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PROPOSAL OF NEW AHP MODEL IN LIGHT OF DOMINANT RELATIONSHIP AMONG ALTERNATIVES

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Abstract The dominant alternatives method by Kinoshita and Nakanishi (1997) is a new type of AHP–Analytic Hierarchy Process–designed to deal with the case in which the weights of criteria vary in accordance with the alternative chosen as the dominant viewpoint.

This study clarifies differences between the general viewpoint and the dominant viewpoint, and features of the relative and the absolute measurements under both views.

When conducting continuous surveys, additional data from the latest survey have to be reflected into the result of the previous survey in a certain scheme. This paper proposes "concurrent convergence" as a processing technique for additional data in an application of the dominant alternative method.

When there are more than one dominant alternative, the technique requires a convergent calculation toward the coincidence among derived weights of criteria on each alternative. By adopting the convergent values in the overall evaluations, every evaluation value on every alternative will be equal. This technique, therefore, enables us to reserve the essential features of the dominant alternative method.

1. Introduction

The analytic hierarchy process (AHP) developed by Thomas L. Saaty is a method which combines subjective judgment and system approach properly. The AHP is widely adopted in Europe and America and other areas in various fields such as economic problems, management problems, energy problems, a policy decision and city planning.

The original AHP of Saaty is called a relative measurement and has shortcomings such as it can not deal with a case involving many alternatives. To overcome the shortcomings, Saaty proposed an absolute measurement (the author has realized a calculation method in this method). The AHP includes two methods, relative measurement and absolute measurement. The relative measurement carries out overall evaluation based on pair-wise comparison of alternatives about each criterion, and is adopted where direct comparison between the alternatives is effective. The latter is adopted where indirect comparison through an evaluation scale is effective. A common feature to the two methods is that criteria are weighted independent of evaluation of the alternatives. The two measurements proposed by Saaty are called a conventional AHP (conventional relative measurement and conventional absolute measurement) by the author.

At first, it was supposed that each criterion is independent of other criteria, each alternative is independent of other alternatives, and each criterion is independent of each alternative in the conventional AHP. However, they are sometimes not independent but dependent.

Therefore, Saaty proposed an inner dependence method for such a case as criteria are dependent or alternatives are dependent to each other. This method is an improved model of separately measuring dependent relationship between the criteria or alternatives pairwise and incorporating the results to the model.

Also Saaty proposed an outer dependence method for such a case as criteria and alternatives are dependent to each other. The feature of this concept is that the weight of each criterion is determined about each alternative and not uniquely according to the overall object. As a result, there may be weights of different values. When there is dependence between hierarchy levels, analysis is carried out by a super matrix (proposed by Saaty) expressing the relationships simultaneously. As a result, weights of each criterion and judgment values of each alternative converge to certain values. This concept is applicable not only to hierarchy elements but also to elements of a network. Saaty has proposed an analytic network process (ANP) as an improved model of the conventional AHP.

There is also a problem how to deal with numbers used for pairwise comparison. This is an interesting theme of the AHP. Saaty proposed a linear scale (1, 3, 5, 7, and 9) but Lootsma an exponent scale. Many other scales are also proposed implying that this problem is an important theme in application of the AHP.

There are also several problems such as a problem in an indirect approximation method of an incomplete pair comparison determinant, inversion of priorities of alternatives by addition of a new alternative, application to group decision, and application to cost, benefit, and risk analysis. The author has conducted research on the improvement of the conventional AHP and combination of the AHP with other models. As a result, the author has realized the necessity of introducing new viewpoints to the conventional AHP.

This paper proposes a new calculation method in the AHP, introducing new viewpoints into the concept of evaluating criterion importance and the concept of evaluating alternatives about the criteria. This paper outlines the conventional AHP proposed by Saaty in Chapter 2, and a dominant AHP proposed by the author in Chapter 3, and proposes concurrent convergence in Chapters 4 and 5, concluding in Chapter 6.

2. Conventional AHP

2.1 Conventional AHP process

The conventional AHP process consists of the following three steps.

(1) Step 1

A problem under complicated situations is decomposed to a hierarchy structure. However, the top level consists of one element, an overall objective. The elements of lower levels are determined subjectively by the decision maker according to the relationships of each element with the elements of the immediately above level. Finally alternatives are listed on the lowest level.

(2) Step 2

The elements of intermediate levels are weighted. Namely, each element on a level is evaluated about the elements, i.e. criteria, on the immediately above level to acquire relative weights (Wi) of the elements on the same level.

(3) Step 3

The weight of the entire hierarchy is acquired using the weights of the elements on each level. According to the weight, priority of each alternative for the overall objective is acquired.

2.2 Conventional relative measurement

The relative measurement of the conventional AHP is hereinafter called the conventional relative measurement. The measurement acquires the weights of criteria and the judgment values of alternatives by pairwise comparison in the step 2. (However, $aij = 1/aij$ in the pairwise comparison matrix.) The weight Wi of each criterion is acquired as an eigenvector for a maximum eigenvalue of the pairwise comparison matrix.

$$wi(k) = \frac{\sum_{j=1}^{n} aij \cdot wj(k-1)}{\sum_{i=1}^{n}\sum_{j=1}^{n} aij \cdot wj(k-1)} \quad ; i = 1, \ldots, n; k = 1, 2, \ldots$$
$$wi = \lim_{k \to \infty} wi(k) \quad ; \ i = 1, \ldots, n \tag{2.1}$$

Priority of alternatives will be acquired on the following assumptions. The weights of criteria are $B1 : B2 = 0.4 : 0.6$. Judgment values of the alternatives about each criterion are shown in Tables 1 and 2.

