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AN OPTIMAL INSPECTION POLICY FOR A STORAGE SYSTEM WITH THREE TYPES OF HAZARD RATE FUNCTIONS

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Abstract A system such as missiles and spare parts of aircrafts has to perform a normal operation in a severe environment at any time when it is used. However, the system is in storage for a long time from the transportation to the usage and its reliability goes down with time. Thus, a system in storage should be inspected and maintained at a periodic time to hold a high reliability. But, we do not wish to inspect a system too often because each inspection involves a cost and sometimes degrades it. Therefore, we have to establish an optimal inspection policy for such a storage system.

This paper considers a periodic inspection policy for a storage system with unit 1 and unit 2: Unit 1 is inspected and maintained at each inspection, however, unit 2 is degraded with time and at each inspection. Three hazard rate functions of the system are defined and its reliability is derived. Using this results, the mean time to failure and the expected number of inspections before failure are computed. Further, the expected total cost until the detection of failure is obtained and an optimal policy to minimize it is discussed. Numerical examples are finally given.

1. Introduction

A system such as missiles and spare parts of aircrafts is in storage for a long time from the transportation to the usage and has to keep a high mission reliability at any time when it is used. However, its reliability goes down with time[1-4] and it is impossible to check whether a system can operate normally or not. We need to inspect and maintain a system in storage at a suitable time to hold a high reliability[5-8]. But, we do not wish to inspect a system too often because each inspection involves a cost and sometimes degrades it[5]. Therefore, we have to establish an optimal inspection policy for such a storage system.

Barlow and Proschan[9] summarized the optimal inspection policy which minimizes the expected total cost until the detection of failure. Luss and Kander[10] and Zacks and Fenske[11] extended to much more complicated systems. Shima and Nakagawa[12] discussed the inspection of a machine with protective device. Martinez[13] considered the periodic test of an electronic equipment in storage for a long period, and showed how to compute its reliability after 10 years of storage.

We have already considered the inspection policies for a storage system which is required to have a higher reliability than a prespecified value q[14]. To keep its reliability, a system is inspected and maintained at a periodic time and overhauled if the reliability becomes lower than q.

Usual storage units are degraded gradually by the power on-off cycles during inspection interval[5]. This paper considers periodic inspections of a storage system with two units, where unit 1 is checked and maintained at time NT ($N = 1, 2, \cdots$), and unit 2 is degraded with time and at each inspection. For such a generalized inspection model, we derive the following two reliability quantities: (i) The mean time to system failure and (ii) the expected number of inspections before failure.

Using these quantities, we obtain the expected total cost $C(T)$ until the detection of failure and discuss an optimal inspection time T^* which minimizes $C(T)$. Numerical examples are given when reliability functions are exponential and Weibull ones.

2. Analysis of model

Consider the following inspection policy for a storage system which has to operate when it is used at any time :

1) The system is new at time 0, and it is checked and maintained if necessary at periodic time NT $(N = 1, 2, \cdots)$, where $T(> 0)$ is constant and previously specified.
2) The system is mainly consisted of two independent units, where unit 1 is like new after every inspection, however, unit 2 does not become like new and is degraded with time and at each inspection. That is, unit 1 is a general term for some parts in the system whose functions can be certified by inspections, and unit 2 is for the other parts whose functions can not be done. As a typical example of a storage system, we give a schematic diagram of missiles in Figure 1.
3) Unit 1 has a hazard rate function h_1, which is given by $h_1(t - NT)$ for $NT < t \leq (N + 1)T$, because it is like new at time NT.
4) Unit 2 has two hazard rate functions h_2 and h_3, which are the hazard rates of system degradations with time and at each inspection, respectively. The hazard rate $h_2(t)$ remains undisturbed by any inspection. Further, since unit 2 is degraded by the power on-off cycles during this interval[5], h_3 increases by the constant rate λ_3 at each inspection, and is defined as

$$h_3(t) = N\lambda_3,$$

for $NT < t \leq (N + 1)T$.

5) The hazard rate function $h(t)$ of the system is, from 3) and 4),

$$h(t) = h_1(t - NT) + h_2(t) + N\lambda_3, \tag{2.1}$$

for $NT < t \leq (N + 1)T$.

Under the assumptions above, we obtain the probability that the system does not fail until time t, i.e., the reliability of the system at time t.

