

**NUMERICAL SOLUTION TECHNIQUE FOR JOINT
CHANCE-CONSTRAINED PROGRAMMING PROBLEM
—AN APPLICATION TO ELECTRIC POWER CAPACITY EXPANSION—**

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Abstract We consider a joint chance-constrained linear programming problem with random right hand side vector. The deterministic equivalent of the joint chance-constraint is already known in the case that the right hand side vector is statistically independent. But if the right hand side vector is correlative, it is difficult to derive the deterministic equivalent of the joint chance-constraint. We discuss two methods for calculating the joint chance-constraint. For the case of uncorrelated right hand side, we try a direct method different from the usual deterministic equivalent, for the correlative right hand side case, we apply numerical integration. In this paper a chance-constrained programming problem is developed for electric power capacity expansion, where the error of forecasted electricity demand is defined by a random variable. Finally we show that this problem can be solved numerically using the trust region method and numerical integration, and we present the results of our computational experiments.

1. Introduction

Charnes and Cooper [3] introduced probabilistic factors into a linear programming problem. Later, Prékopa [13] formulated a chance-constrained programming problem as a nonlinear programming problem as follows.

$$\min \quad h(x) \tag{1.1}$$

$$\text{subject to} \quad h_0(x) = P(g_1(x, \xi) \geq 0, \dots, g_r(x, \xi) \geq 0) \geq p_0, p_0 \in [0, 1] \tag{1.2}$$

$$\begin{aligned} h_1(x) &\geq p_1 \\ &\vdots \end{aligned} \tag{1.3}$$

$$h_m(x) \geq p_m$$

$x \in \mathbb{R}^n$ is a decision variable vector and $\xi \in \Omega \subset \mathbb{R}^q$ is a random vector. We assume that (Ω, \mathcal{F}, P) is a known probability space, where a family \mathcal{F} of events, i.e. subsets of Ω and the probability distribution P on \mathcal{F} , are given. h, h_0, h_1, \dots, h_m and g_1, \dots, g_r are defined on $\mathbb{R}^n, \mathbb{R}^{n+q}$ respectively. p_0 is a prescribed level of reliability. If g_i is a linear function of x and ξ , i.e. $g_i(x, \xi) = T_i x - \xi_i, i = 1, \dots, r$, then the chance-constraint (1.2) is indicated as follows. Here, T_i is the i th row vector of the $r \times n$ matrix T .

$$P(Tx \geq \xi) \geq p_0 \tag{1.4}$$

Next, we define a joint distribution function of ξ as $F(z) = P(\xi \leq z)$, and the chance-constraint becomes as follows.

$$F(Tx) \geq p_0 \tag{1.5}$$

Prékopa [11, 12] introduced a logarithmic concave probabilistic measure in relation to a set of feasible points of the chance-constraint.

Definition 1 Probability measure \mathcal{P} on \mathbb{R}^r is called a logarithmic concave probability measure, if

$$\mathcal{P}(\lambda A + (1 - \lambda)B) \geq \{\mathcal{P}(A)\}^\lambda \{\mathcal{P}(B)\}^{1-\lambda} \quad (1.6)$$

holds $\forall A, B \subseteq \mathbb{R}^r, \forall \lambda : 0 \leq \lambda \leq 1$, where $\lambda A + (1 - \lambda)B = \{\lambda x + (1 - \lambda)y : x \in A, y \in B\}$.

There are many types of probability distribution that have a logarithmic concave probability measure, for instance uniform, normal, Dirichlet, Wishart, Beta, and Gamma distribution. Prékopa [11, 12] proved the next theorem.

Theorem 1 If ξ has a logarithmic concave probability measure on \mathbb{R}^r and has a distribution function F , then the set of feasible points of $F(Tx) \geq p_0$ is convex.

The proof of this theorem is evident from the definition of a logarithmic concave probability measure and concavity of $\log F$. From this theorem if the objective function is convex and the functions $h_i(x), i = 1, \dots, m$ of constraints (1.3) are concave, the chance-constrained programming results in a convex programming and can be handled relatively simply.

We consider a joint chance-constrained linear programming problem with random right hand side vector and apply it to an electric capacity expansion problem in section 3. Watanabe and Ellis [20] reviewed the joint chance-constrained programming problem. The deterministic equivalent of the joint chance-constraint is already known in the case that the right hand side vector is statistically independent. But if the right hand side vector is correlative, it is difficult to derive the deterministic equivalent of the joint chance-constraint. We discuss two methods for calculating the joint chance-constraint in section 3. For the case of uncorrelated right hand side, we try a direct method different from the usual deterministic equivalent, for the correlative right hand side case, we apply numerical integration.

2. Electric Power Capacity Expansion

In an electric power utility, many planning methods are used. For example, maintenance scheduling problems are dealt with as integer programming problems (Shiina and Kubo [19]). In this paper, we consider an application of chance-constrained programming to an electric power capacity expansion problem. This planning aims to maintain stable electric power supply, and manage and develop electric power plants economically. We develop a new mathematical programming model for this problem.