Overall judgment values of each alternative weighted according to the weights of the criteria are given in Table 3. Priority of each alternative is $C1 > C2 > C3$.

Table 1. Alternative values about criterion $B1$ by the pairwise comparison

$B1(0.4)$	$C1$	$C2$	$C3$	Weight
$C1$	1	1/2	1/3	0.167
$C2$	2	1	2/3	0.333
$C3$	3	3/2	1	0.500

Table 2. Alternative values about criterion $B2$ by the pairwise comparison

$B2(0.6)$	$C1$	$C2$	$C3$	Weight
$C1$	1	2	6	0.6
$C2$	1/2	1	3	0.3
$C3$	1/6	1/3	1	0.1

Table 3. Overall judgment and the weights of criteria by the conventional AHP

		Criteria		
		$B1$ (0.4)	$B2$ (0.6)	E Overall
Values of alternatives	$C1$	0.167	0.600	0.427
	$C2$	0.333	0.300	0.313
	$C3$	0.500	0.100	0.260

2.3 Conventional absolute measurement

The absolute measurement of the conventional AHP is hereinafter called the conventional absolute measurement. This method evaluates alternatives by the evaluation scale values of the alternatives acquired according to the evaluation scales provided for each criterion in the step 2, and not by pairwise comparison. The evaluation scale is acquired by pairwise comparison (such as high (H), middle (M), and low (L)) of criteria. The judged level of each alternative about each criterion is converted to an evaluation scale value defined by

Table 4. Evaluation scale of criterion $B1$

$B1(0.4)$	H	M	L	Evaluation scale
H	1	2	4	0.558
M	1/2	1	3	0.320
L	1/4	1/3	1	0.122

Table 5. Evaluation scale of criterion $B2$

$B2(0.6)$	H	M	L	Evaluation scale
H	1	3	5	0.627
M	1/3	1	4	0.279
L	1/5	1/4	1	0.094

each evaluation scale. For example, the case given in the preceding paragraph 2-2 will be evaluated in the conventional absolute method. (Tables 4 and 5)

Evaluation scale values of each alternative about criteria are acquired in the step 2. The evaluation scale values are weighted and totaled, and finally normalized so that the total shall be 1, to acquire the overall judgment value of each alternative, in the step 3. The resultant overall evaluation is shown in Table 6. The priority of the alternatives is $C1 > C2 > C3$, the same as that shown in Table 3.

Table 6. Overall judgment and weights of criteria by the conventional absolute measurement

		Criteria			
		$B1$ (0.4)	$B2$ (0.6)	E Overall	Eg Normalized value
Values of alternatives	$C1$	$L0.122$	$H0.627$	0.425	0.425
	$C2$	$M0.320$	$M0.279$	0.295	0.295
	$C3$	$H0.558$	$L0.094$	0.280	0.280
				1.000	1.000

3. Dominant AHP

The importance of each criterion is uniquely determined top-down from the overall objective in the conventional AHP.

However, there may be such an approach as the importance of a criterion is determined with a particular alternative in mind so that the alternative can be easily evaluated. Such an alternative as governs the importance of a criterion is called the regulating alternative in this paper.

According to the above concept, there are importances of criteria in the same number as that of alternatives. This predicts conflicts among regulating alternatives in determination of criteria importances. Saaty's external dependent method is one proposal for bottom-up approach. The method, however, requires an democratic attitude to input the importances of criteria determined by all the regulating alternatives, increasing analysis load immensely.

However, we are not always taking such a process as scrutinizing and reducing criteria importances. Decision will be made with as less analysis load as possible allowing more or less errors as far as risk is small.

Being considered in this section is an approach to carry out evaluation of criteria according to the regulating alternatives established first as a base as far as the concept of evaluating the importance of criteria according to the regulating alternatives is not disrupted, as a leading method satisfying the desire. However, an alternative serving as a base is not necessarily evaluated at first. The alternative is previously selected as a guidance leading overall evaluation process. The base may be rough, apparent, contrary, or else, and is selected arbitrarily by the evaluator.

In this section, the following evaluation method will be considered. The importances of each alternative may be scattered according to the respective regulating alternatives.

However, it is supposed that the distribution is uniquely determined by the regulating alternative selected arbitrarily by the decision maker. Namely, the importance of each criterion governing other regulating alternative than the regulating alternative serving as a base of evaluation completely complies with the evaluation of the criterion governing the regulating alternative serving as a base. The regulating alternative that dominates is called the dominant alternative and the regulating alternative that complies is called the dependent alternative. Namely, the importance of the criterion of the dependent alternative is automatically derived from the importance of the criterion of the dominant alternative. In the concerned model, the dominant alternative governs not only the importance distribution of criteria but also the overall judgment value induced from each importance distribution.

This, however, does not necessarily mean that the dominant alternative will be the most preferable alternative. The dominant alternative only provides a base for determining the importance of criteria. Namely, the overall judgment value is identical throughout all the importance distributions of all the criteria of the dominant alternatives and the dependent alternatives. The overall judgment value is in fact already determined subconsciously at the stage of evaluation based on the dominant alternative. Evaluation of importance of the criteria by the dependent alternatives is merely serving as a support of the soundness of overall evaluatlon by the dominant alternative from a subordinate standpoint. However, such an evaluation method as above is a common practice in our daily life. The evaluation method based on a new viewpoint, proposed by us, is hereinafter called the dominant alternative method.

3.1 Dominant alternative method

Governing relationship in the relative measurement will be checked first. Unlike the conventional relative measurement, the weight of a criterion varies according to an alternative considered and also it is assumed that a weight in relation to a particular alternative (a dominant alternative) regulates a weight in relation to other alternatives (dependent alternatives). In this paper, the method is called the dominant alternative method.

