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AN ANALYSIS OF NEW PRODUCT DIFFUSION PROCESS

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Abstract This paper clarifies the diffusion mechanism of new product, which is needed under social considerations, from the viewpoints of technological advances (TA), market and public subsidy policies. First, a linear model is constructed and analyzed to grasp a trend of the diffusion process, taking into account pricing for new product and effects of TA on production and market. Next, economic effects of subsidy policies are discussed on the pricing and technical development, and some important viewpoints are presented. Finally, this model is extended to the case where there is a potential demand peculiar to new product and the difference in pricing between the original and extended models is examined.

1. Introduction

The quick spread of socially desirable new products is among the most crucial challenges facing the society today. The electric car is an example. The purpose of this study is to clarify the mechanism of new product diffusion from the viewpoints of technological advances (TA), market and public subsidy policies.

TA on electronics, new materials, high-temperature superconductivity, biotechnologies and so forth have been developed highly, compositely, on a large scale and at increasing speed. Therefore, the advances are giving so great impacts on the economy, society and nature as we have never experienced before. Such TA have been achieved by seeking management efficiency under competitive conditions. It has been the motive force of economic development. On the one hand, serious problems are emerging throughout the world. Environmental destruction, population explosion, poverty and so forth raise the question how to solve the problems on global scale from the viewpoint of balancing efficiency and equity [12]. Under the circumstances, it is considered to be important for us to seek TA or efficiency towards the diffusion of desirable technology which solves various problems for mankind. Conventional technology is only efficiency-oriented, but from now on, it will be needed to change the direction of technological development [5], [10]. The fact that excellent invention has not always practical application shows that "technology as an individual" pursuing efficiency or performance only offers a technological feasibility. An important viewpoint for making use of technology for mankind is to clarify "technology as a field" [11] which concerns an effective use of individual technology, that is, the mechanism of technology diffusion.

Many researches on this subject have been conducted [3], especially focusing on growth curve models. A pioneering model was constructed by Mansfield to explain the difference among innovations in the rate of immitation and to test the model against date [7]. Bass developed a model in which potential adopters of innovation are influenced by two means of communications, namely, mass media and word of mouth [1]. This model includes Mansfield's model which treats the communication of word of mouth. Blackman revised Mansfield's model in terms of market share rather than in terms of the cumulative number of firms adopting new technology [2]. Metcalfe developed a suggestive framework for analyzing the interaction of demand and supply during diffusion [9]. He introduced both pricing decision and profitability concept for adopting and producting the innovation into his model. After that time, refinements and extentions of diffusion models have been extensively carried out [6]. However, most of them, including Metcalfe's, treat technology exogenously or extrapolately in the context of growth curve models. And these do not always clarify how the production cost reduces due to TA, how the improved performance affects on market and how the public subsidy policy speeds up the new technology diffusion. Therefore, it is appropriate to introduce pricing for new product and its profitability into a more general context involving growth curve models for obtaining better perspectives of diffusion process and subsidy policies.

From the above, taking into account two characteristics of production and market under TA, we quantitatively clarify the mechanism in which the existing technology is replaced by new one and new product based on the new technology penetrates into the market. Here, it is assumed that the market of old product based on the existing technology has been already established. The purpose of this study is to show a trend of new product diffusion in an industry. Therefore, we will not focus on individual firms but on the industry as a whole or as a mean trend, assuming that the industry itself posseses a decision making function.

This study is constructed as follows: In Chapter 2, as a preliminary consideration for the study of new product diffusion, a concept of market share of new product is introduced and a linear model is constructed with respect to new product diffusion. Here, it is assumed that industry determines the price of new product so as to maximize the total profit obtained from selling existing and new products. From Chapters 3 through 5, new product diffusion is analyzed by using the model. Chapter 3 discusses how the set price of new product and its market share vary over time by TA. Chapter 4 discusses economic effects and important viewpoints on typical subsidy policies (purchase price subsidy of new product and promotion of technical development) which speed up the new technology diffusion. Chapter 5 gives some numerical examples for Chapters 2 through 4. Chapter 6 presents a more generalized model by relaxing the assumption of total demand being composed of both conventional and replaced demands, and by supposing that there occurs a newly created demand peculiar to new product. We will consider the difference between set price and market share in Chapter 2 and those in Chapter 6. In Chapter 7, some important points of new product diffusion are summarized and the importance of subsidy policies, which change the technology direction from social viewpoints, is pointed out.

2. Diffusion model of new product

2.1 Problem

We will take an example of motor industry. In the industry, the environmental contamination such as air pollution, global warming and so on due to automobile exhaust gas has become actual, so that electric motor car has been developed and some types are put to practical use [4], [8], [13], [14]. However, the number in use is only about fifteen thousand because of its high price (two and a half times as high as gasoline cars), its short running range per one charging due to inferior performance of battery, its inferior acceleration capability, underdeveloped infrastructure of charging stations and so on. Since electric motor car is a new product whose diffusion is desirable from the social standpoint, the necessity of policies for technical development, demand promotion and development of infrastructure has been recognized, and they have been gradually put into practice. However, there has been no research which theoretically clarifies how these policies facilitate the electric car diffusion through its market mechanism. In this study, we generally construct a model in which a new product diffuses in place of existing product, and consider important aspects of its diffusion.

