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DYNAMIC PRODUCTION PLAN OF PROBABILISTIC MARKET DEMAND AND FIXED SELLING TIME WITH UNRELIABLE MACHINES AND OBTAINABLE WORKING HOUR CAPACITY

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Abstract The Dynamic Production Plan (*DPP*) Model for reaching an optimal control of the production undergoing the considerations of probabilistic market demand, future obtainable working hour capacity and unreliable machines is proposed in this paper. It can be applied to evaluate the optimal production rate to reduce the risk in the future of uncertainty. It is also suggested that the time interval of production, maintenance cost of an unreliable machine, transaction, penalty, holding costs, sales price and the machine reliability should be taken into considerations. In addition, sensitivity analyses on the key variables of optimal solution are presented. Actually, this study efficiently provides a dynamic tool capable of controlling the production plan (rate) at any time for the production planner having great insight.

1. Introduction

To control the time interval of production and the production rate at any given time is an efficient way to enlarge the production profits for a firm. This can be suitable to both single-stage and multi-stage manufacturing processes. In general, resources limit the production in each period of time. In fact, the resource limitation makes the production rate restricted. According to Metters [7], the resource limitation is closely related to the capacity of critical production equipment. The capacity is always treated fixed in most studies because adding capacity is expensive. As a matter of fact, the consideration on the finite capacity is a practical problem confronting companies.

In addition, Sox and Muckstadt [8] proposed that an effective plan of production is essential for manufacturing companies to make efficient use of their resources. Traditional production plan models are built around the assumption of infinite capacity. However, this assumption is not appropriate in dynamic environments where adaptability and flexibility are essential.

Market demand can be divided into two different types; one is deterministic demand and the other is probabilistic demand [1, 3]. This paper focuses on the probabilistic demand. In addition, Kalir and Ariz [4] stated that a workstation consisting of several unreliable machines [4, 5] of the same type in parallel is very common in industry. Also, the output rate of this workstation corresponds to the output rate of all these unreliable machines. The unreliable machine is defined as that the machine failure can occur randomly.

In practice, the finite resource capacity, the time interval of production, the relevant costs, the reliability of a machine are considered simultaneously to determine the production rate of production plan under probabilistic market demand, leading to the maximization of total profits. In this paper, the maximum obtainable working hours (depending on the number of machines can be offered) of the workstation at each given time is regarded as the finite resource capacity at that time. Also, the single-stage manufacturing process workstation consisting of several unreliable machines of the same type in parallel is discussed in this study.

2. Notations and Assumptions

For constructing a mathematical model, several assumptions and notations should be stated clearly as follows:

2.1. Assumptions

1. $U(t)$ is a strictly increasing function of t and represents the cumulative maximum available working hours during time interval $[0,t]$. It is evaluated before production and constructed under that all vacant and crashing working hours can be offered. In addition, because the new production plan cannot change the previous plans, $U(t)$ is known before production.
2. Single-stage automatically manufactured products of the same type are discussed in this study.
3. The deterioration of a product is neglected.

2.2. Parameters and notations

T: due date of production (*i.e.* selling time).

P: sales price of a product.

c_s: unit transaction cost. It occurs when the backlog quantity at time T is greater than the demand quantity at time T.

c_p: unit penalty cost. It occurs when the backlog quantity at time T is less than the demand quantity at time T.

L: processing time of one product in workstation.

R: expected reliability of a machine, which is defined by $R = \varepsilon/(\delta+\varepsilon)$; where ε and δ are the mean length of time interval between failures and the mean length of time interval to repair a machine respectively.

c_m: maintenance (repair) cost per unit time. It occurs when the machine fails to work. *i.e.* $c_m\delta$ is the mean maintenance (repair) fees for a single machine.

c_h: holding cost per unit time of a product.

Y: demand quantity of products at time T. Here, Y is a random variable, its probability density function is $f(y)$, and its cumulative distribution function is $F(y)$.

$[d]^+$: $[d]^+ = d$ if $d \geq 0$, and $[d]^+ = 0$ if $d < 0$.

2.3. Decision function

$x(t)$: cumulative operational working hours during time interval $[t_x, T]$, where t_x is the initial time of x for production; that is

$$x(t) = \begin{cases} 0 & \text{if } 0 \leq t \leq t_x, \\ > 0 & \text{if } t_x < t \leq T. \end{cases}$$

2.4. Given functions

$u(t)$: $u(t) = U'(t) > 0$, $u(t)$ means the maximum available working hours at time t; where $u(t)$ is already known by the manufacturer before production.