3.1.1 Process of dominant alternative method

(1) Step 1

A hierarchy consisting of an overall objective, criteria, and alternatives is organized. (Same as the conventional relative measurement.)

(2) Step 2

Assuming that criteria are Bi $(i = 1, \ldots, n)$ and alternatives are Cj $(j = 1, \ldots, m)$, pairwise comparison is carried out between the criteria. However, comparison is carried out in relation to a dominant laternative Cj^*. (Table 7) Since the importance of each alternative is determined according to the ratio of importance to that of the dominant alternative, eigenvector calculation required in the conventional AHP is not necessary. The judgment value of the dependent alternative Cj^{**} acquired by pairwise comparison is weighted by the weight $Bi(Cj^*)$ of the criterion Bi in relation to the dominant alternative and the result is normalized to yield the weight of the criterion, $Bi(Cj^{**})$, in relation to the dependent alternative.

Table 7. Evaluation of each alternative Cj

Cj^*	$B1$	$\cdots$	Bl	$\cdots$	Bn	E
$C1$	$C11$	$\cdots$	$C1l$	$\cdots$	$C1n$	$\sum BlC1l$
$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
Cj	$Cj1$	$\cdots$	Cjl	$\cdots$	Cjn	$\sum BlCjl$
$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
Cj^*	1	$\cdots$	1	$\cdots$	1	1
$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
Cm	$Cm1$	$\cdots$	Cml	$\cdots$	Cmn	$\sum BlCml$

(3) Step 3

Evaluation of each alternative, $Cji_{Bi(Cj^*)}$, is carried out about each criterion Bi. However, the judgment value is compared only with the dominant alternative Cj^*. The judgment values of the dominant alternative Cj^* in relation to the criteria are all assumed to be 1. Accordingly, the overall judgment value Ej of each alternative is acquired.

It is supposed that the weights of criteria evaluated in relation to the dominant alternative $C1^*$ are $B1 : B2 = 0.4 : 0.6$. (Table 8) Weights of criteria in relation to dependent alternatives $C2^{**}$ and $C3^{**}$ are acquired from Table 8 and overall judgment values acquired from those values are shown in Table 9 and 10. The weights of criteria in relation to the dependent alternative $C2^{**}$ are;

$$\frac{B1(C2^{**})}{B2(C2^{**})} = \frac{C12_{B1(C1^*)} \cdot B1(C1^*)}{C22_{B2(C1^*)} \cdot B2(C1^*)} = \frac{2 \times 0.4}{\frac{1}{2} \times 0.6} = \frac{0.727}{0.273}.$$

Table 8. Weights of criteria about the dominant alternative $C1^*$

		Criteria			
Dominant alternatives	$C1^*$	$B1$ (0.4)	$B2$ (0.6)	E Overall	Eg Normalized value
Values of alternative	$C1$	1	1	1	0.294
	$C2$	2	1/2	1.1	0.324
	$C3$	3	1/6	1.3	0.382

The weights of criteria in relation to the dependent alternative $C3^{**}$ are;

$$\frac{B1(C3^{**})}{B2(C3^{**})} = \frac{C13_{B1(C1^*)} \cdot B1(C1^*)}{C23_{B2(C1^*)} \cdot B2(C1^*)} = \frac{3 \times 0.4}{\frac{1}{6} \times 0.6} = \frac{0.922}{0.078}.$$

Normalized overall judgment values of alternatives in Table 8 to 10 are all identical, $C1 : C2 : C3 = 0.294 : 0.324 : 0.382$ respectively and identical. Namely, even if the weights of criteria governed by any dependent alternative are used, the resultant overall judgment

Table 9. Weights of criteria about the dependent alternative $C2^{**}$

		Criteria			
Dependent	$C2^*$	$B1$	$B2$	E	Eg
alternative		(0.727)	(0.273)	Overall	Normalized value
Values of	$C1$	1/2	2	0.909	0.294
alternatives	$C2$	1	1	1	0.324
	$C3$	3/2	1/3	1.181	0.382

Table 10. Weights of criteria about the dependent alternative $C3^{**}$

		Criteria			
Dependent	$C3^*$	$B1$	$B2$	E	Eg
alternative		(0.922)	(0.078)	Overall	Normalized value
Values of	$C1$	1/3	6	0.776	0.294
alternatives	$C2$	2/3	3	0.849	0.324
	$C3$	1	1	1	0.382

values are identical with those acquired according to the weights of criteria governed by the dominant alternative. Therefore, the priority of alternatives are $C1 < C2 < C3$.

To acquire the same overall judgment values as $C1 : C2 : C3 = 0.427 : 0.313 : 0.260$ yielded in the conventional relative measurement, shown in Table 3, the weights of criteria in relation to the dominant alternative in the dominant alternative method should be $B1(C1^*) : B2(C1^*) = 0.156 : 0.844$. On this occasion, the arithmetic mean of the three weights of each criterion in the dominant alternative method is $B1 : B2 = 0.507 : 0.493$, not agreeing with $0.4 : 0.6$. It is clear that the weight of criterion acquired in the conventional relative measurement from general viewpoint is not identical with the arithmetic mean of three weights of criterion.

3.1.2 Differences in viewpoint between dominant alternative method and conventional relative measurement

In the conventional relative measurement, the weights of criteria are uniquely determined according to the objective. In the dominant alternative method, however, the weights serve as a rating standard of scale of each criterion for the dominant alternative. In other words, the weights of criteria $B1 : B2$ are the magnitudes of criterion dominating power. In the dominant alternative method, however, the weights of criteria are relative sizes of scales of criteria when comparing with the dominant alternative.