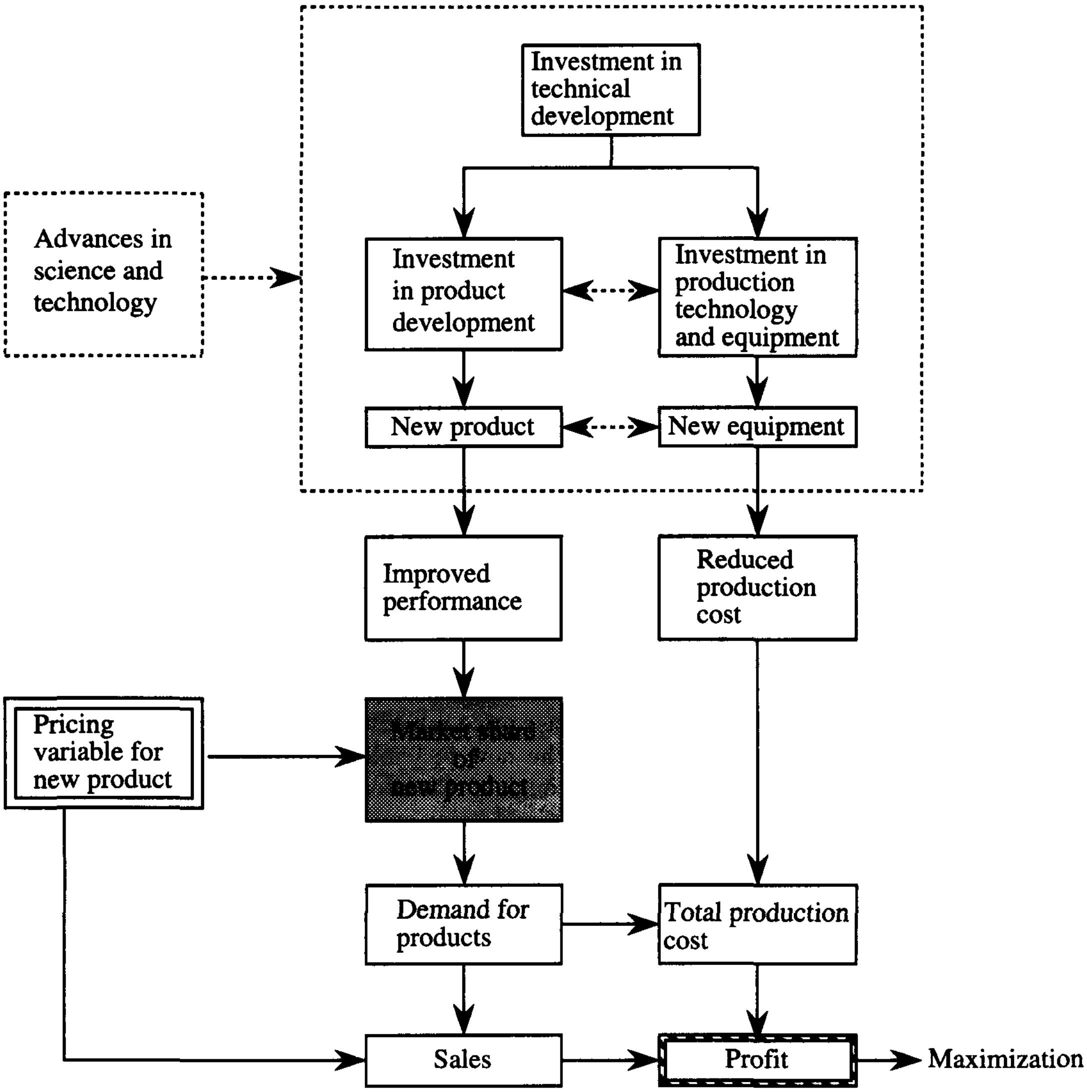


Figure 1 Mechanism of new product diffusion

Suppose a manufacturing industry. Existing product manufactured and sold has attained technical maturity and its price is stable. In this industry, a new product has recently been developed due to TA and expected to replace the existing product. Compared with the existing product, the new product is more desirable with respect to social direction of technology in environmental problems and so on, but it is not technically mature. Its performance will be much improved in future. TA reduce the production cost of new product through higher production efficiency. Of course, the degree of production cost reduction also depends on the production volume. Since the new product price is set by taking its cost into consideration, the cost reduction enables the producer to cut down the price. Both the price and performance of new product determine the degree of substitution of existing product for new one. In these circumstances, it is expected that the industry will gradually increase the proportion of new product while selling both products. It is assumed that the industry sets a new product price so as to maximize at each time the total profit obtained from the mixed production of existing and new products. As a result, the market share of new product will be determined by the price set for new product. This relation is illustrated in Fig. 1. Since we are interested in clarifying the substitution mechanism of new product for existing one, we mainly consider the situation where new product price affects only the market share which determines the substitution between both products. Concretely, we clarify a situation where the market share of new product at set price increases in time, a changing trend of the set price, and the economic effects and important aspects of typical subsidy policies for increasing the market share.

2.2 Description of the model

Notations

j: product series ($j = 1$ and $j = 2$ denote existing and new products, respectively)
p_1: price of existing product
p_2: price of new product
$x(t)$: total demand of both products at time t
$s(p_2, t)$: market share of new product at time t if it is priced at p_2 (the rate of new product demand to total demand composed of both existing and new products; hereafter we merely call it market share)
$x_j(p_2, t)$: demand of product j at time t if new product is priced at p_2
$x_1(p_2, t) = \{1 - s(p_2, t)\}x(t)$; $x_2(p_2, t) = s(p_2, t)x(t)$
$h(u) \uparrow u$ ($\downarrow u$): function $h(u)$ is non-decreasing (non-increasing) in u
$h(u)$: convex(u): function $h(u)$ is downward convex in u

Premises

① Time trends of technical development investment, increasing development of infrastructure and so forth are known.
② Initial investment for producing new products has been decided or put into operation.
③ We only consider the demands for existing product and for new product replacing the existing one, and do not consider the case where new demand peculiar to new product occurs. In Chapter 6, we relax this assumption, and make some considerations.
④ The price of existing product is given and its time trend is known (hereafter, we denote the price at time t by $p_1(t)$). On the other hand, the price of new product can be set freely.
⑤ The market share of new product at any time is determined by both of the price and time (increased performance) and these relations are known.
⑥ Industry optimally sets the price of new product so as to maximize the total profit at each time (hereafter, we simply call it set price).

Formulation

The total demand of both products at any time is considered to increase with time due to TA. Initial investment is made at the starting point of production for each product. Further, additional investment is needed at each time in order to produce existing or new product by the newest technology. Existing product price at each time is stable to the variation in production volume because of the technological maturity and scale merit and is known. The advanced production technology on new product will enable the manufacturer to cut down product cost (variable cost) and set lower price for new product. From the above, TA will determine the market share of new product depending on its performance and price.

Let's consider the total production cost structure by classifying it into fixed and variable costs. Fixed cost is composed of labor cost, expenses (depreciation cost, technical development cost, etc.) and so forth. Variable cost is composed of materials cost, labor cost and utility cost (electricity, gas, oil, etc.). Converting the investment cost at time t into the annual adjusted mean value for operating periods since time t, and adding all the mean values occurring at time t and other fixed expenses, the fixed cost $a_j(t)$ of product j at time t is obtained. Variable cost $b_j(t)$ of product j at time t is considered to depend upon advances in production technology. Here, it can be considered that there hold $a_j(t) \uparrow t$ due to increasing investment for plant and equipment caused by advances in production technology, and $b_j(t) \downarrow t$ and $b_j(t)$: convex(t) due to higher performance of equipment. From the above, the total production cost of product j at time t is expressed by

$$a_j(t) + b_j(t)x_j \tag{2.1}$$

The structure is illustrated in Fig. 2.

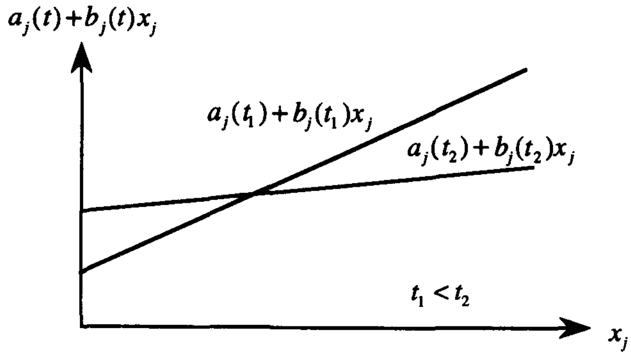


Figure 2 Structure of total production cost

Next, we shall consider the structure of market share $s(p_2, t)$. Denoting the preference of new product to existing one by P_f, the market share is considered to be proportional to P_f. Letting $q_j(t)$ be the performance of product j at time t, P_f will be determined by price ratio $\lambda(p_2,t)(= p_2/p_1(t))$ and performance ratio $\mu(t)(= q_2(t)/q_1(t))$. Therefore, denoting the proportional constant by k, $s(p_2,t)$ can be written as $s(p_2,t) = kP_f(\lambda(p_2,t), \mu(t))$. Clearly, $P_f \downarrow \lambda(p_2,t)$ and $P_f \uparrow \mu(t)$ will hold.

We shall consider changes in $s(p_2,t)$ when p_2 or t changes. Keeping t constant, there holds $\lambda(p_2,t) \uparrow p_2$ from the definition of $\lambda(p_2,t)$. Thus, $P_f \downarrow p_2$ holds. On the other hand, since existing product is technically mature and its price $p_1(t)$ is stable, the change of $\lambda(p_2,t)$ with respect to t is considered to be small in case of keeping p_2 constant. Since the performance of new product increases more than existing one due to TA, $\mu(t) \uparrow t$ is considered to hold and it changes very much. Therefore, we may consider $P_f \uparrow t$. From the above, the relations of $s(p_2,t) \downarrow p_2$ and $s(p_2,t) \uparrow t$ are considered to hold.

