

## A DEA SELECTION SYSTEM FOR SELECTIVE EXAMINATIONS

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*Abstract* This paper presents a DEA (Data Envelopment Analysis) selection system applicable to selective examinations on multiple academic subjects, e.g., university entrance examinations in Japan. Using this system, we can select "masters of an art" (brilliant in some academic subject but not necessarily in all) as well as "jacks of all trades" (scoring quite well across all the subjects) as successful candidates. This is also a proposal for the potential use of DEA in multi-dimensional evaluation other than the standard DEA efficiency analysis. Defining candidates for examination as DMUs (Decision Making Units) and examination scores as outputs, and considering that every DMU has only one input whose amount is unity, we can comprehensively evaluate the candidates in terms of their examination scores using DEA. The results of a selective examination case show good performance of the DEA selection system, i.e., we can select various types of candidates with different featured characteristics so as to fill up the allocation of students exactly. It is concluded that DEA selection, which relatively evaluates candidates in accordance with their individual characteristics, can be considered as an alternative method other than the usual selection on the basis of total score.

### 1. Introduction

Selective (competitive) examinations are widely held in various fields. In Japan, entrance examination for a university is a typical and a representative selective examination. As the Japan society tends to attach importance to school career, people in Japan are interested in matriculating at a university, so that selective examination for entry to a university is an object of public concern.

Entrance to a university in Japan is generally through examination by university department. Most of the examinations are those on paper and on multiple academic subjects, each of which is marked on a maximum of certain (e.g., 100) points. They are not *placement* nor *qualifying* examinations but *selective* examinations, relatively selecting the given definite number (the allocation of students) of candidates for examination.

Relative selection of candidates is on the basis of total of examination scores on multiple subjects, i.e., candidates are selected so as to fill up the allocation in order of the total score. The total here implies a weighted sum, where the weight is the maximum number of marks given for each subject. Selection on the basis of total score has the following properties: Because of the difficulty to define such an *a priori* weighting system, the weights cannot but be arbitrary; When we employ this type of weighting system, resulting selection means uniform evaluation of candidates with varying characteristics. While this total-score selection method seems to be one based on the consideration to select candidates brilliant in multiple academic subjects, there are no other firm foundations to employ it.

On the other hand, there is the following opinion mainly at university in Japan: University students are too uniform in these days, i.e., "unconventional" students are seldom seen; To make university students be rich in diversity or variety, we should admit those who are brilliant in

some subject but not necessarily in all as "masters of an art". However, as long as we employ the total score method, we cannot select such a candidate that is proficient in a single subject but does not score so well in the total.

This paper explores an alternative selection method for selective examinations in the case where the above-stated opinion is taken into consideration. The method is required to be one that can select "masters in an art" as well as "jacks of all trades" scoring quite well across all the subjects. Selecting candidates for entry to a university requires value judgements on the qualifications of candidates based on some evaluation criterion. The criterion has to be fair, but would not necessarily have to be uniform across candidates. We propose the use of *DEA* (*Data Envelopment Analysis*) as such a selection method. DEA can evaluate candidates multi-dimensionally, where the evaluation criterion is not uniform across them.

## 2. DEA for selective examinations

Charnes *et al.* [5] proposed DEA as a method for measuring relative *efficiencies* of *DMUs* (*Decision Making Units*) (see [3, 4, 15] for overviews). The relative efficiency (DEA measure) of target DMU  $j_0$ ,  $h_{j_0}$  ( $0 \leq h_{j_0} \leq 1$ ), can be obtained by solving the following fractional programming problem:

$$(2.1) \quad \text{Maximize} \quad h_{j_0} = \frac{\sum_{r=1}^t u_r y_{rj_0}}{\sum_{i=1}^m v_i x_{ij_0}}$$

$$\text{subject to} \quad \frac{\sum_{r=1}^t u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1, \quad j = 1, \dots, n,$$

$$u_r, v_i > 0, \quad r = 1, \dots, t, \quad i = 1, \dots, m,$$

where  $y_{rj}$  = the amount of *output*  $r$  from DMU  $j$ ;  $x_{ij}$  = the amount of *input*  $i$  to DMU  $j$ ;  $u_r$  = the weight given to output  $r$ ;  $v_i$  = the weight given to input  $i$ ;  $n$  = the number of DMUs;  $t$  = the number of outputs;  $m$  = the number of inputs. We can find DEA measures of all the DMUs by solving problem (2.1)  $n$  times, setting each DMU as target DMU  $j_0$  in turn. Here, DMUs  $j_0$  with maximum  $h_{j_0} = 1$  are judged *DEA efficient*, while the other DMUs are *DEA inefficient*.

