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PERFORMANCE EVALUATION OF COMPUTER SYSTEM WITH FAILURE BASED ON FUZZY SET THEORY

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Abstract The purpose of this paper is to present the performance evaluation of computer system with failure based on fuzzy set theory. The performance evaluation of on-line computer system with failure is fuzzified. We represent the performance measures of the on-line computer system by modeling occurrences of failed computer equipment units based on fuzzy set theory. We are forced to employ subjective possibilities or probabilities when there is no information about a model or some parameters of a model are vague. The information may be obtained subjectively from experts or very little data. We present the fuzzification model and the performance evaluation of on-line computer system with failure based on fuzy set theory. Also, it is shown that this performance evaluation is more flexible and realistic than crisp performance evaluation of conventional method in some numerical examples.

1 Introduction

Computer systems may be failed due to software faults or hardware failures. Software faults are due to error in programming whereas hardware failures are associated with the computer electronic, electrical and mechanical elements. Computer failures occur due to various causes. The major sources of system failures are human errors, power failures, processor and memory failures, communication network failures, peripheral device failures, and mysterious failures etc.[4]. In this paper we consider the on-line computer system. And we does not consider the failures of host computer system. That is, we only consider the terminal failures and the peripheral device failures of terminal. The purpose of this paper is to present the performance evaluation of on-line computer system with failures based on fuzzy set theory.

The performance evaluation of on-line computer system deals with the data which are fuzzy, vague and full of human subjectivity because the environment of the real world is frequently fuzzy. The conventional methods which are used very widely are based upon probabilistic theory, where the probability is expressed in terms of the statistical information of its components. On some real situation, we may sometimes obtain the statistical information subjectively. Particularly the average occurrence rate and the average repair rate of failed computer equipment units can be obtained from experts and it is subjective and fuzzy in this paper.

If we analyze a problem with a crisp values without considering this vagueness, it may be that we spoil some information of a actual problem and we get the fantastic results. Therefore, we will present the fuzzification model and the performance evaluation of on-line computer system with failure based on fuzzy set theory. Also, it is shown that this performance evaluation is more flexible and realistic than crisp that of conventional method in some numerical examples.

There are various types of fuzzy number to express a vagueness[10], the triangular fuzzy number(TFN) among these types is appropriate to express the fuzzy average occurrence rate and the fuzzy average repair rate of failed computer equipment units in this model because a TFN is represented by a triplet, *i.e.*, a minimum value, a modal value and a maximum value. In this model, to use a trapezoidal fuzzy number is seldom appropriate to express the fuzzy average occurrence rate and the fuzzy average repair rate of failed computer equipment units because the modal value of it has a width.

In computer systems, the queueing theory is quite useful to estimate the value of a computer performance evaluation. Two examples of such evaluation are a size of buffer storage requirements at message switching centers, and waiting time to use an on-line terminal[1][2]. Yamazaki, Kobayashi, Takagi, and Wolff[13] represented the performance analysis of a slotted ring local area network by a single-buffer model. Recently, Jo, Tsujimura, Gen, and Yamazaki[6] represented the delay analysis of queueing model for data network using fuzzy set theory. And they represented the characteristic and performance of open central server network model and the closed single class BCMP(Baskett, Chandy, Muntz, and Palacios) network model based on fuzzy set theory[7][8].

2 Preliminaries

Let us introduce the notation needed in the rest of the paper. All our fuzzy numbers will be triangular shaped. We will place a tilde over a capital letter if it represents a TFN so $\tilde{A}$, $\tilde{B}$, $\tilde{C}$, $\tilde{X}$, $\tilde{N}$, and $\tilde{N}_Q$ are all TFNs. We use fuzzy numbers which are characterized by α-cuts and TFNs. The reason for using these characterizations is the simplicity and flexibility of these two concepts: the intervals of confidence and the triangular shapes. It is well known in fuzzy approach that the intervals of confidence associated with monotonic functions can be very convenient. The TFN is extensively used in modeling of systems because fuzzy numbers of this type are very simple to treat.

Let $\tilde{A}$ be a TFN with membership function $\mu_{\tilde{A}}(x)$. We represent an interval of confidence of TFN $\tilde{A}$ for all α-cut level, that is, $A(\alpha) = [a_L{}^{(\alpha)}, a_R{}^{(\alpha)}]$ for $\alpha \in [0,1]$. If $\alpha = 1$, $a_L{}^{(\alpha)}$ is equal to $a_R{}^{(\alpha)}$. And the TFN $\tilde{A}$ is specified by triplet (a_1, a_2, a_3) as follows:

(1) $a_1 < a_2 < a_3$;
(2) $\mu_{\tilde{A}}(x) = 0$ outside (a_1, a_3) and equals one at a_2;
(3) $\mu_{\tilde{A}}(x)$ is continuous and monotonically increasing from zero to one on $[a_1, a_2]$;
(4) $\mu_{\tilde{A}}(x)$ is continuous and monotonically decreasing from one to zero on $[a_2, a_3]$.

For any TFN $\tilde{A}$, we can write as follows:

(1) $\tilde{A} \geq c$ if $a_1 \geq c$;
(2) $\tilde{A} > c$ if $a_1 > c$;
(3) $\tilde{A} \geq c$ if $a_3 \geq c$;
(4) $\tilde{A} < c$ if $a_3 < c$,

where c is a real number constant. The support of $\tilde{A}$ is (a_1,a_3). Comparisons of two TFNs are assumed to be made by Kaufmann and Gupta[9] methodology using linear ordering of fuzzy numbers. In this paper, addition, subtraction, multiplication and division of fuzzy numbers are used to the intervals of confidence for α-cut level[9].