$C_t(Z)$: production cost of the operational working hours Z at time t, $C_t(0) = 0$ and $\frac{d}{dZ}C_t(Z) = g(\frac{Z}{u(t)})$, where $g(\frac{Z}{u(t)})$ is a nonnegative continuous strictly increasing function of $\frac{Z}{u(t)}$ (operational usage rate of working hours at time t), where $\frac{Z}{u(t)} \in [0, 1]$.

In fact, $g(\frac{x'(t)}{u(t)})$ presents the marginal cost of input operational working hours at time t, and g is determined by the value of $\frac{x'(t)}{u(t)}$. Since $u(t)$ is fixed for a given t, the larger $x'(t)$ makes $\frac{x'(t)}{u(t)}$ larger. Practically, the more input of operational working hours gets more marginal cost of input operational working hours under the limited working hours. Therefore, that

$$g \quad \text{is an increasing function of} \quad \frac{x'(t)}{u(t)} \tag{1}$$

is suggested in this study.

3. Model Development

In this study, $\int_{t_x}^{T} C_t(x'(t))dt$ and $\int_{t_x}^{T} c_m(1-R)x'(t)dt$ represent the production cost and the maintenance (repair) cost during time interval $[t_x, T]$ respectively. $\int_{t_x}^{T} \frac{c_h R}{L} x(t)dt$ means the total holding cost during time interval $[t_x, T]$.

Since the quantity of products sold, $\min\left\{\frac{R}{L}x(t), Y\right\}$, is a random variable, $\int_0^{\frac{Rx(T)}{L}} Pyf(y)dy + \int_{\frac{Rx(T)}{L}}^{\infty} P\frac{Rx(T)}{L}f(y)dy$ means the expected revenue and $\int_0^{\frac{Rx(T)}{L}} c_s(\frac{Rx(T)}{L} - y)f(y)dy$ shows the expected transaction cost for surplus products. In addition, $\int_{\frac{Rx(T)}{L}}^{\infty} c_p(y - \frac{Rx(T)}{L})\, f(y)dy$ represents the expected penalty cost for lacking products. Thus, a mathematical model is constructed below:

$$\begin{cases} \max\limits_{x} \left[\int_0^{\frac{Rx(T)}{L}} Pyf(y)dy + \int_{\frac{Rx(T)}{L}}^{\infty} P\frac{Rx(T)}{L}f(y)dy\right] \\ \quad -\left\{\int_{t_x}^{T}\left[C_t(x'(t)) + c_m(1-R)x'(t) + \frac{c_h R}{L}x(t)\right]dt \right. \\ \quad \left. + \int_0^{\frac{Rx(T)}{L}} c_s(\frac{Rx(T)}{L} - y)f(y)dy + \int_{\frac{Rx(T)}{L}}^{\infty} c_p(y - \frac{Rx(T)}{L})f(y)dy\right\} \\ s.t. \quad \text{domain } x = [t_x, T] \text{ with } 0 \le t_x \le T \\ \qquad x(t_x) = 0,\ t_x \text{ and } x(T) \text{ are free, and } 0 \le \frac{x'(t)}{u(t)} \le 1\ \forall t \in [t_x, T]. \end{cases}$$

The above model can be rearranged to be Dynamic Production Plan (DPP) Model. It is described as follows.

$$
DPP \begin{cases} \max\limits_{x} \int_0^{\frac{Rx(T)}{L}} \left[Py - c_s(\frac{Rx(T)}{L} - y) \right] f(y)dy + \\ \quad + \int_{\frac{Rx(T)}{L}}^{\infty} \left[P\frac{Rx(T)}{L} - c_p(y - \frac{Rx(T)}{L}) \right] f(y)dy \\ \quad - \int_{t_x}^{T} \left[C_t(x'(t)) + c_m(1-R)x'(t) + \frac{c_h R}{L} x(t) \right] dt \\ s.t. \quad \text{domain } x = [t_x, T] \text{ with } 0 \le t_x \le T \\ \quad x(t_x) = 0,\ t_x \text{ and } x(T) \text{ are free, and } 0 \le \frac{x'(t)}{u(t)} \le 1\ \forall t \in [t_x, T]. \end{cases}
$$

Note that, $P, L, T, R, U(t), u(t), F(y), f(y), c_p, c_s, c_m$, and c_h are given, and $t_x, x(t)$, and $x'(t)$ are decision variable and functions.