A linear programming (LP) model is often employed for electric power supply planning. In the usual LP model, electric demand is given as deterministic known data from a discretized load duration curve which is a rearrangement of loads in a monotonic decreasing order for 8,760 hours in a year. A reserve capacity is held for unforeseen situations, such as breakdown of power plants or fluctuation of electric demand. Oyama [10] developed a method to measure marginal costs.

Anderson [1] and Sasson [14] reviewed LP models with constraints of demand fulfillment and capacity of plants. The objective function is minimization of the total cost of operation and plant construction. In these formulations the ratio of the reserve capacity of peak time is fixed. We think that probabilistic randomness in electric demand is not reflected sufficiently. Therefore we develop a stochastic programming model for electric power capacity expansion planning that overcomes these disadvantages of former models.

Stochastic programming (Kall and Wallace [8]) solves a decision problem in which some parameters in an objective function or constraints include uncertainty. Uncertainty is characterized by defining some parameters of the model as random variables. Ishii [6] surveyed

a stochastic linear programming and a modeling methodology, and classified solution techniques. Murphy et al. [9] and Sherali et al. [16] applied a recourse model to an electric utility capacity expansion planning problem. But with this approach a decision maker cannot set up a reliability level. That is we cannot estimate the reliability of electric power supply. So we consider that the recourse model is unsuitable especially when a decision maker wishes to set a reliability level. For an electric power system planning Scherer and Joe [15] formulated an integer programming model. In their model loss-of-load probability (LOLP) is used to indicate reliability.

In the next section we apply chance-constrained programming to electric power supply planning, which allows sufficient probability of constraints to be estimated. In our electric power supply planning model a decision maker is able to set up the reliability level of electric supply, and we can derive the solution that minimizes the total supply cost for a certain level of reliability.

3. Chance-Constrained Programming Model for Electric Power Capacity Expansion Planning

3.1. Introduction of approximate daily load curve and workable supply capacity

We consider that fixing the rate of reserve capacity at the peak time is not flexible for economical and reliable electric power supply because in reality electric demand might grow continuously against the scenario. Moreover, holding excessive reserve capacity is expensive for the electric power industry.

So we propose a new stochastic electric power supply planning model that overcomes these problems, and introduce an approximate daily load curve and a workable supply capacity.

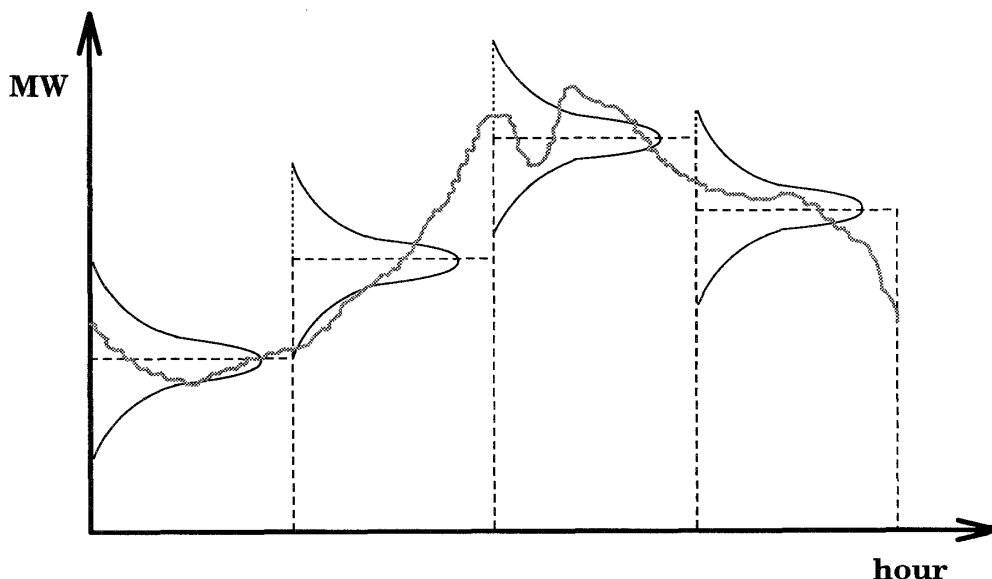


Figure 1: Approximate Daily Load Curve

Approximate Daily Load Curve We divide one day into several time zones and define the set of electric demand in each time zone as a multivariate random vector.

The reason we introduce the approximate load curve is that we can see the times when the different plants are started up, loaded, unloaded, and shut down. Ideally, we wish to

partition one day hourly or more finely, but increasing the number of partitions makes the computation more difficult. From the load duration curve we can find the total operating time of each plant for the period represented by the curve. We describe in a later section how to define a cumulative joint distribution function of electric demand.

