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PARTY POWER IN THE HOUSE OF COUNCILORS IN JAPAN: AN APPLICATION OF THE NONSYMMETRIC SHAPLEY-OWEN INDEX*

Rie Ono
Toyama University

Shigeo Muto†
Tokyo Metropolitan University

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Abstract We evaluate the party power distribution in the Japanese House of Councilors using the nonsymmetric Shapley-Owen index. This index, a generalization of the well-known symmetric Shapley-Shubik power index, takes account of the parties' ideological differences. We first determine the parties' ideological positions by performing the factor analysis of the votes observed during the period of 1989–1992. The data shows that the distribution of bills submitted to the House was very strongly biased; this contradicts random appearance of bills that the index basically assumes. Thus we evaluate the party power without supposing the random appearance of bills; we use just the observed data instead. In our analysis, the index thus calculated gives very convincing results.

1. Introduction

The nonsymmetric Shapley-Owen index is a generalization of the well-known (symmetric) Shapley-Shubik index. The Shapley-Shubik index, defined originally in Shapley and Shubik [11], has been widely used to measure voter's power in various voting systems. See Lucas [4]. The generalized nonsymmetric index, originated by Owen [5] and later developed by Shapley [10], has successfully introduced voters' nonsymmetry into the original symmetric index. The nonsymmetry is observed, in particular, in political voting systems in which voters have their own ideologies. Owen's idea is to place voters as well as issues in a Euclidean space, called an ideology profile space, according to their ideological characteristics; and then calculate each voter's power taking account of his/her position.

The Shapley-Shubik index has been proved useful to evaluate voters' power through its applications to various voting situations in the real world. Regretfully, however, there have been a few applications of its nonsymmetric generalization; those include in part the analyses of the U.S. Supreme Court by Frank and Shapley [3], the Israeli Knesset by Rapoport and Golan [8] and the U.S. Presidential Elections by Rabinowitz and Macdonald [7]. More applications are necessary to examine merits of the generalized index.

This paper has two purposes: the first is to present another application of the nonsymmetric Shapley-Owen index, and the other is to point out its merits and drawbacks revealed through the application. We apply the nonsymmetric Shapley-Owen index to the analysis of party power in the House of Councilors in Japan during the period between 1989 and 1992. In the general election of 1989, the Liberal Democratic Party (LDP) lost their dictatorial position in the House that they had held for more than thirty years. Our aim is to find parties' "true" power. It has

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often been said that small parties which were ideologically located between big parties had strong power in comparison with the numbers of their seats; for example, the Komeito (Komei) located between the LDP and the Social Democratic Party of Japan (SDPJ) had more power than the Japan Communist party (JCP), though they were similar to each other in size. We calculate the nonsymmetric Shapley-Owen indices of these parties. Then, taking into account merits and drawbacks of the generalized index revealed through the analysis, we propose a modification of it and reexamine each party power in terms of the modified index. It will be shown that the modification of the Shapley-Owen index may provide a justification of our usual understanding of power distribution of political parties in the House.

This paper is organized as follows. In section 2, the definition of the symmetric Shapley-Shubik index is briefly given, and then its nonsymmetric generalization due to Owen and Shapley is reviewed by the use of a simple example. In section 3, each party power in the House of Councilors in Japan during the period between 1989 and 1992 is measured in terms of the nonsymmetric Shapley-Owen index. Unfortunately, these power do not agree in the least with our usual understanding, because the assumption of the random appearance of bills required by Owen and Shapley is never met in our case. Then we calculate, in section 4, the index using just the data observed during the period, and show that the calculated index gives very convincing results. We conclude the paper in section 5 with remarks on future studies.

2. The Shapley-Shubik Symmetric Index and its Nonsymmetric Generalization

Let $N = \{1, 2, \cdots, n\}$ be the set of voters and let $v : 2^N \to R$ be a characteristic function, where 2^N is the set of all subsets $S \subseteq N$ and R is the set of real numbers. The game (N, v) is called a voting game, if the following conditions are satisfied:

1. $\forall S \subseteq N : v(S) \in \{0, 1\}$
2. $v(N) = 1$
3. $\forall S, T \subseteq N : (S \subset T \Rightarrow v(S) \leq v(T))$

The first condition implies that v assumes only the values 1 and 0. Sets S with $v(S) = 1$ are called winning coalitions. That is, if members in S agree on a proposed bill, it is approved regardless of the other voters' behavior. Sets S with $v(S) = 0$ are called losing coalitions. Denote the set of winning coalitions by $\mathcal{W}$ and the set of losing ones by $\mathcal{L}$. The grand coalition N is winning by condition 2. The last condition is called the monotonicity; sets including a winning coalition are winning, while sets included in a losing coalition are losing. In addition, we assume the game is proper, that is,

$$\forall S \subset N : v(S) + v(N \setminus S) \leq 1,$$

where $N \setminus S$ is the complement of S with respect to N. This property excludes the case where two or more disjoint coalitions are winning.