The cumulative hazard function $H(t)$ of the system is, from (2.1),

$$\begin{aligned} H(t) &\equiv \int_0^t h(u)du \\ &= NH_1(T) + H_1(t - NT) + H_2(t) \\ &\quad + \sum_{j=0}^{N-1} j\lambda_3 T + N\lambda_3(t - NT), \end{aligned} \tag{2.2}$$

for $NT < t \leq (N + 1)T$, where $H_i(t) \equiv \int_0^t h_i(u)du \quad (i = 1, 2)$. Thus, the reliability $R(t)$ of the system at time t is

$$\begin{aligned} R(t) &= \exp[-H(t)] \\ &= \exp\left[-NH_1(T) - H_1(t - NT) - H_2(t) - N\lambda_3\left(t - \frac{N+1}{2}T\right)\right]. \end{aligned} \tag{2.3}$$

for $NT < t \leq (N + 1)T \quad (N = 0, 1, 2, \cdots)$.

Using this result, we obtain the mean time $\gamma(T)$ to system failure and the expected number $M(T)$ of inspections before failure.

The mean time $\gamma(T)$ is, from (2.3),

$$\gamma(T) = \int_0^\infty R(t)dt$$
$$= \sum_{N=0}^{\infty} \exp\left[-NH_1(T) - \frac{N(N-1)}{2}\lambda_3 T\right] \times \int_0^T \exp[-H_1(t) - H_2(t+NT) - N\lambda_3 t]dt\,. \tag{2.4}$$

It is evident that

$$\lim_{T\to\infty} \gamma(T) = \int_0^\infty \exp[-H_1(t) - H_2(t)]dt\,. \tag{2.5}$$

which represents the mean time in the case where the system is not inspected at all.

The expected number of inspections before failure is

$$M(T) = \sum_{N=1}^{\infty} N\{R(NT) - R[(N+1)T]\}$$
$$= \sum_{N=1}^{\infty} \exp\left[-NH_1(T) - H_2(NT) - \frac{N(N-1)}{2}\lambda_3 T\right] \tag{2.6}$$

Evidently,

$$\lim_{T\to 0} M(T) = \infty\,, \tag{2.7}$$

$$\lim_{T\to\infty} M(T) = 0\,. \tag{2.8}$$

Further, we have, from (2.4),

$$\gamma(T) \geq \sum_{N=0}^{\infty} \exp\left[-NH_1(T) - \frac{N(N-1)}{2}\lambda_3 T\right] \times T\exp\{-H_1(T) - H_2[(N+1)T] - N\lambda_3 T\}$$
$$= TM(T)\,, \tag{2.9}$$

$$\gamma(T) \leq \sum_{N=0}^{\infty} \exp\left[-NH_1(T) - \frac{N(N-1)}{2}\lambda_3 T\right] \times T\exp[-H_2(NT)]$$
$$= T[M(T)+1]\,, \tag{2.10}$$

since cumulative hazard functions are increasing in time t. Thus, we have the inequality

$$TM(T) \leq \gamma(T) \leq T[M(T)+1]\,. \tag{2.11}$$

which is equal to the result in [15].

We have the following three particular cases:

(i) $h_2 = \lambda_3 = 0$

The system is like new after every inspection. Then,

$$\gamma(T) = \frac{\int_0^T \exp[-H_1(t)]dt}{1 - \exp[-H_1(T)]}\,, \tag{2.12}$$

$$M(T) = \frac{\exp[-H_1(T)]}{1 - \exp[-H_1(T)]}\,. \tag{2.13}$$

which are equal to (3) and (6) in [15], respectively.

(ii) $h_1 = \lambda_3 = 0$

The system is not changed at any inspection. Then,

$$\gamma(T) = \int_0^\infty \exp[-H_2(t)]dt\,, \tag{2.14}$$

$$M(T) = \sum_{N=1}^{\infty} \exp[-H_2(NT)]\,. \tag{2.15}$$

(iii) $h_1 = h_2 = 0$

The system is degraded at each inspection by the power on-off cycles. Then,

$$\gamma(T) = T + \sum_{N=1}^{\infty} \frac{1}{N\lambda_3}\left\{\exp\left[-\frac{N(N-1)}{2}\lambda_3 T\right] - \exp\left[-\frac{N(N+1)}{2}\lambda_3 T\right]\right\}\,. \tag{2.16}$$

$$M(T) = \sum_{N=1}^{\infty} \exp\left[-\frac{N(N-1)}{2}\lambda_3 T\right]\,. \tag{2.17}$$

3. Optimal inspection policy

We introduce the inspection costs: A cost c_1 is required for one inspection and a cost c_2 is for time elapsed between failure and its detection per unit time. Having assumed that system failure is detected only by inspection, the expected total cost until the detection of failure is, from (14) in [15],

$$C(T) = (c_1 + c_2 T)[M(T) + 1] - c_2\gamma(T)\,, \tag{3.1}$$

where $\gamma(T)$ and $M(T)$ are given by (2.4) and (2.6), respectively.