Definition 1: For TFNs $\tilde{A}_1 = (a_{11}, a_{12}, a_{13})$, $\tilde{A}_2 = (a_{21}, a_{22}, a_{23})$, $\ldots$, $\tilde{A}_n = (a_{n1}, a_{n2}, a_{n3})$, we define an average TFN $\tilde{A}$ as follows:

$$\tilde{A} = \left(\frac{a_{11} + a_{21} + \cdots + a_{n1}}{n}, \frac{a_{12} + a_{22} + \cdots + a_{n2}}{n}, \frac{a_{13} + a_{23} + \cdots + a_{n3}}{n}\right). \quad (2.1)$$

Dubois and Prade[5] and Zimmermann[14] have defined the fuzzifying function and its integration over a crisp interval as follows:

Definition 2: Let X and Y be universes and $P(Y)$ be the set of all fuzzy sets in Y(power set).

$$\tilde{f} : X \rightarrow \tilde{P}(Y) \text{ is a mapping.}$$
$$\tilde{f} \text{ is a fuzzifying function, iff}$$
$$\mu_{\tilde{f}(x)}(y) = \mu_{\tilde{R}}(x, y), \ \forall (x, y) \in X \times Y,$$

where $\mu_{\tilde{R}}(x,y)$ is the membership function of a fuzzy relation.

Definition 3: Let $\tilde{f}(x)$ be a fuzzifying function from $[a, b] \subseteq R$ to R, such that, $\tilde{f}(x)$, $\forall x \in [a, b]$ is a fuzzy number. And α-level curves $f_\alpha^-(x)$ and $f_\alpha^+(x)$ are the curves which values of membership function of $\tilde{f}(x)$ are $\mu_{\tilde{f}(x)}(f_\alpha^-(x)) = \mu_{\tilde{f}(x)}(f_\alpha^+(x)) = \alpha$ respectively. The integral of $\tilde{f}(x)$ over $[a, b]$ is then defined to be the fuzzy set as follows:

$$\int_a^b \tilde{f}(x)dx = \left\{ \left(\left[\int_a^b f_\alpha^-(x)dx, \int_a^b f_\alpha^+(x)dx \right], \alpha \right) \right\}. \tag{2.2}$$

Definition 4: Let $\tilde{f}(x)$ be a fuzzifying function from $[a,b] \subseteq R$ to R, such that $\forall x \in [a,b]$ $\tilde{f}(x)$ is a fuzzy number. We assume that all α-level curves f_α defined by Definition 3 are differentiable for all $x \in [a, b]$. The differentiation of fuzzifying function $\tilde{f}(x)$ at real point x_0 is then defined as follows:

$$\mu_{\frac{\partial \tilde{f}(x_0)}{\partial x}}(y) = \sup_{y \in \frac{\partial f_\alpha(x_0)}{\partial x}} \mu_{\tilde{f}}(f_\alpha). \tag{2.3}$$

Sanchez[12] presented necessary and sufficient conditions for $\tilde{A} + \tilde{X} = \tilde{C}$ and $\tilde{A} \cdot \tilde{X} = \tilde{C}$ to have solutions for $\tilde{X}$, when $\tilde{A}$ and $\tilde{C}$ are arbitrary fuzzy subsets of the real numbers. The author represented solutions of fuzzy equations using extended operations.

Theorem 1: The fuzzy equation $\tilde{A} + \tilde{X} = \tilde{C}$ has a solution $\tilde{X}$ iff $c_1 - a_1 < c_2 - a_2 < c_3 - a_3$. If there is a solution, then $\tilde{X}$ is the TFN $(c_1 - a_1, c_2 - a_2, c_3 - a_3)$.

Proof. Taking α-cuts we obtain $a_L{}^{(\alpha)} + x_L{}^{(\alpha)} = c_L{}^{(\alpha)}$ and $a_R{}^{(\alpha)} + x_R{}^{(\alpha)} = c_R{}^{(\alpha)}$. Then $x_1 < x_2 < x_3$ and $1/(x_2 - x_1) > 0$, $1/(x_2 - x_3) < 0$ iff $c_1 - a_1 < c_2 - a_2 < c_3 - a_3$.

Definition 5: Using Theorem 1, in order to obtain a solution $\tilde{X}$ for a fuzzy equation $\tilde{A} + \tilde{X} = \tilde{C}$, we define the operator $\ominus$ as follows:

$$\tilde{C} \ominus \tilde{A} = [c_L{}^{(\alpha)} - a_L{}^{(\alpha)}, c_R{}^{(\alpha)} - a_R{}^{(\alpha)}], \ \forall \alpha \in [0,1]. \tag{2.4}$$

Theorem 2: We assume that $\tilde{A} > 0$ and $\tilde{C} \geq 0$. Then there is a solution $\tilde{X}$ of the fuzzy equation $\tilde{A} \cdot \tilde{X} = \tilde{C}$ iff $a_1 c_2 > a_2 c_1$ and $a_3 c_2 < a_2 c_3$ when $\tilde{A} > 0, \tilde{C} \geq 0$.

Proof. Taking α-cuts we obtain $x_L{}^{(\alpha)} = c_L{}^{(\alpha)}/a_L{}^{(\alpha)}$, $x_R{}^{(\alpha)} = c_R{}^{(\alpha)}/a_R{}^{(\alpha)}$ when $\tilde{A} > 0$, $\tilde{C} \geq 0$. We obtain $1/(x_2 - x_1) > 0$ and $1/(x_2 - x_3) < 0$, iff the conditions stated above hold.