We also define a special class of voting games called weighted majority games. Let $w_i > 0$ be the weight of voter $i \in N$, or the number of his votes. All the members in N vote either yea or nay. The issue passes if the number of yea-votes is more than or equal to the quota q, where $\frac{1}{2}\sum_{i \in N} w_i < q \leq \sum_{i \in N} w_i$. The characteristic function v is given as

$$v(S) = \begin{cases} 1 & \text{if } \sum_{i \in S} w_i \geq q, \\ 0 & \text{if } \sum_{i \in S} w_i < q. \end{cases}$$

We denote this game by $[q; w_1, \cdots, w_n]$.

The Shapley-Shubik index can be derived from axioms similar to those for the Shapley value[1]. In the following, however, we will take another approach to define the index. Suppose voters join

[1] For detailed description of the axioms, see Dubey [2].

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a coalition one after another and eventually form the grand coalition. Then there exists a unique voter who joins and thereby turns a losing coalition into a winning one: this voter is called a pivot.

Definition 2.1 *Take an ordering of n voters and voter $i \in N$. Let S be the set of voters preceding i. Then voter i is called a pivot for the ordering if $v(S \cup \{i\}) = 1$ and $v(S) = 0$.*

Each of the $n!$ orderings of n voters has a unique pivot. The Shapley-Shubik index of a voter is the probability of his being a pivot when every ordering is equally probable.

Definition 2.2 *The Shapley-Shubik index of a voting game (N, v) is given as the n-vector $\varphi(v) = (\varphi_1(v), \cdots, \varphi_n(v))$ where*

$$\varphi_i(v) = \sum_{S \in \mathcal{L}, S \cup \{i\} \in \mathcal{W}} \frac{s!(n-s-1)!}{n!}, \quad i = 1, \cdots, n. \tag{2.1}$$

Here, s is the number of members in S.

The Shapley-Shubik index assumes that all orderings are formed with equal probability. But in political voting situations, some orderings could be more probable than the others. For instance, take three voters 1, 2 and 3, who are a liberalist, a centrist and a conservative, respectively. Since the extreme voters 1 and 3 are opposed to each other, ordering 132 is less likely to be formed than 123 or 321. Here ordering ijk implies that i is the first to join, j is the second and k is the last, i.e., the coalition is formed in the order of i, j, k.

To take the nonsymmetry of voters into account, Owen [5] introduced an ideology profile space. It is a multidimensional real space, and each dimension corresponds to a particular ideology; right versus left, conservative versus progressive, and so on. Every voter is placed in this space depending on his ideological position. Each proposed bill has ideologies as well, and is placed in the space. If a bill is proposed, voters whose ideological positions are close to the bill would enthusiastically support it. Owen thus supposed that voters form a coalition according to their Euclidean distances from the proposed bill. More precisely, for each bill, the closest voter joins first, the second-closest joins next, and so on, and the most distant voter joins last.Owen further supposed that bills appear at random in this ideology profile space.

If there is only one ideological axis, say a left-right axis, then the three voters' case above can be depicted in terms of a line as in figure 1: for each $i = 1, 2, 3$, x^i denotes voter i's position. The space is divided into four regions E_1, E_2, E_3 and E_4 by the midpoints of the line segments x^1x^2, x^1x^3 and x^2x^3. For any bill in region E_1, voter 1 is the closest, 2 is the next and 3 is the most distant; and thus the grand coalition is formed in the order of 123. Similarly in regions E_2, E_3, E_4, orderings are 213, 231, 321, respectively. It is to be noted that regions E_2 and E_3 (producing orderings 213 and 231, respectively) are bounded intervals; while regions E_1 and E_4 (producing orderings 123 and 321, respectively) are unbounded. This means that if bills (or issues) arise at random in the whole real line, orderings 231 and 213 appear only in a negligibly few occasions; and the other two orderings 123 and 321 appear with equal probability of 1/2. In a simple majority case, voter 2 is pivotal in both orderings; thus 2 has the whole power. In a unanimous case, on the other hand, 3 is pivotal in 123 and 1 is pivotal in 321; thus voters 1 and 3 have equal power of 1/2 and 2 is powerless.