In the evaluation in the dominant alternative method as shown in Table 8 to 10, the evaluator is not necessarily aware of the effect (attribute) of the evaluating object. However, the fact that comparison with the dominant alternative is possible follows that the evaluator can measure the effect of the object even with a provisional scale. It may be difficult to guess the effects right but it is enough to know the ratio (normalized values) of effects by means of the provisional scale for comparative evaluation.

The dominant alternative method is suitable in such a case as comparison is made where much information is available for a particular alternative but not for other alternatives.

In fact, the weights of criteria in relation to alternatives are nothing but the ratio of effects of alternatives on criteria. (Table 11) Namely, the dominant alternative method is a process of inferring the effect of each alternative according to the information on the dominant alternative as a clue. The reason why the overall evaluation values of alternatives, dominant and dependent, agree is that information on the effect of each alternative becomes unique. It can be said that the dominant alternative method is applicable to such a case as evaluation is made by inferring effect by the evaluator.

Table 11. Inferred effects of alternatives (attributes) by the dominant alternative

		Criteria			
		$B1$ (Weight)	$B2$ (Weight)	E Overall	Eg Normalized value
Effects of alternatives	$C1$	20 (0.400)	30 (0.600)	50	0.294
	$C2$	40 (0.727)	15 (0.273)	55	0.324
	$C3$	60 (0.922)	5 (0.078)	65	0.382

Suppose that the decision maker makes evaluation in the general viewpoint analysis method because he is not aware of the dominant viewpoint analysis method in spite of the fact that the weights of criteria $B1 : B2 = 0.4 : 0.6$ referred to in the case of the general viewpoint analysis are in fact the weights of criteria about the dominant alternative $C1^*$. What results will be brought about by general viewpoint analysis?

Since the weights of criteria are determined with reference to the dominant alternative, the correct judgment values must be $C1 : C2 : C3 = 0.294 : 0.324 : 0.382$ (Table 8) and $C1 < C2 < C3$. However, the results of the general viewpoint analysis are $C1 : C2 : C3 = 0.427 : 0.313 : 0.260$, and $C1 > C2 > C3$.

The decision maker will not satisfy with the results. On such an occasion, he will often adjust the weights of criteria in the general viewpoint analysis process so that the results will come up to his expectation. To acquire the desired overall judgment values, the weights of criteria acquired by general viewpoint analysis should be $B1 : B2 = 0.706 : 0.294$. If the decision maker is unaware of the dominant alternative method, he has to acquire the weights of criteria by trial and error. If he is aware of the dominant viewpoint, he will be able to carry out detailed, efficient overall evaluation.

3.2 Dominant evaluation level method

Governing relationship in the absolute measurement will be examined in this section. Dominant viewpoint in the absolute measurement concerns about governing relationship on the evaluation level.

Unlike the conventional absolute measurement, it is assumed that the weights of criterion are different according to the concerned evaluation level, and also that the weights in accordance with a particular evaluation level (dominant evaluation level) govern the weights in accordance with other evaluation level (dependent evaluation level).

Therefore, the weights of criteria in accordance with the dependent evaluation level can be acquired from the weights of criteria in accordance with the dominant evaluation level in the same calculation method as that in the dominant alternative method. This method is called the dominant evaluation level method.

3.2.1 Example of calculation in dominant evaluation level method

An example of calculation will be presented. Suppose that information on the dominant evaluation level H^* given in Table 12 and 13 are available. The criteria are weighted

Table 12. The evaluation scale on the criterion $B1$

$B1(H^*) = 0.4$	H	M	L	Evaluation scale	H Standard scale
H	1	2	4	0.558	1
M	1/2	1	3	0.320	0.573
L	1/4	1/3	1	0.122	0.219

Table 13. The evaluation scale on the criterion $B2$

$B2(H^*) = 0.6$	H	M	L	Evaluation scale	H Standard scale
H	1	3	5	0.627	1
M	1/3	1	4	0.279	0.445
L	1/5	1/4	1	0.094	0.150

$B1(H^*) : B2(H^*) = 0.4 : 0.6$ according to the dominant evaluation level (H^*, H^*). The evaluation scales of the dependent evaluation levels (L^{**}, L^{**}) and (M^{**}, M^{**}) are directly defined by the ratio to the dominant evaluation level (H^*, H^*) governing the weight of each criterion. The weights of criteria are directly acquired according to the ratio to the dominant criteria, and the calculation of eigenvectors is not necessary.

An alternative $X(H^*, H^*)$ marking the highest values H = dominant evaluation level for all the criteria is supposed (quasi-alternative). As far as a dominant evaluation level has been set by the decision maker, the quasi-alternative may be of any evaluation level value for any criterion (for instance, a quasi-alternative evaluated H^* for $B1$ and L^* for $B2$).

Like the dominant alternative method, the judgment values of every criterion about $X(H^*, H^*)$ are assumed 1, aud each alternative is compared with $X(H^*, H^*)$. The weights of the alternatives are directly acquired according to the ratio to that of the quasi-alternative. After all, calculation of eigenvector is not necessary in the dominant evaluation level method. Finally, each alternative will be evaluated by weighting according to the weights of criteria. The overall judgment values of alternatives with reference to the dominant evaluation level (H^*, H^*) are given in Table 14.

