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A SECRETARY PROBLEM WITH UNCERTAIN EMPLOYMENT WHEN THE NUMBER OF OFFERS IS RESTRICTED

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Abstract We consider the so-called secretary problem, in which an offer may be declined by each applicant with a fixed known probability q $(= 1 - p,\ 0 \leq q < 1)$ and the number of offering chances are at most m (≥ 1). The optimal strategy of this problem is derived and some asymptotic results are presented. Furthermore we briefly consider the case in which the acceptance probability depends on the number m of offering chances.

1. Introduction

We consider a variation of the sequential observation and selection problem, often referred to as secretary problem and studied extensively by Gilbert and Mosteller[3]. The basic framework of the classical secretary problem can be described as follows. N applicants appear one by one in random order with all $N!$ orderings equally likely. We are able, at any time, to rank the applicants that have so far appeared according to some order of preference. As each applicant appears, we must decide whether or not to make an offer to that applicant with the objective of maximizing the probability of choosing the best over all. It is assumed that each applicant accepts an offer of employment with certainty and that previously passed over applicants cannot be recalled later.

Smith[6] generalized the classical secretary problem to allow the applicant the right to refuse an offer of employment with a known fixed probability q $(0 \leq q < 1)$, independent of his/her rank and the arrangement of the other applicants. In other words, each applicant only accepts an offer with probability $p = 1 - q$. Offers can be made indefinitely until an offer is accepted in Smith[6]. To examine the effect of the number of offers, we reconsider here the Smith's problem under the assumption that only a predetermined number m of offers can be made (see Ano and Tamaki[1], and Tamaki[7] for other modifications of the Smith's problem). We call this problem *m-problem.* To solve the m-problem, we must solve the (m-1)-,(m-2)-,$\cdots$, 1-problems sequentially. Hereafter, we call an applicant *candidate* if the applicant is relatively best, i.e., the applicant is more preferred to all those preceding him/her. Obviously the optimal strategy only gives an offer to a candidate. In Section 2, it is shown that the optimal strategy for the m-problem is threshold type and described as follows: pass over the first $s_m - 1$ applicants and give an offer to the first candidate that appears. It is also shown that s_m is non-increasing in m. In Section 3, some asymptotic results will be given. We have so far assumed that the acceptance probability is always constant. In Section 4, we briefly consider the case in which this assumption dose not hold. Let p_m be the acceptance probability when we are allowed to make m more offers. It can be shown that for some particular values of p_m, $m = 1, 2, \cdots$, the optimal strategy gives an offer not only to a candidate but also to a non-candidate.

2. The m-problem

Let X_n denote the relative rank of the nth applicant among the first n applicants(rank 1 being the relatively best). Since the applicants appear in random order, it is easy to see that X_n, $i \le n \le N$, are independent random variables with $P(X_n = i) = 1/n$, $1 \le i \le n$. Note that, if $X_n = 1$, the nth applicant is called *candidate*. Define the state of the process as (n, m), $1 \le n, m \le N$, when we confront the m-problem and observe that the nth applicant is a candidate. In state (n, m), we must decide either to give an offer or not to the current candidate. Our trial is said to be a *success* if we can employ the overall best. Let $w_n{}^{(m)}$ be the probability of success starting from state (n, m). Also let $u_n{}^{(m)}(v_n{}^{(m)})$ be the corresponding probability when we make an offer (when we do not make an offer) to the current candidate and proceed optimally thereafter. Then, by the principle of optimality, we have

$$w_n^{(m)} = \max\{u_n^{(m)},\ v_n^{(m)}\}, \quad n, m = 1, \cdots, N-1, \tag{2.1}$$

$$u_n^{(m)} = p\frac{n}{N} + qv_n^{(m-1)}, \tag{2.2}$$

$$v_n^{(m)} = \frac{1}{n+1}w_{n+1}^{(m)} + (1 - \frac{1}{n+1})v_{n+1}^{(m)}. \tag{2.3}$$