With respect to $s(p_2,t)$ keeping t constant, from the same reasoning as deriving growth curve, we assume that the absolute value of change of $s(p_2,t)$ according to the change of p_2 is proportional to both $s(p_2,t)$, which indicates propagation power of new product, and

$1 - s(p_2, t)$, which expresses the potential propagation capability, and changes in opposite direction to the change of p_2. That is, we assume the following relation:

$$\frac{\partial s(p_2,t)}{\partial p_2} = -\beta_t s(p_2,t)\{1 - s(p_2,t)\}$$

Here, noticing that a function $s'(p_2,t)$ defined by the following relation $\frac{\partial s'(p_2,t)}{\partial p_2} = \beta_t s'(p_2,t)\{1 - s'(p_2,t)\}$ is generally given by the growth curve of $s'(p_2,t) = 1/\{1 + exp(-\beta_t p_2 + \gamma_t)\}$ for any fixed t, $s(p_2,t)$ under the above assumption is symmetric with respect to the inflection point $p_2 = p_r$ of above growth curve $s'(p_2,t)$ as shown in Fig. 3. On the other hand, the change of $s(p_2,t)$ with respect to t for an arbitrary fixed p_2 is considered to be nonlinear as illustrated in typical patterns (i) and (ii) of Fig. 4. (see p. 145 in Jap. ed. of [3]). Patterns (i) and (ii) correspond to the cases where low cost innovations with relatively simple TA and high cost innovations with complicated TA occur, respectively.

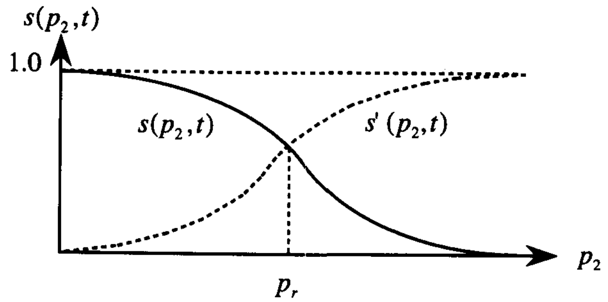


Figure 3 Relation between p_2 and $s(p_2,t)$

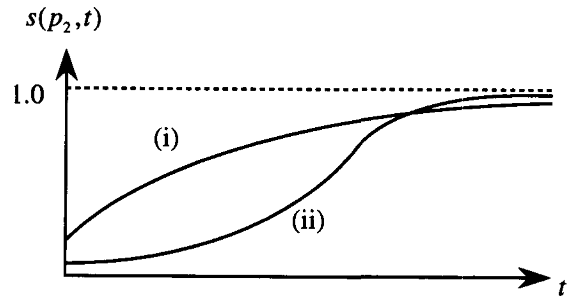


Figure 4 Typical change patterns of $s(p_2,t)$ with respect to t

In this study, we are interested in the variation trend of new product diffusion by pricing policy under TA, which is caused by essential factors. For this purpose, we approximate the change of $s(p_2,t)$ in Fig. 3 by the linear function as illustrated in Fig. 5. That is, we discuss the case of $s(p_2,t)$ given by

$$s(p_2,t) = \begin{cases} 1 & p_2 \le p_L(t) \\ -\dfrac{p_2 - p_U(t)}{\Delta p(t)} & p_L(t) < p_2 < p_U(t) \\ 0 & p_U(t) \le p_2 \end{cases} \tag{2.2}$$
$$\Delta p(t) \equiv p_U(t) - p_L(t) \ge 0,$$

where $p_L(t)$ and $p_U(t)$ respectively denote new product prices corresponding to market share 1 and 0. Hereafter, we call $p_L(t)$ and $p_U(t)$ lower limit price and upper limit price, respectively. In considering new product diffusion, we derive set price (optimal set price) of new product at any time t and then we discuss the change in time of set price. To this end, we assume only the linearity of $s(p_2,t)$ with respect to p_2 for an arbitrary fixed time t. First, we shall derive set price of new product at an arbitrary time. From (2.2), we have

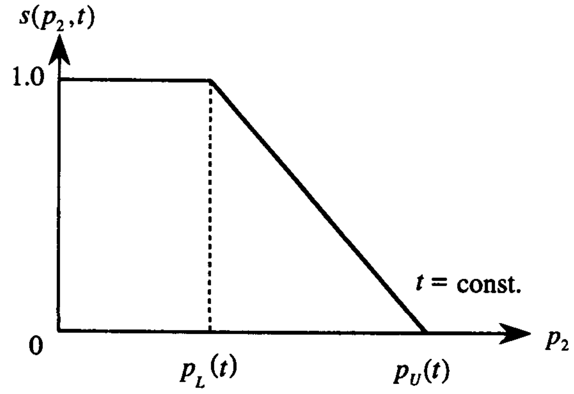


Figure 5 Relation between p_2 and $s(p_2,t)$ with linear structure

$$\begin{aligned} \frac{\partial s(p_2,t)}{\partial p_2} &= \begin{cases} -1/\Delta p(t) & p_L(t) < p_2 < p_U(t) \\ 0 & p_2 \le p_L(t) \text{ or } p_2 \ge p_U(t) \end{cases} \\ \frac{\partial^2 s(p_2,t)}{\partial {p_2}^2} &= 0 \qquad\qquad\qquad p_2 \ne p_L(t), p_U(t). \end{aligned} \tag{2.3}$$

The demands $x_1(p_2,t)$ and $x_2(p_2,t)$ for existing and new products at time t are respectively represented by

$$\begin{aligned} x_1(p_2,t) &= \{1 - s(p_2,t)\}x(t) \\ x_2(p_2,t) &= s(p_2,t)x(t) \end{aligned} \tag{2.4}$$

in the case new product price is p_2.

If new product is priced at p_2 at time t, the total profit $r(p_2,t)$ at time t is given by

$$\begin{aligned} r(p_2,t) &= p_1(t)x_1(p_2,t) + p_2x_2(p_2,t) - \{a_1(t) + b_1(t)x_1(p_2,t) + a_2(t) \\ &\quad + b_2(t)x_2(p_2,t)\} \\ &= m_1(t)x(t) + \{p_2 - \tilde{b}_2(t)\}s(p_2,t)x(t) - \sum_{j=1}^{2} a_j(t) \end{aligned} \tag{2.5}$$

$$m_1(t) \equiv p_1(t) - b_1(t),\ \tilde{b}_2(t) \equiv m_1(t) + b_2(t).$$

Here, $m_1(t)$ is gross profit at time t of existing product, or it can be interpreted as the opportunity cost at the sacrifice of existing production in order to produce new product. $\tilde{b}_2(t)$ is the sum of above opportunity cost and variable cost of new product, which can be interpreted as an equivalent variable cost of new product in substituting existing product for new one. Therefore, the problem of setting new product price p_2 which maximizes (2.5), can be written as follows:

$$\max_{p_2 \ge \tilde{b}_2(t)} \quad r(p_2,t) \qquad \text{for } \forall t = 1, 2 \cdots \tag{2.6}$$

2.3 Price setting for new product at each time

Letting

$$q(p_2,t) \equiv \{p_2 - \tilde{b}_2(t)\}s(p_2,t) \tag{2.7}$$

(2.6) is equivalent to

$$\max_{p_2 \geq \tilde{b}_2(t)} \quad q(p_2,t) \qquad \text{for } \forall t = 1,2,\cdots \tag{2.8}$$

from (2.5). Applying the relation of (2.2) to $s(p_2,t)$ in (2.7), we have

$$q(p_2,t) = \begin{cases} p_2 - \tilde{b}_2(t) & p_2 \leq p_L(t) \\ -\{p_2 - \tilde{b}_2(t)\}\{p_2 - p_U(t)\}/\Delta p(t) & p_L(t) < p_2 < p_U(t) \\ 0 & p_2 \geq p_U(t). \end{cases} \tag{2.9}$$