Definition 6: We assume that $\tilde{A} > 0$ and $\tilde{C} \geq 0$. Using Theorem 2, in order to obtain a solution $\tilde{X}$ for a fuzzy equation $\tilde{A} \cdot \tilde{X} = \tilde{C}$, We define the operator $\oplus$ as follows:

$$\tilde{C} \oplus \tilde{A} = \left[\frac{c_L{}^{(\alpha)}}{a_L^{(\alpha)}}, \frac{c_R{}^{(\alpha)}}{a_R^{(\alpha)}} \right], \quad \forall \alpha \in [0,1] \tag{2.5}$$

$$\text{iff } \tilde{A} > 0, \tilde{C} > 0, \qquad a_1 c_2 > c_1 a_2 \text{ and } a_3 c_2 > c_3 a_2.$$

Theorem 3: We assume that $\tilde{A} > 0$, $\tilde{C} \geq 0$. Then there is a solution $\tilde{X}$ of the fuzzy equation $\tilde{X} \oplus \tilde{A} = \tilde{C}$ iff $c_1 a_3 < c_2 a_1 < c_3 a_1$.

Proof. Taking α-cuts we obtain $x_L{}^{(\alpha)} = c_L{}^{(\alpha)} a_R{}^{(\alpha)}, x_R{}^{(\alpha)} = c_R{}^{(\alpha)} a_L{}^{(\alpha)}$ when $\tilde{A} > 0$, $\tilde{C} \geq 0$. We obtain $1/(x_2 - x_1) > 0$ and $1/(x_2 - x_3) < 0$, iff the condition stated above hold.

Theorem 4: We assume that $\tilde{A} < 1$ and $\tilde{C} \geq 0$. Then there is a solution $\tilde{X}$ of the fuzzy equation $\tilde{X} \div (1 - \tilde{A}) = \tilde{C}$ iff $(1 - a_1)c_1 < (1 - a_2)c_2 < (1 - a_3)c_3$ when $\tilde{A} > 0$ and $\tilde{C} \geq 0$.

Proof. Taking α-cuts we obtain $x_L{}^{(\alpha)} = (1 - a_L{}^{(\alpha)})c_L{}^{(\alpha)}$, $x_R{}^{(\alpha)} = (1 - a_R{}^{(\alpha)})c_R{}^{(\alpha)}$ when $\tilde{A} > 0$, $\tilde{C} \geq 0$. We obtain $1/(x_2 - x_1) > 0$ and $1/(x_2 - x_3) < 0$, iff the condition stated above hold.

By Theorem 4, we obtain a fuzzy number solution $\tilde{X}$ of the fuzzy equation $\tilde{X} \div (1 - \tilde{A}) = \tilde{C}$ and approximate α-cut solution by a approximate TFN, that is,

$$\tilde{X} = (1 \ominus \tilde{A}) \cdot \tilde{C} \tag{2.6}$$

$$\text{iff } (1 - a_1)c_1 < (1 - a_2)c_2 < (1 - a_3)c_3 \text{ when } \tilde{A} > 0 \text{ and } \tilde{C} \geq 0.$$

The following expressions are obtained by above definitions:

$$\tilde{A} \oplus (1 \ominus \tilde{B}) = \frac{\tilde{A}}{1 - \tilde{B}}$$

$$\tilde{A} \oplus (1 \oplus \tilde{B}) = \tilde{A} - \frac{1}{\tilde{B}}$$

$$(\tilde{A} \ominus \tilde{B}) \oplus (\tilde{C} \ominus \tilde{D}) = \frac{\tilde{A} \ominus \tilde{B}}{\tilde{C} - \tilde{D}}$$

In general, the approximation becomes worse according to iteration of the multiplication and the division of TFNs. That is, these operations for TFNs in a computational process may give rise to an error which may then be amplified in subsequent operations. It is important to note that approximation is never affected by a triplet.

The following notations are used throughout this paper:

- $\tilde{N}(t)$: number of failed computer equipment units in the system at time t
- $\tilde{A}(t)$: number of failed computer equipment units which occurred in the interval$[0,t]$
- $\tilde{N}$: average number of failed computer equipment units in the system
- $\tilde{W}$: average sojourn time in the system
- $\tilde{N}_Q$: average number of failed computer equipment units in waiting line
- $\tilde{W}_Q$: average waiting time in waiting line
- $\tilde{P}_n$: fuzzy probability of having n units of failed computer equipment in the system
- $\tilde{\lambda}$: average occurrence rate of failed computer equipment units
- $\tilde{\mu}$: average repair rate of failed computer equipment units.