More orderings may appear in a two-dimensional space. Figure 2 depicts the case with three voters: each x^i denotes voter i's position. Since there are three points, we have three perpendicular bisectors of each pair of points. In this figure, the line ℓ_{ij}-ℓ'_{ij} represents the perpendicular bisector of x^i and x^j, $i, j = 1, 2, 3, i \neq j$. For example, bills in the sector formed by half-lines $O\ell_{13}$ and $O\ell'_{12}$ produce ordering 312; because for any bill in the region voter 3 is the most enthusiastic supporter, voter 1 is the next, and 2 is the least enthusiastic. Assuming issues appear at random in the whole two-dimensional space, we obtain that the ordering 312 is produced with probability $\alpha/2\pi$ where α is the angle formed by two lines $O\ell_{13}$ and $O\ell'_{12}$. For each of the other regions, we can find, in a similar manner, which ordering is produced and how it is probable. These orderings are given in figure 2. Thus in the simple majority case where the second voter is always a pivot, the

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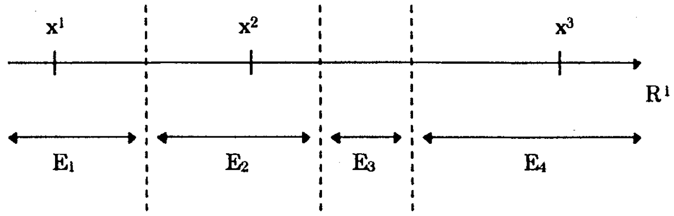

Figure 1: Three voters' locations in a unidimensional profile space

nonsymmetric Shapley-Shubik index is:

$$(\frac{\alpha}{\pi}, \frac{\beta}{\pi}, \frac{\gamma}{\pi}).$$

In the unanimous case, it is:

$$(\frac{\beta+\gamma}{2\pi}, \frac{\alpha+\gamma}{2\pi}, \frac{\alpha+\beta}{2\pi})$$

since the last voter is a pivot. In a space with more dimensions, the same results are obtained. For details, see Owen [5].

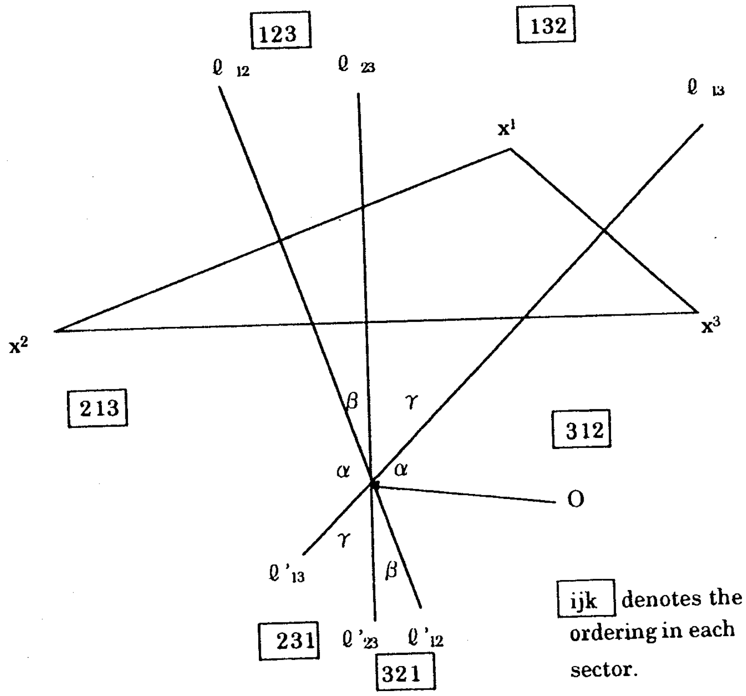

Figure 2: Three voters' locations in a two-dimensional space

Owen's method requires a highly dimensional space; $(n-1)$-dimensions are necessary for the case with n-voters; and thus all perpendicular bisectors (hyperplanes) meet at one point[2]. But this method is not practical for cases with many voters. Shapley [10] developed the following alternative method that enables us to find the nonsymmetric index using a space with less dimensions. See also Owen and Shapley [6]. Take an ideology profile space with any dimension. Similarly to Owen's method, voters are represented by points in the space. Issues are, however, represented not by points but by directed arrows (or vectors) passing through the origin of the space. Now take two

[2]It is possible to partition the space even if the number of dimensions is less than $(n-1)$. But Owen's way to calculate power by angles is no longer available.