4. Optimal Solution

Let $x^*(t), t_x^* \le t \le T$, be the optimal solution of DPP Model and assume that the time interval $[\bar{t}, \hat{t}]$ is a maximum subinterval of $[t_x^*, T]$ satisfying the constraint $0 < \frac{x^{*'}(t)}{u(t)} < 1\ \forall t \in (\bar{t}, \hat{t})$. It is valid that the necessary condition of $x^*(t)$, Euler Equation [2, 6] of DPP Model, is given by

$$
\frac{c_h R}{L} = \frac{d}{dt}\left[C_t'(x^{*'}(t)) + c_m(1-R) \right] = \frac{d}{dt}\left[g(\frac{x^{*'}(t)}{u(t)}) + c_m(1-R) \right]. \tag{2}
$$

Then, there exists a constant k to satisfy

$$
\frac{c_h R}{L} t + k = g(\frac{x^{*'}(t)}{u(t)}) + c_m(1-R) \quad \forall t \in [\bar{t}, \hat{t}] \tag{3}
$$

and hence

$$
x^*(t) = \int_{\bar{t}}^{t} u(s) g^{-1}(\frac{c_h R}{L} s + k - c_m(1-R)ds \quad \forall t \in [\bar{t}, \hat{t}]. \tag{4}
$$

Then, two Properties are proposed and discussed as follows:

Property 1.

(i) $\frac{x^{*'}(t)}{u(t)}$ is strictly increasing for $t \in [\bar{t}, \hat{t}]$.

(ii) If the curve $y = x^{*'}(t)$ touches the curve $y = u(t)$, these two curves should overlap from the touch point to T.

Proof: (i) Eq.(3) yields that $g(\frac{x^{*'}(t)}{u(t)})$ is a strictly increasing function of t, and hence, by (1), the Property 1.(i) can be shown.

(ii) Since Property 1.(i) holds for any subinterval of $[t_{x^*}, T]$ subject to $0 < \frac{x^{*'}(t)}{u(t)} < 1$ for all t belonging this subinterval, the curve in the time interval $(\hat{t}, T)$ (shown in Figure 1) cannot exist because it contradicts the Property 1.(i). Hence, the Property 1.(ii) is verified.

By the choices of $[\bar{t}, \hat{t}]$ and Property 1, it yields that $\bar{t}$ and $\hat{t}$ are uniquely determined by x^* and

$$
0 \le \bar{t} = t_{x^*} \le \hat{t} \le T, \quad (T - \hat{t})(1 - \frac{x^{*'}(\hat{t})}{u(\hat{t})}) = 0. \tag{5}
$$

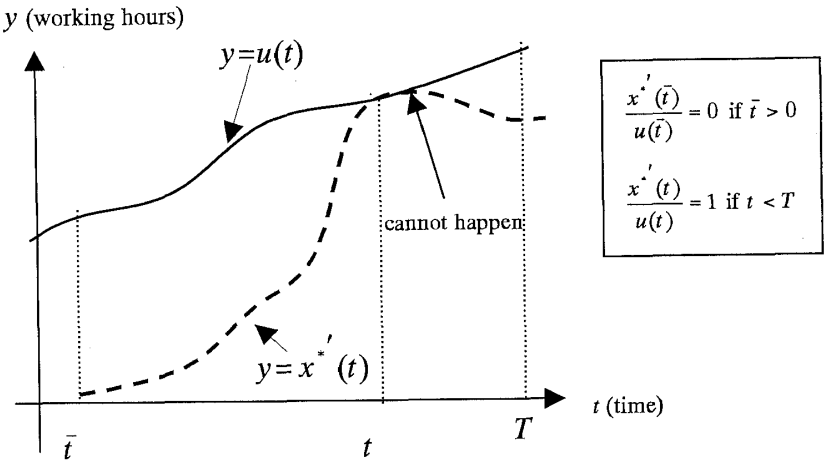

Figure 1: Possible conditions of curve $y = u(t)$ and $y = x^{*'}(t)$

Property 2. t_{x^*}, the initial time of x^* for production, is given by

$$t_{x^*} = \frac{L}{c_h R}[g(0) + c_m(1-R) - k]^+. \tag{6}$$

Proof. (a) If $t_{x^*} = 0$, by Eq.(3), (5), and Property 1.(i)