Table 14. Overall judgment and the weights of criteria by the dominant evaluation levels (H^*, H^*)

		Criteria			
Dominant	(H^*, H^*)	$B1$ (0.4)	$B2$ (0.6)	E Overall	Eg Normalized value
Values of alternatives	$C1$	L 0.219	H 1.000	0.688	0.411
	$C2$	M 0.573	M 0.445	0.496	0.296
	$C3$	H 1.000	L 0.150	0.490	0.293
Quasi alternative	X	$H1$	$H1$	1.000	-

Based on the results of the dominant evaluation level (H^*, H^*), the weights of criteria with reference to other evaluation levels (for instance, dependent evaluation level (L^{**}, L^{**}), (M^{**}, M^{**}), etc.) and overall judgments values will be acquired next. At first, the weights of criteria and overall judgment values in relation to the dependent evaluation level (L^{**}, L^{**}) will be acquired. Comparison is made with reference to a quasi-alternative (L^{**}, L^{**}) of

Table 15. Overall judgment and the weights of criteria by the dependent evaluation levels (L^{**}, L^{**})

Dependent	Criteria (L^{**}, L^{**})	$B1$ (0.493)	$B2$ (0.507)	E Overall	Eg Normalized value
Values of alternatives	$C1$	L 1.000	H 6.667	3.872	0.411
	$C2$	M 2.616	M 2.967	2.794	0.296
	$C3$	H 4.566	L 1.000	2.759	0.293
Quasi alternative	Y	$L1$	$L1$	1.000	-

which the judgment values about every criterion are the lowest L. (Table 15)

$$\frac{B1(L)}{B2(L)} = \frac{L_{B1(H^*)} \cdot B1(H^*)}{L_{B2(H^*)} \cdot B2(H^*)} = \frac{0.219 \times 0,4}{0.150 \times 0.6} = \frac{0.493}{0.507}.$$

Similarly, the weights of criteria and overall judgment values will be acquired with reference to the dependent evaluation level (M^{**}, M^{**}). Comparison will be made with reference to the quasi-alternative Z of which the judgment values of every criterion is medium M (Table 16).

$$\frac{B1(M)}{B2(M)} = \frac{M_{B1(H^*)} \cdot B1(H^*)}{M_{B2(H^*)} \cdot B2(H^*)} = \frac{0.573 \times 0.4}{0.445 \times 0.6} = \frac{0.462}{0.538}.$$

Table 16. Overall judgment and the weights of criteria by the dependent evaluation levels (M^{**}, M^{**})

Dependent	Criteria (M^{**}, M^{**})	$B1$ (0.462)	$B2$ (0.538)	E Overall	Eg Normalized value
Values of alternatives	$C1$	L 0.382	H 2.247	1.386	0.411
	$C2$	M 1.000	M 1.000	1.000	0.296
	$C3$	H 1.754	L 0.337	0.988	0.293
Quasi alternative	Z	$M1$	$M1$	1.000	-

Normalized overall judgment values given in Table 14 to 16 are all $C1$ (0.411), $C2$ (0.296), and $C3$ (0.293), yielding the same results. Overall judgment values are identical with those acquired by the dominant evaluation level even if the weights of criteria governed by any dependent evaluation level are used, and priority is $C1 > C2 \doteqdot C3$.

3.2.2 Differences in viewpoint between dominant evaluation method and conventional absolute measurement

The dominant evaluation level method is suitable for such a case as there is a set of some evaluation levels (quasi-alternatives). In the dominant evaluation level method, each judgment value is an evaluation index of each evaluation level when the judgment value of the dominant evaluation level is assumed 1. The weight of the evaluation level is uniquely determined according to the criteria in the conventional absolute measurement but serves as a standard for rating the scale of each criterion with reference to the dominant evaluation level in the dominant evaluation level method. In other words, the weights of criteria $B1 : B2$ are the magnitudes of the criterion dominating power. In the dominant evaluation level method, however, the weights of criteria are relative sizes of scales of criteria when the alternatives are evaluated by the dominant evaluation level (quasi-alternative). This comes from different view points as in the case of the conventional relative measurement and the dominant alternative method. Overall judgment values acquired by the dominant evaluation level agree with those acquired by the dependent evaluation level in the dominant evaluation level method as in the dominant alternative method, based on the same principle of inferring effect (attribute).

Table 17. Evaluation by the dominant evaluation levels (L^*, M^*) = the dominant alternative $C1^*$

		Criteria			
Dominant	(L^*, H^*)	$B1$ (0.400)	$B2$ (0.600)	E Overall	Eg Normalized value
Values of alternatives	$C1$	L 1.000	H 1.000	1.000	0.236
	$C2$	M 2.616	M 0.445	1.314	0.311
	$C3$	H 4.566	L 0.150	1.916	0.453
Quasi alternative	$W = C1^*$	$L1$	$H1$	1.000	-

If the evaluation level (L, H) of the dominant altenrative C^* is used as a dominant evaluation level, the results $C1 < C2 < C3$ given in Table 17 are the same as those acquired in the dominant alternative method. This means that the model of the dominant alternative method is compatible with that of the dominant evaluation level method. On this occasion, the values of the dependent evaluation level with reference to the dominant evaluation level (quasi-alternative) $W(L^*, H^*)$ are $X(H^*, H^*) = 0.753 : 0.257$, $Y(L^*, L^*) = 0.817 : 0.183$, and $Z(M^*, M^*) = 0.797 : 0.203$.

3.3 Difference between general viewpoint and dominant viewpoint

Existence of the general viewpoint implies that the weights of criteria are not always given transcendentally as Saaty supposes. General viewpoint analysis may not be applicable to some object of analysis. It can be said that the dominant viewpoint analysis and the general viewpoint analysis have their applicable fields respectively (Table 18). We must find out through analysis either the general view point analysis or the dominant view point analysis is preferred according to the object of evaluation concerned.

