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L-SHAPED DECOMPOSITION METHOD FOR MULTI-STAGE STOCHASTIC CONCENTRATOR LOCATION PROBLEM

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Abstract A stochastic version of a concentrator location problem is dealt with in which traffic demand at each terminal location is uncertain. The concentrator location problem is defined as to determine the following: (i) the numbers and locations of concentators that are to be open, and (ii) the allocation of terminals to concentrator sites. The problem is formulated as a stochastic multi-stage integer linear program, with first stage binary variables concerning network design and continuous recourse variables concerning expansion of capacity. Given a first stage decision, the series of realization of traffic demand may possibly imply a violation of the capacity constraint of the concentrator. Therefore from the second stage to the last stage, recourse action is taken to correct the violation. The objective function minimizes the cost of connecting terminals and the cost of opening concentrators and the expected recourse cost of capacity expansion. We propose a new algorithm which combines an L-shaped method and a branch-and-bound method. Under some assumptions it decomposes the problem into a set of problems as many as the number of stages in parallel. Finally we demonstrate the computational efficiency of our algorithm for the multi-stage model.

1. Introduction

This paper focuses on topological design of centralized computer networks (Ahuja [1]). The concentrator location problem (Bertsekas and Gallager [4], Ahuja, Magnanti and Orlin [2]) is defined as determining the following: (i) the number and locations of concentrators that are to be open, and (ii) the allocation of terminals to concentrator sites without violating the capacities of concentrators. Figure 1 illustrates an example of optimal network with 4 potential locations and 10 terminals. In this example the number of concentrators to be open is 2, and for the other 2 potential sites concentrators are not open.

Since the problem belongs to the class of NP-hard, most prior researches have developed heuristic procedures to seek approximate solutions (Mirzaian [15], Pirkul [17], Pirkul et al. [18]). For this problem a new algorithm (fractional cutting plane algorithm/branch-and-bound (Nemhauser and Wolsey [16])) that yields an exact solution was presented in our previous paper (Shiina [20]). In this approach strong valid inequalities are used as cutting planes. The computational results show that this algorithm performs reasonably well for relatively large problems involving up to 100 terminals.

However for many actual problems, the assumption that the traffic demands at each terminal are deterministic known data is often unjustified. These data contain uncertainty and are thus represented as random variables since the data represent information about the future. In this paper a stochastic version of a concentrator location problem is dealt with in which traffic demand at each terminal location is uncertain. Locating too few concentrators may result in shortage of capacity for the future demand. On the other hand excessive investments will cause excess of capacity. Our problem is thus a strategic decision problem

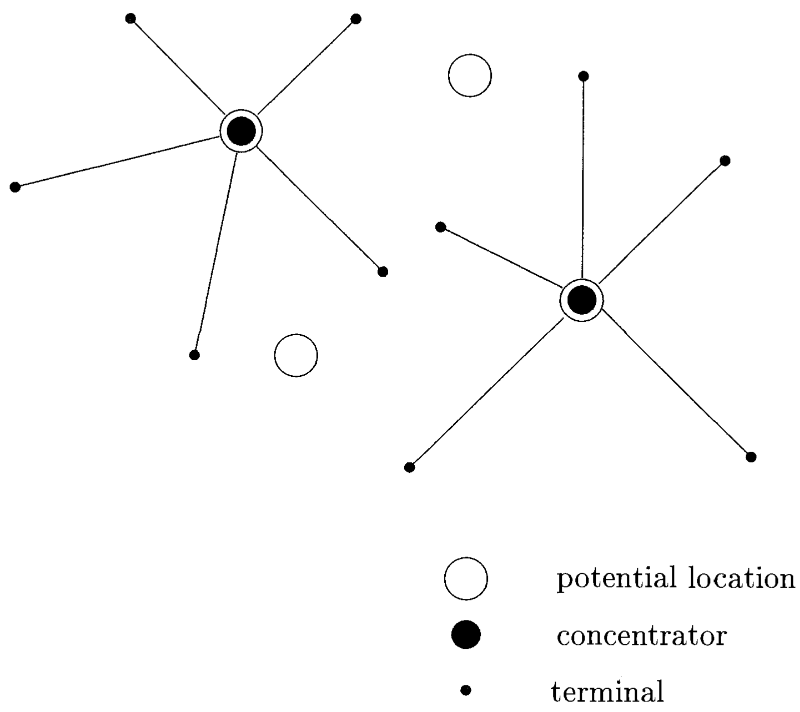

Figure 1: Concentrator Location Problem

under uncertainty and can be viewed as a stochastic programming problem (Kall [9, 10], Kall and Wallace [11], Prékopa [19], Birge and Louveaux [6]).

Stochastic programming dates back to the pioneering study by Dantzig [7]. Stochastic programming problem with recourse is referred to as two-stage stochastic problem. In the first stage, a decision has to be made without complete information on random factors. After the value of random variables are known, recourse action can be taken in the second stage. For the continuous stochastic programming problem with recourse an L-shaped method (Van Slyke and Wets [21]) is well-known. That approach is based upon Benders'[3] decomposition. Stochastic integer programs have both features of integer programs and stochastic programs that are computationally intractable. Wollmer [23] used a cutting plane algorithm for the case that first stage decision variables are restricted to integer. Louveaux and Peeters [14] presented a dual-based heuristic procedure for stochastic facility location. Laporte and Louveaux [12] proposed a branch-and-cut procedure for stochastic integer programs with first stage binary variables. Laporte, Louveaux and Hamme [13] solved a capacitated facility location problem to optimality by means of a branch-and-cut method. In section 2 we define the stochastic programming model with recourse for concentrator location problem.

In section 3 we consider a multi-stage stochastic concentrator location problem with recourse. Many real problems require that decisions are made subsequently over time. Birge [5] extended the L-shaped method to the multi-stage method. Gassmann [8] solved the multi-stage programs by the nested decomposition. The standard formulation for multi-stage stochastic concentrator location problem contains a lot of constraints because the total number of scenarios is very large. So it is necessary to decompose the problem. Our decomposition differs from the nested decomposition. Under some assumptions it decomposes the problem into a set of problems as many as the number of stages in parallel. We propose a new framework of algorithm which combines an L-shaped method and a branch-and-bound method.

In section 4 we demonstrate the computational efficiency of our algorithm for multi-stage models.

2. Stochastic Programming Model with Recourse for Concentrator Location Problem

The following describes the symbols and notations used in the paper.

Table 1: Notation

Symbol	Definition
I	Index set of potential concentrator locations
J	Index set of terminal locations
c_{ij}	Cost connecting terminal j and concentrator i
f_i	Fixed setup cost of concentrator at site i
$a_j(\tilde{\xi})$	Traffic at terminal j (For a given realization ξ of random vector $\tilde{\xi}$, $a_j(\xi)$ becomes known.)
b_i	Capacity of concentrator at site i
Decision Variables	
x_{ij}	$\begin{cases} 1 & \text{if link exists between terminal } j \text{ and location } i, i \in I, j \in J \\ 0 & \text{otherwise} \end{cases}$
y_i	$\begin{cases} 1 & \text{if concentrator is open at location } i, i \in I \\ 0 & \text{otherwise} \end{cases}$

The mathematical formulation of the capacitated concentrator location problem is stated as follows.