$$\frac{c_h R}{L} t_{x^*} + k = g(\frac{x^{*'}(t_{x^*})}{u(t_{x^*})}) + c_m(1-R) \geq g(0) + c_m(1-R)$$

and hence

$$\frac{L}{c_h R}[g(0) + c_m(1-R) - k]^+ = 0 = t_{x^*}.$$

(b) Assume that $t_{x^*} > 0$. We claim that

$$x^{*'}(t_{x^*}) = 0. \tag{7}$$

If $x^{*'}(t_{x^*}) > 0$, the DPP Model meets the transversality condition of free $x^*(t_{x^*})$ [2, 6], and hence

$$0 = \left[\frac{\partial C_t(x'(t))}{\partial x'(t)} + c_m(1-R)\right]_{t=t_{x^*}} = g(\frac{x^{*'}(t_{x^*})}{u(t_{x^*})}) + c_m(1-R). \tag{8}$$

Eq. (8) contradicts that g is a nonnegative function and $c_m(1-R) \geq 0$.

Therefore, $x^{*'}(t_{x^*}) = 0$ is asserted.

Here, together with Eq.(3) and (7), they yield that

$$\frac{c_h R}{L} t_{x^*} + k = g(0) + c_m(1-R)$$

and hence

$$t_{x^*} = \frac{L}{c_h R}[g(0) + c_m(1-R) - k] = \frac{L}{c_h R}[g(0) + c_m(1-R) - k]^+.$$

Combine (a) and (b), then the Property 2 is verified.

Now, we claim that $\hat{t} = T$ (the proof is shown in Appendix).
Let

$$G(x) = \int_0^{\frac{Rx}{L}} \left[Py - c_s(\frac{Rx}{L} - y)\right] f(y)dy + \int_{\frac{Rx}{L}}^{\infty} \left[P\frac{Rx}{L} - c_p(y - \frac{Rx}{L})\right] f(y)dy$$

and

$$\bar{F}(x, x') = -\left[C_t(x') + c_m(1-R)x' + \frac{c_h R}{L}x\right].$$

Then, the necessary conditions, Euler Equation and transversality conditions [2, 6], of *DPP* Model are listed below. From Eq.(3), (5), and $\hat{t} = T$ the following property can be formed.

$$g(\frac{x^{*'}(t)}{u(t)}) = \frac{c_h R}{L}t + k - c_m(1-R), \quad \text{it is a linear function of } t \text{ in } [t_{x^*}, T]. \tag{9}$$

From the transversality condition of salvage value for free $x(T)$ [2, 6], $(\bar{F}_{x'} + G_x)\mid_T = 0$, then the following equation is obtained.

$$-g(\frac{x^{*'}(T)}{u(T)}) - c_m(1-R) + (P + c_p)\frac{R}{L} - (P + c_p + c_s)\frac{R}{L}F(\frac{R}{L}x^*(T)) = 0. \tag{10}$$

Combine (9) and (10), we have

$$-\frac{c_h R}{L}T - k + (P + c_p)\frac{R}{L} - (P + c_p + c_s)\frac{R}{L}F(\frac{R}{L}x^*(T)) = 0. \tag{11}$$

Substitute t_{x^*} into (9), then

$$k = -\frac{c_h R}{L}t_{x^*} + g(\frac{x^{*'}(t_{x^*})}{u(t_{x^*})}) + c_m(1-R). \tag{12}$$

Using (12), rearrange Eq.(11), then the following equation can be formed.

$$\begin{aligned} &\frac{R}{L}[P + c_p - c_h(T - t_{x^*})]\left[1 - F(\frac{R}{L}x^*(T))\right] - \frac{R}{L}[c_s + c_h(T - t_{x^*})]F(\frac{R}{L}x^*(T)) \\ &\quad = g(\frac{x^{*'}(t_{x^*})}{u(t_{x^*})}) + c_m(1-R). \end{aligned} \tag{13}$$

In addition, from (4), (5), and (6), the following equation can be obtained.

$$x^*(t) = \int_{\frac{L}{c_h R}[g(0)+c_m(1-R)-k]^+}^{t} u(s)g^{-1}(\frac{c_h R}{L}s + k - c_m(1-R))ds. \tag{14}$$

Substitute T into Eq.(14), we have

$$x^*(T) = \int_{\frac{L}{c_h R}[g(0)+c_m(1-R)-k]^+}^{T} u(s)g^{-1}(\frac{c_h R}{L}s + k - c_m(1-R))ds. \tag{15}$$

Together with Eq.(11) and (15), the values of k and $x^*(T)$ can be determined because of two equations with two unknown values. After the determination of k, substituting into Eq.(6) and (14), t_{x^*} and $x^*(t)$ are determined. Also, differentiate Eq.(14) with respect to t, then the $x^{*'}(t)$ is obtained.

