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AN OPTIMAL CURTAILMENT STRATEGY FOR CATALOG DELIVERY IN DIRECT MAIL

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Abstract This study proposes a curtailment strategy for catalog delivery to direct mail for customers, where catalog delivery to a customer is curtailed when he/she shows no response for the last K issues of catalogs successively delivered to the customer. For the purpose of determining the value of K, the expected catalog profit per customer is defined, which is to be maximized. Two types of models for the expected catalog profit are formulated; (i) a model for a customer who has responded to catalogs only a few times up to this point, and (ii) a model for a customer who has responded many times. The conditions under which an optimal curtailment time exists are then clarified for both models. The optimal curtailment strategy for catalog delivery is also discussed based on the above two models. Numerical illustrations are presented to illustrate the proposed optimal curtailment strategy.

1. Introduction

Catalogs are an essential medium for direct mail business to inform customers of the merchandise they offer. Considerable cost is, for this reason, spent in designing and publishing catalogs as well as in delivering them.

Some customers, however, do not respond to catalogs they receive, i.e., they make no purchases even when they receive a series of issues successively. For such customers, further catalog delivery should be curtailed since the cost per catalog is high. It is, however, a problem how the timing to curtail the catalog delivery should be determined.

For problems associated with advertisement, many theoretical models have, so far, been developed[1-15]. These models can be grouped into two broad classes; (1) selling models and (2) consumer behavior models. Selling models focus on the need of the seller to convert the product into cash and are discussed mainly in terms of his/her sales and are concerned in whether sales are going up or going down. The consumer behavior models focus upon the marketing manager's concern for delivering product or service benefit, changing brand attitudes and influencing consumer perceptions. No models have, however, been reported for the curtailment problem mentioned above.

This study proposes a curtailment strategy for catalog delivery to a customer, where the catalog delivery is curtailed if the customer has not responded for the last K successive issues of catalogs. With a view to determining the value of K, the expected catalog profit per customer is introduced. For this expected catalog profit, two types of models are proposed; (i) a model for a customer who has responded only a few times up to this point, and (ii) a model for a customer who has responded many times. The conditions under which an optimal curtailment time exists are then clarified for the two model types. The optimal curtailment strategy for catalog delivery is also discussed on the basis of these models. Numerical examples are presented to illustrate the theoretical underpinnings of the curtailment strategy formulation in this study.

2. Model 1

Model 1 considers customers in the mailing list, who have just responded for the i-th time to catalogs ($i = 0, 1, 2, \cdots$). Let G_i denote the group of such customers. For the customers in G_i, this section discusses the curtailment time for catalog delivery, where catalog delivery is curtailed if they do not respond to K_i successive issues of catalogs delivered to them hereafter.

2.1 Expected catalog profit

Let p_{ij} denote the purchase probability that the customers in G_i respond to the j-th ($j = 0, 1, 2, \cdots$) issue of catalogs immediately after their i-th ($i = 0, 1, 2, \cdots$) response. It is further assumed that $p_{i0} = 0, 1 \geq p_{i1} \geq p_{i2} \geq \cdots$. Such a tendency is easily verified by observing actual data.

Let Q_{ij} express the probability that a customer in G_i responds to none of the j successive issues delivered to them after their i-th response, then Q_{ij} is written as

$$Q_{ij} = \prod_{k=0}^{i} (1 - p_{ik}), \quad j = 0, 1, 2, \cdots. \tag{1}$$

Let a_i denote the profit per purchase by a customer in G_i, which does not consider the cost spent in catalog delivery to the customer. Let $b(< a_i)$ express the cost per catalog, which considers all the costs concerning catalog delivery, e.g., cost for publishing the issue, the cost for delivering catalogs and so forth. When the customers in G_i show their $(i+1)$-st response for some merchandise listed in the j-th issue, which they receive immediately after their i-th response, let us define *the catalog profit* by $a_i - jb$.

Under the above notation and nomenclature, model 1 defines the expected catalog profit per customer by

$$P_i(K_i) = \sum_{j=1}^{K_i} [a_i - j_b + P_{i+1}(K_{i+1})][Q_{i(j-1)} - Q_{ij}] - K_i b Q_{iK_i}, \quad K_i = 1, 2, \cdots. \tag{2}$$

The first term in the right-hand-side of Eq. (2) signifies the conditional expected catalog profit per customer when the customers show their $(i+1)$-st response at the j-th ($j = 1, 2, \cdots, K_i$) issue since their i-th response. The second term expresses the conditional expected catalog profit in case the catalog delivery is curtailed to the customers because they show no response for the K_i successive issues of catalogs delivered after their i-th response.