As is well known, DEA examines how efficiently DMUs convert multiple inputs into multiple outputs. That is, any DMU producing more outputs with fewer inputs is judged DEA efficient. However, DEA models do not necessarily assume such organic relationships between inputs and outputs as those in production [see model (2.1)]. Thus, replacing inputs with *negative* evaluation items (the smaller the value, the better) and outputs with *positive* evaluation items (the greater the value, the better), yields a combined evaluation of these items. This is a comprehensive evaluation different from traditional ones in that it replaces a uniform evaluation using an *a priori* weighting system with a flexibly defined weighting system corresponding to each DMU [9, 11].

We should note that DEA model (2.1) in the case where every DMU has only one "unit input" (the amount of input is unity) can be written as follows:

$$(2.2) \quad \text{Maximize} \quad h_{j_0} = \sum_{r=1}^t u_r y_{rj_0}$$

$$\begin{aligned} \text{subject to } & \sum_{r=1}^t u_r y_{rj} \leq 1, \quad j = 1, \dots, n, \\ & u_r > 0, \quad r = 1, \dots, t. \end{aligned}$$

Using this *pure output* DEA model [1], we can do a DEA analysis in that any DMU with more outputs is judged DEA efficient. Defining candidates as DMUs and examination scores on multiple academic subjects as outputs, we can evaluate the candidates in terms of the examination scores using DEA. Here, as evaluation criterion, we use the weighted sum of examination scores like the total score method, but unlike that, the weights are not fixed but can vary by candidate. That is, every candidate can be relatively evaluated in terms of weights that best suit himself/herself. This can be considered to guarantee fair competition among candidates. Therefore, we think it fair and reasonable to use DEA for selecting candidates, though the evaluation criterion is not uniform across candidates.

From a methodological point of view, this study is a proposal for the potential use of DEA in multi-dimensional evaluation other than the standard DEA efficiency analysis. While it should be possible to apply DEA to fields beyond efficiency analysis, we found but a few "non-standard" applications. These included preferential rankings aggregation (using the pure output model) [6, 10], computer printer comparison [8], living desirability assessment [11, 12] and baseball batters evaluation [9]. In the current paper, we focus on DEA as a multi-dimensional evaluation tool, and seek its application to a selection system for selective examinations.

### 3. DEA selection system

In order to illustrate the performance of DEA selection system, we suppose the following selective examination like entrance examination in Japan: The examination covers Mathematics, Science, Japanese (as the national language), Social studies and English (as a foreign language), which are called the "five key academic subjects" in Japan, and the maximum number of marks for every subject is 100; Through the examination, we seek to select 25 (the allocation) as successful candidates; The number of candidates taking the examination is 50.

#### 3.1. Data

Table 1 shows examination scores on the five subjects for the 50 candidates. Because of the same maximum number of marks for every subject, the total here implies the equally weighted sum of five examination scores. These data consist of 45 real records for some examination and five hypothetical records. The latter are added so as to demonstrate the performance of DEA selection system clearly, and candidate name  $C^{**}H$  denotes the hypothetical candidates. They are, as extreme cases, supposed to be brilliant only in Mathematics or only in Japanese. Their examination scores are arbitrarily assumed, but the Mathematics score of candidate C46H is more than, and the Japanese score of candidate C48H is equal to the highest points on the respective subjects of the real 45 candidates.

Moreover, Table 1 lists records by the total scores, and candidates are named in accordance with the ranking. For example, candidate C06b means one of those ranking sixth because symbols  $a, b$ , etc. are attached in the case of same rank. Here, we should note the following: When we employ the total score method, candidates C01 to C25 would be selected as successful candidates; For the hypothetical five candidates, which might include "masters of an art", none of them would be selected because their total scores are very low.