Workable Supply Capacity A workable supply capacity is composed of electric load and a reserve capacity. In our model a workable supply capacity is provided for any unforeseen increase of electric demand.

In the usual model the total plant capacity is decided by adding the reserve capacity to the peak time load. On the other hand, in our model the workable supply capacity is decided stochastically by the joint chance-constraints. By subtracting electric load from the workable supply capacity we get the reserve capacity. We consider that the plants for which the workable supply capacity equals zero are not operating, that is, they might be undergoing periodic inspection or repair.

3.2. Formulation

We apply chance-constrained programming to electric power supply planning. First we define the notations.

Table 1: Sets of Indices

Sets of Indices	Meaning
I	Set of new plants
J	Set of existing plants
K	Set of planning period
S	Set of seasons
T	Set of time zone in approximate load curve
M	Set of fuels

Table 2: Variables of the Model

Variable	Indices	Meaning
z_{jkst}	$j \in J, k \in K, s \in S, t \in T$	Electric Energy Supplied by Existing Plant (MWh)
y_{ikst}	$i \in I, k \in K, s \in S, t \in T$	Electric Energy Supplied by New Plant (MWh)
w_{jkst}	$j \in J, k \in K, s \in S, t \in T$	Workable Supply Capacity of Existing Plant (MW)
v_{ikst}	$i \in I, k \in K, s \in S, t \in T$	Workable Supply Capacity of New Plant (MW)
x_{ik}	$i \in I, k \in K$	Capacity of New Plant (MW)
g_{jkms}	$j \in J, k \in K, m \in M, s \in S$	Fuel Consumption of Existing Plant (MJ)
f_{ikms}	$i \in I, k \in K, m \in M, s \in S$	Fuel Consumption of New Plant (MJ)

Constraints are described as follows.

(a) Workable Supply Capacity and Level of Reliability

$$\sum_{i \in I} d_t v_{ikst} + \sum_{j \in J} d_t w_{jkst} \geq P_{kst} + \varepsilon_{kst} \quad k \in K, s \in S, t \in T \tag{3.1}$$

Table 3: Parameters of the Model

Parameter	Indices	Meaning
C_{jk}	$j \in J, k \in K$	Capacity of Existing Plant (MW)
\dot{K}_j	$j \in J$	Fuel Consumption Rate of Existing Plant (MJ/MWh)
K_i	$i \in I$	Fuel Consumption Rate of New Plant (MJ/MWh)
Γ_{kms}	$k \in K, m \in M, s \in S$	Upper Bound of Fuel Consumption (MJ)
A_{ik}	$i \in I, k \in K$	Investment Cost of New Plant (yen/MW)
B_{km}	$k \in K, m \in M$	Fuel Cost (yen/MJ)
V_{ik}	$i \in I, k \in K$	Operating Cost of New Plant (yen/MWh)
\dot{V}_{jk}	$j \in J, k \in K$	Operating Cost of Existing Plant (yen/MWh)
U_{ik}	$i \in I, k \in K$	Workable Supply Capability Cost of New Plant (yen/MW)
\dot{U}_{jk}	$j \in J, k \in K$	Workable Supply Capability Cost of Existing Plant (yen/MW)
W_{ik}	$i \in I, k \in K$	Cost per Capacity of New Plant (yen/MW)
\dot{W}_{jk}	$j \in J, k \in K$	Cost per Capacity of Existing Plant (yen/MW)
d_t	$t \in T$	Duration of Time Zone (h)
p_{kst}	$k \in K, s \in S, t \in T$	Mean Value of Electric Demand (MWh)
P_{kst}	$k \in K, s \in S, t \in T$	Electric Demand (Estimated Value) (MWh)
σ_{kst}	$k \in K, s \in S, t \in T$	Standard Deviation of Error of Estimated Demand
α_{ks}	$k \in K, s \in S$	Level of Reliability

Workable supply capacities are greater than estimated electric demand plus error of the estimation. We define the error ε_{kst} as a random variable, and apply the joint-chance constraint. Let $(\varepsilon_{ks1}, \dots, \varepsilon_{ks|T|})$ be a multivariate random variable vector. We want the next $|T|$ constraints to be satisfied simultaneously with prescribed probability α_{ks} .

$$\sum_{i \in I} d_1 v_{iks1} + \sum_{j \in J} d_1 w_{jks1} \geq P_{ks1} + \varepsilon_{ks1}$$

$$\vdots$$

$$(3.2)$$

$$\sum_{i \in I} d_{|T|} v_{iks|T|} + \sum_{j \in J} d_{|T|} w_{jks|T|} \geq P_{ks|T|} + \varepsilon_{ks|T|}$$

We regard α_{ks} as a daily reliability level in season s and period k .