(Concentrator Location Problem Prototype (CLP))

$$\min \quad \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{i\in I} f_i y_i \tag{2.1}$$

$$\text{subject to} \quad \sum_{j\in J} a_j(\tilde{\xi})x_{ij} \le b_i y_i, \quad i \in I \tag{2.2}$$

$$\sum_{i\in I} x_{ij} = 1, \quad j \in J \tag{2.3}$$

$$x_{ij} \le y_i, \quad i \in I, j \in J \tag{2.4}$$

$$x_{ij}, y_i \in \{0,1\}, \ i \in I, j \in J \tag{2.5}$$

The objective function (2.1) minimizes the cost of connecting terminals to concentrators and the cost of opening concentrators. Constraint (2.2) represents capacities of concentrators. Constraint (2.3) ensures that each terminal is assigned to a single concentrator. Constraint (2.4) assures that no terminals are connected to a concentrator that is not open. We assume the random vector $\tilde{\xi}$ is defined on a known probability space $(\Omega, \mathcal{F}, P)$ and has a discrete distribution. Therefore possible finite scenarios can occur. Let Ξ be the support of $\tilde{\xi}$, i.e. the smallest closed set such that $P(\Xi) = 1$. Given a decision x, y, the realization of traffic demand $a_j(\xi), j \in J$ of $a_j(\tilde{\xi}), j \in J$ may imply a violation of the capacity constraint of the concentrator. Therefore after observing the realization ξ, the corrective action $w(\xi)$ is taken to compensate the violation. $w_i(\xi), i \in I$ represent the amount of expanded capacity at concentrator site i and are assumed to cause penalty of $q_i(\geq 0)$ per unit.

The problem is formulated as a stochastic integer program, with first stage binary variables x, y concerning network design and second stage continuous variables $w_i(\xi), i \in I$ concerning expansion of capacity as follows.

$$\left(\begin{array}{c}\text{Stochastic}\\ \text{Concentrator}\\ \text{Location}\\ \text{Problem}\\ \text{with}\\ \text{Recourse}\\ \text{(SCLP)}\end{array}\right) \quad \begin{array}{rl} \min & \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{i\in I} f_i y_i + \mathcal{Q}(x,y) \\ \text{subject to} & \sum_{i\in I} x_{ij} = 1, \quad j \in J \\ & x_{ij} \leq y_i, \quad i \in I, j \in J \\ & x_{ij}, y_i \in \{0,1\}, \quad i \in I, j \in J \\ \text{where} & \mathcal{Q}(x,y) = E_{\tilde{\xi}}[Q(x,y,\tilde{\xi})] \\ & Q(x,y,\xi) = \min_w \{\sum_{i\in I} q_i w_i(\xi) | \sum_{j\in J} a_j(\xi) x_{ij} \leq b_i y_i + w_i(\xi), \\ & \quad w_i(\xi) \geq 0, i \in I\}, \xi \in \Xi \end{array}$$

The symbol $E_{\tilde{\xi}}$ represents the mathematical expectation with respect to $\tilde{\xi}$, and $Q(x, y, \xi)$, $\mathcal{Q}(x, y)$ are called the recourse function in state ξ and the expected recourse function, respectively. The objective function minimizes the cost of connecting terminals and the cost of opening concentrators and the expected recourse cost of capacity expansion.

3. Multi-Stage Model

3.1. Decision-observation scheme in multi-stage model

The previous section concerned stochastic programs with two stages. However most practical decision problems involve a sequence of decisions that react to outcomes periodically over time. We define $t = 1, \ldots, T$ as planning periods and call them *stages*. Let $\tilde{\xi}^1, \tilde{\xi}^2, \ldots, \tilde{\xi}^T$ be a sequence of random vectors that are observed as $\xi^1, \xi^2, \ldots, \xi^T$, where ξ^t is observed in stage t. We assume the random vector $\tilde{\xi}^t$ is defined on a known probability space $(\Xi^t, \mathcal{F}, P)$ and has a discrete distribution like the two-stage model. We describe a decision-observation scheme as follows.

Decision-Observation Scheme

- initial investment x, y concerning network design
- observation of ξ^1
- corrective action $w^1(\xi^1)$ concerning capacity expansion in stage 1
- observation of ξ^2
- corrective action $w^2(\xi^1, \xi^2)$ concerning capacity expansion in stage 2

⋮

- observation of ξ^T
- corrective action $w^T(\xi^1, \ldots, \xi^T)$ concerning capacity expansion in stage T

We have to make a decision over a finite number of periods. Having fixed the initial investment x, y and observed the random vector ξ^1, we have to make the decision on $w^1(\xi^1)$ concerning capacity expansion so that the constraints involving $x, y, w^1(\xi^1)$ are satisfied. Similarly, having also fixed $w^{t-1}(\xi^1, \ldots, \xi^{t-1})$ and observed ξ^t, we decide on $w^t(\xi^1, \ldots, \xi^t)$ so that the constraints involving $x, y, w^1(\xi^1), \ldots, w^t(\xi^1, \ldots, \xi^t)$ are satisfied for $t = 1, \ldots, T$. The recourse variables $w^t(\xi^1, \ldots, \xi^t), i \in I, t = 1, \ldots, T$ cause penalty of q_i^t per unit. We make the next two assumptions on the traffic demands and the recourse costs.

Assumption 1 *We assume the traffic demands are monotone nondecreasing in t.*

$$a_j(\xi^t) \le a_j(\xi^{t+1}), \forall \xi^t \in \Xi^t, \forall \xi^{t+1} \in \Xi^{t+1}, j \in J, t = 1, \dots, T-1$$

Assumption 2 *We assume the recourse costs are monotone nonincreasing in t.*

$$q_i^t \ge q_i^{t+1} \ge 0, i \in I, t = 1, \dots, T-1$$

Assumption 1 expresses an ideal situation in which the traffic demand grows monotonically and all of the realizations of the traffic demand in some stage are greater than or equal to those in former stages. Taking the present worth factor into account, Assumption 2 is true of the case that the recourse costs are constant. These Assumptions are required to characterize the optimal solution of the recourse problem and to prove that the problem (MS-CLP) are equivalent to the reformulated prolem (RMS-CLP).

And we define $q_i^0, q_i^{T+1} = 0, i \in I$. The sets of possible sequences of observations $(\xi^1, \dots, \xi^T)$ are called scenarios. The scenarios are often described using an event tree as shown in Figure 2.