With respect to the right hand side of (2.9), we let $q_1(p_2,t) \equiv p_2 - \tilde{b}_2(t), q_2(p_2,t) = \{p_2 - \tilde{b}_2(t)\}/ \triangle p(t)$ and $q_3(p_2,t) \equiv -\{p_2 - \tilde{b}_2(t)\}\{p_2 - p_U(t)\}/ \triangle p(t)$. Therefore, noticing the relation $p_2 \geq \tilde{b}_2(t)$, the value $p^\circ(t)$ of new product price p_2 achieving (2.8), and $s(p^\circ(t),t)$ are determined as follows corresponding to cases (i) through (iii) in Fig. 6:

Case (i) $\tilde{b}_2(t) \geq p_U(t)$

Since there holds $s(p_2,t) \equiv 0$ for $\forall p_2 \geq \tilde{b}_2(t)$, we may produce only existing product. In this case, though set price $p^\circ(t)$ of new product is meaningless, we shall conveniently write $s(p_2,t)$ as

$$s(p^\circ(t),t) \equiv 0 \tag{2.10}$$

Case (ii) $p_L(t) - \Delta p(t) < \tilde{b}_2(t) < p_U(t)$

The value $p^\circ(t)$, which is the value of p_2 such that maximizes $q(p_2,t)$, needs to satisfy the following relation obtained from the equation $\partial q(p_2,t)/\partial p_2 = 0$ and (2.2):

$$s(p_2,t) = \frac{p_2 - \tilde{b}_2(t)}{\Delta p(t)} \qquad (p_2 \geq \tilde{b}_2(t))$$

The above relation implies that "set price $p^\circ(t)$ of new product is determined as price p_2 at which the market share is equal to the ratio of equivalent gross profit $p_2 - \tilde{b}_2(t)$ of new product to the difference $\Delta p(t)$ between upper and lower limit prices". Applying the relation (2.2) to the left hand side of above equation, we have following equations:

$$\begin{aligned} p^\circ(t) &= \frac{p_U(t) + \tilde{b}_2(t)}{2} \\ s(p^\circ(t),t) &= \frac{p_U(t) - \tilde{b}_2(t)}{2 \triangle p(t)} \end{aligned} \tag{2.11}$$

Case (iii) $\tilde{b}_2(t) \leq p_L(t) - \Delta p(t)$

In this case, $q(p_2,t)$ is shown in Fig. 6(iii) and we have

$$\begin{aligned} p^\circ(t) &= p_L(t) \\ s(p^\circ(t),t) &\equiv 1. \end{aligned} \tag{2.12}$$

Since we have the relation of $x(t) = x_2(p^\circ(t),t)$, only new product is produced.

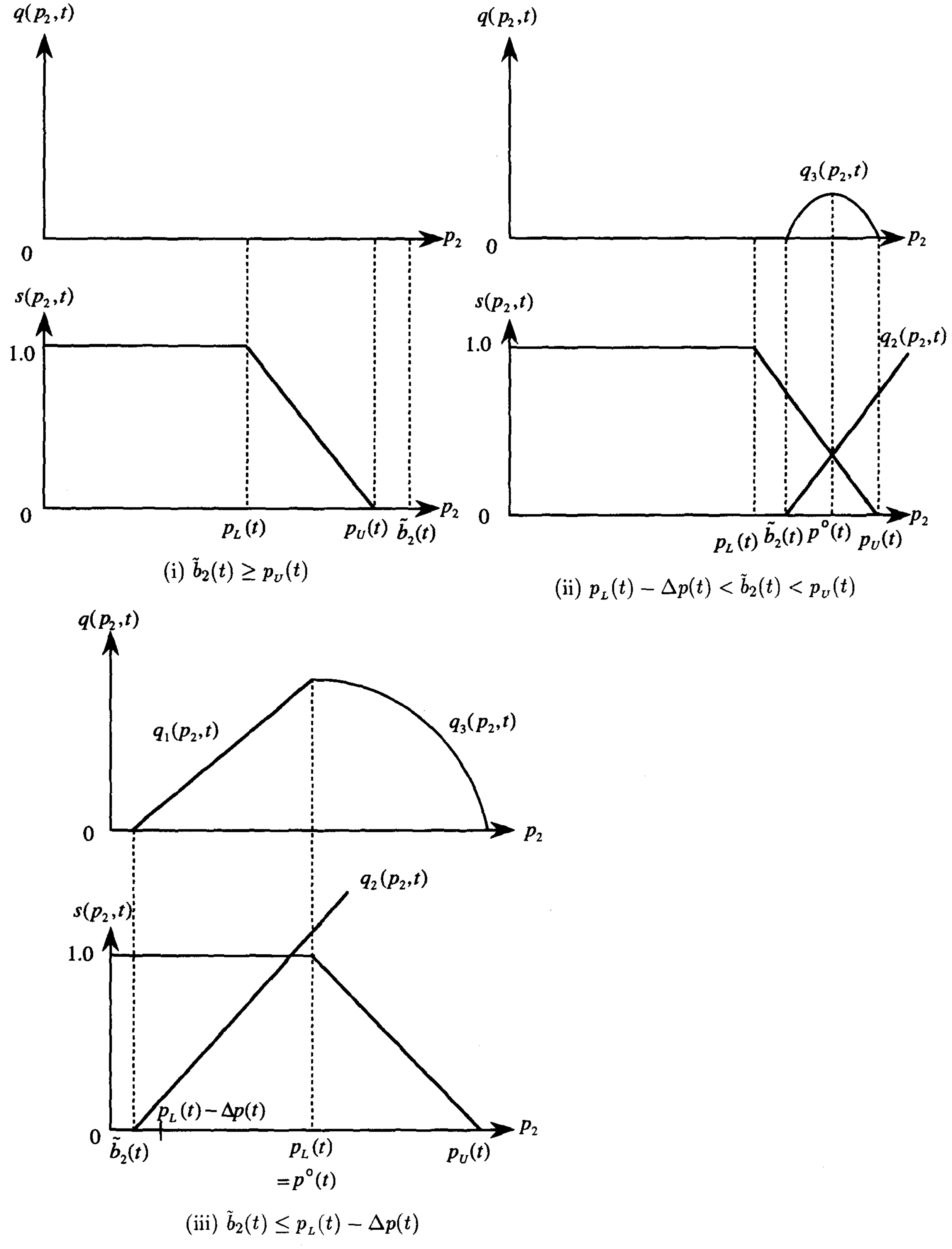


Figure 6 Three cases which maximize $q(p_2, t)$

3. Set price of new product and change of market share with time

In this chapter, we focus on how the set price $p^\circ(t)$ of new product and its market share $s(p^\circ(t), t)$ vary with time.

First, we consider variations with time of both $s(p_2, t)$ (changes of $p_L(t), p_U(t)$, $\triangle p(t)$ and $\tilde{b}_2(t)$. As stated in Section 2.2, it is considered to hold $s(p_2, t) \downarrow p_2$ and $s(p_2, t) \uparrow t$. And $p_L(t+1) - p_L(t) > 0$ and $p_U(t+1) - p_U(t) > 0$ hold from $s(p_2, t) \uparrow t$. We denote $s(p_2, t)$ change from time t to time $t+1$ by

$$\begin{aligned} \delta s(p_2, t) &\equiv s(p_2, t+1) - s(p_2, t) \\ &= k\{P_f(\lambda(p_2, t+1), \mu(t+1)) - P_f(\lambda(p_2, t), \mu(t))\}. \end{aligned}$$