We easily have, from (2.11),

$$C(T) \geq c_1\frac{\gamma(T)}{T} \geq c_1 M(T)\,.$$

Thus, from (2.5), (2.7) and (2.8),

$$\lim_{T\to 0} C(T) = \lim_{T\to\infty} C(T) = \infty\,. \tag{3.2}$$

Therefore, there exists a positive and finite T^* which minimizes the expected cost $C(T)$ in (3.1).

Consider two particular cases where the hazard rates functions are exponential and Weibull ones.

3.1 Exponential case

When $h_i(t) = \lambda_i (i = 1, 2)$, the expected cost is

$$\begin{aligned} C(T) &= (c_1 + c_2 T)\sum_{N=0}^{\infty} \exp\left[-\left(\lambda_1 + \lambda_2 + \frac{N-1}{2}\lambda_3\right)NT\right] \\ &\quad - c_2 \sum_{N=0}^{\infty} \frac{1 - \exp[-(\lambda_1 + \lambda_2 + N\lambda_3)T]}{\lambda_1 + \lambda_2 + N\lambda_3} \\ &\quad \times \exp\left[-\left(\lambda_1 + \lambda_2 + \frac{N-1}{2}\lambda_3\right)NT\right] \end{aligned} \tag{3.3}$$

In particular case of $\lambda_3 = 0$, i.e., the system is not degraded at each inspection,

$$C(T) = \frac{c_1 + c_2 T}{1 - e^{-(\lambda_1+\lambda_2)T}} - \frac{c_2}{\lambda_1 + \lambda_2}, \tag{3.4}$$

which is equal to (19) in [15]. In this case, there exists a unique T^* which satisfies

$$\exp[(\lambda_1 + \lambda_2)T] - [1 + (\lambda_1 + \lambda_2)T] = \frac{(\lambda_1 + \lambda_2)c_1}{c_2}. \tag{3.5}$$

Further, in the case of $\lambda_1 = \lambda_2 = 0$,

$$\begin{aligned} C(T) &= (c_1 + c_2 T) \sum_{N=0}^{\infty} \exp\left[-\frac{N(N-1)}{2}\lambda_3 T\right] \\ &- c_2 \left\{T + \sum_{N=1}^{\infty} \frac{1}{N\lambda_3}\left\{\exp\left[-\frac{N(N-1)}{2}\lambda_3 T\right] - \exp\left[-\frac{N(N+1)}{2}\lambda_3 T\right]\right\}\right\}. \end{aligned} \tag{3.6}$$

To minimize $C(T)$, we put its derivative equal to zero. Then, we have

$$\frac{\sum_{N=0}^{\infty} \exp\left[-\frac{N(N-1)}{2}\lambda_3 T\right]}{\sum_{N=0}^{\infty} N(N-1)\lambda_3 \exp\left[-\frac{N(N-1)}{2}\lambda_3 T\right]} - T = \frac{c_1}{c_2}. \tag{3.7}$$

An optimal time T^* is given by a solution of (3.7). since equation (3.7) is a function from zero to infinity. It can be easily seen that when $\lambda_3 = \lambda_1 + \lambda_2$, the expected cost becomes $C(T) - c_1$, where $C(T)$ is given by (3.6). Thus, an optimal time T^* which satisfies (3.7), is also equal to that of the case $\lambda_3 = \lambda_1 + \lambda_2$.

3.2 Weibull case

When $H_i(t) = \lambda_i t^m (i = 1, 2)$, the expected cost is

$$\begin{aligned} C(T) &= (c_1 + c_2 T) \sum_{N=0}^{\infty} \exp\left[-N\lambda_1 T^m - \lambda_2 (NT)^m - \frac{N(N-1)}{2}\lambda_3 T\right] \\ &- c_2 \sum_{N=0}^{\infty} \exp\left[-N\lambda_1 T^m - \frac{N(N-1)}{2}\lambda_3 T\right] \\ &\times \int_0^T \exp[-\lambda_1 t^m - \lambda_2 (t + NT)^m - N\lambda_3 t]dt. \end{aligned} \tag{3.8}$$

Changing T, we can numerically obtain an optimal time T^* which minimizes $C(T)$.