Table 18. Difference between general viewpoint and dominant viewpoint in AHP

View point / Approach	The general viewpoint (The conventional AHP)	The dominant viewpoint (The dominant AHP)
Relative measurement	Conventional relative measurement	Dominant alternative method
Absolute measurement	Conventional absolute measurement	Dominant evaluation method

4. Processing Additional Data in Dominant Alternative Method

4.1 Problem of additional data in dominant alternative method

Being discussed is a processing method in such a case as the results given below are acquired by the latest survey though the results shown in Table 8–10 were acquired by the previous survey.

The relative judgments of the alternatives about $B1$ and $B2$ remain the same, but the dominant alternative is changed to $C2$ in the latest survey and the weights of criteria are;

$$B1(C2) : B2(C2) = 0.8 : 0.2.$$

If the values of criteria are;

$$B1(C2) : B2(C2) = 0.727 : 0.273,$$

the weights of criteria will remain the same through the successive surveys even though the dominant alternative had been changed. However, the values of criteria change in the above case.

If the response of the evaluator straightly reflects the change of his value sense, the results of the previous survey should be discarded and those of the latest survey should be adopted as the current judgment. However, if the values reflect his straying viewpoint, the previous results and the latest results must be merged in a certain method.

Such a merging method as stated below is proposed. The concurrent convergence is a concrete method of merging.

Proposal: If two or more different judgments are yielded in the dominant alternative method, the results derived by respective evaluations are merged to yield an overall judgment retaining the properties of the dominant alternative.

4.2 Concurrent convergence

It is assumed that there are s dominant alternatives, $Cj^*(k)$ $(k = 1, \ldots, s)$, including dominant alternatives given as additional data, in relation to the evaluation of m alternatives, Cj $(j = 1, \ldots, m)$, and alternative evaluation with a viewpoint on each dominant alternative, $[Cji]j^*(k)$, $j = 1, \ldots, m; i = 1, \ldots, n$ is given. It is also supposed that the dominant alternatives do not change but evaluation information may change. If two or more different judgment values are yielded by evaluation of alternatives about criteria, the judgment values are geometrically averaged to acquire representative judgment values Cji. For each dominant alternative, n criteria, Bi $(i = 1, \ldots, n)$, are provided with respective peculiar criterion values.

The ratios given below are solved according to the criterion value $Bj(Cj^*(k))$ for each dominant alternative $Cj^*(k)$ to derive the criterion value $Bi(Cj(k))$ for other alternatives.

$$\begin{aligned} &B1(Cj)_{(k)} : B2(Cj)_{(k)} \cdots : Bn(Cj)_{(k)} \\ = \; &B1(Cj^*)_{(k)} \cdot \hat{C}j1_{(j^*)} : B2(Cj^*)_{(k)} \cdot \hat{C}j2_{(j^*)} \cdots : Bn(Cj^*)_{(k)} \cdot \hat{C}jn_{(j^*)}. \end{aligned}$$

However,

$$\sum_{i=1}^{n} Bi(Cj)_{(k)} = 1. \tag{4.1}$$

The criterion values for other alternative, derived from a dominant alternative, generally do not agree with those derived from other dominant alternative.

$$Bi(Cj)_{(1)} \neq Bi(Cj)_{(2)} \neq \cdots \neq Bi(Cj)_{(s)}. \tag{4.2}$$

Therefore, criterion values are averaged to acquire a primary composite criterion values $Bj(Cj)^{(1)}$.

$$Bi(Cj)^{(1)} = \frac{1}{S} \sum_{k=1}^{S} Bi(Cj)_{(k)}. \tag{4.3}$$

Starting with the primary composite criterion values, similar derivation and averaging are repeated as expressed by the following recurrence formula, eventually derived values agree converging to composite criterion values $Bj(Cj)$. (Number of repetitions P)

$$Bi(Cj)^{(P+1)} = \frac{1}{S}\left[\sum_{k=1}^{S} \frac{B_i(C_j)^{(P)}_{(k)} \cdot Cji_{(k)}}{\sum_{i=1}^{n} Bi(Cj)^{(P)}_{(k)} \cdot Cji_{(k)}}\right]. \tag{4.4}$$

The composite criterion values are a solution of the following equation.

$$\hat{B}i(Cj) = \frac{1}{S}\left[\sum_{k=1}^{S} \frac{\hat{B}_i(C_j)_{(k)} \cdot Cji_{(k)}}{\sum_{i=1}^{n} \hat{B}i(Cj)_{(k)} \cdot Cji_{(k)}}\right]. \tag{4.5}$$

If the composite criterion values are used, the criterion values derived about respective alternatives all agree, and the overall judgment values of each alternative acquired from respective criterion values also all agree.

Such a method as the weights of other criteria are concurrently derived from the weights of respective criteria and derivation is repeated till the mean values converge is called concurrent convergence. Converging efficiency of the concurrent convergence is very high, and previous data and latest data converge in several repetitions even though they considerably disagree each other.

A case including a sole dominant alternative can be covered by this model as a particular case ($P = 0$) of the concurrent convergence because derived values of criteria agree from the beginning.

5. Example of Concurrent Convergence Calculation

The additional data processing method in the concurrent convergence will be explained in two simple cases. It is supposed that additional data of which $C2$ is a dominant alternative is given and two dominant alternatives $C1$ and $C2$ are brought about ($C3$ is a dependent alternative).

5.1 Case of difference only in criterion values

Explanation will be made on the case given in Table 19 at first.