From Eq. (2), we have

$$P_i(K_i) = a_i'(1 - Q_{iK_i}) - b \sum_{j=1}^{K_i} Q_{i(j-1)}, \tag{3}$$

where

$$a_i' = a_i + P_{i+1}(K_{i+1}). \tag{4}$$

It should be noticed in Eq. (4) that $P_{i+1}(K_{i+1})$ has been treated as a constant value since we can maximize it using the purchase probability $p_{(i+1)j}$ independently of $P_i(K_i)$ (not independently of $P_{i+2}(K_{i+2})$).

We have formulated the expected catalog profit in the above. If $K_i = K_i^*$ maximizes $P_i(K_i)$ in Eq. (3), it presents an optimal curtailment time for catalog delivery to the customers in G_i. In the following section, we examine the conditions under which such an optimal curtailment time exists.

2.2 Analysis

From Eq. (4), we have

$$\begin{aligned}\triangle P_i(K_i) &\equiv P_i(K_i) - P_i(K_i - 1) \\ &= (a_i' - b)Q_{i(K_i-1)} - a_i' Q_{iK_i}, \quad K_i = 2, 3, \cdots.\end{aligned} \tag{5}$$

Hence, $\triangle P_i(K_i) \geq 0$ agrees with

$$\frac{Q_{i(K_i-1)}}{Q_{iK_i}} = \frac{1}{1 - p_{iK_i}} \geq \frac{a_i'}{a_i' - b}, \quad K_i = 2, 3, \cdots, \tag{6}$$

which is equivalent to

$$a_i' p_{iK_i} \geq b, \quad K_i = 2, 3, \cdots. \tag{7}$$

Since $p_{i1} \geq p_{i2} \geq \cdots$, the left-hand-side of Eq. (7) is non-increasing in K_i. Hence if there exists K_i satisfying Eq. (7), the maximum of such K_i is optimum.

These observations reveal that the necessary and sufficient condition under which $K_i = K_i^*(\geq 1)$ is optimum is

$$a_i' p_{iK_i^*} \geq b > a_i' p_{iK_i^*+1}, \tag{8}$$

which is intuitively understandable.

3. Model 2

Model 2 considers customers who have a long history of responses to catalogs and whose purchase probability satisfies $p_j = \lim_{i\to+\infty} p_{ij}$ for $j = 0, 1, 2, \cdots (p_0 = 0, 1 \geq p_1 \geq p_2 \geq \cdots)$. The group of such customers is denoted by G_∞.

3.1 Additional notation

The following notation is used in this section:

Q_j	$= \Pi_{k=0}^{j}(1 - p_k)$ $(j = 0, 1, 2, \cdots)$.
a	Profit per purchase, which does not consider the cost spent in catalog delivery.
b	Cost per catalog.
K_∞	Curtailment time for catalog delivery to the customers in G_∞.
$P(K_\infty)$	Expected catalog profit per customer in G_∞.

3.2 Expected catalog profit

Under the above conditions and notation, the process behavior generates a renewal process[16]. Hence, the expected catalog profit satisfies the following renewal equation:

$$P(K_\infty) = \sum_{j=1}^{K_\infty}[a - jb + P(K_\infty)](Q_{j-1} - Q_i) - K_\infty b Q_{K_\infty}, \quad K_\infty = 1, 2, \cdots. \tag{9}$$

The first term in the right-hand-side of Eq. (9) means the conditional expected catalog profit per customer in G_∞ when the customers respond for the first time after their previous response, and the second term the conditional expected catalog profit when the catalog delivery is curtailed to the customers.

By solving the above renewal equation, we have

$$P(K_\infty) = a\Big(\frac{1}{Q_{K_\infty}} - 1\Big) - b\sum_{j=1}^{K_\infty} \frac{Q_{j-1}}{Q_{K_\infty}}. \tag{10}$$

We have formulated the expected catalog profit $P(K_\infty)$ for the customers in G_∞. If $K_\infty = K_\infty^*$ maximizes $P(K_\infty)$, it provides us with the optimum curtailment time.