As for the data in Table 1, we have positive correlations among scores on the five subjects across the real 45 candidates. The greatest correlation coefficient is 0.704 between Mathematics and Science, and half the ten correlation coefficients have values more than 0.5. In general, we

Table 1. Candidate data and selection results

Candidate	Mathematics	Science	Japanese	Social studies	English	Total	Stage selected
C01	84	87*	67*	74*	79*	391*	1
C02	86*	66	56	59	77	344	1
C03	65	67	66	73	72	343	2
C04	77	81	56	60	66	340	2
C05	78	68	62	60	66	334	2
C06a	66	62	50	55	65	298	3
C06b	58	48	64	73	55	298	3
C08	65	57	45	60	69	296	3
C09	69	55	60	50	60	294	3
C10	59	64	43	50	61	277	3
C11a	66	54	52	56	47	275	4
C11b	55	61	56	58	45	275	3
C11c	55	50	55	50	65	275	3
C14	59	59	49	47	50	264	4 <sup>+</sup>
C15	54	51	54	53	51	263	4 <sup>+</sup>
C16a	60	52	50	50	48	260	
C16b	59	56	50	30	65	260	4
C16c	48	45	53	54	60	260	4
C19a	56	35	52	46	68	257	3
C19b	40	56	55	49	57	257	4
C21	46	62	43	57	48	256	3
C22a	66	55	44	50	40	255	
C22b	49	56	45	61	44	255	4
C24	55	57	47	45	50	254	
C25	62	49	59	38	45	253	4 <sup>+</sup>
C26	51	42	55	50	54	252	4 <sup>++</sup>
C27	48	54	43	47	53	245	
C28a	67	57	31	37	52	244	4
C28b	53	50	46	44	51	244	
C30	54	58	52	51	28	243	4 <sup>++</sup>
C31	46	46	44	54	52	242	
C32a	58	41	44	50	48	241	
C32b	42	41	55	42	61	241	4 <sup>++</sup>
C34	52	43	46	50	45	236	
C35a	52	46	39	49	49	235	
C35b	50	50	39	45	51	235	
C35c	50	44	48	40	53	235	
C38	55	49	43	46	38	231	
C39	55	43	43	41	48	230	
C40	56	48	32	49	44	229	
C41	52	38	47	55	33	225	
C42	48	49	40	45	42	224	
C43a	46	35	47	41	45	214	
C43b	37	40	48	47	42	214	
C45	46	42	37	36	44	205	
C46H	90	0	0	0	0	90	1
C47H	70	0	0	0	0	70	3
C48H	0	0	67	0	0	67	2
C49aH	50	0	0	0	0	50	
C49bH	0	0	50	0	0	50	

\* The highest point of the real 45 records

▨ Candidates left out in the select lists of the selection stages up to the fourth

†† Left out due to the Case A<sup>+</sup> (Case B<sup>+</sup>) prioritization model

observe high correlations among scores on entrance examination subjects [13]. Therefore, any entrance examination on multiple subjects cannot but depend on such correlated score data. Then, the total score method presupposes repetition of information because it might be enough to conduct an examination on a single subject. For the DEA selection in such a case, we would judge relatively few candidates DEA efficient at a time. It is considered that successful candidates in terms of the respective selection methods would not be so different in the case of high correlations.

### 3.2. Selection method

In the DEA selection system, defining candidates as DMUs and their examination scores as outputs, and regarding that every DMU has only one input whose amount is unity, we apply DEA to the data in Table 1. Then, DEA discriminates some *frontier* (DEA efficient) candidates from DEA inefficient candidates as well as computes DEA measures for the respective candidates. But we must consider how to select 25 candidates from among 50.

We propose the following selection method: First, we select frontier candidates as successful candidates (of the first selection stage) at a time. They would include not only "jacks of all trades" but also "masters of an art". These candidates are *DEA brilliant* in the sense that each of them can rank top in terms of the weights optimal to himself/herself, i.e., no other candidates dominate them. This is the criterion to identify DEA brilliant candidates. We also employ this criterion, not DEA measures for DEA inefficient candidates, to select the subsequent successful candidates when the allocation is not yet filled up. Because DEA measures are the values compared to different *reference points* on the frontier, indiscriminate comparisons of DEA measures for DEA inefficient DMUs are not meaningful [11], i.e., we do not think it appropriate to use DEA measures to judge the subsequent DEA brilliance. Therefore, removing the successful candidates and applying DEA to the remaining candidates, we select frontier candidates at that time as successful candidates of the second selection stage. They can be regarded DEA brilliant next to the successful candidates of the first stage because no candidates but those selected at the first stage dominate them. We repeat this selection up to the stage at which the cumulative number of successful candidates is equal to the allocation or more.