Now, we introduce fuzzy set theory into Little's law. The fuzzy number of failed computer equipment units in the system observed up to time t is as follows:

$$\tilde{N}_t = \frac{1}{t}\int_0^t \tilde{N}(\tau)\mathrm{d}\tau \tag{2.7}$$

which we call the time average of $\tilde{N}(\tau)$ up to time t, where a fuzzifying function $\tilde{N}(\tau)$ and a integration of fuzzifying function are defined by Definitions 2 and 3, respectively. Of course, $\tilde{N}_t$ depends on t, but $\tilde{N}_t$ tends to a steady-state $\tilde{N}$ according to increment of t, that is,

$$\tilde{N} = \lim_{t\to\infty} \tilde{N}_t. \tag{2.8}$$

The fuzzy average occurrence rate over the interval[0,t] is as follows:

$$\tilde{\lambda}_t = \frac{\tilde{A}(t)}{t}. \tag{2.9}$$

The steady-state fuzzy occurrence rate is defined as

$$\tilde{\lambda} = \lim_{t\to\infty} \tilde{\lambda}_t \tag{2.10}$$

assuming that the limit exist. The fuzzy average time of the failed computer equipment unit delay up to time t is similarly defined as

$$\tilde{W}_t \div \frac{1}{\tilde{A}(t)}\int_0^t \tilde{N}(\tau)\mathrm{d}\tau, \tag{2.11}$$

that is, the fuzzy average time spent in the system per failed computer equipment unit up to time t. The steady-state fuzzy average time of the failed computer equipment unit delay is defined with assuming that the limit exists as follows:

$$\tilde{W} = \lim_{t\to\infty} \tilde{W}_t. \tag{2.12}$$

And, we have

$$\tilde{W} \div \frac{\tilde{N}}{\tilde{\lambda}}. \tag{2.13}$$

Eq.(2.13) is represented using α-cut level as follows:

$$W(\alpha) = \left[\frac{n_L{}^{(\alpha)}}{\lambda_R^{(\alpha)}}, \frac{n_R{}^{(\alpha)}}{\lambda_L^{(\alpha)}}\right], \quad \forall\alpha \in [0,1]. \tag{2.14}$$

From eq.(2.14) we can compute $\tilde{N}$ with interval of confidence as follows:

$$N(\alpha) = [\lambda_R^{(\alpha)} w_L{}^{(\alpha)}, \lambda_L^{(\alpha)} w_R{}^{(\alpha)}]. \tag{2.15}$$

Similar formulas exist for $\tilde{N}_Q$ and $\tilde{W}_Q$, that is

$$\tilde{W}_Q \div \frac{\tilde{N}_Q}{\tilde{\lambda}}. \tag{2.16}$$

By deconvolution of eq.(2.16) we have

$$N_Q(\alpha) = [\lambda_R^{(\alpha)} w_{QL}{}^{(\alpha)}, \lambda_L^{(\alpha)} w_{QR}{}^{(\alpha)}], \quad \forall\alpha \in [0,1]. \tag{2.17}$$

3 Performance Evaluation of On-line Terminal System with Failure

3.1 Model I

We consider the on-line terminal system such as on-line real time system of a bank. In this model we consider only the terminal failure and the peripheral device failure of terminal. This model is known as the single-channel and single-phase structure. An example of such a model is a single-facility computer equipment unit repair shop. The model is subject to the following assumptions.

I. Occurrences of failed computer equipment units and its repair are both a Poisson processes with fuzzy parameters[11].

II. Unlimited source of occurrences.

III. Infinite waiting line.

IV. The waiting line discipline is first come and first repaired.

V. The occurrence rate of failed computer equipment units is less than its repair rate.

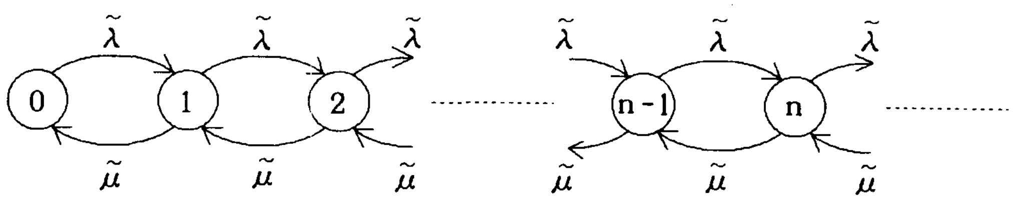
When both the occurrence and repair rates are not fuzzy, the crisp ρ is given as follows:

$$\rho = \frac{\lambda}{\mu}.$$

For fuzzifying of this relation, we may use our defined operator $\bigcirc\!\!\!\!\!\div$ in Definition 5 and can get as follows:

$$\tilde{\rho} = \tilde{\lambda} \oplus \tilde{\mu} \tag{3.1}$$

The state-transition diagram for this model is given by Figure 1. In Figure 1, the circle n represents state n, where state n means that the number of failed computer equipment units is n in the system.



Fiqure 1: State-transition rate diagram for Model I

According to above assumption V we know that $\tilde{\rho} < 1$, therefore a steady state solution exist. From following balance equations we can get following equations:

$$\begin{aligned}
\tilde{P}_0\tilde{\lambda} &= \tilde{P}_1\tilde{\mu}. \\
\tilde{P}_1 &\div \left(\frac{\lambda_1}{\mu_1}p_{01}, \frac{\lambda_2}{\mu_2}p_{02}, \frac{\lambda_3}{\mu_3}p_{03}\right) \\
&= \left(\tilde{\lambda} \bigcirc\!\!\!\!\!\div\, \tilde{\mu}\right)\tilde{P}_0 \qquad (3.2) \\
&= \tilde{\rho}\tilde{P}_0, \\
\text{if } \mu_1\lambda_2 p_{02} &> \mu_2\lambda_1 p_{01} \text{ and } \mu_3\lambda_2 p_{02} > \mu_2\lambda_3 p_{03}.
\end{aligned}$$

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$$\begin{aligned}
\tilde{P}_n &\div (\rho_1, \rho_2, \rho_3)^n\,\tilde{P}_0 \qquad (3.3) \\
&= \tilde{\rho}^n\tilde{P}_0.
\end{aligned}$$

The fuzzy probabilities $\tilde{P}_n$ are positive and sum up as follows:

$$\begin{aligned}
\sum_{n=0}^{\infty}\tilde{P}_n &\div \sum_{n=0}^{\infty}\tilde{\rho}^n\tilde{P}_0 \\
&\div \frac{\tilde{P}_0}{1-\tilde{\rho}} \qquad (3.4) \\
&\div \left(\frac{p_{01}\mu_1}{\mu_1-\lambda_1}, \frac{p_{02}\mu_2}{\mu_2-\lambda_2}, \frac{p_{03}\mu_3}{\mu_3-\lambda_3}\right)
\end{aligned}$$

where $\tilde{\rho} < 1$.