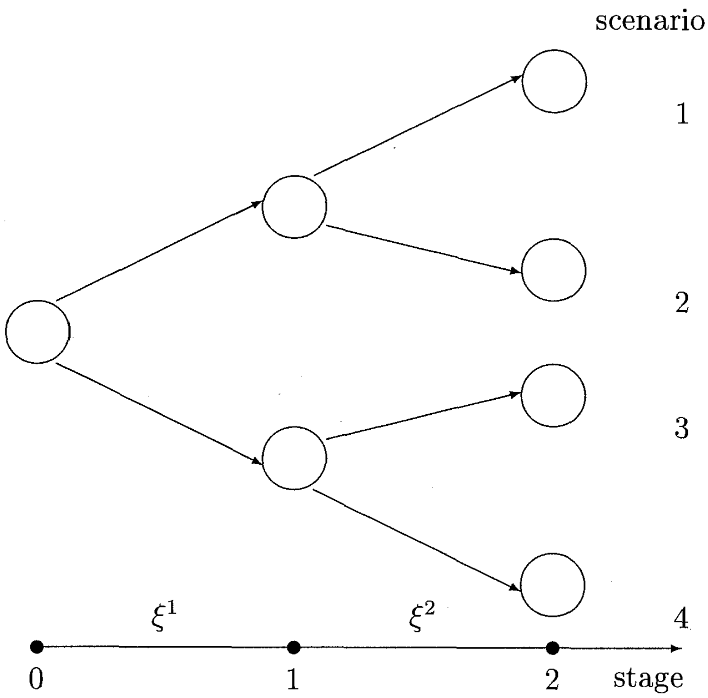

Figure 2: Event Tree

Figure 2 illustrates the event tree with $T = 2$ and $|\Xi^1| = |\Xi^2| = 2$. This tree represents 4 scenarios, because the tree has $|\Xi^1| \times |\Xi^2| = 4$ leaves.

We have a recourse function $Q^t(x, y, w^1, \dots, w^{t-1}, \xi^1, \dots, \xi^t), t = 1, \dots, T$ as follows.

$$\begin{aligned} &Q^t(x, y, w^1, \dots, w^{t-1}, \xi^1, \dots, \xi^t) \\ &= \min_{w^t} \{ \sum_{i \in I} q_i^t w_i^t(\xi^1, \dots, \xi^t) | \sum_{j \in J} a_j(\xi^t) x_{ij} \le b_i y_i + w_i^1(\xi^1) + \dots + w_i^t(\xi^1, \dots, \xi^t), i \in I \\ &\quad w_i^t(\xi^1, \dots, \xi^t) \ge 0, i \in I \\ &\quad w^k(\xi^1, \dots, \xi^k) = \text{argmin } Q^k(x, y, w^1, \dots, w^{k-1}, \xi^1, \dots, \xi^k), k = 1, \dots, t-1 \} \end{aligned} \quad (3.1)$$

3.2. Formulation of the multi-stage model

The multi-stage concentrator location problem with recourse takes the following form.

$$\left(\begin{array}{c}\text{Multi-Stage}\\ \text{Concentrator}\\ \text{Location}\\ \text{Problem}\\ \text{(MS-CLP)}\end{array}\right) \left| \begin{array}{rl} \min & \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{i\in I} f_i y_i \\ & + \sum_{t=1}^{T} E_{\tilde{\xi}^1,\dots,\tilde{\xi}^t}[Q^t(x,y,w^1,\dots,w^{t-1},\tilde{\xi}^1,\dots,\tilde{\xi}^t)] \\ \text{subject} & \sum_{i\in I} x_{ij} = 1, \quad j\in J \\ \text{to} & x_{ij} \le y_i, \quad i\in I, j\in J \\ & x_{ij}, y_i \in \{0,1\}, \quad i\in I, j\in J \\ & Q^t(x,y,w^1,\dots,w^{t-1},\xi^1,\dots,\xi^t) \\ & \quad = \min_{w^t}\{\sum_{i\in I} q_i^t w_i^t(\xi^1,\dots,\xi^t) | \\ & \sum_{j\in J} a_j(\xi^t)x_{ij} \le b_i y_i + w_i^1(\xi^1) + \dots + w_i^t(\xi^1,\dots,\xi^t), i\in I \\ & w_i^t \ge 0, i\in I \\ & w^k(\xi^1,\dots,\xi^k) = \text{argmin } Q^k(x,y,w^1,\dots,w^{k-1},\xi^1,\dots,\xi^k), \\ & \quad k=1,\dots,t-1\}, \xi^1\in\Xi^1,\dots,\xi^t\in\Xi^t, t=1,\dots,T \end{array}\right.$$

The mathematical programming problem (MS-CLP) is equivalent to the next program.

$$\left(\begin{array}{c}\text{Multi-Stage}\\ \text{Concentrator}\\ \text{Location}\\ \text{Problem-}\\ \text{Mixed}\\ \text{Integer}\\ \text{Problem}\\ \text{(MS-CLP-MIP)}\end{array}\right) \left| \begin{array}{rl} \min & \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{i\in I} f_i y_i + \sum_{t=1}^{T}\theta^t \\ \text{subject to} & \sum_{i\in I} x_{ij} = 1, \quad j\in J \\ & x_{ij} \le y_i, \quad i\in I, j\in J \\ & x_{ij}, y_i \in \{0,1\}, \quad i\in I, j\in J \\ & \theta^t \ge E_{\tilde{\xi}^1,\dots,\tilde{\xi}^t}[\theta_{\tilde{\xi}^1,\dots,\tilde{\xi}^t}], t=1,\dots,T \\ & \theta_{\xi^1,\dots,\xi^t} \ge \sum_{i\in I} q_i^t w_i^t(\xi^1,\dots,\xi^t), \xi^1\in\Xi^1,\dots,\xi^t\in\Xi^t, \\ & \qquad t=1,\dots,T \\ & \sum_{j\in J} a_j(\xi^t)x_{ij} \le b_i y_i + w_i^1(\xi^1) + \dots + w_i^t(\xi^1,\dots,\xi^t), \\ & \qquad \xi^1\in\Xi^1,\dots,\xi^t\in\Xi^t, t=1,\dots,T, i\in I \end{array}\right.$$

Because the feasible solution of (MS-CLP) satisfies the constraints of (MS-CLP-MIP) to set $\theta_{\xi^1,\dots,\xi^t} = Q^t(x,y,w^1,\dots,w^{t-1},\xi^1,\dots,\xi^t)$ and $\theta^t = E_{\tilde{\xi}^1,\dots,\tilde{\xi}^t}[Q^t(x,y,w^1,\dots,w^{t-1},\tilde{\xi}^1,\dots,\tilde{\xi}^t)]$, the optimal objective value of (MS-CLP) is greater than or equal to the one of (MS-CLP-MIP). But by the definition of the problem (MS-CLP-MIP) the optimal value of (MS-CLP-MIP) is the upper bound for that of (MS-CLP). Accordingly both of them coincide with each other. Though the problem (MS-CLP-MIP) does not contain optimization over multi-stages, it contains many constraints. Therefore we consider the problem (MS-CLP).