Table 19. Additional data processed by the dominant alternative method C_{2*}

		Criteria		
Dominant alternative	$C2^*$	$B1$ (0.8)	$B2$ (0.2)	E Overall
Evaluation	$C1$	1/2	2	0.8
	$C2$	1	1	1
	$C3$	3/2	1/3	1.267

Differences from Table 9 in evaluation is only the weights of criteria about $C2$ ($k = 2$). Figure 1 shows the converging process in the non-weighted concurrent convergence with deriving sources of $C1$ and $C2$.

C1*dominant (k=1)			C2*dominant (k=2)			C3**dependent of C1*, C2*		
Criterion	B1(C1)	B2(C1)	Criterion	B1(C2)	B2(C2)	Criterion	B1(C3)	B2(C3)
Derived source k1	0.4000	0.6000	Derived source k2	0.8000	0.2000			
			k1 Deriving source	0.7273	0.2727	k1 Deriving source	0.9231	0.0769
k2 Deriving source	0.5000	0.5000				k2 Deriving source	0.9474	0.0526
Average	0.4500	0.5500	Average	0.763	0.2364	Average	0.9352	0.0648
			k1 Deriving source	0.7660	0.2340	k1 Deriving source	0.9364	0.0636
k2 Deriving source	0.4468	0.5532				k2 Deriving source	0.9356	0.0644
Average	0.4484	0.5516	Average	0.7648	0.2352	Average	0.9360	0.0640
			k1 Deriving source	0.7648	0.2352	k1 Deriving source	0.9360	0.0640
k2 Deriving source	0.4484	0.5516				k2 Deriving source	0.9360	0.0640
Average	0.4484	0.5516	Average	0.7648	0.2352	Average	0.9360	0.0640
	⋮			⋮			⋮	
Average	0.4484	0.5516	Average	0.7648	0.2352	Average	0.9360	0.0640

Figure 1. Processing the dominant alternative (The non-weighted concurrent convergence)

The weights of criteria about $C1$ and $C3$, derived from the weights of criteria about $C2$ in Table 19 in the method outlined above (at first derivation, $P = 1$) are:

$$B1(C1)^{(1)}_{(2)} : B2(C1)^{(1)}_{(2)} = 0.5000 : 0.5000$$
$$B1(C3)^{(1)}_{(2)} : B2(C3)^{(1)}_{(2)} = 0.9474 : 0.0526.$$

The above values are different from the following values acquired by the previous survey (the dominant alternative is $C1$ and $k = 1$).

$$B1(C1^*)_{(1)} : B2(C1^*)_{(1)} = 0.4000 : 0.6000$$
$$B1(C3)_{(1)} : B2(C3)_{(1)} = 0.9231 : 0.0769.$$

The weights are averaged about $C1, C2$, and $C3$ respectively. (First averaging)

$$B1(C1)^{(1)} = (0.4000_{(1)} + 0.5000_{(2)})/2 = 0.4500$$
$$B2(C1)^{(1)} = (0.6000_{(1)} + 0.5000_{(2)})/2 = 0.5500$$
$$B1(C2)^{(1)} = (0.7273_{(1)} + 0.8000_{(2)})/2 = 0.7636$$
$$B2(C2)^{(1)} = (0.2727_{(1)} + 0.2000_{(2)})/2 = 0.2364$$
$$B1(C3)^{(1)} = (0.9231_{(1)} + 0.9474_{(2)})/2 = 0.9352$$
$$B2(C3)^{(1)} = (0.0769_{(1)} + 0.0526_{(2)})/2 = 0.0648.$$

When the weights of $B1$ and $B2$ about $C1, C2$, and $C3$ are derived (2nd derivation, $P = 2$) from the values of $C1$ and $C2$ as a deriving source, the derived values converge greatly but do not agree yet. Therefore, the second averaging of respective derived values is conducted to carry out the third derivation.

$$B1(C1)^{(2)} = (0.4500_{(1)} + 0.4468_{(2)})/2 = 0.4484$$
$$B2(C1)^{(2)} = (0.5500_{(1)} + 0.5532_{(2)})/2 = 0.5516$$
$$B1(C2)^{(2)} = (0.7636_{(1)} + 0.7660_{(2)})/2 = 0.7648$$
$$B2(C2)^{(2)} = (0.2363_{(1)} + 0.2340_{(2)})/2 = 0.2352$$
$$B1(C3)^{(2)} = (0.9364_{(1)} + 0.9356_{(2)})/2 = 0.9360$$
$$B2(C3)^{(2)} = (0.0636_{(1)} + 0.0644_{(2)})/2 = 0.0640.$$

Disagreements in the results of the 3rd derivation are:

$$B1(C3)^{(3)}_{(1)} : B2(C3)^{(3)}_{(1)} = 0.936031 : 0.063969$$
$$B1(C3)^{(3)}_{(2)} : B2(C3)^{(3)}_{(2)} = 0.936030 : 0.063970.$$

Disagreements are all less than 0.00001. In a few succeeding processes, the values of the recurrence formula converges completely and the following criterion values making all derived values agree are acquired.

$$\hat{B}1(C1) = 0.4484 \qquad \hat{B}2(C1) = 0.5516$$
$$\hat{B}1(C2) = 0.7648 \qquad \hat{B}2(C2) = 0.2352$$
$$\hat{B}1(C3) = 0.9360 \qquad \hat{B}2(C3) = 0.0640.$$

Using the weights, the overall judgment values of alternatives are acquired about $C1, C2$, and $C3$ respectively.

$$C1 : C2 : C3 = 1.000 : 1.173 : 1.437$$
$$C1 : C2 : C3 = 0.853 : 1.000 : 1.226$$
$$C1 : C2 : C3 = 0.696 : 0.816 : 1.000.$$

Normalizing the above values so that the total shall be 1, the following values are acquired in relation to any alternative,

$$C1 : C2 : C3 = 0.277 : 0.325 : 0.398 \qquad (5.1)$$

proving that the results of convergence retains the properties of the dominant alternatives. (The properties are not obtained by using the midway values of convergence.) The overall judgments value acquired by the previous survey, $C1 : C2 : C3 = 0.294 : 0.324 : 0.382$ are hanged to the above values by the additional data.