The boundary conditions are $v_n{}^{(0)} = 0$ for all n, $v_N{}^{(m)} = 0$ for $m \ge 1$ and $w_N{}^{(m)}{=}u_N{}^{(m)}{=}p$ for $m \ge 1$. The first term of right hand side in Eq.(2.2) follows since when the current ca ndidate accepts the offer, the probability of success is the probability that all of the $(n+1)th$,$(n+2)th$,$\cdots$,Nth applicants are not candidate, that is, $P(X_{n+1} > 1, \cdots, X_N > 1) = n/N$. The second term of right hand side in Eq.(2.2) follows since when the offer is declined, the m-problem enters into the $(m-1)$-problem. Now repeated use of (2.3) yields

$$v_n^{(m)} = \sum_{j=n+1}^{N} \frac{n}{j(j-1)} w_j^{(m)}. \tag{2.4}$$

Throughout this paper, the vacuous sum is assumed to be zero. Let B_m be the one-stage look-ahead stopping region for the m-problem, that is, B_m is the set of state (n, m) for which giving an offer immediately to the current candidate is at least as good as waiting for the next candidate to appear to whom an offer is given. Thus

$$B_m = \{(n, m) : u_n^{(m)} \ge \sum_{j=n+1}^{N} \frac{n}{j(j-1)} u_j^{(m)}\}.$$

Let $A_n{}^{(m)} = \{u_n^{(m)} - \sum_{j=n+1}^{N}(n/j(j-1))u_j^{(m)}\}\{N/n\}$. Then $B_m = \{(n, m) : A_n^{(m)} \ge 0\}$ and $A_n^{(m)}$ can be written as follows from (2.2) and (2.4):

$$\begin{aligned} A_n^{(m)} &= p + \frac{qN}{n}v_n^{(m-1)} - \sum_{j=n+1}^{N} \frac{1}{j(j-1)}\{p\frac{j}{N} + qv_j^{(m-1)}\} \\ &= A_n^{(1)} + q\sum_{j=n+1}^{N} \frac{N}{j(j-1)}\{w_j^{(m-1)} - v_j^{(m-1)}\}. \end{aligned} \tag{2.5}$$

It is well known that if B_m is closed, e.g., $B_m = \{(n, m) : n \ge s_m^*\}$ for some specified integer s_m^*, then B_m gives the optimal offering region for the m-problem (see,e.g., Ross[4]). The following theorem is the main result of this section.

Theorem 1 *Let* $s^*_m = \{min\{n : A^{(m)}_n \geq 0\}$. *Then* B_m *is written as* $B_m = \{(n,m) : n \geq s^*_m\}$ *and gives an optimal offering region for the m-problem. Moreover* s^*_m *is non-increasing in m.*

Proof. It suffices to show that (i) for fixed k, if $A^{(k)}_n \geq 0$ for some n then $A^{(k)}_j \geq 0$ for all $j \geq n+1$, and (ii) for all n, $A^{(k+1)}_n \geq A^{(k)}_n$. We show these by induction on k. The assertion for $k=1$ is immediate since we have from (2.5)

$$A^{(1)}_n = p(1 - \sum_{j=n+1}^{N} \frac{1}{j-1}),$$

which is obviously increasing in n and

$$A^{(2)}_n - A^{(1)}_n = q \sum_{j=n+1}^{N} \frac{N}{j(j-1)} \{w^{(1)}_j - v^{(1)}_j\} \geq 0.$$

Assume both (i) and (ii) hold for $k = m$. Then the optimal strategy for the m-problem gives an offer to the candidate which appears after or on s^*_m where $s^*_m = \min\{n : A^{(m)}_n \geq 0\}$. Therefore for $j \geq s^*_m$,

$$\begin{aligned} w^{(m)}_j &= u^{(m)}_j, \\ w^{(m)}_j - v^{(m)}_j &= u^{(m)}_j - \sum_{l=j+1}^{N} \frac{j}{l(l-1)} u^{(m)}_l, \end{aligned}$$

and for $j < s^*_m$,

$$w^{(m)}_j = v^{(m)}_j.$$

Consequently we have

$$w^{(m)}_j - v^{(m)}_j = \begin{cases} 0, & j < s^*_m \\ u^{(m)}_j - \sum_{l=j+1}^{N} \frac{j}{l(l-1)} u^{(m)}_l = \frac{j}{N} A^{(m)}_j, & j \geq s^*_m. \end{cases} \tag{2.6}$$

Substituting (2.6) into (2.5), we have

$$A^{(m+1)}_n = A^{(1)}_n + q \sum_{j=\max(n+1, s^*_m)}^{N} \frac{1}{j-1} A^{(m)}_j. \tag{2.7}$$