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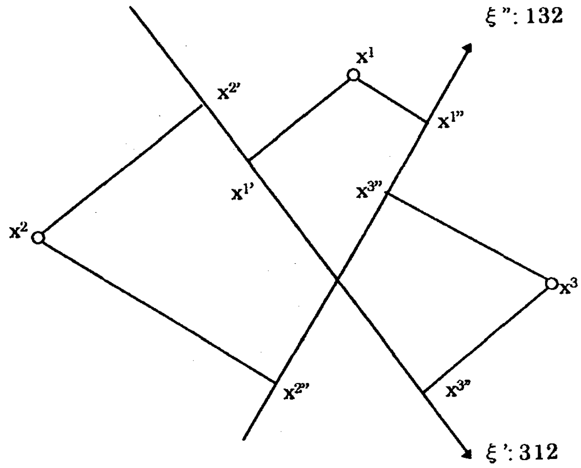

Figure 3: Shapley's construction of orderings

voters, i and j, and an issue ξ. Positions of the two voters are represented by points x^i and x^j. Drop perpendiculars from x^i and x^j to ξ; and denote their foots by x'^i and x'^j, respectively. Shapley assumed voter i prefers ξ more than j does, or i precedes j with respect to ξ, if $||\vec{Ox'^i}|| > ||\vec{Ox'^j}||$ where O is the origin of the space and $||\vec{Ox'^i}||$ (resp. $||\vec{Ox'^j}||$) is the signed distance along the arrow ξ between O and x'^i (resp. x'^j). The inequality means that voter i's projection to ξ is greater; and thus he/she supports the issue more enthusiastically. In figure 3, for issue ξ' we have $||\vec{Ox'^3}|| > ||\vec{Ox'^1}|| > ||\vec{Ox'^2}||$; thus ordering 312 is produced. Similarly we obtain 132 for issue ξ''. If we turn the arrow around the origin assuming issues arise at random, we find for each ordering the sector in which it is produced. For each ordering the proportion of the corresponding angle to 2π gives the probability in which the ordering appears. We thus obtain the nonsymmetric index by finding a pivotal voter in each ordering. It should be noted that (1) in Shapley's method we may put the origin in an arbitrary position. The same indices are obtained even if the origins are different; (2) Shapley's method gives the same result as Owen's nonsymmetric index if it is applied to the case with $(n-1)$-dimensional space (n is the number of voters); hence we may say that Shapley's device is a generalization of Owen's method. These two facts are shown without much difficulty. The nonsymmetric index found by Shapley's method is often called the Shapley-Owen index.

The example given in figure 4 may help us better understand Shapley's method. The configuration of voters 1,2 and 3 is the same as in figure 2, voter 4 is newly added. Owen's method is not applicable because the perpendicular bisectors of each pair of points no longer intersect at one point. Even in this case, Shapley's method works well. Let O be the origin where O is exactly the same as in figure 2, then all we have to do is to drop a perpendicular from the origin to each line segment connecting two of the four x^i's. In figure 4, in addition to the three lines already given in figure 2, three dotted lines are newly drawn. Thus the space is divided into twelve regions. For example, the dotted line ℓ_{24}-ℓ'_{24} is perpendicular to the line x^2x^4. Issues in the sector formed by half-lines $O\ell_{13}$ and $O\ell'_{24}$ produce ordering 3142; and those in the sector formed by $O\ell'_{24}$ and $O\ell'_{12}$ produce 3124. We can then find which ordering is produced in each of the other ten regions.

3. Power Distribution in the House of Councilors in Japan - A Direct Application of the Nonsymmetric Shapley-Owen Index

In this section, we will measure party influences on decisions in the House of Councilors in Japan during the period between 1989 and 1992.

In Japan, the Liberal Democratic Party (LDP) held a majority and thus a dictatorial position

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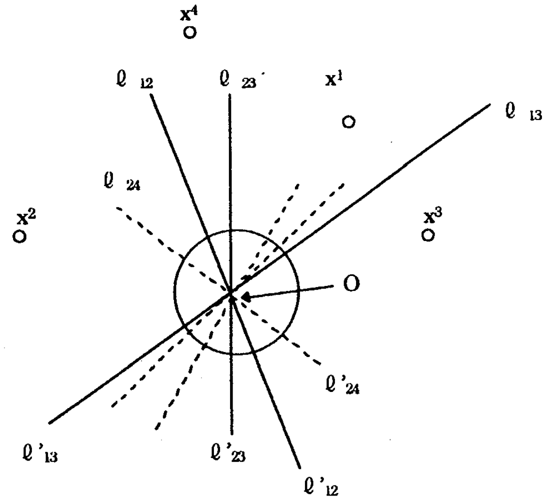

