mathbf{0}, \\
& \mathbf{r}_l \ge \mathbf{0}.
\end{aligned}$$

The dual problem of (31) is as follows:

$$\min_{\mathbf{z}} \quad [\mathbf{c}_1'\mathbf{c}_2'\mathbf{c}_3'\mathbf{c}_4'\mathbf{c}_1'\mathbf{c}_2'\mathbf{c}_3'\mathbf{c}_4'\mathbf{c}_5']'\mathbf{z} \tag{32}$$

$$\text{s. t.} \quad \begin{bmatrix} -\mathbf{B}\,\mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & -\mathbf{B}\,\mathbf{B} & -\mathbf{I} \end{bmatrix} \mathbf{z} \ge \begin{bmatrix} -\mathbf{w} \\ \mathbf{w} \end{bmatrix},$$

where

$$\begin{aligned}
& \mathbf{c}_1' = [c_{11}', c_{21}', \dots, c_{m_1 1}'], \\
& \mathbf{c}_2' = [c_{(m_1+1)2}', c_{(m_1+2)2}', \dots, c_{m2}'], \\
& \mathbf{c}_3' = [c_{12}', c_{22}', \dots, c_{m_1 2}'], \\
& \mathbf{c}_4' = [c_{(m_1+1)1}', c_{(m_1+2)1}', \dots, c_{m1}'], \\
& \mathbf{c}_5' = [0]_{1\times n}, \\
& \mathbf{B} = [\mathbf{b}_1, \dots, \mathbf{b}_{m_1}, \mathbf{b}_{m_1+1}, \dots, \mathbf{b}_m], \\
& \dim(\mathbf{I}) = n,
\end{aligned}$$

$c_{i1}' = (c_i - (1-h)d_i - \mathbf{b}_i'\mathbf{a}_0)/(1-h)$, $i = 1,\dots,m_1$,

$c_{i2}' = (c_i + (1-h)d_i - \mathbf{b}_i'\mathbf{a}_0)/(1-h)$, $i = 1,\dots,m_1$,

$c_{j1}' = -(c_j - (1-h)d_j - \mathbf{b}_j'\mathbf{a}_0)/(1-h)$, $j = m_1+1,\dots,m$,

$c_{j2}' = -(c_j + (1-h)d_j - \mathbf{b}_j'\mathbf{a}_0)/(1-h)$, $j = m_1+1,\dots,m$.

Considering the theorem of complementary slackness, it is convenient to know which constraints in (31) are active by solving the dual problem (32).

Definition 1 *Fuzzy numbers C_i in (1) is called critical possibilistic information for fusion if any inequality for describing the possibilistic constraint associated with C_i is active in the LP problem (31).*

Theorem 2 *Denote the optimal solution obtained from (32) as $\mathbf{z}^* = [z_1^*,\dots,z_{4m+n}^*]'$ and the index set of positive elements of $\mathbf{z}^*$ as $D = \{i \mid z_i > 0, i \le 4m\}$. The index set of critical possibilistic informations for fusion, denoted as D_r, is*

$$D_r = \{j \mid j = i - m\delta(i), i \in D\} \tag{33}$$

where

$$\delta(i) = \begin{cases} 0; if & i \le m \\ 1; if & m < i \le 2m \\ 2; if & 2m < i \le 3m \\ 3; if & 3m < i \le 4m \end{cases},$$

and it should be noted that j in (33) denotes the jth constraint in (1).

Proof: The constraints of the dual problem of the problem (32) are as follows.