5.2 Case of difference also in alternative judgment values

Being explained is a processing method in such a case as data including different alternative judgment values are added. On this occasion, both Step 1 and Step 2 are adopted.

Step 1)

The criteria values as well as the alternative judgment values shown in Table 20 are different from those given in Table 9. At first, criterion values about the dependent alternative will be calculated based on the criterion values about the dominant alternatives.

Table 20. Dependent alternative judgment values based on the dominant alternative $C2^*_{(2)}$

	Criteria			
Dominant alternative	$C2^*$	$B1$ (0.8)	$B2$ (0.2)	E Overall
	$C1$	1/4	3	0.8
Evaluation	$C2$	1	1	1
	$C3$	3/2	1/3	1.267

Table 21. The dependent alternative $C1^{**}_{(2)}$ based on the dominant alternative $C2^*_{(2)}$

	Criteria		
Dependent alternative	$C1$	$B1$	$B2$
	$C1$	1	1
Evaluation	$C2$	4	1/3
	$C3$	6	1/9

Table 22. The dependent alternative $C3^{**}_{(2)}$, based on the dominant alternative $C2^*_{(2)}$

	Criteria		
Dependent alternative	$C3$	$B1$	$B2$
	$C1$	1/6	9
Evaluation	$C2$	2/3	3
	$C3$	1	1

Since judgment values about the dominant alternative $C1$ ($k = 1$) have been derived as shown in Table 8 to 10, judgment values about the two dominant alternatives $C1^{**}_{(2)}$ and $C3^{**}_{(2)}$ will be derived. (Table 21 and 22)

Next, a pair of Table 8 and 21, Table 9 and 20, and Table 10 and 22 will be merged respectively. Composite values are acquired by geometrically averaging the values in the respective tables. As a result, derived values agree respectively providing the representative alternative judgment values shown in Table 23 to 25.

Table 23. Merging of Table 8 and Table 21

	Criteria		
Alternative	$C1$	$B1$	$B2$
	$C1$	1	1
Evaluation	$C2$	2.828	0.408
	$C3$	4.243	0.136

Step 2)

Criterion values about the respective alternatives $C1$ and $C2$ as a dominant alternative will be acquired by the merging process of the concurrent convergence to acquire the composite criterion values of criteria $B1$ and $B2$ about the representative judgment values of alternatives $C1, C2$, and $C3$ respectively.

Table 24. Merging of Table 9 and Table 20

Alternative	$C2$	Criteria $B1$	$B2$
	$C1$	0.354	2.449
Evaluation	$C2$	1	1
	$C3$	1.5	0.333

Table 25. Merging of Table 10 and Table 22

Alternative	$C3$	Criteria $B1$	$B2$
	$C1$	0.236	7.348
Evaluation	$C2$	0.667	3
	$C3$	1	1

Criterion values about dominant alternative $C1$ ($k = 1$)

$$B1(C1^*)_{(1)} : B2(C1^*)_{(1)} = 0.400 : 0.600.$$

Criterion values about dominant alternative $C2$ ($k = 2$)

$$B1(C2^*)_{(2)} : B2(C2^*)_{(2)} = 0.800 : 0.200.$$

As a result, the following composite criterion values are acquired.

$$\begin{array}{ll} \hat{B}1(C1) = 0.383 & \hat{B}2(C1) = 0.617 \\ \hat{B}1(C2) = 0.811 & \hat{B}2(C2) = 0.189 \\ \hat{B}1(C3) = 0.951 & \hat{B}2(C3) = 0.049. \end{array}$$

Overall judgment values of the alternatives with reference to each alternative are calculated with the weights of criteria given above. The results are;

$$\begin{array}{ll} \text{With reference to } C1 & C1 : C2 : C3 = 1.000 : 1.335 : 1.708 \\ \text{With reference to } C2 & C1 : C2 : C3 = 0.749 : 1.000 : 1.280 \\ \text{With reference to } C3 & C1 : C2 : C3 = 0.586 : 0.781 : 1.000. \end{array}$$

The above values are normalized so that the total shall be 1 and the results given below are acquired. The results with the additional data taken into consideration are identical in relation to any alternative.

$$C1 : C2 : C3 = 0.247 : 0.330 : 0.422. \tag{5.2}$$

The overall judgment values are changed further when the additional data includes changes in the judgment values of alternatives.

6. Conclusion

This paper proposes the dominant AHP different from the Saaty type AHP (the conventional AHP). The paper also explains that the viewpoint of the former is different from that of the latter. The proposed model is applicable to the two approaches (relative measurement and absolute measurement) proposed by Saaty, and is designated dominant relative measurement and dominant absolute measurement respectively. Further it is found that the AHP can be summarized as shown in Table 18 according to the two viewpoints and two approaches. The paper also proposes "concurrent convergence", an additional data processing method in accordance with the dominant AHP. The concurrent convergence can process additional data without loosing the nature of the dominant AHP. This method can be used each time data are added. Also the method can process many additional data at the same time. The weights adopted for assessing the additional data are dependent on the analytical viewpoint of the surveyor and analyzer.

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