The recourse function of (MS-CLP) $Q^t(x,y,w^1,\dots,w^{t-1},\xi^1,\dots,\xi^t)$ indicates that the optimal recourse action $w^t(\xi^1,\dots,\xi^t)$ at stage t depends on the previous decisions and the realizations observed until stage t, i.e.

$$w^t(\xi^1,\dots,\xi^t) = w^t(x,y,w^1,\dots,w^{t-1},\xi^1,\dots,\xi^t).$$

Definition 1 *In the history of $(\xi^1,\dots,\xi^t)$ up to stage t, we define $u_i(x,y,\xi^1,\dots,\xi^t)$ as the minimum k $(1\le k\le t)$ such that $\sum_{j\in J} a_j(\xi^k)x_{ij} \ge b_i y_i$ and $y_i = 1$. Else if $\sum_{j\in J} a_j(\xi^t)x_{ij} < b_i y_i$ or $y_i = 0$, then we define $u_i(x,y,\xi^1,\dots,\xi^t) = T+1$.*

$u_i(x, y, \xi^1, \ldots, \xi^t)$ represents the stage in which it becomes necessary for concentrator i to expand the capacity.

Proposition 1 *The solution of the recourse problem in stage t*

$$
\begin{aligned}
&Q^t(x, y, w^1, \ldots, w^{t-1}, \xi^1, \ldots, \xi^t) \\
&= \min_{w^t} \ \{\sum_{i\in I} q_i^t w_i^t(\xi^1, \ldots, \xi^t) | \sum_{j\in J} a_j(\xi^t)x_{ij} \leq b_i y_i + w_i^1(\xi^1) + \ldots + w_i^t(\xi^1, \ldots, \xi^t), i \in I \\
&\qquad w_i^t(\xi^1, \ldots, \xi^t) \geq 0, i \in I \\
&\qquad w^k = \text{argmin } Q^k(x, y, w^1, \ldots, w^{k-1}, \xi^1, \ldots, \xi^k), k = 1, \ldots, t-1\}, \\
&\qquad \xi^t \in \Xi^t, t = 1, \ldots, T
\end{aligned}
$$

is given as follows.

$$
w_i^t(\xi^1, \ldots, \xi^t) = \begin{cases} 0 & \text{if } t < u_i(x, y, \xi^1, \ldots, \xi^t) \\ \sum_{j\in J} a_j(\xi^t)x_{ij} - b_i y_i & \text{if } t = u_i(x, y, \xi^1, \ldots, \xi^t) \\ \sum_{j\in J} a_j(\xi^t)x_{ij} - \sum_{j\in J} a_j(\xi^{t-1})x_{ij} & \text{if } t > u_i(x, y, \xi^1, \ldots, \xi^t) \end{cases}
$$

Proof. Because of $q_i^t \geq 0$, from the definition of $Q^t(x, y, w^1, \ldots, w^{t-1}, \xi^1, \ldots, \xi^t)$,

$$
w_i^t(\xi^1, \ldots, \xi^t) = \max\{0, \sum_{j\in J} a_j(\xi^t)x_{ij} - b_i y_i - w_i^1(\xi^1) - \ldots - w_i^{t-1}(\xi^1, \ldots, \xi^{t-1})\}.
$$

The results are immediate if $t \leq u_i(x, y, \xi^1, \ldots, \xi^t)$ from Assumption 1. We assume $w_i^l = \sum_{j\in J} a_j(\xi^l)x_{ij} - \sum_{j\in J} a_j(\xi^{l-1})x_{ij}$ for $l > u_i(x, y, \xi^1, \ldots, \xi^t)$. Then for $l+1$

$$
\begin{aligned}
&\sum_{j\in J} a_j(\xi^{l+1})x_{ij} - b_i y_i - w_i^1 - \ldots - w_i^l \\
&= \sum_{j\in J} a_j(\xi^{l+1})x_{ij} - b_i y_i - (\sum_{j\in J} a_j(\xi^{u_i(x,y,\xi^1,\ldots,\xi^t)})x_{ij} - b_i y_i) \\
&\quad - \ldots - (\sum_{j\in J} a_j(\xi^l)x_{ij} - \sum_{j\in J} a_j(\xi^{l-1})x_{ij}) \\
&= \sum_{j\in J} a_j(\xi^{l+1})x_{ij} - \sum_{j\in J} a_j(\xi^l)x_{ij} \\
&\geq 0
\end{aligned}
$$

Therefore by mathematical induction the result follows. ∎

From Proposition 1 the recourse function is described as follows.

$$
\begin{aligned}
&Q^t(x, y, w^1, \ldots, w^{t-1}, \xi^1, \ldots, \xi^t) \\
&= \sum_{i\in\{i|t=u_i(x,y,\xi^1,\ldots,\xi^t)\}} (\sum_{j\in J} a_j(\xi^t)x_{ij} - b_i y_i)q_i^t \\
&\quad + \sum_{i\in\{i|t>u_i(x,y,\xi^1,\ldots,\xi^t)\}} (\sum_{j\in J} a_j(\xi^t)x_{ij} - \sum_{j\in J} a_j(\xi^{t-1})x_{ij})q_i^t
\end{aligned}
\tag{3.2}
$$

In a two-stage model with continuous first stage variables the recourse function is a piecewise linear convex function. But in this case it is difficult to derive optimality cuts to the recourse function because the structure of (3.2) is so complex that its analysis is very difficult.

Here we reformulate (MS-CLP) to obtain (RMS-CLP). We introduce variables $v_i^t(\xi^t), i \in I, \xi^t \in \Xi^t, t = 1, \ldots, T$ equal to the amount of expanded capacity that were added from stage 1 to stage t.

$$\left(\begin{array}{c}\text{Reformulated}\\ \text{Multi-Stage}\\ \text{Concentrator}\\ \text{Location}\\ \text{Problem}\\ \text{(RMS-CLP)}\end{array}\right) \left| \begin{array}{ll} \min & \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{i\in I} f_i y_i + \sum_{t=1}^{T} E_{\tilde{\xi}^t}[R^t(x,y,\tilde{\xi}^t)] \\ \text{subject to} & \sum_{i\in I} x_{ij} = 1, \quad j \in J \\ & x_{ij} \le y_i, \quad i \in I, j \in J \\ & x_{ij}, y_i \in \{0,1\}, \quad i \in I, j \in J \\ & R^t(x,y,\xi^t) \\ & \quad = \min_{v^t}\{\sum_{i\in I}(q_i^t - q_i^{t+1})v_i^t(\xi^t) \mid \\ & \qquad \sum_{j\in J} a_j(\xi^t)x_{ij} \le b_i y_i + v_i^t(\xi^t), v_i^t(\xi^t) \ge 0, i \in I\}, \\ & \qquad \xi^t \in \Xi^t, t = 1, \ldots, T \end{array}\right.$$