$$P\left(\sum_{i \in I} d_1 v_{iks1} + \sum_{j \in J} d_1 w_{jks1} - P_{ks1} \geq \varepsilon_{ks1}, \dots, \sum_{i \in I} d_{|T|} v_{iks|T|} + \sum_{j \in J} d_{|T|} w_{jks|T|} - P_{ks|T|} \geq \varepsilon_{ks|T|}\right) \geq \alpha_{ks} \quad k \in K, s \in S$$

$$(3.3)$$

Let F_{ks} be a joint distribution function of $(\varepsilon_{ks1}, \dots, \varepsilon_{ks|T|})$ in period k and season s .

$$F_{ks}\left(\sum_{i \in I} d_t v_{ikst} + \sum_{j \in J} d_t w_{jkst} - P_{kst}, t \in T\right) \geq \alpha_{ks} \quad k \in K, s \in S$$

$$(3.4)$$

(b) Balance of Electric Demand and Supply

$$\sum_{i \in I} y_{ikst} + \sum_{j \in J} z_{jkst} = p_{kst} \quad k \in K, s \in S, t \in T$$

$$(3.5)$$

(c) Supplied Electric Energy, Workable Supply Capacity and Capacity

$$w_{jkst} \leq C_{jk} \quad j \in J, k \in K, s \in S, t \in T \quad (3.6)$$

$$v_{ikst} \leq x_{ik} \quad i \in I, k \in K, s \in S, t \in T \quad (3.7)$$

$$z_{jkst} \leq d_t w_{jkst} \quad j \in J, k \in K, s \in S, t \in T \quad (3.8)$$

$$y_{ikst} \leq d_t v_{ikst} \quad i \in I, k \in K, s \in S, t \in T \quad (3.9)$$

(d) Fuel Consumption

$$\sum_{t \in T} K_j z_{jkst} \leq \sum_{m \in M} g_{jkms} \quad j \in J, k \in K, s \in S \quad (3.10)$$

$$\sum_{t \in T} K_i y_{ikst} \leq \sum_{m \in M} f_{ikms} \quad i \in I, k \in K, s \in S \quad (3.11)$$

(e) Upper Bound of Fuel Consumption

$$\sum_{i \in I} f_{ikms} + \sum_{j \in J} g_{jkms} \leq \Gamma_{kms} \quad k \in K, m \in M, s \in S \quad (3.12)$$

Under the above constraints the problem is to minimize the objective function described below.

$$\begin{aligned} \min \quad & \left(\sum_{i \in I} \sum_{k \in K} A_{ik} x_{ik} \right. \\ & + \sum_{i \in I} \sum_{k \in K} \sum_{m \in M} \sum_{s \in S} B_{km} f_{ikms} + \sum_{j \in J} \sum_{k \in K} \sum_{m \in M} \sum_{s \in S} B_{km} g_{jkms} \\ & + \sum_{i \in I} \sum_{k \in K} \sum_{s \in S} \sum_{t \in T} V_{ik} y_{ikst} + \sum_{j \in J} \sum_{k \in K} \sum_{s \in S} \sum_{t \in T} \dot{V}_{jk} z_{jkst} \\ & + \sum_{i \in I} \sum_{k \in K} \sum_{s \in S} \sum_{t \in T} U_{ik} v_{ikst} + \sum_{j \in J} \sum_{k \in K} \sum_{s \in S} \sum_{t \in T} \dot{U}_{jk} w_{jkst} \\ & \left. + \sum_{i \in I} \sum_{k \in K} W_{ik} x_{ik} + \sum_{j \in J} \sum_{k \in K} \dot{W}_{jk} C_{jk} \right) \end{aligned} \quad (3.13)$$

Each row of the objective function corresponds to the cost of plant construction, fuel consumption, operation, reserve capacity, and capacity, respectively.

3.3. Probability distribution of electricity demand

In this section we show how to define the cumulative distribution function of the error of estimated demand. The Japan Electric Power Survey Committee [7] considers that a reserve capacity is necessary to handle long-term demand fluctuations such as the business cycle or short-term demand fluctuations such as changes of the weather. Their method of defining the distribution function of electricity demand is as follows.

- The deviation between the forecasted electricity demand value and the actual demand value in a daily operation is defined as a random variable.
- It is assumed that this random variable has a normal distribution with cumulative probability 0.99 at the point of 6% of maximum electricity demand based upon the forecasted results in 1964.

We define the error vector $(\varepsilon_{ks1}, \dots, \varepsilon_{ks|T|})$ to have a multivariate normal distribution with mean value $\mathbf{0}$. Thus, the set of solutions that satisfy the chance-constraint is convex from Theorem 1. But the computation of F_{ks} is very difficult as the cumulative distribution function of the normal distribution contains multivariate integration. If the error vector $(\varepsilon_{ks1}, \dots, \varepsilon_{ks|T|})$ is uncorrelated, the deterministic equivalent of the chance-constraint is known.