Figure 4: Three voters' locations in a two-dimensional space

both in the House of Representatives and in the House of Councilors for several decades up until 1989. Hence the LDP held the whole power and the other parties were completely powerless both in terms of the symmetric Shapley-Shubik index and of the nonsymmetric Shapley-Owen index. But in the election of July 1989, some opposition parties made remarkable progress, and the LDP lost their dictatorial power in the House of Councilors, though they were still the largest party. We analyze party power in the House of Councilors during the period 1989-1992[3] by using the data concerning parties "yea/nay" patterns in nonunanimous votes. During this period, the Komeito (Komei), the Democratic Socialist Party (DSP), and the Japan Communist Party (JCP) were similar to each other in size. It has often been claimed, however, the former two parties had much stronger power compared to the JCP, because they are ideologically located between the two big parties: the LDP and the Social Democratic Party of Japan (SDPJ). We will see if this fact is proved true in terms of the power index.

Table 1: The House of Councilors

	July, 1989
Liberal Democratic Party(LDP)	109
Social Democratic Party of Japan(SDPJ)	66
Komeito(Komei)	20
Japan Communist Party(JCP)	14
Democratic Socialist Party(DSP)	8
Rengo	12
Zeikin Party	3
Niin Club	2
Shaminren	0
Sports Peace Party	1
Others	2
Independents	15
	252

Sources: Asahi-Shimbun [1]

Parties' names and the numbers of their seats in July 1989 (just after the election) are given

[3]In the House of Councilors, half of the members are up for election every three years.

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in table 1. Though each member of the House has the right to vote of his own free will, usually members belonging to the same party vote jointly. Thus we formulate the voting system in the House as a weighted majority game in which voters and weights are parties and the numbers of their seats, respectively. In what follows, we choose the six largest parties as voters of the game for simplifying the analysis. Some members in the other mini parties and independents are included in one of the six parties if they always follow that party's decisions. Others are eliminated from the game; but the quota of the game is given as the number of "yea" votes needed to win even if all the eliminated members vote "nay". The weighted majority game, based on the data of 1989, is given as follows.

quota	*LDP*	*SDPJ*	*Komei*	*JCP*	*DSP*	*Rengo*
127;	109	74	21	14	10	12

The symmetric Shapley-Shubik index of this game is given as

LDP	*SDPJ*	*Komei*	*JCP*	*DSP*	*Rengo*
0.567	0.117	0.117	0.067	0.067	0.067.

It gives equal power to the JCP, the DSP, and the Rengo.

Following the studies of Frank and Shapley [3] and Rabinowitz and Macdonald [7], we construct an ideology profile space performing the factor analysis of the 133 nonunanimous votes that occurred in the House of Councilors during the period of 1989-1992[4]. Table 2 gives patterns of party yea/nay combinations and their frequencies. For example, type A is the pattern in which only the JCP voted nay; 85 votes out of 133 were of this type.

Table 2: Pattern of yea/nay combinations (1989–1992)

type	LDP	SDPJ	Komei	JCP	DSP	Rengo	number
A	Y	Y	Y	N	Y	Y	85
B	Y	N	N	N	N	N	18
C	Y	N	N	N	Y	N	9
D	Y	N	Y	N	Y	Y	6
E	N	Y	Y	Y	Y	Y	6
F	Y	N	Y	N	Y	N	5
G	Y	N	Y	Y	Y	Y	3
H	Y	Y	Y	N	Y	N	1

Sources: Sangiin Kaigiroku [9]

The factor analysis shows that the first main factor accounts for 70.9 per cent of the variance in the 133 votes; and the second factor accounts for 18.4 per cent. Together about 90 per cent of the variance is accounted for. Thus, it is enough to take only the first factor or at most the first two factors to estimate parties' positions.

First, take only the first common factor so that the profile space is unidimensional. See figure 5. Parties' positions in this figure are determined by the first column in table 3,the factor score matrix. The factor score matrix gives a placement of the parties. They are aligned in the order of LDP, DSP, Komei, Rengo, SDPJ and JCP (from right to left). The order agrees with what one would expect from journalistic accounts of their ideological characteristics. On this basis only, we would claim that the first factor is interpreted as an ideological left-right measure. It should be noted that five parties are positioned very close to each other, and the JCP is far away from them. This reflects the fact that issues of type A (85 out of 133) were supported by the five parties. In

[4]In this period, the numbers of the party seats changed slightly because of several by-elections. For simplicity, however, we only consider the game based on the data of 1989.