From result of deconvolution eq.(3.4), the fuzzy probability of having no failed computer equipment units in the system is

$$\tilde{P}_0 \doteqdot (1 \ominus \tilde{\rho}) \sum_{n=0}^{\infty} \tilde{P}_n \tag{3.5}$$

iff $\frac{c_2}{1-\rho_1} > \frac{c_1}{1-\rho_2}$ and $\frac{c_2}{1-\rho_3} < \frac{c_3}{1-\rho_2}$ where $\tilde{C} = \sum_{n=0}^{\infty} \tilde{P}_n$.

Thus, the fuzzy probability of having k units of failed computer equipment(*i.e.* waiting unit plus those in repair) in the system is

$$\tilde{P}_k \doteqdot \tilde{\rho}^k (1 \ominus \tilde{\rho}) \sum_{n=0}^{\infty} \tilde{P}_n. \tag{3.6}$$

Similarly, the fuzzy probability of having m or more failed computer equipment units in the system is

$$\begin{aligned} \tilde{P}_{k \geq m} &= \sum_{k=m}^{\infty} \tilde{P}_k \\ &= \sum_{k=0}^{\infty} \tilde{P}_k - \sum_{k=0}^{m-1} \tilde{P}_k \\ &\doteqdot \frac{\tilde{P}_0}{1-\tilde{\rho}} - \frac{\tilde{P}_0}{1-\tilde{\rho}} (1 \ominus \tilde{\rho}^m). \end{aligned} \tag{3.7}$$

Using eq.(3.6) the fuzzy average number of failed computer equipment units in the system is obtained as follows:

$$\begin{aligned} \tilde{N} &\doteqdot \frac{\sum_{n=0}^{\infty} n\tilde{P}_n}{\sum_{n=0}^{\infty} \tilde{P}_n} \\ &= \frac{\sum_{n=0}^{\infty} n\tilde{\rho}^n (1 \ominus \tilde{\rho}) \sum_{n=0}^{\infty} \tilde{P}_n}{\sum_{n=0}^{\infty} \tilde{P}_n} \\ &= \frac{\left(\frac{\lambda_1}{\mu_1-\lambda_1}, \frac{\lambda_2}{\mu_2-\lambda_2}, \frac{\lambda_3}{\mu_3-\lambda_3}\right) \sum_{n=0}^{\infty} \tilde{P}_n}{\sum_{n=0}^{\infty} \tilde{P}_n}. \end{aligned} \tag{3.8}$$

By using eq.(2.12), the fuzzy sojourn time in the system is given as follows:

$$\begin{aligned} \tilde{W} &\doteqdot \frac{\tilde{N}}{\tilde{\lambda}} \\ &\doteqdot \left(\frac{\lambda_1}{\lambda_3(\mu_1-\lambda_1)}, \frac{1}{\mu_2-\lambda_2}, \frac{\lambda_3}{\lambda_1(\mu_3-\lambda_3)}\right) \frac{\sum_{n=0}^{\infty} \tilde{P}_n}{\sum_{n=0}^{\infty} \tilde{P}_n}. \end{aligned} \tag{3.9}$$

And the fuzzy average waiting time is given as follows:

$$\tilde{W}_Q \doteqdot \tilde{W} - \frac{1}{\tilde{\mu}} = \tilde{W} \ominus (1 \oslash \tilde{\mu})$$

$$= \left(\frac{\lambda_1 c_1}{\lambda_3 c_3(\mu_1 - \lambda_1)} - \frac{1}{\mu_1}, \frac{1}{\mu_2 - \lambda_2} - \frac{1}{\mu_2}, \frac{\lambda_1 c_1}{\lambda_3 c_3(\mu_3 - \lambda_3) - \frac{1}{\mu_3}}\right) \quad (3.10)$$

where $\tilde{C} = \sum_{n=0}^{\infty} \tilde{P}_n.$

Similarly, from eq.(2.16)

$$\tilde{W}_Q \doteqdot \frac{\tilde{N}_Q}{\tilde{\lambda}} \quad (3.11)$$

where $\tilde{N}_Q$ is the fuzzy average number of failed computer equipment units in waiting line. By deconvoluting eq.(3.11), the fuzzy average number of failed computer equipment units of in the waiting line is given as follows:

$$\begin{aligned}\tilde{N}_Q &\doteqdot (\lambda_3 w_{Q1}, \lambda_2 w_{Q2}, \lambda_1 w_{Q3}) \\ &\doteqdot \left(\frac{\lambda_1 c_1}{c(\mu_1 - \lambda_1)} - \frac{\lambda_3}{\mu_1}, \frac{\lambda_2}{\mu_2 - \lambda_2} - \frac{\lambda_2}{\mu_2}, \frac{\lambda_3 c_3}{c(\mu_3 - \lambda_3)} - \frac{\lambda_1}{\mu_3}\right),\end{aligned} \quad (3.12)$$

where $\tilde{C} = \sum_{n=0}^{\infty} \tilde{P}_n.$

3.2 Model II

This model is basically same as Model I but with one exception, *i.e.*, the presence of finite waiting line. More clearly, in this case, the waiting line cannot have an unlimited length. One practical example of a limited waiting line is the length of the driveway preceding a drive-in bank teller window. Such a driveway can only practically accommodate a handful of vehicles. Similar limitations can be found in computer systems. In the this model, m is system capacity, that is, the maximum number of failed units in the system. When there are m units in the system, occurring failed computer equipment units are turned away. In this model, the assumption of occurrence rate being less than the repair rate is no longer required. Figure 2 is the state-transition diagram for this model.