Table 1 also shows the results of selections that a total of 29 candidates are selected as successful candidates of the selection stages up to the fourth. That is, at the first stage, three candidates (C01, C02 and C46H) are selected; the second stage, four; the third stage, ten; and the fourth stage, twelve. We should here note that the selected candidates using this DEA method are not so different from those using the total score method except for the hypothetical candidates, because of the earlier-stated high correlations among examination scores. Moreover, those who score quite well in the total are also selected in terms of the DEA because they are apt to be on the frontier.

Table 2 shows *virtual* examination scores [3] for frontier candidates at each selection stage. A virtual score is the product of examination score on a subject and the corresponding optimal weight. For frontier candidates, the sum of virtual scores is one [see model (2.2)], so that individual virtual scores show the contribution rates to being DEA efficient. Therefore, in terms of the virtual examination scores in Table 2, we can see *feature* examination subjects for successful candidates.

At the first stage, candidate C01 is selected being evaluated in terms of Science, while candidate C46H achieves this in terms of Mathematics, because their examination scores on the respective subjects are the highest points of all the 50 candidates. Candidate C02 can rank top by weighting Mathematics much and Science a little. Candidate C48H, who scores top in Japanese, is not selected at the first stage, dominated by C01 also scoring top in Japanese. At the second stage, there is not C01, so that C48H is selected being evaluated in terms of

Table 2. Virtual examination scores for the selected 29 candidates

Selection stage	Candidate	Mathematics	Science	Japanese	Social studies	English
1	C01	0	1	0	0	0
	C02	0.932	0.068	0	0	0
	C46H	1	0	0	0	0
2	C03	0	0	0	1	0
	C04	0.180	0.337	0.483	0	0
	C05	0.182	0.283	0.535	0	0
	C48H	0	0	1	0	0
3	C06a	0.713	0.287	0	0	0
	C06b	0.019	0.424	0	0.531	0.026
	C08	0	0.473	0	0.433	0.094
	C09	0.608	0.103	0.059	0.231	0
	C10	0.209	0.792	0	0	0
	C11b	0	0.448	0.382	0.085	0.085
	C11c	0.060	0.042	0.296	0	0.602
	C19a	0	0.038	0.256	0	0.706
	C21	0	0.769	0.029	0.202	0
C47H	1	0	0	0	0	
4	C11a	0.964	0	0.036	0	0
	C14	0.026	0.560	0.199	0.179	0.036
	C15	0.083	0.069	0.552	0.157	0.139
	C16b	0.328	0	0	0.129	0.543
	C16c	0.267	0	0	0.232	0.502
	C19b	0.072	0.300	0	0.232	0.396
	C22b	0.191	0	0	0.809	0
	C25	0.059	0	0.759	0.182	0
	C26	0.037	0.019	0.667	0.184	0.093
	C28a	0.389	0.474	0.057	0	0.080
	C30	0.024	0.551	0.211	0.195	0.020
	C32b	0	0	0.655	0.098	0.248

Japanese. In this way, Table 2 shows that successful candidates are evaluated on a variety of attributes. Here, we can see a property peculiar to DEA vs other such comprehensive evaluation tools, i.e., multiple candidates with various featured characteristics can attain the maximum measure, one. However, we should also note that the DEA model may have alternative optimal solutions, which would lead to alternative virtual examination scores.

As shown in Tables 2 and 1, three of the hypothetical candidates, who must have been left out in the select list on the basis of total score, are selected as "masters of an art" being evaluated in terms of the respective well-scored subjects. On the contrary, candidates C16a, C22a and C24, who would succeed in terms of the total score method, are not selected because they cannot rank top with any weights optimal to them even at the fourth stage. We can here see the difference between the DEA and the total-score selection methods.

It is noteworthy that the DEA method can select not only candidates proficient in single subjects but also those scoring well across all the subjects and those being somewhere in between. This is possible due to the flexible weighting system that can vary by candidate. That

is, DEA can avoid uniform evaluation of candidates with different characteristics, which would bring diversity in the selected candidates.

It is by chance that candidate C16a is not selected while candidate C32b is selected. In any selective examination (relatively selecting candidates up to the allocation), whether a candidate would be selected or not is determined by the number of those who are superior to him/her. In the current examination case, by chance, there would be relatively many candidates being evaluated superior to C16a while there would be few candidates corresponding to C32b, though the respective evaluation criteria are not the same.