3.3 Optimal curtailment time

From Eq. (10), we have

$$\begin{aligned}\triangle P(K_\infty) &\equiv P(K_\infty) - P(K_\infty - 1)\\ &= \frac{1}{Q_{K_\infty}}\Big[ap_{K_\infty} - b\Big(\sum_{j=1}^{K_\infty - 1} Q_{j-1}p_{K_\infty} + Q_{K_\infty - 1}\Big)\Big], \quad K_\infty = 2, 3, \cdots. \end{aligned} \tag{11}$$

Since $Q_{K_\infty} > 0, \triangle P(K_\infty) \geq 0$ agrees with

$$\sum_{j=1}^{K_\infty - 1} Q_{j-1} + \frac{Q_{K_\infty - 1}}{p_{K_\infty}} \leq \frac{a}{b}. \tag{12}$$

Let $L(K_\infty)$ denote the left-hand-side of Eq. (12), and we obtain

$$\begin{aligned}\triangle L(K_\infty) &\equiv L(K_\infty + 1) - L(K_\infty)\\ &= \frac{Q_{K_\infty}(p_{K_\infty} - p_{K_\infty + 1})}{p_{K_\infty}p_{K_\infty + 1}}, \quad K_\infty = 2, 3, \cdots. \end{aligned} \tag{13}$$

Since $1 \geq p_{K_\infty} \geq p_{K_\infty + 1} > 0$, we have $\triangle L(K_\infty) \geq 0$, and consequently the left-hand-side of Eq. (12) is non- decreasing in K_∞. Hence, if there exists K_∞ satisfying Eq. (12), the maximum of such K_∞ is the optimal curtailment time. Based on the abobe results, the optimal curtailment time can be discussed for the following cases:

(i) If $(1 - p_1 + p_2)/p_2 \geq a/b, P(K_\infty)$ is decreasing in K_∞, and therefore $K_\infty^* = 1$. This suggests curtailing catalog delivery to the customer who did not respond to the previous issue of catalogs delivered to him/her.

(ii) If $(1 - p_1 + p_2)/p_2 < a/b$, there exists $K_\infty^* \geq 2$, which satisfies Eq. (12).

4. Optimal Curtailment Strategy

As observed in Section 2, the curtailment time for a customer in G_i can be optimized according to his/her purchase history. The optimal curtailment strategy $(K_1^*, K_2^*, \cdots, K_\infty^*)$ for catalog delivery can, therefore, be obtained by the following backward procedure:

(1) Estimate purchase probability $p_{ij} (j = 1, 2, \cdots)$ for $(i = 0, 1, 2, \cdots, I)$, where I is such a large number that it satisfies $p_{Ij} = p_{(I+1)j}$.

(2) Let $i = I$. Obtain $K_\infty^* (= K_I^*)$, which maximizes $P(K_\infty)(= P(K_I))$ in Model 2.

(3) Let $i = i - 1$. If $i < 0$ then go to (5), otherwise (4).

(4) Obtain K_i^* which maximizes $P_i(K_i)$ in Model 1 by letting $a_i' = a_i + P_{i+1}(K_{i+1}^*)$. Go to (3).

(5) Stop.

5. Numerical Examples

Figure 1 reveals the purchase probability p_{ij} for $i = 1, 2, 3, 4$, which have been estimated using actual data of 500 customers. In this set of data, $i > 0$ since the purchase history of each customer starts at the time when all the customers purchased some merchandise. For $i \geq 4$, it can be considered that $p_{ij} = p_{(i+1)j}$ has been satisfied.

Figure 2 shows the expected catalog profits for the customers in G_i where $a_1 = \cdots = a_4 = 4000$, and $b = 550$. Table 1 summarizes the optimal curtailment strategy.

It is seen in Table 1 that the optimal curtailment time increases with i. This is because the purchase probability tends to increase with the number of purchases as observed in Fig. 1.