Theorem 1 *Problem (RMS-CLP) is equivalent to problem (MS-CLP) and the optimal value of (RMS-CLP) equals to that of (MS-CLP).*

Proof. Let $(\hat{x}, \hat{y}, \hat{w}^1, \ldots, \hat{w}^T)$ be the optimal solution for (MS-CLP). From Proposition 1 we obtain the next relation.

$$\sum_{k=1}^{t} \hat{w}_i^k(\xi^1, \ldots, \xi^k) = \begin{cases} 0 & \left(\text{if } t < u_i(\hat{x}, \hat{y}, \xi^1, \ldots, \xi^t)\right) \\ \sum_{j\in J} a_j(\xi^t)x_{ij} - b_i y_i \ge 0 & \left(\text{if } t \ge u_i(\hat{x}, \hat{y}, \xi^1, \ldots, \xi^t)\right) \end{cases}$$

As we can see $\sum_{k=1}^{t} \hat{w}_i^k(\xi^1, \ldots, \xi^k)$ does not depend on the history $(\xi^1, \ldots, \xi^t)$ up to period t but only on the realization ξ^t of period t. If we set $\hat{v}_i^t(\xi^t) = \sum_{k=1}^{t} \hat{w}_i^k(\xi^1, \ldots, \xi^k)$, then we prove $(\hat{x}, \hat{y}, \hat{v}^1, \ldots, \hat{v}^T)$ is the feasible solution for (RMS-CLP) and the optimal value of (MS-CLP) is greater than or equal to that of (RMS-CLP). Feasibility of $(\hat{x}, \hat{y}, \hat{v}^1, \ldots, \hat{v}^T)$ is evident because the objective coefficient $(q_i^t - q_i^{t+1})$ of R^t is nonnegative from Assumption 2. The recourse cost of (RMS-CLP) is shown as follows.

$$\begin{aligned} \sum_{t=1}^{T} R^t(\hat{x}, \hat{y}, \xi^t) &= \sum_{i\in I}\sum_{t=1}^{T}(q_i^t - q_i^{t+1})\hat{v}_i^t = \sum_{i\in I}\sum_{t=1}^{T}(q_i^t - q_i^{t+1})\sum_{k=1}^{t}\hat{w}_i^k \\ &= \sum_{i\in I}\sum_{t=1}^{T} q_i^t \sum_{k=1}^{t}\hat{w}_i^k - \sum_{i\in I}\sum_{t=2}^{T+1} q_i^t \sum_{k=1}^{t-1}\hat{w}_i^k = \sum_{i\in I}\sum_{t=1}^{T} q_i^t \hat{w}_i^t \\ &= \sum_{t=1}^{T} Q^t(\hat{x}, \hat{y}, \hat{w}^1, \ldots, \hat{w}^{t-1}, \xi^1, \ldots, \xi^t) \end{aligned}$$

Taking the expectation of both sides we have the next relation.

$$\begin{aligned} E_{\tilde{\xi}^1,\ldots,\tilde{\xi}^T}[\sum_{t=1}^{T} R^t(\hat{x}, \hat{y}, \tilde{\xi}^t)] &= \sum_{t=1}^{T} E_{\tilde{\xi}^t}[R^t(\hat{x}, \hat{y}, \tilde{\xi}^t)] \\ &= E_{\tilde{\xi}^1,\ldots,\tilde{\xi}^T}[\sum_{t=1}^{T} Q^t(\hat{x}, \hat{y}, \hat{w}^1, \ldots, \hat{w}^{t-1}, \tilde{\xi}^1, \ldots, \tilde{\xi}^t)] \\ &= \sum_{t=1}^{T} E_{\tilde{\xi}^1,\ldots,\tilde{\xi}^t}[Q^t(\hat{x}, \hat{y}, \hat{w}^1, \ldots, \hat{w}^{t-1}, \tilde{\xi}^1, \ldots, \tilde{\xi}^t)] \end{aligned}$$

As the optimal objective value of (MS-CLP) equals the objective value of the feasible solution of (RMS-CLP), the optimal objective value of (MS-CLP) is greater than or equal to the optimal objective value of (RMS-CLP).

Conversely let $(\hat{x}, \hat{y}, \hat{v}^1, \ldots, \hat{v}^T)$ be the optimal solution for (RMS-CLP). $\hat{v}_i^t(\xi^t)$ is shown as follows.

$$\hat{v}_i^t(\xi^t) = \begin{cases} 0 & \text{if } \sum_{j\in J} a_j(\xi^t)\hat{x}_{ij} - b_i\hat{y}_i < 0 \\ \sum_{j\in J} a_j(\xi^t)\hat{x}_{ij} - b_i\hat{y}_i \geq 0 & \text{if } \sum_{j\in J} a_j(\xi^t)\hat{x}_{ij} - b_i\hat{y}_i \geq 0 \end{cases}$$

Next we consider $\hat{v}_i^t(\xi^t) - \hat{v}_i^{t-1}(\xi^{t-1})$. We have the next relation.

$$\hat{v}_i^t(\xi^t) - \hat{v}_i^{t-1}(\xi^{t-1}) = \begin{cases} 0 & \left(\text{if } \sum_{j\in J} a_j(\xi^t)\hat{x}_{ij} - b_i\hat{y}_i < 0\right) \\ \sum_{j\in J} a_j(\xi^t)\hat{x}_{ij} - b_i\hat{y}_i \geq 0 & \left(\begin{array}{l}\text{if } \sum_{j\in J} a_j(\xi^t)\hat{x}_{ij} - b_i\hat{y}_i \geq 0 \\ \text{and } \sum_{j\in J} a_j(\xi^{t-1})\hat{x}_{ij} - b_i\hat{y}_i < 0\end{array}\right) \\ \sum_{j\in J} a_j(\xi^t)\hat{x}_{ij} - \sum_{j\in J} a_j(\xi^{t-1})\hat{x}_{ij} \geq 0 & \left(\begin{array}{l}\text{if } \sum_{j\in J} a_j(\xi^t)\hat{x}_{ij} - b_i\hat{y}_i \geq 0 \\ \text{and } \sum_{j\in J} a_j(\xi^{t-1})\hat{x}_{ij} - b_i\hat{y}_i \geq 0\end{array}\right) \end{cases}$$

As shown here $\hat{v}_i^t(\xi^t) - \hat{v}_i^{t-1}(\xi^{t-1})$ depends on the history $(\xi^1, \ldots, \xi^t)$ up to stage t. We set $\hat{w}_i^t(\xi^1, \ldots, \xi^t) = \hat{v}_i^t(\xi^t) - \hat{v}_i^{t-1}(\xi^{t-1})$. $\hat{w}^t(\xi^1, \ldots, \xi^t) \geq 0, t = 1, \ldots, T$ are feasible solutions to (MS-CLP) from Assumption 1. The recourse cost of (MS-CLP) is shown as follows.