Candidates C46H, C47H and C48H, who are selected as successful candidates, might be brilliant in one subject but score zeroes on other subjects. Normally, they would not be desired for the candidates to be admitted. However, we should remember that they are extreme cases for the hypothetical candidates. They would scarcely occur. If we would like to reject those extreme cases at any cost, we can preliminarily screen out candidates in terms of the requisite minimum points for single subjects. On the other hand, in the case of examination with large numbers of both candidates and the allocation, we may select some candidates on the basis of total score to fill part of the allocation in advance, because they would anyhow be selected even if in terms of the DEA. These possibilities of incorporating other selection methods into the system strengthen applicability of the DEA selection.

The DEA method has selected 29 as successful candidates because DEA usually judges multiple DMUs DEA efficient at a time. But the given allocation of successful candidates is here supposed 25. If we have flexibility as to gap between the actual number and the allocation of successful candidates, and if the gap is permitted, we are through this study. However, there would be the case in which we must adjust the number of successful candidates exactly to the allocation. In the next subsection, we consider the case.

### 3.3. Adjusting to the allocation

We here consider how to adjust the number of successful candidates just to the allocation, 25, i.e., how to leave out four from among 29. We determine such rejects amongst candidates selected at the last stage. Because the selected candidates of the earlier stage are more DEA brilliant than those of the later stage, the former should be fixed successful prior to the latter. Therefore, we must here prioritize the twelve selected candidates (frontier DMUs) of the fourth stage.

In DEA, there are several techniques for discriminating frontier DMUs. We here consider two models [7] shown in the Appendix. These two prioritization models can be applied to the current examination case as they are. *Case A* model (A.2) is corresponding to the case where ordinal relations as to the importance of examination subjects are given, while *Case B* model (A.4) is to the case where examination subjects are viewed as being equal importance.

Applying the respective prioritization models to the twelve selected candidates of the fourth stage, we obtained the results shown in Table 3. For Case A, we arbitrarily assumed the ordinal relations of examination subjects as Mathematics, Science, English, Social studies and Japanese (in order of the relative importance). In either case, the top eight candidates shown in Table 3 are selected and added as final successful candidates. In Case A, candidate C28a best fits the assumed ordinal relations of examination subjects among the twelve. In Case B, candidate C11a can be DEA efficient even in terms of the weights equal throughout five examination subjects. The right end column of Table 1 also shows these results.

We cannot say which prioritization model to be used, or it might be better to use alternative techniques for discriminating frontier DMUs (e.g., the DEA exclusion model [2, 10]). We may employ one adjusting method in accordance with examination policy at that time. This is a problem to be considered corresponding to what maximum number of marks for each subject

Table 3. Prioritization measures for candidates selected at the fourth selection stage

Ranking	Case A		Case B	
	Candidate	$\delta_{j_0}$	Candidate	$\gamma_{j_0}$
1	C28a	0.0017	C11a	0
2	C11a	0.0016	C16b	0.0021
3	C16b	0.0010	C16c	0.0027
4	C16c	-0.0013	C19b	0.0027
5	C19b	-0.0016	C14	0.0045
6	C22b	-0.0020	C22b	0.0047
7	C14	-0.0025	C28a	0.0051
8	C15	-0.0039	C25	0.0060
9	C30	-0.0053	C30	0.0066
10	C25	-0.0057	C32b	0.0069
11	C32b	-0.0062	C15	0.0088
12	C26	-0.0072	C26	0.0111

to be given in the total score selection. What we should here note is that we can adjust the number of successful candidates exactly to the given allocation using some prioritization technique.

#### 4. Summary and conclusions

This study proposed a DEA selection system for selective examinations. We presented a DEA selection method and demonstrated its desirable performance, i.e., we can avoid uniform evaluation of candidates and can select various types of candidates with different featured characteristics.

If we would like to do such a DEA selection *a priori* discriminating the importance of examination subjects, e.g., "science-oriented" or "art-oriented" DEA selection, constraints in the shape of bounding ratios of output weights can be added to DEA model (2.2). That is *DEA/AR* (*DEA/ Assurance Region*) [11, 14] model for selective examinations. The possibility of this *DEA/AR* selection, together with the earlier-mentioned possibilities of combinations with other selection methods and exact adjustment to the allocation, strengthens availability and applicability of the DEA selection system.