More directly, Shiina [17] used the approximate formula of the normal distribution in the case of uncorrelated right hand side. From the Williams-Yamauchi formula [22] the standard normal distribution function $\Phi(u)$ of $\mathcal{N}(0, 1^2)$ is approximated as follows.

$$\Phi(u) \approx \tilde{\Phi}(u) = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \exp\left(\frac{-2u^2}{\pi}\right)}, u \geq 0 \quad (3.14)$$

The absolute error of this formula is 3.2×10^{-3} . In Shiina [17] the error vector $(\varepsilon_{ks1}, \dots, \varepsilon_{ks|T|})$ is assumed to be uncorrelated, so $\varepsilon_{kst}, t \in T$ are assumed to be probabilistically independent. The distribution function F_{ks} in period k season s becomes the product of the distribution function F_{kst} of each time zone.

$$F_{ks} = \prod_{t \in T} F_{kst} \quad (3.15)$$

As $\frac{\varepsilon_{kst}}{\sigma_{kst}}$ has standard normal distribution $\mathcal{N}(0, 1^2)$ approximately, the chance-constraints in section 3.2 result in the following inequalities.

$$\prod_{t \in T} \tilde{\Phi} \left(\frac{\sum_{i \in I} d_t v_{ikst} + \sum_{j \in J} d_t w_{jkst} - P_{kst}}{\sigma_{kst}} \right) \geq \alpha_{ks} \quad k \in K, s \in S \quad (3.16)$$

But omitting the correlation in this way, we cannot grasp the relationship between the probability of sufficiency and electricity demand correctly. Moreover, when we apply a nonlinear programming technique (Bersekas [2] and Prékopa [13]), it is necessary to calculate a gradient vector and a Hessian matrix of a function. In this calculation, using differential calculus might cause cancellation.

We therefore introduce numerical integration into the calculation of the chance-constraints, and so can handle the case of the correlative right hand side vector. Drezner [5] calculated multivariate normal integration using a Gauss quadrature formula weighted by e^{-x^2} . The density function of random variables $\mathbf{x} = (x_1, \dots, x_m)^t \sim \mathcal{N}(\mathbf{0}, R)$ is multiple-integrated for each x_i over the interval from $-\infty$ to a_i . Let this definite integral be $\Phi_m(\mathbf{a}, R)$, where the mean value of \mathbf{x} is $\mathbf{0}$ and R is the correlation matrix of \mathbf{x} .

$$\Phi_m(\mathbf{a}, R) = \int_{-\infty}^{a_m} \dots \int_{-\infty}^{a_1} \frac{1}{(2\pi)^{\frac{m}{2}} (\det R)^{\frac{1}{2}}} \exp\left(-\frac{1}{2} \mathbf{x}^t R^{-1} \mathbf{x}\right) dx_1 \dots dx_m \quad (3.17)$$

Now adopting the Gauss quadrature formula, it is said empirically that errors might be small for the same number of sample points when $\mathbf{a} \leq \mathbf{0}$. So Drezner [5] used the following relation recursively, where R_{m-1}^i and R_m^{-i} are the correlation matrices of $(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_m)^t$ and $(x_1, \dots, x_{i-1}, -x_i, x_{i+1}, \dots, x_m)^t$, respectively.

$$\begin{aligned} \Phi_m(\mathbf{a}, R) &= \Phi_{m-1}((a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_m)^t, R_{m-1}^i) \\ &\quad - \Phi_m((a_1, \dots, a_{i-1}, -a_i, a_{i+1}, \dots, a_m)^t, R_m^{-i}) \end{aligned} \quad (3.18)$$

The following transformations are then employed, where r_{ij} is the (i, j) component of R^{-1} .

$$y_i = (a_i - x_i) \sqrt{\frac{r_{ii}}{2}} \quad (3.19)$$

Now all integral intervals of y_i are $\infty \rightarrow 0$, so the resulting integral calculus becomes as follows in the case of $m = 4$. The integral is calculated by the extended formula of the Gauss quadrature formula on $[0, \infty]^4$. A_{i_1}, \dots, A_{i_4} are weights of $e^{-y^t y}$.

$$\begin{aligned} \Phi_4(\mathbf{a}, R) &= \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \left\{ \frac{1}{(2\pi)^{\frac{4}{2}} (\det R)^{\frac{1}{2}}} \prod_{i=1}^4 \left(-\frac{1}{\sqrt{\frac{r_{ii}}{2}}} \right) \exp(-y^t y) \right. \\ &\quad \left. \exp\left(y^t y - \sum_{i=1}^4 \sum_{j=1}^4 \left(a_i \sqrt{\frac{r_{ii}}{2}} - y_i \right) \frac{r_{ij}}{r_{ii} r_{jj}} \left(a_j \sqrt{\frac{r_{jj}}{2}} - y_j \right) \right) \right\} dy_1 dy_2 dy_3 dy_4 \\ &\approx \frac{1}{(2\pi)^2 (\det R)^{\frac{1}{2}}} \prod_{i=1}^4 \left(-\frac{1}{\sqrt{\frac{r_{ii}}{2}}} \right) \sum_{i_1=1}^k \sum_{i_2=1}^k \sum_{i_3=1}^k \sum_{i_4=1}^k A_{i_1} A_{i_2} A_{i_3} A_{i_4} g(y_{i_1}, y_{i_2}, y_{i_3}, y_{i_4}) \end{aligned} \quad (3.20)$$