When $n+1 \leq s^*_m$ since the summation in the right hand side of the above equation is non-negative constant from the definition of s^*_m, and $A^{(1)}_n$ is increasing in n, $A^{(m+1)}_n$ is increasing in n. That is,

$$A^{(m+1)}_1 \leq A^{(m+1)}_2 \leq \cdots \leq A^{(m+1)}_{s^*_m - 1}. \tag{2.8}$$

When $n+1 > s^*_m$, from the definition of s^*_m we have $A^{(m)}_j \geq 0$ for $j = n+1, \cdots, N$. Thus from the hypothesis (ii) with $k = m$,

$$0 \leq A^{(m)}_n \leq A^{(m+1)}_n. \tag{2.9}$$

Inequalities (2.8) and (2.9) imply that (i) holds for $k = m+1$.

Now s^*_{m+1} can be written as $s^*_{m+1} = \min\{1 \le n \le s^*_m : A_n^{(m+1)} \ge 0\}$ and $A_n^{(m+2)}$ can be written as

$$A_n^{(m+2)} = A_n^{(1)} + q \sum_{j=\max(n+1,s^*_{m+1})}^{N} \frac{1}{j-1} A_j^{(m+1)}. \tag{2.10}$$

Therefore we have from (2.7) and (2.10)

$$\begin{aligned} A_n^{(m+2)} - A_n^{(m+1)} &= q\{ \sum_{j=\max(n+1,s^*_{m+1})}^{N} \frac{1}{j-1} A_j^{(m+1)} - \sum_{j=\max(n+1,s^*_m)}^{N} \frac{1}{j-1} A_j^{(m)} \} \\ &\ge q \sum_{j=\max(n+1,s^*_m)}^{N} \frac{1}{j-1} \{A_j^{(m+1)} - A_j^{(m)}\} \\ &\ge 0, \end{aligned}$$

where the first inequality follows from $s^*_{m+1} \le s^*_m$ and the second follows from the induction hypothesis. Thus (ii) for $k = m+1$ is established, which completes the proof.

Tables 1,2 and 3 give the values of s^*_1, s^*_2, s^*_3 and the maximum probabilities of success of the 1-,2-,3-problems for various values of p and N. Tables 4 and 5 give the values of $s^*_4, \cdots, s^*_7$ and the maximum probabilities of success of the 4-,5-,6-,7-problems for specified values of p and N. Note that the probability of success for the m-problem can be given by $w_1^{(m)}$ for each $m = 1, 2, \cdots$.

Table 1. $p = 0.3$

N	s^*_1	s^*_2	s^*_3	$w_1^{(1)}$	$w_1^{(2)}$	$w_1^{(3)}$
2	1	1		0.1500000	0.2550000	
3	2	1	1	0.1500000	0.2050000	0.2295000
4	2	2	1	0.1375000	0.1900000	0.2080000
5	3	2	2	0.1300000	0.1862500	0.1985000
10	4	2	2	0.1196071	0.1686506	0.1860433
50	19	13	10	0.1122825	0.1576602	0.1745811
100	38	26	21	0.1113128	0.1563364	0.1731780
1000	369	259	211	0.1104587	0.1551576	0.1719363

Table 2. $p = 0.5$

N	s^*_1	s^*_2	s^*_3	$w_1^{(1)}$	$w_1^{(2)}$	$w_1^{(3)}$
2	1	1		0.2500000	0.3750000	
3	2	1	1	0.2500000	0.2916667	0.3125000
4	2	2	1	0.2291667	0.2916667	0.2968750
5	3	2	1	0.2166667	0.2812500	0.2916667
10	4	3	3	0.1993452	0.2547098	0.2678205
50	19	14	13	0.1871375	0.2392014	0.2512869
100	38	28	26	0.1855214	0.2371828	0.2493212
1000	369	286	259	0.1840978	0.2354182	0.2475618

Table 3. $p = 0.9$

N	s_1^*	s_2^*	s_3^*	$w_1^{(1)}$	$w_1^{(2)}$	$w_1^{(3)}$
2	1	1		0.4500000	0.4950000	
3	2	1	1	0.4500000	0.4650000	0.4650000
4	2	2	1	0.4125000	0.4350000	0.4353750
5	3	3	3	0.3900000	0.4035000	0.4036500
10	4	4	4	0.3588214	0.3787527	0.3793534
50	19	18	10	0.3368475	0.3539776	0.3545874
100	38	35	35	0.3339385	0.3510086	0.3516011
1000	369	350	349	0.3313761	0.3489551	0.3489711