4. Numerical example

Note from [5] that the degradation hazard rate λ_3 at each inspection is given by

$$\lambda_3 = N_c K \lambda_{SE},$$

where N_c is the ratio of total cycles to inspection time, K is the ratio of cyclic hazard rate to storage hazard rate, λ_{SE} is the storage hazard rate of electronic parts, and their numerical values are $N_c = 2.3 \times 10^{-4}$, $K = 270$, $\lambda_{SE} = 14.88 \times 10^{-6}/hour$ from [5]. Hence, $\lambda_3 = 9.24 \times 10^{-7}/hour$.

First, consider the case where $H_i(t) = \lambda_i t$ $(i = 1, 2)$. When $\lambda_3 = 0$. Table 1 gives the optimal time T^* which satisfies equation (3.5) and the resulting cost $C(T^*)$ for $\lambda = \lambda_1 + \lambda_2 = 29.24 \times 10^{-6}$, 58.48×10^{-6}, $c_1 = 10, 15, 20, 25, 30$ and $c_2 = 1$, where λ is given from [6]. This shows that both T^* and $C(T^*)$ increase when c_1 and $1/\lambda$ increase, and that the system should be inspected once about a month.

Table 2 gives the optimal time T^* which satisfies (3.7) and $C(T^*)$ for $c_1 = 10, 15, 20, 25, 30$ and $c_2 = 1$, when $\lambda_1 = \lambda_2 = 0$ and $\lambda_3 = 9.24 \times 10^{-7}, 18.48 \times 10^{-7}/hour$. This shows that both T^* and $C(T^*)$ increase when c_1 and $1/\lambda_3$ increase.

Next, consider the case where $H_i(t) = \lambda_i t^m$ $(i = 1,2)$, $\lambda_3 = 9.24 \times 10^{-7}/hour$ and $a = 0.9$. Suppose that $\lambda_1 = a\lambda$ and $\lambda_2 = (1-a)\lambda$ $(0 < a < 1)$, where λ is the storage hazard rate of the system and a is the efficiency of inspection. Table 3 gives the optimal time T^* and the resulting cost $C(T^*)$ for $c_1 = 10, 15, 20, 25, 30$, $c_2 = 1$ and $m = 1.0, 1.2$, when $\lambda = 29.24 \times 10^{-6}, 58.48 \times 10^{-6}/hour$. This shows the same tendency as Tables 1 and 2, however, T^*'s are shorter than those in Tables 1 and 2 because the probability of system failure is greater than that of the cases in Tables 1 and 2. It is of interest that T^*'s in the case of $m = 1.2$ are much shorter than those in $m = 1.0$ because the system deteriorates with time. Figure 2 draws the optimal time T^* for variables λ_3 when $\lambda = 29.24 \times 10^{-6}$ and $m = 1.0$, and shows clearly that T^* decreases with λ_3.

5. Conclusion

We have considered the optimal inspection policy for a storage system which is degraded at inspections. The mean time to system failure and the expected number of inspections before failure have been obtained. Using these results, the expected total cost until the detection of failure has been derived. Numerical examples have been given when the hazard rate functions are exponential and Weibull ones. These examples have shown that the system should be inspected much early, compared with the case of no degradation at inspections and that its optimal inspection time decreases with the hazard rates.

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Table 1. Optimal inspection time T^* and its associated expected cost $C(T^*)$ when $\lambda_3 = 0$ and $c_2 = 1$

λ	c_1	T^*	$C(T^*)$
29.24×10^{-6}	10	824	834
	15	1009	1023
	20	1164	1183
	25	1300	1324
	30	1423	1453
58.48×10^{-6}	10	582	592
	15	712	726
	20	821	841
	25	917	941
	30	1004	1033

Table 2. Optimal inspection time T^* and its associated expected cost $C(T^*)$ when $\lambda_1 = \lambda_2 = 0$ and $c_2 = 1$