where $g(y) = \exp\left(y^t y - \sum_{i=1}^4 \sum_{j=1}^4 \left(a_i \sqrt{\frac{r_{ii}}{2}} - y_i \right) \frac{r_{ij}}{r_{ii} r_{jj}} \left(a_j \sqrt{\frac{r_{jj}}{2}} - y_j \right) \right)$,

and $y_{i_1}, y_{i_2}, y_{i_3}, y_{i_4}$ are particular values of the random variables y_1, y_2, y_3, y_4 .

To calculate the probability of (3.3) we substitute $\frac{\sum_{i \in I} d_j v_{iks_j} + \sum_{j \in J} d_j w_{jks_j} - P_{ks_j}}{\sigma_{ks_j}}$ into a_j of (3.20).

3.4. Numerical experiments

We use the following electricity demand data based on a record of an electric power company. It is assumed that the estimated value of electric demand is equal to the mean value of the demand. We omit description of the data of the covariance here.

Table 4: Electricity Demand—mean values

Season	Jan, Feb, Mar	Apr, May, Jun	Jul, Aug, Sep	Oct, Nov, Dec
0000-0600	10779.6	10221.1	10450.6	10896.6
0600-1200	14337.1	13089.8	14633.0	14208.0
1200-1800	15132.1	14177.4	16816.0	15361.6
1800-2400	13107.4	12179.3	13643.4	13266.4

(MW × 6h)

Table 5: Standard Deviations of Error of Estimated Demand

Season	Jan, Feb, Mar	Apr, May, Jun	Jul, Aug, Sep	Oct, Nov, Dec
0000-0600	969.9	592.4	733.5	796.0
0600-1200	2158.8	1537.5	1910.1	1861.4
1200-1800	2288.0	1847.4	2421.8	2104.2
1800-2400	1333.4	928.7	1274.3	1249.1

We assume there are 18 power generating plants and 7 new plants to be built. The size of this stochastic electric power planning problem amounts to 825 variables, 34 equality constraints, and 1,204 inequality constraints. In Shiina[17, 18] SQP (Sequential Quadratic Programming) is introduced for nonlinear optimization. But for large-scale sparse matrices, approximating hessian by the quasi-Newton method is not efficient (Yamashita[21]), so we apply the Trust Region Method (package NUOPT in Yamashita [21]) to the problem.

Based on the above conditions we report the results of numerical experiments on HI-TACHI9000 V725/100. In numerical integration we vary the number of sample points in each dimension from 2 to 10 until the difference of the calculated value reaches 1×10^{-4} . When we set the level of reliability as 0.95 and 0.99, the values of numerical integration converge within 10 sample points in each dimension.

We indicate the ratio of the optimal function value to the one in the case without chance-constraints. We also indicate the results in the case of omitting correlation.

Table 6: Ratio of Optimal Value(considering correlation)

Level of Reliability	Ratio of Optimal Value	Iterations	Residual of KKT Condition	Computational Time (sec)
Without Chance-Constraints	1	21	2.283×10^{-11}	73.95
0.1	1.0000	17	3.493×10^{-9}	59.36
0.3	1.0000	18	9.103×10^{-9}	60.82
0.5	1.0099	45	2.682×10^{-9}	137.2
0.7	1.0672	26	6.346×10^{-9}	88.20
0.9	1.1755	32	8.768×10^{-9}	113.5
0.95	1.2328	21	9.753×10^{-9}	73.12
0.99	1.3500	29	7.336×10^{-10}	104.3

(We let the optimal value in the case without chance-constraints be 1.)

Table 7: Ratio of Optimal Value(omitting correlation)

Level of Reliability	Ratio of Optimal Value	Iterations	Residual of KKT Condition	Computational Time (sec)
Without Chance-Constraints	1	16	3.065×10^{-9}	43.99
0.1	1.0008	19	2.182×10^{-9}	54.39
0.3	1.0393	27	8.543×10^{-11}	94.24
0.5	1.0826	26	1.782×10^{-9}	89.22
0.7	1.1401	25	1.116×10^{-9}	87.37
0.9	1.2361	27	5.092×10^{-10}	87.22
0.95	1.2876	25	6.946×10^{-10}	81.76
0.99	1.3960	29	2.082×10^{-11}	97.12

(We let the optimal value in the case without chance-constraints be 1.)