$$\begin{aligned} \sum_{t=1}^{T} Q^t(\hat{x}, \hat{y}, \hat{w}^1, \ldots, \hat{w}^{t-1}, \xi^1, \ldots, \xi^t) &= \sum_{t=1}^{T}\sum_{i\in I} q_i^t \hat{w}_i^t(\xi^1, \ldots, \xi^t) = \sum_{i\in I}\sum_{t=1}^{T} q_i^t(\hat{v}_i^t(\xi^t) - \hat{v}_i^{t-1}(\xi^{t-1})) \\ &= \sum_{i\in I}\sum_{t=1}^{T} q_i^t \hat{v}_i^t(\xi^t) - \sum_{i\in I}\sum_{t=0}^{T-1} q_i^{t+1}\hat{v}_i^t(\xi^t) \\ &= \sum_{i\in I}\sum_{t=1}^{T}(q_i^t - q_i^{t+1})\hat{v}_i^t(\xi^t) \\ &= \sum_{t=1}^{T} R^t(\hat{x}, \hat{y}, \xi^t) \end{aligned}$$

Taking the expectation of both sides the optimal objective value of (RMS-CLP) is greater than or equal to the optimal objective value of (MS-CLP).

$$\begin{aligned} \sum_{t=1}^{T} E_{\tilde{\xi}^1,\ldots,\tilde{\xi}^t}[Q^t(\hat{x}, \hat{y}, \hat{w}^1, \ldots, \hat{w}^{t-1}, \xi^1, \ldots, \xi^t)] &= E_{\tilde{\xi}^1,\ldots,\tilde{\xi}^T}[\sum_{t=1}^{T} R^t(\hat{x}, \hat{y}, \xi^t)] \\ &= \sum_{t=1}^{T} E_{\tilde{\xi}^t}[R^t(\hat{x}, \hat{y}, \xi^t)] \end{aligned}$$

Therefore the optimal objective value of (MS-CLP) equals that of (RMS-CLP) which completes the proof. ∎

3.3. L-shaped decomposition algorithm

We consider the problem (RMS-CLP) that is equivalent to (MS-CLP). The standard mixed integer programming formulation (MS-CLP-MIP) contains $\prod_{t=1}^{T}|\Xi^t|$ scenarios. The algorithm of integer-L-shaped method decomposes the problem (RMS-CLP) into T subproblems in which we have $|\Xi^t|$ scenarios.

The master problem for (RMS-CLP) is stated as follows.

$$
\left(\begin{array}{c}\text{Master}\\ \text{Problem}\\ \text{for}\\ \text{(RMS-CLP)}\end{array}\right)
\left|
\begin{array}{rl}
\min & \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{i\in I} f_i y_i + \sum_{t=1}^{T}\theta^t \\
\text{subject to} & \sum_{i\in I} x_{ij} = 1, \quad j \in J \\
& x_{ij} \le y_i, \quad i \in I, j \in J \\
& x_{ij}, y_i \in \{0,1\}, \quad i \in I, j \in J \\
& \theta^t \ge \sum_{\tilde{\xi}^t \in \Xi^t} P(\tilde{\xi}^t = \xi^t)\theta_{\xi^t} \\
& \theta^t, \theta_{\xi^t} \ge 0, \xi^t \in \Xi^t, t = 1,\dots,T \\
& \theta_{\xi^t} \ge R^t(x,y,\xi^t), \xi^t \in \Xi^t, t = 1,\dots,T
\end{array}
\right.
$$

The recourse function $R^t(x,y,\xi^t), t = 1,\dots,T$ are not given explicitly in advance. Therefore we add the cut that corresponds to the constraint $\theta_{\xi^t} \ge R^t(x,y,\xi^t)$ sequentially. The next theorem gives a family of optimality cuts.

Theorem 2 *Let* $(\hat{x},\hat{y})$, $\hat{\pi}_i^t$ *be the feasible first stage solution of (RMS-CLP) and the optimal solution of the program* $\max\{\sum_{i\in I}(\sum_{j\in J} a_j(\xi^t)\hat{x}_{ij} - b_i\hat{y}_i)\pi_i^t | 0 \le \pi_i^t \le q_i^t - q_i^{t+1}, i \in I\}$, *respectively. Furthermore, let* θ_{ξ^t} *be the upper bound of* $R^t(x,y,\xi^t)$. $\theta_{\xi^t} \ge \sum_{i\in I}\sum_{j\in J}\hat{\pi}_i^t a_j(\xi^t)x_{ij} - \sum_{i\in I}\hat{\pi}_i^t b_i y_i$ *is a valid inequality for the feasible solution set of (RMS-CLP).*

Proof. Because of $\theta_{\xi^t} \ge R^t(x,y,\xi^t)$ for any feasible solution (x,y) of (RMS-CLP) we have the next inequality.

$$
\begin{aligned}
\theta_{\xi^t} \ge\ & R^t(x,y,\xi^t) \\
=\ & \min\{\sum_{i\in I}(q_i^t - q_i^{t+1})v_i^t(\xi^t) | \sum_{j\in J} a_j(\xi^t)x_{ij} \le b_i y_i + v_i^t(\xi^t), v_i^t(\xi^t) \ge 0, i \in I\} \\
=\ & \max\{\sum_{i\in I}(\sum_{j\in J} a_j(\xi^t)x_{ij} - b_i y_i)\pi_i^t | 0 \le \pi_i^t \le q_i^t - q_i^{t+1}, i \in I\} \\
\ge\ & \sum_{i\in I}\sum_{j\in J}\hat{\pi}_i^t a_j(\xi^t)x_{ij} - \sum_{i\in I}\hat{\pi}_i^t b_i y_i
\end{aligned}
$$

The last inequality holds from the fact that $\hat{\pi}^t$ is the optimal solution of $\max\{\sum_{i\in I}(\sum_{j\in J} a_j(\xi^t)x_{ij} - b_i y_i)\pi_i^t | 0 \le \pi_i \le q_i^t - q_i^{t+1}, i \in I\}$ only when $x = \hat{x}, y = \hat{y}$. ∎

We propose a new framework of algorithm which combines an L-shaped method and a branch-and-bound method. The algorithm repeats the iteration until the difference between the upper bound and the lower bound for the optimal value reaches the tolerance we set beforehand. In **Step 1** we solve a master problem yielding the first stage solution. This first stage solution is feasible because the problem has complete recourse. In **Step 4** we add the optimality cut to approximate the recourse function. Since the framework is simpler than the branch-and-cut method, it is easy to implement the algorithm.