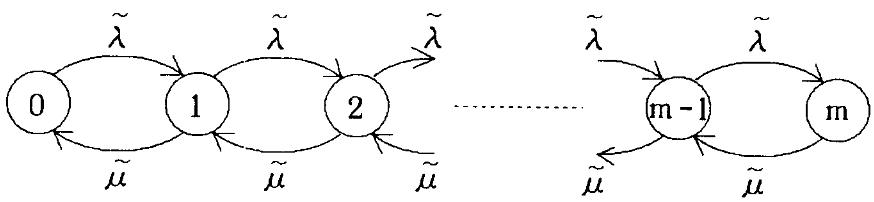

Figure 2: State-transition rate diagram for Model II

In Figure 2 from balance equation

$$\tilde{\lambda}\tilde{P}_{n-1} = \tilde{\mu}\tilde{P}_n \quad \text{for} \quad 1 \leq n \leq m, \quad (3.13)$$

we can obtain the fuzzy probability of having k units in the system, that is,

$$\tilde{P}_k \doteqdot \tilde{\rho}^k \{(1 \ominus \tilde{\rho}) \oplus (1 \ominus \tilde{\rho}^{m+1})\} \sum_{n=0}^{m} \tilde{P}_n$$

$$= \tilde{\rho}^k \frac{1 \ominus \tilde{\rho}}{1 - \tilde{\rho}^{m+1}} \sum_{n=0}^{m} \tilde{P}_n \qquad \text{for } k \leq m, \tag{3.14}$$

where $\tilde{\rho} = \tilde{\lambda} \oplus \tilde{\mu}$.

The fuzzy probability of having zero units in the system is

$$\tilde{P}_0 \doteq \frac{1 \ominus \tilde{p}}{1 - \tilde{\rho}^{m+1}} \sum_{n=0}^{m} \tilde{P}_n \tag{3.15}$$

$$\text{iff} \quad \frac{1 - \rho_1^{m+1}}{1 - \rho_1} c_2 > \frac{1 - \rho_2^{m+1}}{1 - \rho_2} c_1 \quad \text{and} \quad \frac{1 - \rho_3^{m+1}}{1 - \rho_3} c_2 < \frac{1 - \rho_2^{m+1}}{1 - \rho_2} c_3$$

where $\tilde{C} = \sum_{n=0}^{m} \tilde{P}_n$.

The fuzzy average number of units in the system is expressed as follows:

$$\begin{aligned} \tilde{N} &\doteq \frac{\sum_{n=0}^{m} n\tilde{P}_n}{\sum_{n=0}^{m} \tilde{P}_n} \\ &= \frac{\sum_{n=0}^{m} n\tilde{\rho}^n \frac{1 \ominus \tilde{\rho}}{1 - \tilde{\rho}^{m+1}} \sum_{n=0}^{m} \tilde{P}_n}{\sum_{n=0}^{m} \tilde{P}_n} \\ &= \frac{\frac{\tilde{\rho}(1-\tilde{\rho}^{m+1})-(m+1)\tilde{\rho}^{m+1}(1-\tilde{\rho})}{(1-\tilde{\rho}^{m+1})(1-\tilde{\rho})} \sum_{n=0}^{m} \tilde{P}_n}{\sum_{n=0}^{m} \tilde{P}_n}. \end{aligned} \tag{3.16}$$

Similarly, the fuzzy average number of units in the waiting line is given as follows:

$$\tilde{N}_Q \doteq \tilde{N} - \frac{\sum_{n=0}^{m} \tilde{P}_n - \tilde{P}_0}{\sum_{n=0}^{m} \tilde{P}_n}. \tag{3.17}$$

The fuzzy average rate of failed computer equipment units in the system is given as follows:

$$\tilde{\lambda_m} \doteq \tilde{\lambda} \cdot \left(\frac{\sum_{n=0}^{m} \tilde{P}_n - \tilde{P}_m}{\sum_{n=0}^{m} \tilde{P}_n} \right). \tag{3.18}$$

Finally, the fuzzy average sojourn time in the system and the fuzzy average waiting time are expressed as follows:

$$\tilde{W} \doteq \frac{\tilde{N}}{\tilde{\lambda}_m}, \tag{3.19}$$

and

$$\tilde{W}_Q \doteq \tilde{W} - \frac{1}{\tilde{\mu}}. \tag{3.20}$$

In general, the approximation becomes worse according to iteration of the multiplication and the division of TFNs. Every multiplication and division of TFNs in a computational process may give rise to an error which may be amplified in subsequent operations. Therefore, we want to analyze a problem after the vagueness of it consider sufficiently. It is important to note that approximation is never affected by a triplet, *i.e.*, a_1,a_2 and a_3.