5. Sensitivity Analysis

5.1. The effect on changing T, c_s, c_p, P

First, we claim that k should decrease while T, c_s is increasing or c_p, P is decreasing. It is shown as follows: for a given t, if T, c_s increases or c_p, P decreases, and k is assumed to be increasing, from (9), $x^{*'}(t)$ is increasing. When $x^{*'}(t)$ is increasing for every given t, $x^*(T)$ becomes larger. As a result, it contradicts Eq.(11) and that T, c_s increases or c_p, P decreases, then k is decreasing is asserted. Oppositely, while T, c_s is decreasing or c_p, P is increasing, k is increasing. The sensitivity analyses of the decision variables with respect to the parameters, T, c_s, c_p, and P, are presented and shown in Table 1 and Figure 2.

5.2. The effect on changing c_m

Second, we also claim that $[k - c_m(1-R)]$ should decrease while c_m is increasing. It is shown as follows: for a given t, if c_m increases and $[k - c_m(1-R)]$ is assumed to be increasing (this implies that k is increasing), from (9), $x^{*'}(t)$ is increasing. When $x^{*'}(t)$ increases for every given t, $x^*(T)$ becomes larger. As a result, it contradicts Eq.(11) and that c_m increases, then $[k - c_m(1-R)]$ is decreasing is asserted. On the other hand, when c_m decreases, $[k - c_m(1-R)]$ is increasing. The sensitivity analyses of the decision variables with respect to c_m are described and shown in Table 1 and Figure 2.

5.3. The effect on changing L, R, c_h

From (9), the character of the optimal solution is that the marginal cost of operational working hours at time t is a linear function of t with slope $\frac{c_h R}{L}$. When c_h, R increases or L decreases, the slope becomes larger (shown in Figure 3). Oppositely, when c_h, R decreases or L increases, the slope becomes smaller. However, from (9) and (11), the sensitivity analyses of decision variables with respect to the parameter, L, R, c_h, are hard to obtain.

Table 1: The sensitivity analyses of parameters and decision variables.

Parameter / Decision Variables	c_s	c_m	T	c_p	P	reference
k	$-$	$\sharp$	$-$	$+$	$+$	9 and 11
t_{x^*} if $t_{x^*} > 0$	$+$	$+$	$+$	$-$	$-$	Figure 2
t_{x^*} if $t_{x^*} = 0$	nc	nc	nc	nc	nc	Figure 2
$x^*(T)$	$-$	$-$	$-$	$+$	$+$	9 and 11
$x^{*'}(t)$	$-$	$-$	$-$	$+$	$+$	9 and 11

"$+$": Decision variable is an increasing function of the parameter
"$-$": Decision variable is a decreasing function of the parameter
"$\sharp$": Decision variable depends on the changes of other relevant parameters.
"nc": Decision variable does not change.

6. Conclusions

The maximum available working hours at any given time t, the probabilistic market demand, the penalty, transaction, holding, maintenance (repair) costs, the processing time of a product, and the reliability of a machine are considered simultaneously to determine the optimal production quantity (rate) and the initial time of production. Definitely, this