$-\mathbf{b}_i'\mathbf{r}_l \le c_{i1}'$, $i = 1,\dots,m_1$,

$-\mathbf{b}_j'\mathbf{r}_l \le c_{j2}'$, $j = m_1+1,\dots,m$,

$\mathbf{b}_i'\mathbf{r}_l \le c_{i2}'$, $i = 1,\dots,m_1$,

$\mathbf{b}_j'\mathbf{r}_l \le c_{j1}'$, $j = m_1+1,\dots,m$,

$-\mathbf{b}_i'\mathbf{r}_u \le c_{i1}'$, $i = 1,\dots,m_1$,

$-\mathbf{b}_j'\mathbf{r}_u \le c_{j2}'$, $j = m_1+1,\dots,m$,

$\mathbf{b}_i'\mathbf{r}_u \le c_{i2}'$, $i = 1,\dots,m_1$,

$\mathbf{b}_j'\mathbf{r}_u \le c_{j1}'$, $j = m_1+1,\dots,m$,

$\mathbf{r}_l - \mathbf{r}_u \le \mathbf{0}$,

$\mathbf{r}_l \ge \mathbf{0}$,

where i and j indicate the ith possibility constraints of $b_{i1}x_1 + \cdots + b_{in}x_n \approx^- C_i$ and jth possibilistic constraint of $b_{j1}x_1 + \cdots + b_{jn}x_n \approx^+ C_j$ in (1), respectively. It can be seen that one possibilistic constraint in (1) correspond to four constraints in the above constraints. With considering the theorem of complementary slackness and Definition 1, the index set of critical possibilistic informations for fusion can be obtained as D_r. □

Definition 2 *Denote the fused feasible sets of decision variables obtained from the upper and lower possibility distributions with a possibility level h as S_u^h and S_l^h, respectively. The sets S_u^h and S_l^h are defined as follows:*

$$S_u^h = \{\mathbf{x} \in R^n \mid \Pi_u(\mathbf{x}) \ge h\}, \tag{34}$$

$$S_l^h = \{\mathbf{x} \in R^n \mid \Pi_l(\mathbf{x}) \ge h\}. \tag{35}$$

Theorem 3 *For a possibility level h, the relation $S_l^h \subseteq S_u^h$ holds.*

It is trivial to prove this theorem with considering constraint condition $\mathbf{r}_u - \mathbf{r}_l \ge \mathbf{0}$ in (31). This theorem means that the fused region of decision vector covered by the upper possibility distribution leaves more rooms than the one by the lower possibility distribution.

4. Numerical Example

Let us consider the following possibilisitc constraints,

$$2x_1+x_2 \approx^{-} \tilde{15}, \quad (36)$$

$$x_2 \approx^{-} 4.\tilde{5},$$

$$3x_1+4x_2 \approx^{-} 3\tilde{4},$$

$$x_1 \approx^{-} 5.\tilde{5},$$

$$x_1+2x_2 \approx^{+} 1\tilde{3},$$

where

$$1\tilde{5}=(15.,2.)_T,\ 4.\tilde{5}=(4.5,1.)_T,\ 3\tilde{4}=(34.,4.)_T,\ 5.\tilde{5}=(5.5,0.9)_T,\ 1\tilde{3}=(13.,2.)_T. \quad (37)$$

Assume that the possibility distribution of decision variables x_1 and x_2 is $X=(\mathbf{a},\mathbf{r})_T$ with $\mathbf{a}=[a_1,a_2]'$, $\mathbf{r}=[r_1,r_2]'$.

In (36) each possibilistic constraint characterizes one information aspect related to decision variables and leaves a feasible region. Our concern is to obtain the fused feasible region of decision variables satisfying all possibilistic constraints of (36). By setting $h=0.5$, a center vector was obtained with $\beta=0.01$ and $g(\mathbf{a})=2x_1+3x_2$ as follows:

$$\mathbf{a}=[5.28,4.29]'.$$

Upper and lower spread vectors were obtained by (31) with $\mathbf{w}=[1,1]'$ as follows:

$$\mathbf{r}_u=[0.64,1.02]',$$

$$\mathbf{r}_l=[0.56,0.58]'.$$

The fused region of decision variables left by the possibilistic constraints of (36) is characterized by the possibility distribution of decision variables. The upper and the lower possibility distributions of fuzzy decision variables with h=0.5 are shown in Figure 3. In Figure 3 the regions inside the two contours are feasible sets $S_u^{0.5}$ and $S_l^{0.5}$, respectively. It is obvious that the solution obtained from the upper possibility distribution leaves more room than the one from the lower possibility distribution. Solving the dual problem (32), we had $D=\{2,6,11,13\}$. Using (33), we obtained $D_r=\{1,2,3\}$ with m=5. It means that the first three fuzzy numbers are critical for obtaining the fused feasible region. Figure 4 shows the difference between the first and the fourth constraints. It can be seen that there is some margin for the fourth constraint but not for the first one corresponding to critical possibilistic information.