Then, as shown in Fig. 7, we consider the size relation between $\delta s(p_2, t)$'s at $p_2 = p_{21}$ and $p_2 = p_{22}(p_{21} < p_{22})$. For the same increase $\mu(t+1) - \mu(t)$ in performance of both $\delta s(p_{21}, t)$ and $\delta s(p_{22}, t)$, the former with lower price p_{21} has greater increase $P_f(\lambda(p_2, t+1), \mu(t+1)) - P_f(\lambda(p_2, t), \mu(t))$ in preference than the latter, so that there holds

$$\delta s(p_{21}, t) > \delta s(p_{22}, t).$$

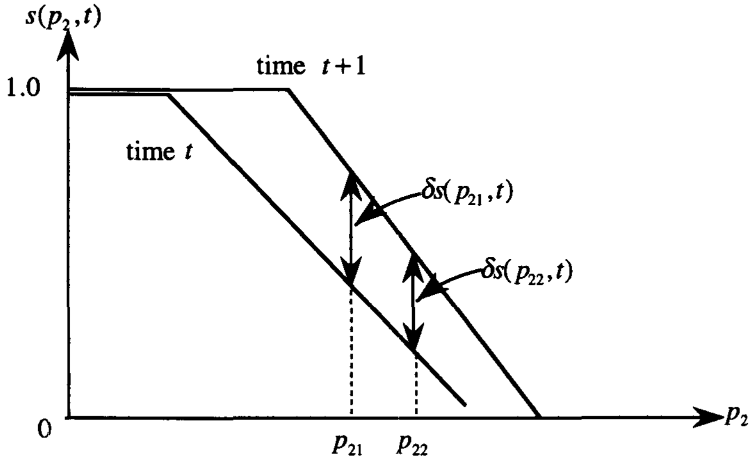


Figure 7 Change of $s(p_2, t)$ with respect to t

This implies, as shown in Fig. 7, that the slope of $s(p_2, t+1)$ becomes greater than that of $s(p_2, t)$ if we compare the decreasing parts of both slopes. Therefore, there holds the relation of $\triangle p(t) = p_U(t) - p_L(t) \downarrow t$, that is, the variation of market share with time is greater for lower price in case of keeping price p_2 constant. The less $\triangle p(t)$ becomes due to TA, the narrower the choice of prices becomes. On the other hand, as defined in (2.5), equivalent variable cost $\tilde{b}_2(t)$ of new product has a relation of $\tilde{b}_2(t) = p_1(t) + b_2(t) - b_1(t)$. In the relation, though price $p_1(t)$ and variable cost $b_1(t)$ of existing product are considered to decrease under TA, these changes will be small since the product is mature. On the other hand, it is considered that the variable cost per unit of new product will be reduced sharply by TA. Hence, we will be able to consider the relation of $b_2(t) - b_1(t) \downarrow t$. That is, it is considered that $\tilde{b}_2(t)$ decreases with time, that is, $\tilde{b}_2(t) \downarrow t$. After all, changes with time of $p_L(t), p_U(t), \triangle p(t)$ and $\tilde{b}_2(t)$ can be described as the following assumptions:

Assumption 1 $p_L(t+1) - p_L(t) > 0,\ p_U(t+1) - p_U(t) > 0$ for $\forall t \in \{1, 2, \cdots\cdots\}$
Assumption 2 $\triangle p(t+1) - \triangle p(t) < 0$ for $\forall t \in \{1, 2, \cdots\cdots\}$
Assumption 3 $\tilde{b}_2(t+1) - \tilde{b}_2(t) < 0$ for $\forall t \in \{1, 2, \cdots\cdots\}$

Lemma

We shall consider in what order does each of cases (i) through (iii) in Fig. 6 occur with time when Assumptions 1 through 3 hold. The cases (i), (ii) and (iii) occur in one direction in that order. Here, any special case which does not occur is excluded.

Proof.

With respect to the conditions of cases (i) through (iii) under Assumptions 1 through 3, there hold

$$\begin{aligned} &p_L(t) - \triangle p(t) < p_L(t) < p_U(t) \\ &p_U(t) - \triangle p(t) \uparrow t \\ &p_L(t) \uparrow t \qquad\qquad\qquad\qquad \text{for } \forall t \in \{1, 2, \cdots\cdots\} \\ &p_U(t) \uparrow t \\ &\tilde{b}_2(t) \downarrow t. \end{aligned}$$

From this, it is known that the cases (i) through (iii) occur in that order.

□

Lemma implies that cases (i) ($s(p^{\circ}(t), t) \equiv 0$), (ii) ($0 < s(p^{\circ}(t), t) < 1$) and (iii) ($s(p^{\circ}(t), t) \equiv 1$) in Fig. 6 occur irreversibly in that order.

Proposition 1

If Assumptions 1 through 3 hold, then the market share $s(p^{\circ}(t), t)$ at set price $p^{\circ}(t)$ increases with time, that is, $s(p^{\circ}(t), t) \uparrow t$ for $\forall t \in \{1, 2, \cdots,\}$.

Proof.

In considering the change of $s(p^{\circ}(t), t)$ with time t, it is seen that we may discuss case (ii) ($0 < s(p^{\circ}(t), t) < 1$) at any time t and time $t+1$, without loss of generality, from Lemma. In this case, $s(p^{\circ}(t), t)$ is given by (2.11). There hold $\tilde{b}_2(t) \downarrow t, p_U(t) \uparrow t$ and $\triangle p(t) \downarrow t$ under Assumptions 1 through 3. Applying these relations to (2.11), the result is derived. □

Proposition 1 implies that new product diffuses under Assumptions 1 through 3 which are considered to be valid for producing new product under remarkable TA, and set price $p^{\circ}(t)$. This may be considered to be selfevident from $s(p^{\circ}(t), t) \uparrow t$. However, this proposition deserves a special mention considering that either of $p^{\circ}(t) \uparrow t$ or $p^{\circ}(t) \downarrow t$ may occur generally with respect to $p^{\circ}(t)$ in (2.11). That is to say, this implies that the set price of new product is not always increasing monotonously but the new product diffuses if we kept optimally setting its price. The problem is the diffusion speed of new product. In case where the rapid diffusion of new product is desired from the social point of view, public subsidy policies will be needed. This will be discussed in Chapter 4.

Next, we shall discuss how the set price of new product $p^{\circ}(t)$ changes with time, that is, the relation between $p^{\circ}(t)$ and $p^{\circ}(t+1)$. From Lemma, cases (i) through (iii) occur one after another as the time passes. In the case (i), price setting for new product is meaningless, because it is not produced. In the case (iii), the market share itself, which describes the substitutional relationship between existing and new products, has no meaning since only new product is produced. For this reason, we will restrict our discussion to the essential case where both $p^{\circ}(t)$ and $p^{\circ}(t+1)$ occur in case (ii) with the competing products.

In case (ii), there hold $p^{\circ}(t) = \{p_U(t)+\tilde{b}_2(t)\}/2$ and $p^{\circ}(t+1) = \{p_U(t+1)+\tilde{b}_2(t+1)\}/2$. Therefore, under Assumptions 1 and 3, we have the following relation:

$$\tilde{b}_2(t) - \tilde{b}_2(t+1) \geq p_U(t+1) - p_U(t) \Leftrightarrow p^{\circ}(t) \geq p^{\circ}(t+1) \text{ for one } t \in \{1, 2, \cdots\cdots\} \quad (3.1)$$

Here, $\tilde{b}_2(t) - \tilde{b}_2(t+1)$ represents a reduction in equivalent variable cost of new product according to advances in production technology. Noticing the relation $\tilde{b}_2(t) - \tilde{b}_2(t+1) = \{m_1(t) - m_1(t+1)\} + \{b_2(t) - b_2(t+1)\}$ from (2.5), $\tilde{b}_2(t) - \tilde{b}_2(t+1)$ increases as an decrease in gross profit due to technical maturity or a reduction in variable cost of new product due to TA becomes large. On the other hand, $p_U(t+1) - p_U(t)$ represents a rise in upper limit price of new product due to increasing performance. The relation (3.1) implies that the changing trend in set price of new product with time is determined by a relative relation between the impact of advances in manufacturing technique on the production (a reduction in equivalent variable cost) and the impact of technological advances in product on the market (a rise in upper limit price). Since new product is usually produced by multiple firms under competitive conditions, they will be obliged to reduce the price of new product. If we can grasp the performance of new product at time t and time $t+1$, we will be able to catch by marketing research the trend of upper limit prices $p_U(t)$ and $p_U(t+1)$ that customers are willing to pay. From this, we obtain a guideline for degree of innovation that manufacturing department should achieve in order to reduce the price of new product. Further, since a price reduction of new product is necessary from the viewpoint of its diffusion, it is important for industry to promote innovation from two aspects of improving performance and manufacturing technology. Especially, manufacturing TA corresponding to performance improvement will be an important point for new product diffusion.