Table 4. $p = 0.3$

N	s_4^*	s_5^*	s_6^*, s_7^*	$w_1^{(4)}$	$w_1^{(5)}$	$w_1^{(6)}$	$w_1^{(7)}$
5	2	2		0.1994	0.1996		
10	2	2	2	0.1897	0.1909	0.19107	0.19108
50	10	10	10	0.1799	0.1812	0.18147	0.18159
100	20	19	19	0.1786	0.1800	0.18025	0.18029
1000	191	183	180	0.1774	0.1788	0.17913	0.17920

Table 5. $p = 0.9$

N	s_4^*, s_5^*	s_6^*, s_7^*	$w_1^{(4)}, w_1^{(5)}$	$w_1^{(6)}, w_1^{(7)}$
5	3		0.40365	
10	4	4	0.37936	0.3793641
25	10	10	0.36025	0.3602451
50	18	18	0.35460	0.3546028
100	36	36	0.35162	0.3516163
1000	349	349	0.34897	0.3489715

3. Asymptotic Results

It is of interest to investigate the asymptotic behavior of s_m^*, when N tends to infinity. If we let $n/N \to x$ as $N \to \infty$, then, from (2.7), $A_n^{(m)}$ becomes an Riemann approximation to

$$A^{(m)}(x) = p(1 + \log x) + q \int_{\max(x,\bar{s}_{m-1}^*)}^{1} \frac{1}{y} A^{(m-1)}(y)dy, \tag{3.1}$$

where $\bar{s}_{m-1}^* = \lim_{N\to\infty} s_{m-1}^*/N$. Since $\bar{s}_m^*$ is the unique root $x \in (0, \bar{s}_{m-1}^*)$ of the equation $A^{(m)}(x) = 0$, we have

$$\bar{s}_m^* = \exp\{-(1 + \frac{q}{p} C^{(m)}(q))\}, \tag{3.2}$$

where

$$C^{(m)}(q) = \int_{\bar{s}^*_{m-1}}^{1} \frac{1}{y} A^{(m-1)}(y)dy, \ m \geq 2, \ (C^{(1)}(q) \equiv 0). \tag{3.3}$$

Then $\bar{s}^*_1 = \exp\{-1\}$ and

$$C^{(2)}(q) = \int_{e^{-1}}^{1} \frac{1}{y} p(1+\log y)dy = \frac{1}{2}p.$$

From (3.2), we get $\bar{s}^*_2 = \exp\{-(1+q/2)\}$. Substituting $A^{(1)}(x) = p(1+\log x)$ into (3.1) with $m = 2$, we have

$$A^{(2)}(x) = \begin{cases} p(1+\log x) + \frac{1}{2}pq, & 0 < x \leq \bar{s}^*_1 \\ p(1+p\log x) - \frac{1}{2}pq\log^2 x, & \bar{s}^*_1 \leq x < 1. \end{cases}$$

Applying the above expression to (3.3) for $m = 3$ yields

$$\begin{aligned} C^{(3)}(q) &= \int_{\bar{s}^*_2}^{\bar{s}^*_1} \frac{p}{y}(1+\log y + \frac{1}{2}q)dy + \int_{\bar{s}^*_1}^{1} \frac{p}{y}(1+p\log y - \frac{1}{2}q\log^2 y)dy \\ &= p(\frac{1}{2} + \frac{1}{3}q + \frac{1}{8}q^2). \end{aligned}$$

Thus from (3.2), we have

$$\bar{s}^*_3 = \exp\{-(1+\frac{q}{2}+\frac{q^2}{3}+\frac{q^3}{8})\}.$$

In a similar way, we have

$$\bar{s}^*_4 = \exp\{-(1+\frac{q}{2}+\frac{q^2}{3}+\frac{q^3}{4}+\frac{q^4}{6}+\frac{7q^5}{72}+\frac{q^6}{24}+\frac{q^7}{128})\}.$$

It is conjectured from Smith's result that as $m \to \infty$,

$$\bar{s}^*_m \to p^{1/q} = \exp\{-(\frac{-\log(1-q)}{q})\} = \exp\{-(1+\frac{q}{2}+\frac{q^2}{3}+\frac{q^3}{4}+\cdots)\}.$$

The limiting probability of success for the m-problem is expressed by $\bar{s}^*_1, \cdots, \bar{s}^*_m$ as follows.