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Table 3: Parties' Positions

party	1st factor	2nd factor	3rd factor	4th factor	5th factor
LDP	0.656	1.852	-0.513	-0.199	-0.034
SDPJ	0.239	-0.874	-1.570	0.924	-0.167
Komei	0.370	-0.512	0.635	-0.709	-1.692
JCP	-2.020	-0.286	0.071	0.009	-0.013
DSP	0.451	-0.078	1.365	1.330	0.569
Rengo	0.303	-0.675	0.013	-1.355	1.336

Table 4: Issues' Directions

type	1st factor	2nd factor	3rd factor	4th factor	5th factor
A	0.989	-0.140	-0.035	-0.004	0.006
B	0.322	0.908	-0.252	-0.097	-0.016
C	0.429	0.687	0.330	0.438	0.208
D	0.690	0.228	0.581	-0.361	0.070
E	-0.322	-0.908	0.252	0.097	0.016
F	0.540	0.461	0.543	0.154	-0.422
G	-0.117	0.428	0.769	-0.452	0.082
H	0.665	0.150	-0.032	0.521	-0.512

fact, the first common factor loads very high on issues of type A as seen in the first column of table 4.

As for issues, the factor pattern matrix (table 4) gives their directions. Issues of type A,B,C,D,F and H are positive; and those of type E and G are negative. See the first column in table 4. If we suppose issues appear at random in the whole real line as required in the definition of the nonsymmetric Shapley-Owen index, only two orders, the order LDP←DSP←Komei←Rengo←SDPJ←JCP (for issues with a positive direction) and its reverse (for issues with a negative direction) arise with equal probability. The pivots are the Komei in the former and the DSP in the latter. Thus these two parties equally share the whole power; and all of the other parties are completely powerless.

The two-dimensional space with the first and the second common factors is shown in figure 6. The horizontal and the vertical axes correspond to the first and the second factors, respectively. Positions of the parties are determined by the first two columns of table 3. The horizontal axis is, as remarked above, interpreted as an ideological left-right measure. The vertical axis is not that easy to identify. On this dimension, the LDP has the highest score, the SDPJ has the lowest, and the others are in between; further the LDP is far distant from the other five parties. This reflects

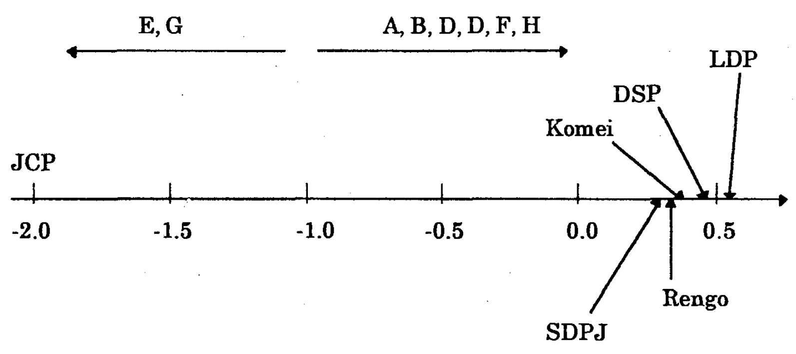

Figure 5: Unidimensional profile space

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the fact that issues of type B (18 out of 133) were supported only by the LDP. Thus the axis might be interpreted as a measure to distinguish a governing party from the Oppositions: the LDP, as a governing party, on occasions submits bills that they do not expect to get other parties' support.

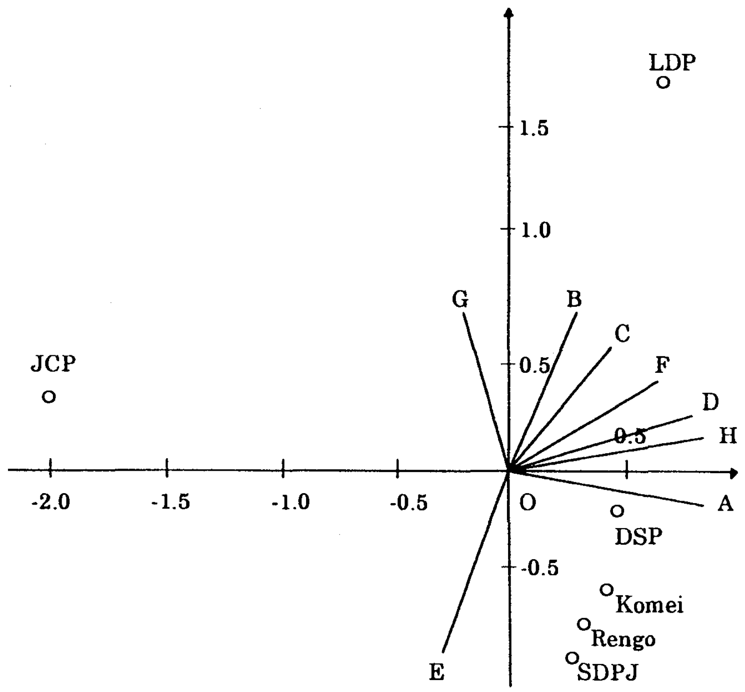

Figure 6: Two dimensional profile space

Directions of issues are determined by the first two columns of the factor pattern matrix, table 4. Eight rays emanating from the origin depict directions of eight types of issues: OA is the direction of issues of type A, etc. Supposing issues arise at random in the whole plane, we may calculate the nonsymmetric Shapley-Owen index.