Algorithm of Integer L-Shaped Method for (RMS-CLP)

- **Step 0. Initialization**
  Set temporary objective value $\bar{z} = +\infty$ and lower bound for optimal objective value $\underline{z} = 0$.
- **Step 1. Solve Master Problem for (RMS-CLP)**
  Solve Master Problem for (RMS-CLP) to obtain optimal solution $(\hat{x}, \hat{y}, \hat{\theta}^t, \hat{\theta}_{\xi^t}(\xi^t \in \Xi^t, t = 1, \ldots, T))$.
- **Step 2. Refinement of temporary objective value and lower bound for optimal objective value**
  If $\sum_{i\in I}\sum_{j\in J} c_{ij}\hat{x}_{ij} + \sum_{i\in I} f_i\hat{y}_i + \sum_{t=1}^{T}\hat{\theta}^t > \underline{z}$, then $\sum_{i\in I}\sum_{j\in J} c_{ij}\hat{x}_{ij} + \sum_{i\in I} f_i\hat{y}_i + \sum_{t=1}^{T}\hat{\theta}^t = \underline{z}$. If $\sum_{i\in I}\sum_{j\in J} c_{ij}\hat{x}_{ij} + \sum_{i\in I} f_i\hat{y}_i + \sum_{t=1}^{T}\mathcal{R}^t(\hat{x}, \hat{y}) < \bar{z}$, then $\sum_{i\in I}\sum_{j\in J} c_{ij}\hat{x}_{ij} + \sum_{i\in I} f_i\hat{y}_i + \sum_{t=1}^{T}\mathcal{R}^t(\hat{x}, \hat{y}) = \bar{z}$, where $\mathcal{R}^t(\hat{x}, \hat{y}) = E_{\tilde{\xi}^t}[R^t(\hat{x}, \hat{y}, \tilde{\xi}^t)]$.
- **Step 3. Check of convergence**
  If $\bar{z} \le (1 + \varepsilon)\underline{z}$, then stop. ($\varepsilon$: tolerance)
- **Step 4. Add optimality cuts**
  If $\hat{\theta}_{\xi^t} < R^t(\hat{x}, \hat{y}, \xi)$ for $\xi^t \in \Xi^t, t = 1, \ldots, T$, then add the optimality cuts to Master Problem for (RMS-CLP) and go to Step 1.

The L-shaped method (Van Slyke and Wets [21]) is a linearlization method using cutting planes (optimality cuts). The optimality cut is derived based upon the Benders' decomposition. Benders' decomposition has been applied to deterministic mixed integer programming problems or stochastic programming problems with continuous variables.

Our algorithm solves the integer master problem using a branch-and-bound method repeatedly. It is not very difficult to implement the algorithm since it does not require the branching operation and complicated fathoming rules.

4. Numerical Experiments

We utilized the integer L-shaped method to solve the problem. The whole framework of the algorithm was coded in Perl (Wall and Schwartz [22]) and XPRESS-MP [24] was used as a linear programming/branch-and-bound solver. Computational experiments were carried out on a SPARC Station 2.

Coordinates for sites of terminals and concentrator locations were generated from a uniform distribution over a rectangle of $[0, 100] \times [0, 100]$, $[10, 90] \times [10, 90]$, respectively. The Euclidean distance between terminal j and concentrator i was used to define cost coefficient c_{ij}.

$$c_{ij} = \lfloor(\text{distance between } i \text{ and } j) \times 0.15\rfloor + 1 \tag{4.1}$$

In the multi-stage model we assume $\tilde{\xi}^t, t = 1, \ldots, T$ has a discrete distribution and set $|\Xi^t| = 3$, where $\Xi^t = \{1, 2, \ldots, |\Xi^t|\}, t = 1, \ldots, T$ is a support of the probability measure P. We define $P(\tilde{\xi}^t = k)$ as follows.

$$P(\tilde{\xi}^t = k) = \frac{1}{|\Xi^t|}, k = 1, 2, \ldots, |\Xi^t|, t = 1, \ldots, T \tag{4.2}$$

Traffic data from every terminal were defined as:

$$a_j(\tilde{\xi}^t = k) = \lfloor U(150) + 50\rfloor, k = 1, 2, \ldots, |\Xi^t|, t = 1, \ldots, T. \tag{4.3}$$

where $U(150)$ was a number drawn from a uniform distribution between 0 and 150. These data $\lfloor U(150)+50 \rfloor, k=1,2,\ldots,|\Xi^t|, t=1,\ldots,T$ are sorted so as to satisfy Assumption 1.

$$a_j(\xi^t) \leq a_j(\xi^{t+1}), \forall \xi^t \in \Xi^t, \forall \xi^{t+1} \in \Xi^{t+1}, j \in J, t=1,\ldots,T-1 \tag{4.4}$$

Recourse costs are defined as follows, where $\frac{1}{(1+\alpha)^{t-1}}$ is a present worth factor. We set $\alpha = 0.01$.

$$q_i^t = \frac{q_i}{(1+\alpha)^{t-1}}, i \in I, t=1,\ldots,T \tag{4.5}$$

As we can see recourse costs $q_i^t, i \in I, t=1,\ldots,T$ satisfy Assumption 2. The number of concentrators to which each terminal can connect is restricted to 10 in the case that the number of terminal is 40 or 8 in the case of 60. The capacity of every concentrator was set as $b_i = 1600, i \in I$. The fixed setup cost was set as $f_i = 180, i \in I$.

We compare the integer L-shaped method (I-L-Shaped) for (RMS-CLP) with the branch-and-bound method (B&B) for (MS-CLP-MIP) with $T=3,4, |J|=40,60$ for the next two cases. In I-L-Shaped we set the tolerance $\varepsilon = 1(\%)$.

- **Low Recourse Cost Case** $q_i = \frac{180\times 4}{1600\times 3} = 0.15,\ i \in I$
  Recourse cost q_i is $\frac{4}{3}$ times larger than the initial investment cost of concentrator per capacity.

- **High Recourse Cost Case** $q_i = \frac{180\times 2}{1600} = 0.225, i \in I$
  Recourse cost q_i is twice as large as the initial investment cost of concentrator per capacity.

Computational time includes the time for problem generaton in XPRESS-MP [24]. The error of the I-L-Shaped algorithm for (RMS-CLP) is defined as follows.

Error of I-L-Shaped algorithm for (RMS-CLP)
= {Approximated objective value for (RMS-CLP) by (I-L-Shaped)
−Optimal objective value of (MS-CLP-MIP) by (B&B)}
/(Optimal objective value of (MS-CLP-MIP) by (B&B)) × 100(%)

The problems considered in this paper consist of 40 to 60 terminal sites and 20 potential concentrator locations. The results show that the integer L-shaped performs reasonably well on relatively large problems. The computing time tends to rise as the size of the problem increases. It is observed that in all cases the computing time of integer L-shaped are less than that of the usual B&B. Especially in the high recourse cost case with 20 potential sites and 60 terminals the computing time of integer L-shaped is nearly $13.5 \approx \frac{7418}{549}$ times faster than that of B&B (Table 2). Table 2 indicates that problems become more difficult when recourse cost q becomes larger. This can be explained as follows. When q is large, the relative importance of recourse function $\mathcal{R}^t(x,y)$ increases in the total objective function. Because the function $\mathcal{R}^t(x,y)$ is convex, convexity of the total cost rises. Though we set the tolerance $\varepsilon = 1(\%)$ in (I-L-Shaped), it should be noted that our algorithm yields good solutions. From Table 3 the error of integer L-shaped ranges from 0.0 to 0.85.