Corollary 2. The limiting probability of success for the m -problem is given by

$$p\bar{s}^*_m + pq\bar{s}^*_{m-1} + pq^2\bar{s}^*_{m-2} + \cdots + pq^{m-1}\bar{s}^*_1.$$

Proof. We have from (2.1),(2.2) and (2.4)

$$v_n^{(m)} = \sum_{j=n+1}^{N} \frac{n}{j(j-1)} \max\{p\frac{j}{N} + qv_j^{(m-1)}, v_j^{(m)}\}. \tag{3.4}$$

If we let $n/N \to x$ as $N \to \infty$, then $v_n^{(m)}$ becomes a Riemann approximation to

$$v^{(m)}(x) = \begin{cases} a^{(m)}, & 0 < x \leq \bar{s}^*_m, \\ \int_x^1 \frac{x}{y^2}(py + qv^{(m-1)}(y))dy, & \bar{s}^*_m \leq x < 1, \end{cases}$$

where

$$a^{(m)} = v^{(m)}(0+) = v^{(m)}(\bar{s}_m^*) = \int_{\bar{s}_m^*}^1 \frac{\bar{s}_m^*}{y^2}(py + qv^{(m-1)}(y))dy. \tag{3.5}$$

On the other hand, $\bar{s}_m^*$ satisfies $A^{(m)}(\bar{s}_m^*) = 0$. Therefore as a Riemann approximation to the equation (2.5), we have

$$A^{(m)}(\bar{s}_m^*) = p + \frac{q}{\bar{s}_m^*}v^{(m-1)}(\bar{s}_m^*) - \int_{\bar{s}_m^*}^1 \frac{1}{y^2}(py + qv^{(m-1)}(y))dy = 0. \tag{3.6}$$

Thus substituting (3.5) into (3.6),

$$\frac{a^{(m)}}{\bar{s}_m^*} = p + \frac{q}{\bar{s}_m^*}v^{(m-1)}(\bar{s}_m^*).$$

Since $a^{(m-1)} = v^{(m-1)}(\bar{s}_m^*)$ for $\bar{s}_m^* \le \bar{s}_{m-1}^*$, we obtain

$$a^{(m)} = qa^{(m-1)} + p\bar{s}_m^*. \tag{3.7}$$

Staring $a^{(1)} = v^{(1)}(\bar{s}_1^*) = p\bar{s}_1^*$ and using (3.7) repeatedly, we reach the desired result.

We see from this corollary that

$$\begin{aligned} a^{(1)} &= pe^{-1}, \\ a^{(2)} &= pe^{-(1+q/2)} + pqe^{-1}, \\ a^{(3)} &= pe^{-(1+q/2+q^2/3+q^3/8)} + pqe^{-(1+q/2)} + pq^2e^{-1}, \end{aligned}$$

$$a^{(4)} = pe^{-(1+q/2+q^2/3+q^3/4+q^4/6+\cdots+q^7/128)} + pqe^{-(1+q/2+q^2/3+q^3/8)} + pq^2e^{-(1+q/2)} + pq^3e^{-1}.$$

Now because $\bar{s}_m^* \le \bar{s}_{m-1}^*$ and $v^{(m-1)}(\bar{s}_m^*) = v^{(m-1)}(\bar{s}_{m-1}^*)$ we have from equation (3.6)

$$p + \frac{q}{\bar{s}_m^*}v^{(m-1)}(\bar{s}_{m-1}^*) + p\log \bar{s}_m^* - q\{\int_{\bar{s}_m^*}^{\bar{s}_{m-1}^*} \frac{v^{(m-1)}(\bar{s}_{m-1}^*)}{y^2}dy + \int_{\bar{s}_{m-1}^*}^1 \frac{v^{(m-1)}(y)}{y^2}dy\} = 0.$$

Solving the above equation, we obtain as another expression for $\bar{s}_m^*$

$$\bar{s}_m^* = \exp\{-(1 + \frac{q}{p}\{\frac{v^{(m-1)}(\bar{s}_{m-1}^*)}{\bar{s}_{m-1}^*} - \int_{\bar{s}_{m-1}^*}^1 \frac{v^{(m-1)}(y)}{y^2}dy\})\}. \tag{3.8}$$