λ_3	c_1	T^*	$C(T^*)$
9.24×10^{-7}	10	570	843
	15	749	1101
	20	898	1343
	25	1039	1560
	30	1171	1764
18.48×10^{-7}	10	449	672
	15	586	882
	20	709	1065
	25	820	1244
	30	922	1407

Table 3. Optimal inspection time T^* and its associated expected cost $C(T^*)$ when $\lambda_3 = 9.24 \times 10^{-7}$, $c_2 = 1$ and $a = 0.9$.

m	λ	c_1	T^*	$C(T^*)$
1.0	29.24×10^{-6}	10	510	603
		15	670	764
		20	800	903
		25	920	1027
		30	1020	1140
1.0	58.48×10^{-6}	10	460	490
		15	590	613
		20	680	718
		25	790	811
		30	840	897
1.2	29.24×10^{-6}	10	430	377
		15	540	461
		20	630	531
		25	670	593
		30	760	648
1.2	58.48×10^{-6}	10	350	286
		15	390	347
		20	470	399
		25	510	445
		30	550	487

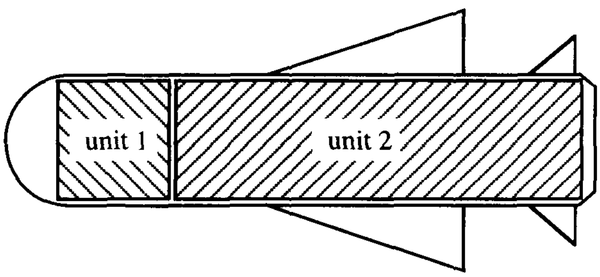

Figure 1. Schematic diagram of a storage system

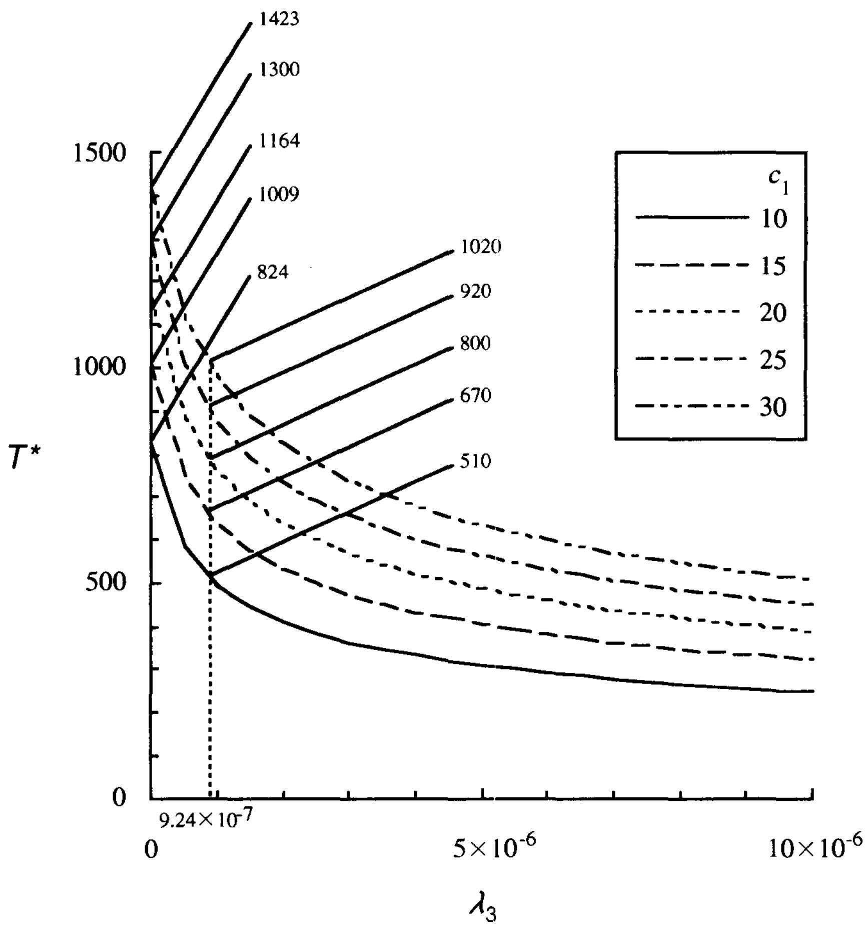

Figure 2. Optimal inspection time T^* for λ_3 when $\lambda = 29.24\times10^{-6}$, $c_2 = 1$ and $m = 1.0$.