From a viewpoint of DEA methodology, this study explored a field application of DEA beyond the standard efficiency analysis, applying DEA to multi-dimensional evaluation of examination scores. As shown in the results of analysis, DEA, which can be considered as a comprehensive evaluation tool with the flexible criterion capable of varying by candidate, evaluates candidates differently from the total score method. We think that the DEA selection system provides the condition to guarantee fair competition among candidates. Therefore, the DEA selection can be considered as an alternative method other than the usual total-score selection.

#### Appendix

Cook *et al.* [7] proposed two models for prioritizing frontier DMUs corresponding to the following cases as to assumptions of management about the importance of outputs. Here, the scales on which outputs  $y_j$  are measured are assumed all roughly the same. These similar scales can be attained through some normalization if necessary.



**Case A: Ordinal relations of outputs are given**

The relative importance of outputs are incorporated into the DEA model in the form of a set of ordinal relations on the output weights. For every frontier DMU  $j_0$ , model (2.2) is equivalent to

$$\begin{aligned}
 \text{(A.1) Maximize} \quad & h_{j_0} = \sum_{r=1}^t u_r y_{rj_0} \\
 \text{subject to} \quad & \sum_{r=1}^t u_r y_{rj} \leq 1, \quad \forall j \in J, \\
 & u_{r_k} - u_{r_{k+1}} \geq \delta, \quad k = 1, \dots, t-1, \\
 & u_r > 0, \quad r = 1, \dots, t, \\
 & (\delta \text{ unconstrained}),
 \end{aligned}$$

where  $J$  = the set of frontier DMUs;  $u_{rk}$  = the weight given to the  $k$ -th ranked output;  $\delta$  = the gap between successively ranked weights.

Then, the following model determines the maximum value of gap  $\delta_{j_0}$ , for which frontier DMU  $j_0$  remains DEA efficient:

$$\begin{aligned}
 \text{(A.2) Maximize} \quad & \delta_{j_0} = \delta \\
 \text{subject to} \quad & \sum_{r=1}^t u_r y_{rj_0} = 1, \\
 & \sum_{r=1}^t u_r y_{rj} \leq 1, \quad \forall j \in J - \{j_0\}, \\
 & u_{r_k} - u_{r_{k+1}} - \delta \geq 0, \quad k = 1, \dots, t-1, \\
 & u_r > 0, \quad r = 1, \dots, t, \\
 & (\delta \text{ unconstrained}).
 \end{aligned}$$

The greater the value of  $\delta_{j_0}$ , the more strongly DMU  $j_0$  fits the given ordinal relations of outputs. Therefore, the higher priority is assigned to the DMU with the greater  $\delta_{j_0}$  value.

**Case B: Outputs are viewed equal in importance**

This is the case where no information is available as to the relative importance of outputs. For every frontier DMU  $j_0$ , model (2.2) is equivalent to

$$\begin{aligned}
 \text{(A.3) Maximize} \quad & h_{j_0} = \sum_{r=1}^t u_r y_{rj_0} \\
 \text{subject to} \quad & \sum_{r=1}^t u_r y_{rj} \leq 1, \quad \forall j \in J, \\
 & \alpha \geq u_r \geq \beta, \quad r = 1, \dots, t, \\
 & u_r, \alpha, \beta > 0, \quad r = 1, \dots, t,
 \end{aligned}$$

where  $\alpha, \beta$  = the upper and the lower limits of weights.

Then, the following model strives to obtain the weight vector whose components are most

homogeneous in size, for which frontier DMU  $j_0$  remains DEA efficient:

$$\begin{aligned}
 \text{(A.4) Minimize} \quad & \gamma_{j_0} = \alpha - \beta \\
 \text{subject to} \quad & \sum_{r=1}^t u_r y_{rj_0} = 1, \\
 & \sum_{r=1}^t u_r y_{rj} \leq 1, \quad \forall j \in J - \{j_0\}, \\
 & \alpha - u_r \geq 0, \quad r = 1, \dots, t, \\
 & u_r - \beta \geq 0, \quad r = 1, \dots, t, \\
 & u_r, \alpha, \beta > 0, \quad r = 1, \dots, t.
 \end{aligned}$$

The greater the value of  $\gamma_{j_0}$ , the more extreme circumstances DEA efficient status of DMU  $j_0$  depends on. Therefore, the higher priority is assigned to the DMU with the smaller  $\gamma_{j_0}$  value.

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