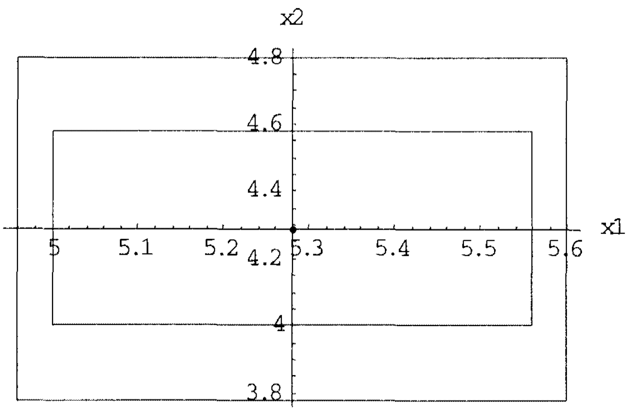

Figure 3. $S_u^{0.5}$ and $S_l^{0.5}$ (the outer is from the upper possibility distribution and the inner is from

the lower possibility distribution).

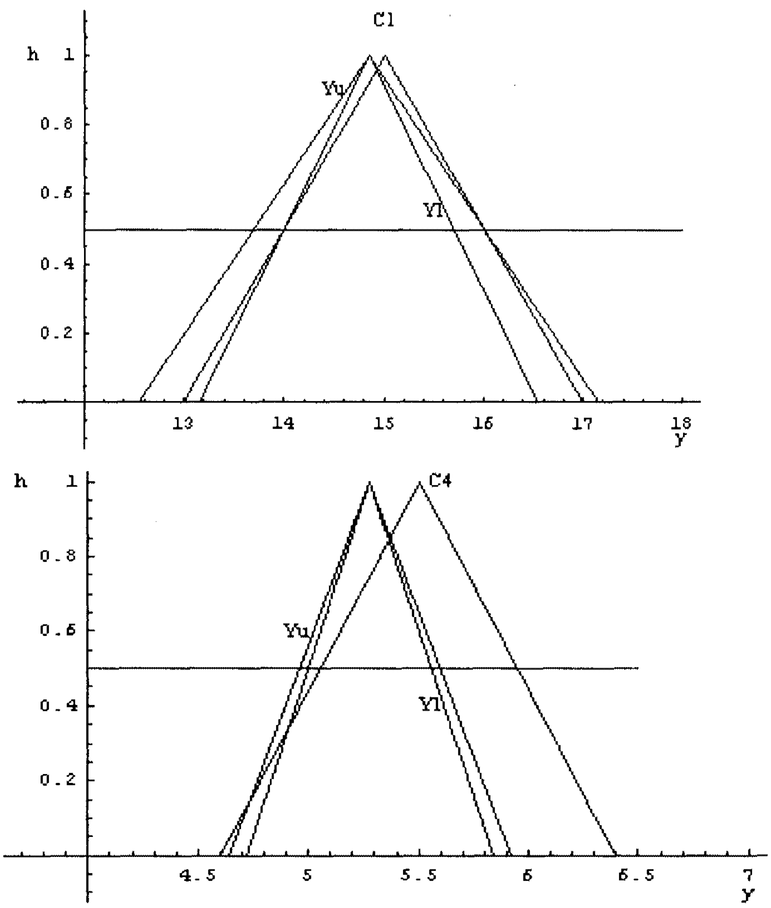

Figure 4. Illustration of the first and fourth constraints

5. Conclusions

In this paper, multi-source information on decision variables is characterized by a set of possibilistic constraints, which characterize decision variables from different information aspects. Two kinds of the fused regions of decision variables are obtained by possibilistic linear programming where the region governed by the upper possibilistic distribution leaves more room than that by the lower possibilistic distribution.