Let $(\hat{x}, \hat{y}, \hat{\theta}, \hat{\theta}_\xi(\xi \in \Xi))$ be the optimal solution of the master problem for (RMS-CLP), then the upper and lower bound for the optimal objective value of (RMS-CLP) are shown as follows.

$$\text{Upper Bound} = \sum_{i\in I}\sum_{j\in J} c_{ij}\hat{x}_{ij} + \sum_{i\in I} f_i \hat{y}_i + \sum_{t=1}^{T} \mathcal{R}^t(\hat{x}, \hat{y}) \tag{4.6}$$

Table 2: Computational Results

Number of Concentrator Locations $\lvert I\rvert$	Number of Terminals $\lvert J\rvert$	Number of Stages T	Number of Scenarios $\lvert\Xi^t\rvert$	Recourse Cost q_i	(I-L-Shaped) Computing Time (sec)	(B&B) Computing Time (sec)
20	40	3	3	0.15	100	194
20	40	4	3	0.15	149	201
20	40	3	3	0.225	197	329
20	40	4	3	0.225	246	1138
20	60	3	3	0.15	213	1691
20	60	4	3	0.15	348	3193
20	60	3	3	0.225	292	2912
20	60	4	3	0.225	549	7418

Table 3: Objective Values

Number of Concentrator Locations $\lvert I\rvert$	Number of Terminals $\lvert J\rvert$	Number of Stages T	Number of Scenarios $\lvert\Xi^t\rvert$	Recourse Cost q_i	(I-L-Shaped) Objective Value (Error(%)) (first stage/recourse)	(B & B) Objective Value (first stage/recourse)
20	40	3	3	0.15	709.6(0.65) (671/38.6)	705.0 (672/34.0)
20	40	4	3	0.15	745.9(0.00) (695/50.9)	745.9 (695/50.9)
20	40	3	3	0.225	723.0(0.00) (672/51.0)	723.0 (672/51.0)
20	40	4	3	0.225	769.4(0.00) (699/70.4)	769.4 (699/70.4)
20	60	3	3	0.15	1126.0(0.63) (902/226.0)	1119.0 (902/217.0)
20	60	4	3	0.15	1175.0(0.43) (1088/87.0)	1170.0 (1088/82.0)
20	60	3	3	0.225	1164.5(0.85) (1072/92.5)	1154.6 (1072/82.6)
20	60	4	3	0.225	1213.3(0.19) (1088/125.3)	1211.0 (1088/123.0)

$$\text{Lower Bound} = \sum_{i \in I} \sum_{j \in J} c_{ij} \hat{x}_{ij} + \sum_{i \in I} f_i \hat{y}_i + \sum_{t=1}^{T} \hat{\theta}^t \tag{4.7}$$

In the algorithm of I-L-Shaped we adopt the best upper bound up to the current iteration as the temporary solution.

Table 4: Iteration of I-L-Shaped ($|J| = 40, T = 3$)

Low Recourse Cost Case $q_i = 0.15$				High Recourse Cost Case $q_i = 0.225$			
Iteration	Lower Bound	Upper Bound	Added Cuts	Iteration	Lower Bound	Upper Bound	Added Cuts
1	389	836.0	9	1	389	1059.5	9
2	695.0	741.1	4	2	712.0	766.1	4
3	695.5	741.1	4	3	712.8	766.1	4
4	705.0	709.6	1	4	722.0	730.4	1
				5	722.0	729.9	1
				6	723.0	723.0	0

Table 5: Iteration of I-L-Shaped ($|J| = 40, T = 4$)

Low Recourse Cost Case $q_i = 0.15$				High Recourse Cost Case $q_i = 0.225$			
Iteration	Lower Bound	Upper Bound	Added Cuts	Iteration	Lower Bound	Upper Bound	Added Cuts
1	451	924.7	12	1	451	1161.7	12
2	734.0	795.0	8	2	757.4	849.5	8
3	740.0	784.4	7	3	763.4	830.1	7
4	740.0	753.1	4	4	763.4	808.5	6
5	745.9	745.9	0	5	769.4	769.4	0

Table 6: Iteration of I-L-Shaped ($|J| = 60, T = 3$)

Low Recourse Cost Case $q_i = 0.15$				High Recourse Cost Case $q_i = 0.225$			
Iteration	Lower Bound	Upper Bound	Added Cuts	Iteration	Lower Bound	Upper Bound	Added Cuts
1	660	1297.0	7	1	660	1616.0	7
2	1118.0	1140.6	4	2	1147.6	1232.3	4
3	1119.0	1126.0	2	3	1152.6	1175.6	2
				4	1152.6	1260.4	3
				5	1154.6	1164.5	2

Because the standard mixed integer programming formulation (MS-CLP-MIP) contains $\prod_{t=1}^{T} |\Xi^t|$ scenarios the computational time of (B&B) rises rapidly when the number of stages increases. The algorithm of integer L-shaped decomposes the problem (RMS-CLP) into T subproblems in which we have $|\Xi^t|$ scenarios. Further, these networks would represent a large computer network in which each one of these terminals is likely to be a cluster of connected smaller computer systems. It is thus obvious that we can treat larger network problems as well.

Table 7: Iteration of I-L-Shaped ($|J| = 60, T = 4$)

Low Recourse Cost Case $q_i = 0.15$				High Recourse Cost Case $q_i = 0.225$			
Iteration	Lower Bound	Upper Bound	Added Cuts	Iteration	Lower Bound	Upper Bound	Added Cuts
1	780	1236.4	8	1	780	1464.5	8
2	1165.0	1260.5	5	2	1206.0	1323.3	5
3	1169.0	1215.8	4	3	1210.0	1274.2	5
4	1170.0	1264.5	5	4	1211.0	1213.3	1
5	1170.0	1175.0	2				

5. Concluding Remarks

In this paper a stochastic version of a concentrator location problem is dealt with in which traffic demand at each terminal location is uncertain. The problem is formulated as a stochastic multi-stage integer linear program, with first stage binary variables concerning network design and continuous recourse variables concerning expansion of capacity. Given a first stage decision, the series of realizations of traffic demand may possibly imply a violation of the capacity constraint of the concentrator. Therefore the recourse action is taken to correct the violation. The objective function minimizes the cost of connecting terminals and the cost of opening concentrators and the expected recourse cost of capacity expansion. We proposed a new algorithm which combines an L-shaped method and a branch-and-bound method. Finally we demonstrated the computational efficiency of our algorithm. Computer codes for the algorithm were applied to a set of problems. The results show that our method solves these problems in less time than a standard mixed integer programming approach.

The following points are left as future problems. In real problems there might be some possibility that Assumption 1 does not hold true. The measures which decompose scenarios without Assumption 1 will have to be taken. It seems to be a wothwhile problem to investigate.

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