4. Economic effects of subsidy policies for new product diffusion and its point of view

For example, in motor industry, the development of electric car which substitutes gasoline car is urgent under environmental destruction such as global warming, air pollution and so forth, by exhaust gas. As indicated in this example, let's consider a case where new product diffusion is needed from social standpoints. Here, it is necessary to promote technical development at the sacrifice of short-term business profits and to perform policies for its diffusion. In performing these policies, there occurs a problem of external economy: "who should pay social cost associated with these policies?". Since this cost is immense, it is often too hard for only one firm to bear this. There is a limit for firms to bear such social costs under cutthroat competitions. It becomes important to incorporate the cost bearing system into the mechanism of free competition through public subsidy policies by central government, local public authorities and other public entities. In fact, various public subsidy policies have been performed such as purchase price subsidy, reduction and exemption of taxes, promotion of technical development, positive utilization of new products, arrangement of infrastructure and so forth.

In this chapter, we discuss economic effects of typical subsidy policies for purchase price and technical development on new product, and its point of view on performing these policies, considering that such policies will be more and more important from the social viewpoint. Purchase price subsidy is a quick-acting policy, which increases the market share by price reduction under the same market share function. On the other hand, the subsidy for technical development is a drastic policy which intends to shift market share function by increasing performance under TA, but it is a slow-acting policy whose effect will appear a few years later.

4.1 Purchase price subsidy policy for new product

In the sense that we consider a policy facilitating new product diffusion, we discuss case (i) $\tilde{b}_2(t) \geq p_U(t)$ with $s(p_2,t) \equiv 0$ and case (ii) $p_L(t) - \Delta p(t) < \tilde{b}_2(t) < p_U(t)$ with $0 < s(p_2,t) < 1$ in Fig. 6, whose market shares are not equal to 1.

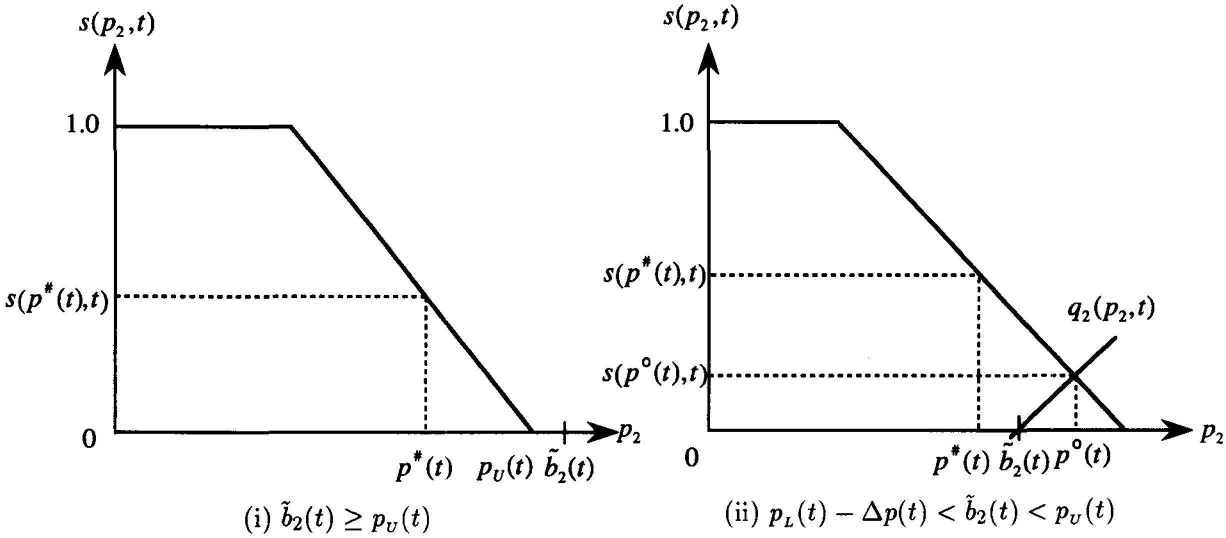


Figure 8 Price subsidy policy for purchase of new product

Case (i) $\tilde{b}_2(t) \geq p_U(t)$

Suppose that investment in plant and equipment has already been executed for manufacturing new product. Then, in order to ensure profits by manufacturing only existing products, it is shown from (2.5) that it is at least necessary to sell new product at price higher than $\tilde{b}_2(t)$ in case of considering the recovery of fixed cost. In order to have a market share more than 0, we shall substantially reduce new product price at time t from equivalent variable cost $\tilde{b}_2(t)$ to $p^\sharp(t)$ ($p^\sharp(t) < p_U(t)$). Then, purchase price subsidy per new product becomes $\tilde{b}_2(t) - p^\sharp(t)$. The market share $s(p^\sharp(t),t)$ at real price $p^\sharp(t)$ of new product has the relation of $0 < s(p^\sharp(t),t) < 1$ from Fig. 8(i). Total amount of price subsidy for new product also becomes $\{\tilde{b}_2(t) - p^\sharp(t)\}s(p^\sharp(t),t)x(t)$. Denoting the increment of industry's total profit induced under price subsidy policy by $\Delta r(p^\sharp(t),t), \Delta r(p^\sharp(t),t)$ is clearly equal to 0. In this case the price subsidy policy has the effect of only increasing market share of new product. The relation between the total amount of subsidy and market share, for various values of new product price $p^\sharp(t)$, gives a base for determining the total amount of subsidy.

Case (ii) $p_L(t) - \Delta p(t) < \tilde{b}_2(t) < p_U(t)$

Suppose the price of new product at time t is substantially reduced from $p^\circ(t)$ to $p^\sharp(t)$ by price subsidy $p^\circ(t) - p^\sharp(t)$ per unit product. Then, market share of new product increases from $s(p^\circ(t),t)$ to $s(p^\sharp(t),t)$. Price subsidy policy in this case is illustrated in Fig. 8(ii). The total amount of price subsidy becomes $\{p^\circ(t) - p^\sharp(t)\}s(p^\sharp(t),t)x(t)$. Noticing gross profit

$p^\circ(t) - \tilde{b}_2(t)$ per unit new product does not change even if new product price is substantially reduced to $p^\sharp(t)$, an incremental profit $\triangle r(p^\sharp(t), t)$ due to increased market share becomes

$$\triangle r(p^\sharp(t), t) = \{p^\circ(t) - \tilde{b}_2(t)\}\{s(p^\sharp(t), t) - s(p^\circ(t), t)\}x(t)$$