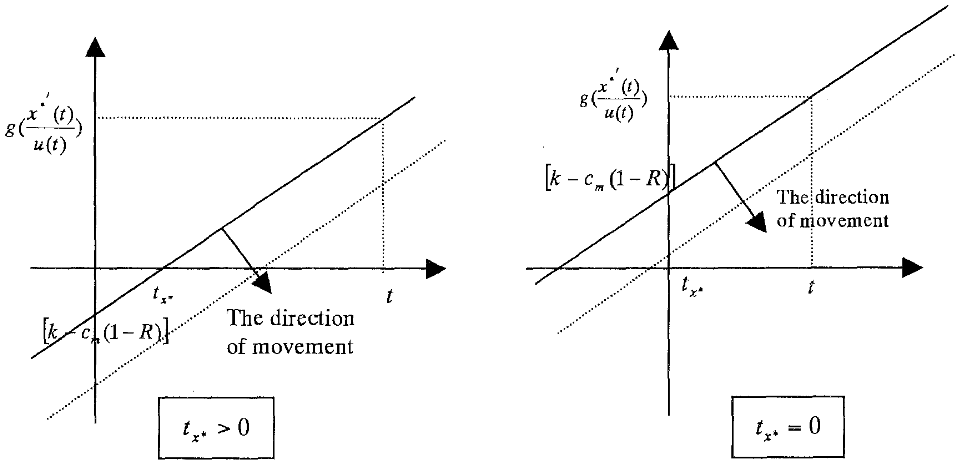

Figure 2: The effect of increasing T, c_s, c_m, or decreasing c_p, P

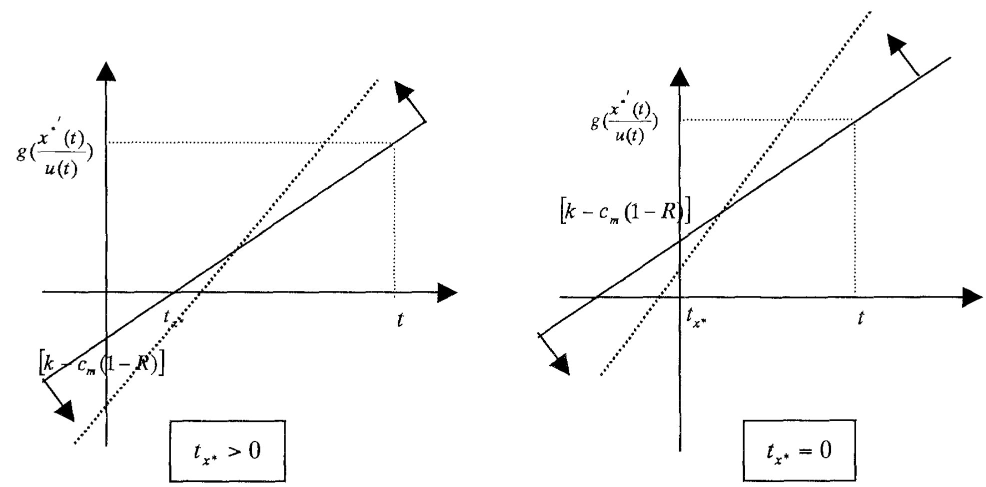

Figure 3: The effect of increasing R, c_h, or decreasing L

is a complicated and hard-solving issue. However, through DPP Model, the above issue becomes concrete and easy-to-solve.

In this study, the finite capacity of obtainable working hours at any given time t is proposed as the production limitation at time t. Therefore, the applicability of DPP Model is significantly extended. In addition, two characters of optimal solution are as follows: First, by (9), the marginal cost of operational working hours at time t is increasing linearly by t. Second, by (13), that the expected loss per unit time of lacking product, $\frac{R}{L}[c_p + P - c_h(T - t_{x^*})]\left[1 - F(\frac{R}{L}x^*(T))\right]$, minus the expected cost per unit time of surplus product, $\frac{R}{L}[c_s + c_h(T - t_{x^*})]F(\frac{R}{L}x^*(T))$, should be equal to the marginal cost of the operational working hours at the initial time of production, $g(\frac{x^{*'}(t_{x^*})}{u(t_{x^*})})$, plus the expected maintenance (repair) cost per unit time $c_m(1-R)$. The viewpoint above shows that the expected loss per unit time of lacking product should be greater than the expected cost per unit time of surplus product at reaching the optimal solution.

Moreover, that the sensitivity analyses on the key variables of optimal solution are fully discussed to arrive at several useful characteristics is also presented in this study. Furthermore, this study presents that the optimal operational working hours at any given time cannot reach the maximum obtainable working hours before selling time under the probabilistic market demand (c.f. Appendix). This result offers significant information to the production planner for production control. In sum, this study shows that time is an important factor and the determination of the production time interval is critical for the production planner.

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Appendix: The proof of $\hat{t} = T$.