4 Numerical Examples

4.1 Example of Model I

A single-channel and single-phase computer equipment repair facility has a Poisson occurrences and the exponential repair time with fuzzy parameter as shown in Figure 3. This model involves a single waiting line. The failed computer equipment units enter to the waiting line with the average occurrence rate $\tilde{\lambda}$. The server repairs to the failed computer equipment units with the average repair rate $\tilde{\lambda}$. The average occurrence rate of failed computer equipment units has minimum 3.8 units/day, maximum 5 units/day and modal value 4 units/day. The fuzzy average occurrence rate and the fuzzy average repair rate are $\tilde{\lambda} = (3.8, 4, 5)$/day and $\tilde{\mu} = (9.6, 10, 12)$/day, respectively. Let $\sum \tilde{P}_n$ be $(0.95, 1, 1.1)$. The information $\tilde{\lambda}$, $\tilde{\mu}$ and $\sum \tilde{P}_n$ are obtained by experiment of experts or very little data. Therefore, the information is presented by fuzzy numbers.

Calculate the fuzzy probability of having no failed computer equipment units in the system and the fuzzy average number of failed computer equipment units in the system and in the waiting line, respectively and the fuzzy average sojourn time in the system and the fuzzy average waiting time, if

I. the waiting line is unlimited.

II. the occurrence source population is unlimited.

III. the waiting line discipline is first come and first served.

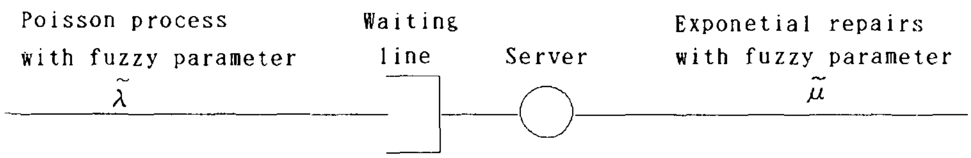

Figure 3: Single-channel and single-phase computer equipment repair facility model

By substituting the above data and eq.(3.1) into eq.(3.5), we get

$$\tilde{P}_0 \doteqdot (0.574, 0.600, 0.642).$$

Thus the fuzzy probability of having no failed computer equipment unit in the system is (0.574, 0.600, 0.642). Substituting the given data into eq.(3.8) we get

$$\tilde{N} \doteqdot (0.566, 0.600, 0.827).$$

It means that there are (0.566, 0.667, 0.827) failed computer equipment units in the system. Substituting $\tilde{N}$ and $\tilde{\lambda}$ into eq(3.9) we get

$$\tilde{W} \doteqdot (0.113, 0.167, 0.218).$$

It means that the fuzzy average sojourn time in the system is (0.113, 0.167, 0.218) day. Substituting $\tilde{W}$ and $\tilde{\mu}$ into eq.(3.10) we get

$$\tilde{W}_Q \doteqdot (0.009, 0.067, 0.134).$$

It means that the fuzzy average waiting time in the waiting line is (0.009, 0.067, 0.134) day. Again, substituting the given data into eq.(3.12) yields it means that there are (0.045, 0.267, 0.510) failed computer equipment units in the waiting line.

When the crisp input data of the conventional model are the modal values of the input data, *i.e.*, TFNs of above example, the performance measures of this model get the modal values of above results, *i.e.*, TFNs. That is, when crisp average occurrence rate λ and crisp average repair rate μ of failed computer equipment units are 4 units/day and 10 units/day respectively in the conventional model, the number of average failed computer equipment units in the system is 0.667 units, the average sojourn time in the system is 0.167 days and so on. Therefore, we can also apply this method to the conventional model.

In the above numerical example, we can verify the reality and the flexibility because the results, *i.e.*, the performance measures maintain the fuzziness of input information.

4.2 Example of Model II

In the Model II above, the average occurrence rate of the failed computer equipment units and the average repair rate are $\tilde{\lambda}$ = (5, 6, 7)/day and $\tilde{\mu}$ = (3.4, 4, 4.6)/day, respectively. Let system capacity, m be 5 and also let $\sum \tilde{P}_n$ be (0.91, 1, 1.1). We calculate the probability of having no failed computer equipment units in the system and the average number of failed computer equipment units in the waiting line and in the system, respectively and fuzzy average sojourn time in the system and fuzzy average waiting time.

By substituting the above data into eq.(3.15) we get

$$\tilde{N}_0 \doteqdot (0.047, 0.048, 0.050).$$

Thus, the fuzzy probability of having no failed computer unit in the system is (0.047, 0.048, 0.050). Substituting the given data into eq.(3.16) we get

$$\tilde{N} = (3.524, 3.577, 3.611).$$

It means that there are (3.524, 3.577, 3.611) failed computer equipment units in the system. Substituting the given data and $\tilde{P}_0$ and $\tilde{N}$ into eq.(3.17) we get

$$\tilde{N}_Q \doteqdot (2.367, 2.626, 2.829).$$

It means that there are (2.367, 2.626, 2.829) failed computer equipment units in the waiting line. Substituting eq.(3.17) and eq.(3.18) into eq.(3.20) we get

$$\tilde{W} = (0.590, 0.940, 1.589).$$

Thus the fuzzy average sojourn time in the system is (0.590, 0.940, 1.589) day. And substituting $\tilde{W}$ and $\tilde{\mu}$ into eq.(3.20) we get

$$\tilde{W}_Q = (0.296, 0.0690, 1.372).$$

It means that the fuzzy average waiting time in the waiting line is (0.296, 0.690, 1.372) day.

In the analysis of the conventional model, because data has been treated as a crisp value in spite of fuzzy data frequently, it is not validation to analyze a model when some parameters of that contain vague information. We introduced the fuzzy set theory into a conventional model, then we can solve such problem well under fuzzy environment.