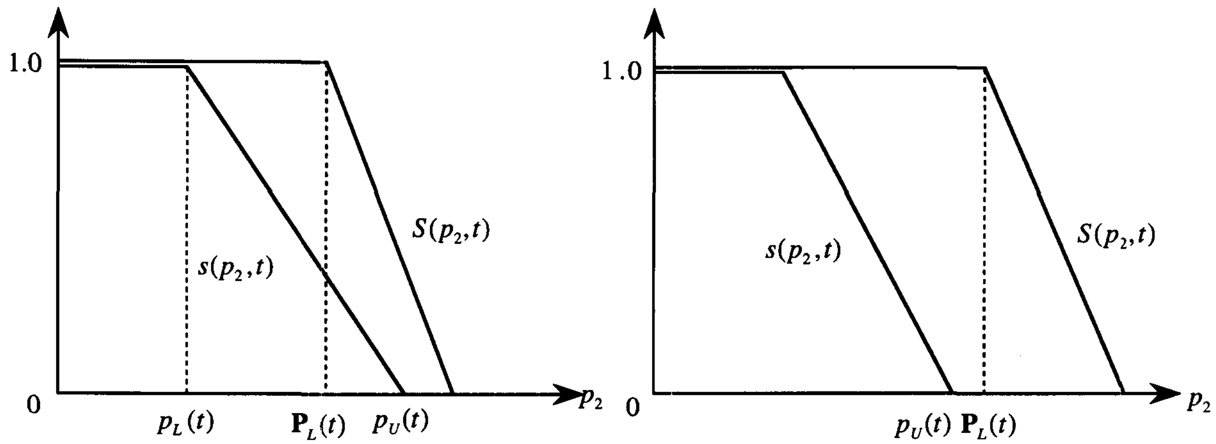
from (2.5). Price subsidy policy in this case contributes to increase in both market share and industry profit. As subsidy policy is basically financed by tax, $\triangle r(p^\sharp(t), t)$ should not be utilized for any purpose other than new product diffusion. From this viewpoint, it is necessary to construct a system in which $\triangle r(p^\sharp(t), t)$ is re-invested as a fund for spreading new product or is allotted for cutting down subsidy (hereafter, we call it differential subsidy). As the case (ii) with $0 < s(p_2, t) < 1$ are realistic, it is especially important to construct such a system. Following differential subsidy policy, the total net amount of price subsidy becomes

$$\begin{aligned}&\{p^\circ(t) - p^\sharp(t)\}s(p^\sharp(t), t)x(t) - \triangle r(p^\sharp(t), t)\\ &= [\{\tilde{b}_2(t) - p^\sharp(t)\}s(p^\sharp(t), t) - \{p^\circ(t) - \tilde{b}_2(t)\}s(p^\circ(t), t)]x(t).\end{aligned}$$

In case of changing $p^\sharp(t)$, the relation between the total net amount of price subsidy and market share gives a base for determining the total amount of subsidy.

4.2 Subsidy policy for technical development

As another effective policy for promoting new product diffusion, there is a subsidy policy for technical development which concerns the development and production of new products. Suppose a series of multi-period subsidy policies and corresponding investments for speeding up the technical development for certain periods. An improved performance of new product achieved by subsidy policies changes the total demand from $x(t)$ to $X(t)$ and increases market share of new product from $s(p_2, t)$ to $S(p_2, t)$ through strengthening competitive power against existing product at the same price. The relation between $s(p_2, t)$ and $S(p_2, t)$ is illustrated in Fig. 9.



case I: $p_L(t) < \mathbf{P}_L(t) < p_U(t)$ case II: $p_U(t) < \mathbf{P}_L(t)$

Figure 9 Relation between subsidy policy for technical development and market share of new product

Since the investment in technical development is incorporated into annual fixed cost, fixed cost at time t increases from $a_2(t)$ to $A_2(t)$ under subsidy policies. Further, variable cost will decrease from $b_2(t)$ to $B_2(t)$ through increasing production efficiency of new product under technical development. Thus, equivalent variable cost is also reduced from $\tilde{b}_2(t) \equiv m_1(t)+b_2(t)$ to $\tilde{B}_2(t) \equiv m_1(t)+B_2(t)$. For the above reason, it is considered to be valid to assume the following conditions:

Assumption 4 $a_2(t) < A_2(t)$
Assumption 5 $p_L(t) < \mathbf{P}_L(t), p_U(t) < \mathbf{P}_U(t)$
Assumption 6 $p_U(t) - p_L(t) > \mathbf{P}_U(t) - \mathbf{P}_L(t)$
Assumption 7 $\tilde{b}_2(t) > \tilde{B}_2(t)$

In the same way as (2.2), $S(p_2,t)$ at any time t is expressed by

$$S(p_2,t) = \begin{cases} 1 & p_2 \le \mathbf{P}_L(t) \\ -\dfrac{p_2 - \mathbf{P}_U(t)}{\Delta\mathbf{P}(t)} & \mathbf{P}_L(t) < p_2 < \mathbf{P}_U(t) \\ 0 & \mathbf{P}_U(t) \le p_2 \end{cases} \tag{4.1}$$
$$\Delta\mathbf{P}(t) \equiv \mathbf{P}_U(t) - \mathbf{P}_L(t)(\ge 0).$$

Further, letting

$$\begin{aligned} &\triangle a_2(t) \equiv A_2(t) - a_2(t) \\ &\triangle s(p_2,t) \equiv S(p_2,t) - s(p_2,t) \\ &\triangle \tilde{b}_2(t) \equiv -\{\tilde{B}_2(t) - \tilde{b}_2(t)\} \\ &\triangle x(t) \equiv X(t) - x(t) \end{aligned} \tag{4.2}$$

we have the relations of

$$\triangle a_2(t) > 0, \triangle s(p_2,t) \ge 0, \triangle \tilde{b}_2(t) > 0$$

from Assumptions 4, 5 and 7. Then the profit at time t, $R(p_2,t)$, under subsidy policy becomes

$$\begin{aligned} R(p_2,t) &= m_1(t)X(t) + \{p_2 - \tilde{B}_2(t)\}S(p_2,t)X(t) - \{a_1(t)+A_2(t)\} \\ &= r(p_2,t) + m_1(t)\triangle x(t) + \{p_2 - \tilde{b}_2(t)\}\triangle s(p_2,t)X(t) \\ &+ \triangle \tilde{b}_2(t)S(p_2,t)X(t) - \triangle a_2(t) \end{aligned} \tag{4.3}$$

from (2.5). The total profit at time t changes from $r(p_2,t)$ to $R(p_2,t)$. From (4.3), an increment in profit $\triangle r(p_2,t)$ at time t under subsidy policy becomes

$$\begin{aligned} \triangle r(p_2,t) &= m_1(t)\triangle x(t) + \{p_2 - \tilde{b}_2(t)\}\triangle s(p_2,t)X(t) \\ &+ \triangle \tilde{b}_2(t)S(p_2,t)X(t) - \triangle a_2(t). \end{aligned} \tag{4.4}$$

At the right hand side of (4.4), the first, second, third and fourth terms are concerned with variations in demand, market share, production cost of new product and investment for subsidy policy (hereafter, we call $\triangle a_2(t)$ additional investment). The first through third terms usually become positive, because it is considered that there hold $\triangle x(t) > 0$, $\triangle S(p_2,t) > 0$ and $\triangle \tilde{b}_2(t) > 0$ under subsidy policy and $p_2 - \tilde{b}_2(t) > 0$ at setting new product price. The forth term is clearly negative. In case where the effect of additional investment under subsidy policy has occurred on and after time t_0, we may consider that there hold $\triangle x(t) = 0$, $\triangle s(p_2, t) = 0$ and $\triangle \tilde{b}_2(t) = 0$ at any t with $t < t_0$. Then, we have $\triangle r(p_2, t) = -\triangle a_2(t)$. Since the amount of additional investment usually becomes large, it is highly possible that the amount exceeds the sum of increases in profit from the first term through the third. From the social viewpoint that the diffusion of new product is desirable, industry should bear such social cost incurred in technical development for diffusion even if $\triangle r(p_2, t)$ is negative. However, in case where the cost is great, it is necessary that the technical development is subsidized by the government and other public organizations. Such a technical development is shown in semiconductor industry, and it is also going to be introduced for developing a new storage battery for electric car and for recycling technology. Here, an important point is that any subsidy policies should be executed based not on additional investment $\triangle a_2(t)$ but on $\triangle r(p_2, t)$ under the concept of differential subsidy. In case where there is a delay in effects of additional investment, it is highly possible that $\triangle r(p_2, t) < 0$ at earlier time while $\triangle r(p_2, t) > 0$ at later time. It is necessary to construct a system which enables public organization to execute a flexible subsidy policy, grasping continuously and correctly the change of $\triangle r(p_2, t)$. Next, we will consider how set price of new product and its market share at set price change under subsidy policy.