In the case without chance-constraints, workable supply capacities in each time zone remained equal to the average value of electricity demand, so power plants are operated with no reserve capacity. This operation corresponds to the solution in the case of omitting correlation with the level of reliability $0.0625 = 0.5^4$. The ratio in the case of omitting correlation is larger than that in the case of considering correlation with the equal reliability level. This is because the correlation coefficients between electric demand are close to 1. Therefore it is essential to consider the correlation for reliable and economical operation. If we set the level of reliability as $0.1 \sim 0.7$, there is a possibility of a shortage of electricity supply, so in a real plan we must set the level of reliability close to 1.

The optimal value of the objective function tends to diverge to infinity as the level of reliability approaches 1. We express the relation between the reliability level and the ratio of the optimal value on a bi-logarithmic scale.

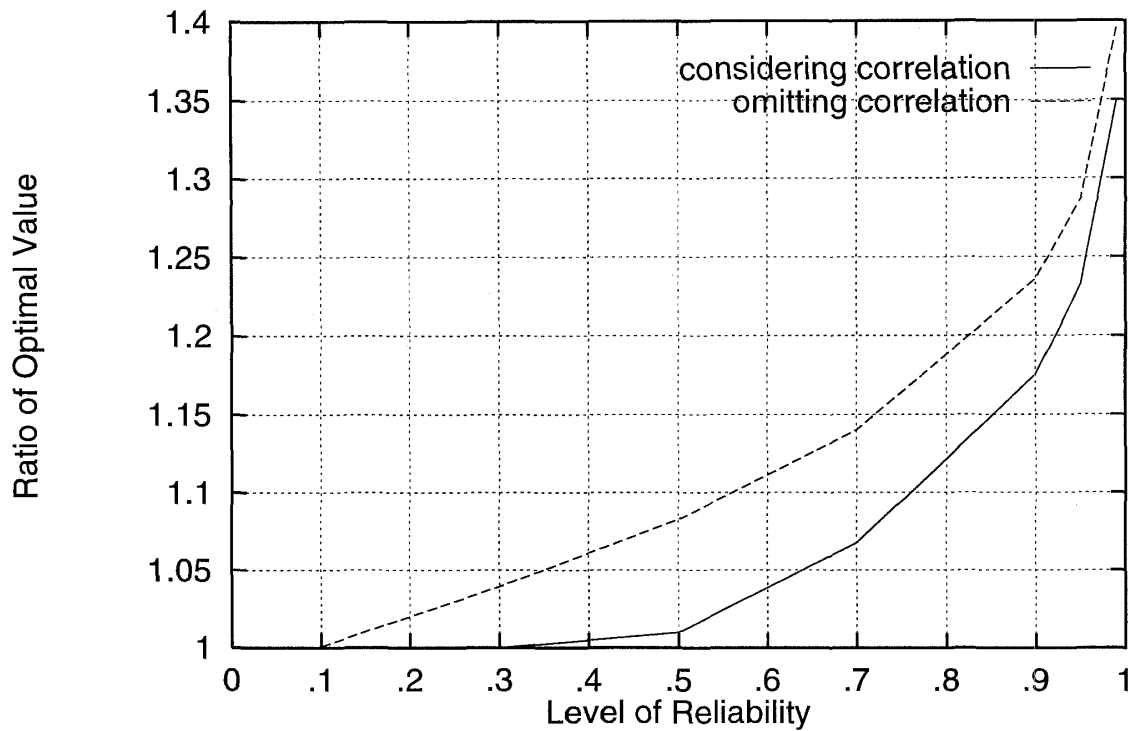


Figure 2: Ratio of Optimal Value

Figure 3 shows logarithmic linearity and we can estimate the tradeoff of reliability and economical efficiency of electric supply. From figure 3 we can see that when the level of reliability rises from 0.9 to 0.99, the ratio of the optimal value rises from 1.18 to 1.35 and consequently the optimal value increases by $\frac{1.35}{1.18} \approx 1.15$ times.

Table 8 shows workable supply capacities for the level of reliability 0.5 ~ 0.99. Although we can calculate the utility rate of every facility and in every time zone, we omit them. This table illustrates the tendency of the workable supply capacity to grow remarkably as the level of reliability increases. We consider it is a merit of our model that the workable supply capacity can be decided in each time zone, which gives a guide to hold an operating reserve or a cold reserve, because it is not enough for supply reliability to consider supply shortages only during peak times. In the method we have developed, first a decision maker sets up a level of reliability, then an optimal workable supply capacity is obtained.

4. Concluding Remarks

A stochastic programming model has been investigated theoretically but it has not been applied to actual problems in many fields. In this paper we demonstrated that a stochastic programming model is one of the encouraging methods that can be solved analytically without the Monte-Carlo simulation. A chance-constrained programming model is expected to be applied to many problems in the electric power industry. Though it is usual to formulate an electric power supply planning problem as a static mathematical programming problem, the supply and demand of electric power include fluctuating factors especially at peak times in summer. In this paper we attempted to create an electric power supply planning model under such uncertainty.