When the crisp input data of the conventional model are the modal values of the input data, *i.e.*, TFNs of above example, the performance measures of this model get the modal values of above results, *i.e.*, TFNs. That is, when crisp average occurrence rate $\lambda = 6$ units/day, crisp average repair rate $\mu = 4$ units/day and system capacity m is 5 in the conventional model, the number of average failed computer equipment units in the system is 3.577 units, the average sojourn time is 0.940 days and so on. Therefore we can also apply this method to the conventional method.

We can verify that the performance evaluation based on fuzzy set theory has the reality and flexibility as shown in the numerical examples.

5 Conclusion

We presented performance evaluation of on-line terminal system with failure based on fuzzy set theory. For the fuzzification models, the equations to derive performance measures such as the fuzzy probability of having no failed units in the system, the fuzzy average sojourn time in the system, and the fuzzy average waiting time were presented by fuzzy set theory. Therefore it is shown that the performance evaluation is more flexible and realistic than crisp that of conventional method in some numerical examples.

Every multiplication and division of TFNs in a computational process may give rise to an error which may be amplified in subsequent operations. Therefore, we want to analyze a problem after the vagueness of it consider sufficiently.

The deconvolution of fuzzy equations is too restrictive because often there is no solution or very strong conditions must be placed on the fuzzy equations so that there will be a solution. Therefore, a new solution concept is needed so that fuzzy equations has always a solution.

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Appendix

We show the process of development of eq.(3.8) and (3.16) down to the last detail respectively as follows:

$$
\begin{aligned}
\sum_{n=0}^{\infty} \tilde{P}_n &= \frac{\sum_{n=0}^{\infty} n\tilde{\rho}^n (1 \ominus \tilde{\rho}) \sum_{n=0}^{\infty} \tilde{P}_n}{\sum_{n=0}^{\infty} \tilde{P}_n} \\
&= \frac{\left((1 \ominus \tilde{\rho}) \sum_{n=0}^{\infty} \tilde{P}_n\right) \sum_{n=0}^{\infty} n\tilde{\rho}^n}{\sum_{n=0}^{\infty} \tilde{P}_n} \\
&= \frac{\left((1 \ominus \tilde{\rho}) \sum_{n=0}^{\infty} \tilde{P}_n\right) \tilde{\rho}\frac{\partial}{\partial \tilde{\rho}} \left(\sum_{n=0}^{\infty} \tilde{\rho}^n\right)}{\sum_{n=0}^{\infty} \tilde{P}_n} \\
&= \frac{\left((1 \ominus \tilde{\rho}) \sum_{n=0}^{\infty} \tilde{P}_n\right) \tilde{\rho}\frac{1}{(1-\tilde{\rho})^2}}{\sum_{n=0}^{\infty} \tilde{P}_n} \\
&= \frac{\frac{\tilde{\rho}}{1-\tilde{\rho}} \sum_{n=0}^{\infty} \tilde{P}_n}{\sum_{n=0}^{\infty} \tilde{P}_n} \qquad (3.8) \\
&= \frac{\left(\frac{\rho_1}{1-\rho_1}, \frac{\rho_2}{1-\rho_2}, \frac{\rho_3}{1-\rho_3}\right) \sum_{n=0}^{\infty} \tilde{P}_n}{\sum_{n=0}^{\infty} \tilde{P}_n} \\
&= \frac{\left(\frac{\lambda_1}{\mu_1-\lambda_1}, \frac{\lambda_2}{\mu_2-\lambda_2}, \frac{\lambda_3}{\mu_3-\lambda_3}\right) \sum_{n=0}^{\infty} \tilde{P}_n}{\sum_{n=0}^{\infty} \tilde{P}_n}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{N} &\doteqdot \frac{\sum_{n=0}^{m} n\tilde{P}_n}{\sum_{n=0}^{m} \tilde{P}_n} \\
&= \frac{\sum_{n=0}^{m} n\tilde{\rho}^n \frac{1 \ominus \tilde{\rho}}{1-\tilde{\rho}^{m+1}} \sum_{n=0}^{m} \tilde{P}_n}{\sum_{n=0}^{m} \tilde{P}_n}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left(\dfrac{1 \ominus \tilde{\rho}}{1-\tilde{\rho}^{m+1}} \sum_{n=0}^{m} \tilde{P}_n\right) \tilde{\rho} \dfrac{\partial}{\partial \tilde{\rho}} \left(\sum_{n=0}^{m} \tilde{\rho}^n\right)}{\sum_{n=0}^{m} \tilde{P}_n} \\
&= \frac{\left(\dfrac{1 \ominus \tilde{\rho}}{1-\tilde{\rho}^{m+1}} \sum_{n=0}^{m} \tilde{P}_n\right) \tilde{\rho} \dfrac{\partial}{\partial \tilde{\rho}} \left(\dfrac{1-\tilde{\rho}^{m+1}}{1-\tilde{\rho}}\right)}{\sum_{n=0}^{m} \tilde{P}_n} \qquad (3.16) \\
&= \frac{\dfrac{\tilde{\rho}\left(1-\tilde{\rho}^{m+1}\right) - (m+1)\,\tilde{\rho}^{m+1}\left(1-\tilde{\rho}\right)}{\left(1-\tilde{\rho}^{m+1}\right)\left(1-\tilde{\rho}\right)} \sum_{n=0}^{m} \tilde{P}_n}{\sum_{n=0}^{m} \tilde{P}_n}
\end{aligned}
$$