The nonsymmetric Shapley-Owen indices in the cases of unidimensional and two-dimensional spaces are summarized as follows.

dimension	*LDP*	*SDPJ*	*Komei*	*JCP*	*DSP*	*Rengo*
1	0	0	0.5	0	0.5	0
2	0.155	0.032	0.211	0.144	0.458	0.000

In the unidimensional case, the calculated indices do not agree with our understanding; no one would accept the fact that the LDP is completely powerless. In the two-dimensional case, four parties, the LDP, the Komei, the JCP and the DSP, have significant positive power. The fact that the DSP had nearly half of the whole power and that the LDP was less powerful than the Komei and the DSP would never be accepted.

Why did the analysis produce the outcomes contradicting our usual understanding? One of the reasons is that we assume the uniform appearance of issues to calculate the index, though we use a particular (and biased) distribution of issues observed in 1989-1992 to determine parties' positions. Thus the analysis carried out above contains a self-contradiction, which leads to counter-intuitive outcomes. In the next section, we will avoid the self-contradiction in a reasonable manner.

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4. Power Distribution in the House of Councilors in Japan - The Nonsymmetric Shapley-Owen Index Based on the Data of 1989-1992

The assumption that any issue may arise originated from the symmetric Shapley-Shubik index. The Shapley-Shubik index assumes two sorts of symmetry: the symmetry of voters and the symmetry of issues. In the nonsymmetric Shapley-Owen index, only the symmetry of voters is relaxed.

The symmetry of issues, or the assumption that any issue may arise, must be appropriate in some cases. A typical example is the analysis of justices' power in the U.S. Supreme Court done by Frank and Shapley [3]; because any issue may be brought into the Court. Frank and Shapley first found a transformation that makes a distribution of observed issues uniform; then they adjusted positions of justices by performing the same transformation. Finally they calculated the nonsymmetric Shapley-Owen index on the basis of the adjusted positions.

In our case, however, observed issues are by no means uniformly distributed as figure 6 shows. The non-uniformity of issues is intrinsic to our study since bills are submitted mainly by a cabinet, most of its members belong to the governing party. In the study of the U.S. Presidential Elections, Rabinowitz and Macdonald [7] also found non-uniformity; directions for all elections after 1964 lie within a sector of about 90 degrees. They calculated the index assuming possible directions of issues to be uniformly distributed within this sector. Unfortunately in our case the distribution of observed issues is more strongly biased: *e.g.* 85 bills out of 133 have the same direction. Thus we calculate the index using only the observed data, believing that this is the best way to evaluate the party power in our case. Hence the obtained index gives only a power distribution during the period of 1989-1992: not a prediction for the future. But we would claim that if a similar structure concerning the parties' platform, the parties seats, etc. continued in the House, the index might serve as an approximation of power distribution in the future[5].

In the unidimensional case, the Komei is pivotal in issues of types A,B,C,D,F and H, i.e., in 124 issues out of 133; while the DSP is pivotal in the rest. Thus the index is $124/133 = 0.932$ for the Komei, $9/133 = 0.068$ for the DSP, and 0 for the others.

In the two-dimensional case, we first find out which party is pivotal in each of the eight types. The LDP is pivotal for 85 issues of type A, the Komei for types B,C,D,F,H, and the DSP for E,G. Thus the index is $85/133 = 0.639$ for the LDP; $(18+9+6+5+1)/133 = 0.293$ for the Komei; $(6+3)/133 = 0.068$ for the DSP, and 0 for the others. It is somewhat interesting that bills of type A (85 out of 133) induce the order, DSP←Komei ←Rengo←LDP←SDPJ←JCP. Thus we may claim that in more than 60 per cent of the bills, the LDP compromised with the DSP, the Komei and the Rengo to pass them; but it should be noted that the LDP was in a pivotal position in those bills.

The index in the two-dimensional case shows that the LDP has the largest power, and the Komei and the DSP have some power, which, we think, approaches our usual understanding. The indices in one- and two-dimensional cases are summarized in table 5 with those in case of more dimensions.