4. When the acceptance probability is not constant.

Here we consider the case where the acceptance probability is not constant. In this case, it may occur that an optimal strategy makes an offer to a non-can didate for some values of p_m. Thus, to describe the evolution of the process completely, we must introduce, in addition to state (n, m), additional state $< n, m >$, where we confront the m-problem and observe that the nth applicant is a non-candidate. Let $\tilde{w}_n^{(m)}$, $\tilde{u}_n^{(m)}$ and $\tilde{v}_n^{(m)}$ be defined for state $< n, m >$, as quantities corresponding to $w_n^{(m)}, u_n^{(m)}$ and $v_n^{(m)}$. Then letting $q_m = 1 - p_m$, we have, from the principle of optimality

$$\begin{aligned} w_n^{(m)} &= \max\{u_n^{(m)}, v_n^{(m)}\}, \\ \tilde{w}_n^{(m)} &= \max\{\tilde{u}_n^{(m)}, \tilde{v}_n^{(m)}\}, \qquad 2 \le n \le N, \\ u_n^{(m)} &= p_m\frac{n}{N} + q_m v_n^{(m-1)}, \\ \tilde{u}_n^{(m)} &= q_m v_n^{(m-1)}, \\ v_n^{(m)} &= \tilde{v}_n^{(m)} = \frac{1}{n+1}w_{n+1}^{(m)} + \frac{n}{n+1}\tilde{w}_{n+1}^{(m)}. \end{aligned}$$

Since $u_n^{(m)} \ge \tilde{u}_n^{(m)}$, it is easy to see that

(i) If it is optimal to make an offer in state $< n,m >$, it is also optimal to make an offer in state (n,m).

(ii) If it is optimal to make no offer in state (n,m), it is also optimal to make no offer in state $< n,m >$.

We give some numerical examples for the 3-problem assuming $N = 100$. To describe the optimal offering region, we use the following notations.

$$\begin{aligned} G_m &= \{(n,m) : u_n^{(m)} \geq v_n^{(m)}\}, \quad 1 \leq m \leq 3, \\ \tilde{G}_m &= \{< n,m >: \tilde{u}_n^{(m)} \geq \tilde{v}_n^{(m)}\}, \quad 1 \leq m \leq 3. \end{aligned}$$

Example 1 $(p_1 = 0.9,\ p_2 = 0.6,\ p_3 = 0.3)$

$$\begin{aligned} &G_1 = \{(1,n) : n \geq 38\}, G_2 = \{(2,n) : n \geq 28\}, \\ &G_3 = \{(3,n) : n \geq 14\}, \tilde{G}_m,\ 1 \leq m \leq 3,\ \text{are empty}, \\ &w_1^{(3)} = 0.248302. \end{aligned}$$

Example 2 $(p_1 = 0.3,\ p_2 = 0.6,\ p_3 = 0.9)$

$$\begin{aligned} &G_1 = \{(1,n) : n \geq 38\}, G_2 = \{(2,n) : n \geq 34\}, \\ &G_3 = \{(3,n) : n \geq 36\}, \tilde{G}_m,\ 1 \leq m \leq 3,\ \text{are empty}, \\ &w_1^{(3)} = 0.346008. \end{aligned}$$

Example 1 treats the case where p_i is decreasing, while Example 2 treats the case where p_i is increasing. The optimal strategy for Example 2 can be summarized as follows : pass over the first 35 applicants and then give an offer to a candidate successively. The next example gives a case where the set $\tilde{G}_m$ is not empty. It seems that such a thing can occur where the values of p_2 or p_3 are small compared with the value of p_1.

Example 3 $(p_1 = 0.9,\ p_2 = p_3 = 0.1)$

$$\begin{aligned} G_1 &= \{(1,n) : n \geq 38\}, \quad \tilde{G}_1 \text{ is empty}, \\ G_2 &= \{(2,n) : n \geq 15\}, \quad \tilde{G}_2 = \{< 2,n >: 41 \leq n \leq 99\}, \\ G_3 &= \{(3,n) : n \geq 10\}, \quad \tilde{G}_3 = \{< 3,n >: 41 \leq n \leq 98\}, \\ w_1^{(3)} &= 0.293058. \end{aligned}$$

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