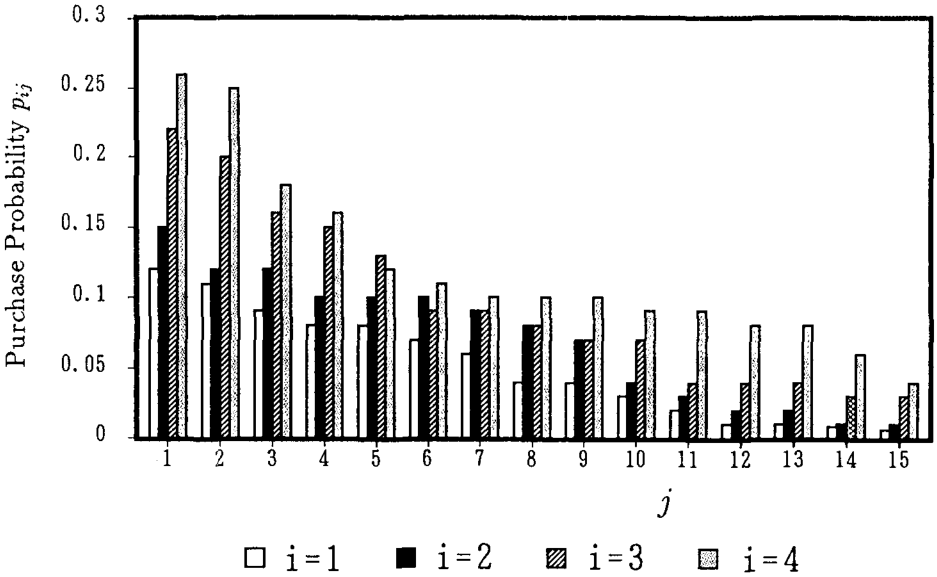

Fig. 1 Purchase probability.

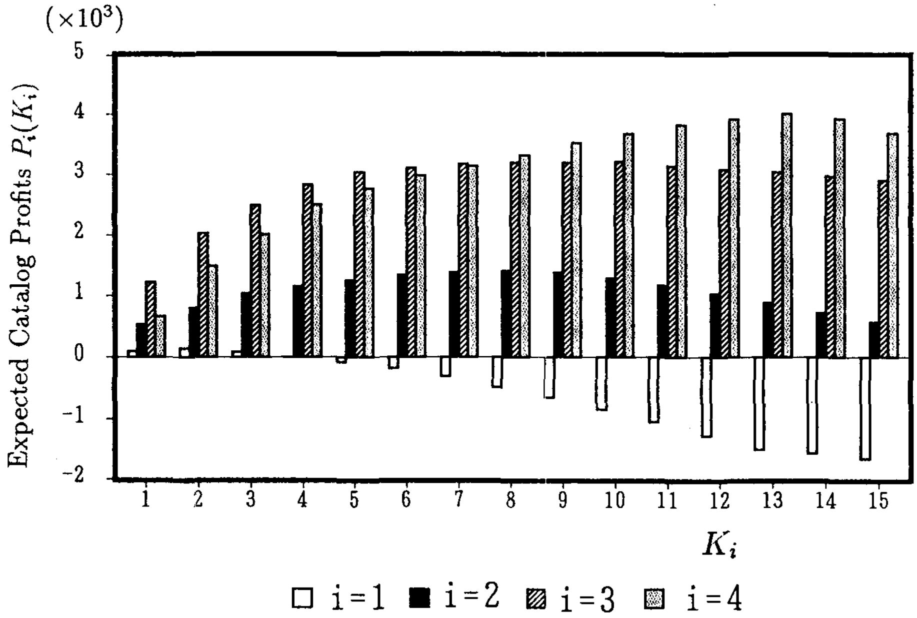

Fig. 2 Expected catalog profits.

Table 1: Optimal curtailment strategy.

i	K_i^*	$P_i(K_i^*)$
1	2	137
2	8	1401
3	10	3213
4	13	4012

6. Concluding Remarks

This study proposed an optimal curtailment strategy for catalog delivery in the direct mail business. Under this strategy, catalog delivery to a customer is curtailed if there is no response to K successive issues of catalogs. For the purpose of determining an optimal value of K, the expected catalog profit per customer was introduced. Two types of models were developed for the expected catalog profit; (i) a model for a customer who has a short purchase history and (ii) a model for a customer who has a long purchase history. For both models, the conditions under which an optimal curtailment time exists were clarified. The optimal curtailment strategy for catalog delivery was then discussed along with numerical examples.

This study assumed that the purchase probability p_{ij} can be estimated for $i = 0, 1, 2, \cdots$ and $j = 1, 2, \cdots$. This estimation might require an enormous data set associated with purchase histories and considerable effort in calculation. In such a case, it would be possible to build another model which uses a discrete-type probability distribution, e.g., a discrete Weibull distribution, for $Q_{i(j-1)}(1 - p_{ij})(j = 1, 2, \cdots)$. This will be discussed a forthcoming paper.

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