Table 5: The nonosymmetric Shapley-Owen indices

dim.	LDP	SDPJ	Komei	JCP	DSP	Rengo
1	0.000	0.000	0.932	0.000	0.068	0.000
2	0.639	0.000	0.293	0.000	0.068	0.000
3	0.639	0.135	0.180	0.000	0.045	0.000
4	0.707	0.007	0.105	0.000	0.135	0.045
5	0.511	0.166	0.202	0.000	0.049	0.072

We could get more detailed outcomes if we developed the analysis in spaces with more di-

[5] Unfortunately in the summer of 1993 drastic political reforms, triggered by a split in the LDP, started.

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mensions. If the assumption of random appearance of issues were removed, we might carry out the analysis without much difficulty, even in spaces with many dimensions[6]. Since there are six parties, we can use at most five factors; that is, parties' positions are fully characterized in five-dimensional space. Indices corresponding to each space are summarized in table 5. In section 3, we claimed two-dimensional space is sufficient to estimate parties' positions. But it does not claim that two-dimensional space is sufficient to obtain reasonable power indices. In fact, table 5 shows that values change drastically according to the number of dimensions[7]. In our model, in five-dimensional space, where parties are fully characterized, power index agrees with our usual understanding. In the five-dimensional analysis, it may be the case wherein some of the parties are equally distant from a given issue. For example, the distances of the five parties except the JCP from the issue of type A are the same. In this case, each of the 5! orderings of the five parties is assumed to appear with equal probability. Other ties are treated similarly. Table 5 shows the indices in one- to five-dimensional spaces. This index shows that (1) the LDP has more than half of the whole power; (2) the Komei is more powerful than the SDPJ, but the DSP and the Rengo are less powerful; and (3) the JCP has no power. We conclude that our usual understanding about power is theoretically justfied by this nonsymmetric Shapley-Owen power index.

5. Concluding Remarks

In this paper we have studied the political party power in the House of Councilors in Japan. We first showed that a direct application of the nonsymmetric Shapley-Owen index gives an outcome contradicting our usual understanding. The assumption of the random appearance of possible issues that the index requires is not very reasonable in our case, because bills were usually submitted by a cabinet consisting mainly of members belonging to the LDP, the largest party. Distribution of bills is by no means uniform but rather biased. We thus calculated the index just by using the data observed in the period of 1989-1992; the calculation is not that complicated even in multidimensional spaces. We obtained, in the five-dimensional case, that the LDP has most power, then the Komei, the SDPJ, the Rengo, and finally the DSP in this order; the JCP is powerless. Thus we would claim that our modification of the Shapley-Owen power index may give a theoretical justification of our recognition of power distribution of the political parties in the House of Councilors in Japan.

In this paper, we used the factor analysis to process the data since it gives projections of the parties' positions to issues' directions, and thus a good statistical tool for calculating the nonsymmetric Shapley-Owen index. Owen's original method, however, requires us to place both parties and issues in the same space and to measure their distances. Recall section 2. In order to calculate Owen's original index, we would propose to use instead the quantification method of the third. If we remove the assumption that issues appear at random and calculate the index just using the observed data, we may develop the analysis without much difficulty even in spaces with many dimensions. It should be interesting to compare the index with the nonsymmetric Shapley-Owen index obtained through the factor analysis. This study will be done in a future paper.

We conclude this paper with one last remark. Throughout this paper, we have focused only on voting decisions on submitted issues in a conference. In this sense, negotiations on submitted issues among the parties in the conference were taken into consideration. Some issues might be influenced by prenegotiations before being submitted. Our analysis could also cover such influences. However, there may be some issues which are withdrawn without being submitted to the conference as the result of prenegotiations. Parties influences in those negotiations were neglected as far as we stick to the voting games and existing power indices. A new type of game as well as a new power index must be developed since the traditional voting game and the power indices never assume such

[6]Frank and Shapley [3] illustrates difficulties that may arise in an analysis with a three-dimensional space in case the assumption of uniform distribution of possible issues is kept.

[7]Drastic change according to dimensions can happen even in case of uniformly distributed issues.

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prenegotiations. It needs further and elaborate consideration[8].

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Rie Ono
Faculty of Economics
Toyama University
3190 Gofuku Toyama 930 Japan
E-mail: onorie@eco.toyama-u.ac.jp

Shigeo Muto
Faculty of Economics
Tokyo Metropolitan University
1-1 Minami-Ohsawa Hachiohji Tokyo 192-03 Japan
E-mail: muto@bcomp.metro-u.ac.jp

[8]We are grateful to one of the referees for making suggestion on this point.

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