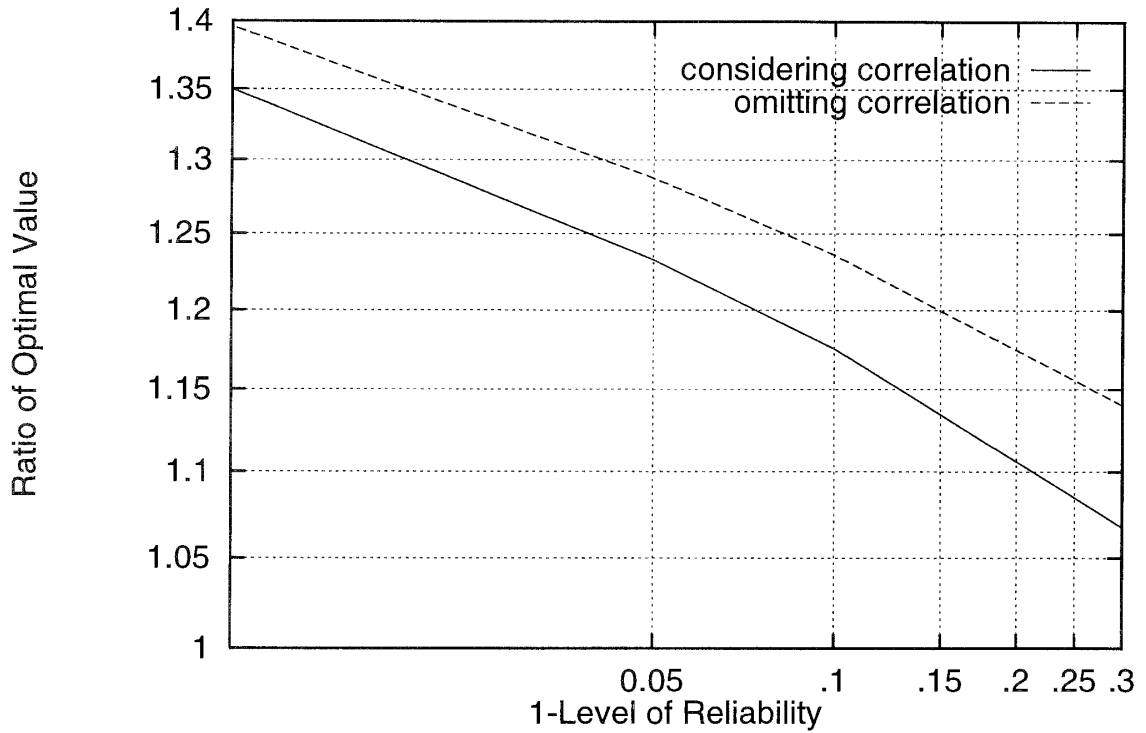


Figure 3: Ratio of Optimal Value(bi-logarithmic scale)

Table 8: Optimal Workable Supply Capacities

Level of Reliability	Without Chance-Constraints	0.5	0.7	0.9	0.95	0.99
Season	Time Zone					
Jan, Feb, Mar	0000-0600	10780	12102	12629	12990	13670
	0600-1200	14337	14536	15752	17298	19613
	1200-1800	15132	15329	16465	18261	20719
	1800-2400	13107	13367	14128	15167	16481
Apr, May, Jun	0000-0600	10221	11193	11443	11875	12395
	0600-1200	13090	13284	14087	15343	16852
	1200-1800	14177	14360	15325	16675	18651
	1800-2400	12179	12678	12990	13611	14591
Jul, Aug, Sep	0000-0600	10451	11498	12017	12429	12990
	0600-1200	14633	14633	15811	17268	19358
	1200-1800	16816	16816	17971	20032	22642
	1800-2400	13643	13744	14574	15665	16913
Oct, Nov, Dec	0000-0600	10897	11991	12411	12941	13272
	0600-1200	14208	14421	15440	16763	18758
	1200-1800	15362	15572	16588	18221	20486
	1800-2400	13266	13525	14219	15186	16421

(MW × 6h)

A remaining problem to be solved is how to introduce a random variable to this model except for electric demand. As various costs are based on forecasted values, we can derive a more detailed model by randomizing these parameters. Further, when we increase the number of time zones, we should examine the quasi-Monte-Carlo method (Deák[4]) for numerical integration. Another problem is how to establish a level of reliability for real supply planning. Also, as we adopt a multivariate normal distribution, it is necessary to analyze the real demand distribution in detail.

Because of maintenance schedules (Shiina and Kubo[19]), the table of available plant capacities will differ between weeks. Thus, different approximate load curves for each week and for weekdays or weekends are required when applying the model of this study to practical problems.

It is also necessary to develop a more efficient algorithm for nonlinear programming and numerical integration.

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