Suppose that $\hat{t} < T$, then by Property 1.(ii), DPP Model can be rewritten as follows:

$$\underset{\hat{t}<T}{DPP}\begin{cases} \max\limits_{x} G(\hat{t}, x(\hat{t})) + \int_{t_x}^{\hat{t}} \bar{F}(x(t), x'(t))dt \\ s.t. \quad \text{domain } x = [t_x, \hat{t}] \text{ with } 0 \le t_x \le \hat{t} \le T \\ \qquad x(t_x) = 0,\ t_x \text{ and } x(\hat{t}) \text{ are free, and } 0 \le \dfrac{x'(t)}{u(t)} \le 1\ \forall t \in [t_x, \hat{t}] \end{cases}$$

where

$$\begin{aligned} G(t,x) &= \int_0^{\frac{R}{L}\left[\int_t^T u(t)dt + x\right]} \left[Py - c_s\left(\frac{R\left[\int_t^T u(t)dt + x\right]}{L} - y\right)\right] f(y)dy \\ &+ \int_{\frac{R}{L}\left[\int_t^T u(t)dt + x\right]}^{\infty} \left[P\frac{R\left[\int_t^T u(t)dt + x\right]}{L} - c_p\left(y - \frac{R\left[\int_t^T u(t)dt + x\right]}{L}\right)\right] f(y)dy \\ &- \int_t^T \left[C_t(u(t)) + c_m(1-R)u(t) + \frac{c_h R}{L}\left[\int_t^T u(t)dt + x\right]\right] dt \end{aligned}$$

and $\bar{F}(x, x') = -\left[C_t(x') + c_m(1-R)x' + \frac{c_h R}{L}x\right]$.

The necessary conditions, Euler Equation and transversality conditions [2, 6], of $\underset{\hat{t}<T}{DPP}$ Model are listed below.

From Euler Equation and $\dfrac{x^{*\prime}(\hat{t})}{u(\hat{t})} = 1$, the following equation is gained.

$$k = g(1) + c_m(1-R) - \frac{c_h R}{L}\hat{t}. \tag{16}$$

Substitute Eq.(16) into Eq.(6), then

$$t_{x^*} = \frac{L}{c_h R}\left[\frac{c_h R}{L}\hat{t} - [g(1) - g(0)]\right]^+. \tag{17}$$

From the transversality condition of salvage value for free $x(\hat{t})$ [2, 6], $(\bar{F}_{x'} + G_x)\ |_{\hat{t}} = 0$, then the following equation is obtained.

$$-g(1) - c_m(1-R) + \frac{R}{L}(P + c_p) - \frac{R}{L}(P + c_p + c_s)F\left(\frac{R}{L}\left[\left(\int_{\hat{t}}^T u(t)dt\right) + x^*(\hat{t})\right]\right) - \frac{c_h R}{L}(T - \hat{t}) = 0. \tag{18}$$

From the transversality condition of salvage value for free $\hat{t}$ [2, 6], $(\bar{F} - x'\bar{F}_{x'} + G_t)\mid_{\hat{t}} = 0$, then the following equation is obtained.

$$u(\hat{t})[g(1) + c_m(1-R)] + \frac{R}{L}u(\hat{t})(P + c_p + c_s)F(\frac{R}{L}\left[(\int_{\hat{t}}^{T} u(t)dt) + x^*(\hat{t})\right]) - \frac{R}{L}u(\hat{t})(P + c_p)$$
$$+\frac{c_h R}{L}(\int_{\hat{t}}^{T} u(t)dt) + \frac{c_h R}{L}u(\hat{t})(T - \hat{t}) = 0. \tag{19}$$

Both sides of Eq.(19) can be divided by $u(\hat{t})$ to form the following equation.

$$[g(1) + c_m(1-R)] + \frac{R}{L}(P + c_p + c_s)F(\frac{R}{L}\left[(\int_{\hat{t}}^{T} u(t)dt) + x^*(\hat{t})\right]) - \frac{R}{L}(P + c_p)$$
$$+\frac{c_h R}{Lu(\hat{t})}(\int_{\hat{t}}^{T} u(t)dt) + \frac{c_h R}{L}(T - \hat{t}) = 0. \tag{20}$$

Combine Eq.(18) and (20) to yield

$$\frac{c_h R}{Lu(\hat{t})}(\int_{\hat{t}}^{T} u(t)dt) = 0. \tag{21}$$

From Eq.(21) and $u(t) > 0, \hat{t} = T$ is asserted. This result gets contradiction and means that $\hat{t} < T$ cannot happen. Hence, that $\hat{t} = T$ is verified.

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