Acknowledgements

This research was supported in part by the Japanses Ministry of Education, Sports, Science and Technology (Research-in-aid Program No. 12680444). The authors wish to acknowledge two anonymous referees for their helpful comments.

References

[1] D. Dubois and H. Prade: Possibility theory and data fusion in poorly informed environment. *Control Engineering Practice*, **2** (1994) 811-823.

[2] D. Dubois and H. Prade: An introductory survey of possibility theory and its recent developments. *Journal of Japan Society for Fuzzy Theory and Systems*, **10** (1998) 21-42.

[3] P. Guo: *Mathematical Approaches to Knowledge Representation, Fusion and Decision Based on Possibility Theory* (Doctoral Dissertation, Osaka Prefecture University, Japan, 2000).

[4] P. Guo, H. Tanaka: Fuzzy decision in possibility programming problems. *Proceedings of Asian Fuzzy System Symposium*, (1996) 278-283.

[5] P. Guo and H. Tanaka: Possibilistic data analysis and its application to portfolio selection problems. *Fuzzy Economic Review*, **3/2** (1998) 3-23.

[6] P. Guo and H. Tanaka: Possibilisitc information fusion. *Proceedings of The Eighth IEEE International Conference on Fuzzy Systems*, **2** (1999) 819-823.

[7] P. Guo and H. Tanaka, M. Inuiguchi: Self-organizing fuzzy aggregation models to rank the objects with multiple attributes. *IEEE Transactions on SMC, Part A: Systems and Humans*, **30** (2000) 573-580.

[8] P. Guo, H. Tanaka and H.-J. Zimmermann: Upper and lower possibility distributions of fuzzy decision variables in upper level decision problems. *Fuzzy Sets and Systems*, **111** (2000) 71-79.

[9] H. Ishii, M. Tada, T. Masuda: Two scheduling problems with fuzzy due-dates. *Fuzzy Sets and Systems*, **46** (1992) 339-347.

[10] M. Inuiguchi, H. Ichihashi and H. Tanaka: Fuzzy Programming: A survey of recent developments. In R. Slowinski and J. Teghem (eds.): *Stochastic Versus Fuzzy Approaches to Multiobjective Mathematical Programming under Uncertainty* (Kluwer Academic Publishers, Netherlands, 1990), 45-68.

[11] M. Inuiguchi and J. Ramik: Possibilistic linear programming: a brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problem. *Fuzzy Sets and Systems*, **111** (2000) 3-28.

[12] Z. Pawlak: *Rough Sets* (Kluwer Academic Publishers, Netherlands, 1991).

[13] J. Pearl: *Possibilistic Reasoning in Intelligent Systems: Networks of Plausible Inference* (Morgan Kaufman, California, 1993).

[14] H. Rommelfange: Fuzzy linear programming and applications. *European Journal of Operational Research*, **92** (1996) 512-527.

[15] M. Sakawa: *Fuzzy Sets and Interactive Multiobjective Optimization* (Plenum Press, New York, 1993).

[16] G. Shafer: *A Mathematical Theory of Evidence* (Princeton University Press, Princeton, 1976).

[17] H. Tanaka and K. Asai: Fuzzy solution in fuzzy linear programming problems. *IEEE Transaction on System, Man, and Cybernetics*, **14** (1984) 325-328.

[18] H. Tanaka andP. Guo: *Possibilistic Data Analysis for Operations Research* (Physica-Verlag, Heidelberg, 1999)

[19] R. R. Yager: Information fusion in multiple database environments. *Proceeding of Seventh IFSA World Congress*, (1997) 261-266.

[20] R. R. Yager and A. Kelman: Fusion of fuzzy information with considerations for compatibility, partial aggregation, and reinforcement. *International Journal of Approximate Reasoning*, **15** (1996) 93-122.

[21] H. -J. Zimmermann: Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, **1** (1978) 45-55.

Peijun Guo
Faculty of Economics
Kagawa University
Takamatsu 760-8523, JAPAN
guo@ec.kagawa-u.ac.jp