Letting

$$Q(p_2, t) \equiv \{p_2 - \tilde{B}_2(t)\} S(p_2, t), \tag{4.5}$$

the problem of setting new product price which maximizes (4.3) is equivalent to

$$\max_{p_2 \geq \tilde{B}_2(t)} Q(p_2, t) \tag{4.6}$$

Substituting (4.1) for $S(p_2, t)$ in (4.5), we obtain

$$Q(p_2, t) = \begin{cases} p_2 - \tilde{B}_2(t) & p_2 \leq \mathbf{P}_L(t) \\ -\{p_2 - \tilde{B}_2(t)\}\{p_2 - \mathbf{P}_U(t)\}/\triangle\mathbf{P}(t) & \mathbf{P}_L(t) < p_2 < \mathbf{P}_U(t) \\ 0 & p_2 \geq \mathbf{P}_U(t). \end{cases} \tag{4.7}$$

Therefore, with respect to the value $\mathbf{P}^\circ(t)$ of price $p_2(t)$ which satisfies (4.6) and market share $S(\mathbf{P}^\circ(t), t)$, the following relations hold by substituting $\mathbf{P}^\circ(t)$ for $p^\circ(t)$, $\mathbf{P}_L(t)$ for $p_L(t)$, $\mathbf{P}_U(t)$ for $p_U(t)$, $\tilde{B}_2(t)$ for $\tilde{b}_2(t)$ and $\triangle\mathbf{P}(t)$ for $\triangle p(t)$ in (2.10) through (2.12):

Case (i') $\tilde{B}_2(t) \geq \mathbf{P}_U(t)$

$$S(\mathbf{P}^\circ(t), t) \equiv 0 \tag{2.10$'$}$$

Case (ii') $\mathbf{P}_L(t) - \triangle\mathbf{P}(t) < \tilde{B}_2(t) < \mathbf{P}_U(t)$

$$\begin{aligned} \mathbf{P}^\circ(t) &= \frac{\mathbf{P}_U(t) + \tilde{B}_2(t)}{2} \\ 0 < S(\mathbf{P}^\circ(t), t) &= \frac{\mathbf{P}_U(t) - \tilde{B}_2(t)}{2\triangle\mathbf{P}(t)} < 1 \end{aligned} \tag{2.11$'$}$$

Case (iii') $\tilde{B}_2(t) \leq \mathbf{P}_L(t) - \triangle\mathbf{P}(t)$

$$\begin{aligned} \mathbf{P}^\circ(t) &= \mathbf{P}_L(t) \\ S(\mathbf{P}^\circ(t), t) &\equiv 1 \end{aligned} \tag{2.12$'$}$$

Proposition 2.

If Assumptions 5 through 7 hold at any time t, there holds the relation of

$$S(\mathbf{P}^\circ(t),t) \geq s(p^\circ(t),t)$$

with respect to market share $s(p^\circ(t),t)$ without subsidy policy and market share $S(\mathbf{P}^\circ(t),t)$ under subsidy policy. Here, we let $p^\circ(t) = \tilde{b}_2(t)$ and $\mathbf{P}^\circ(t) = \tilde{B}_2(t)$ in the cases $s(p_2,t) \equiv 0$ and $S(p_2,t) \equiv 0$, respectively.

Proof.

We discuss the relation between $s(p_2,t)$ and $S(p_2,t)$ by classifying it into case I $p_L(t) < \mathbf{P}_L(t) < p_U(t)$ and case II $p_U(t) < \mathbf{P}_L(t)$ in Fig. 9. Case II is self-evident from Fig. 9 and Assumptions 6 and 7. Therefore we discuss only case I.

(i') $\tilde{B}_2(t) \geq \mathbf{P}_U(t)$

There holds $S(p_2,t) \equiv 0$ in (2.10') from the relation $\tilde{B}_2(t) \geq \mathbf{P}_U(t)$. Then, since we have the relation $\tilde{b}_2(t) > \tilde{B}_2(t) \geq \mathbf{P}_U(t) > p_U(t)$ from Assumption 5 and 7, there holds $s(p_2,t) \equiv 0$ in (2.10). Therefore, the result follows.

(ii') $\mathbf{P}_L(t) - \triangle\mathbf{P}(t) < \tilde{B}_2(t) < \mathbf{P}_U(t)$

(a) $p_U(t) \leq \tilde{b}_2(t)$

There holds the relation $0 < S(\mathbf{P}^\circ(t),t) < 1$ in (2.11') from the relation $\mathbf{P}_U(t) - \triangle\mathbf{P}(t) < \tilde{B}_2(t) < \mathbf{P}_U(t)$. Further, since we have $s(p_2,t) \equiv 0$ in (2.10) from the relation $p_U(t) \leq \tilde{b}_2(t)$, there holds $p^\circ(t) = \tilde{b}_2(t)$. From this, the result follows.

(b) $\tilde{b}_2(t) < p_U(t)$

In this case, $S(\mathbf{P}^\circ(t),t)$ is given by (2.11'), since we have the relation of $\mathbf{P}_L(t) - \triangle\mathbf{P}(t) < \tilde{B}_2(t) < \tilde{b}_2(t) < p_U(t) < \mathbf{P}_U(t)$ from Assumptions 5 and 7. And $s(p^\circ(t),t)$ is given by (2.11), since we have the relation of $p_L(t) - \triangle p(t) < \tilde{B}_2(t) < \tilde{b}_2(t) < p_U(t)$ by applying Assumptions 5 through 7 to the above relation. Applying Assumptions 5 through 7 to (2.11) and (2.11'), there holds $S(\mathbf{P}^\circ(t),t) > s(p^\circ(t),t)$. Thus the result follows.

(iii') $\tilde{B}_2(t) \leq \mathbf{P}_L(t) - \triangle\mathbf{P}(t)$

In this case, there hold the relations of $S(\mathbf{P}^\circ(t),t) \equiv 1$ and $\mathbf{P}^\circ(t) = \mathbf{P}_L(t)$. Therefore, the result follows.

From (i') through (iii'), the relation of $S(\mathbf{P}^\circ(t),t) \geq s(p^\circ(t),t)$ has been proved for Case I. Thus, the proof is complete. □

We have mentioned that Assumptions 4 through 7 with respect to variable cost, market share, etc. under subsidy policy are considered to be natural. Proposition 2 implies that market share under new subsidy policy for technical development is greater than or equal to that without subsidy policy so long as Assumptions 5 through 7 hold. In other words, it is shown that new product diffusion depends only on changes in market share and equivalent variable cost and is not affected by changes in fixed cost and demand.

Next, we shall compare price changes of new product under the cases with and without subsidy policy. Restricting our argument to Case I (ii') (b) in Proposition 2 from the same reason as stated in the relation (3.1), it is easily shown that the following relation holds.

$$\tilde{b}_2(t) - \tilde{B}_2(t) \geq \mathbf{P}_U(t) - p_U(t) \Leftrightarrow \mathbf{P}^\circ(t) \leq p^\circ(t) \text{ for one } t \in \{1,2,\cdots\} \tag{4.8}$$

The relation (4.8) implies that the decrease in set price of new product under subsidy policy is determined by the size relation that the acceleration impact of progress in manufacturing technique on production (decrease in equivalent variable cost) exceeds the acceleration impact of progress in product technology on the market (increases in upper limit of product price).

5. Numerical examples

We shall present some numerical examples with respect to main results of anayses from Chapters 2 through 4. It is assumed that variable cost $b_j(t)$ of project j $(j = 1.2)$, quantities of $s(p_2, t)$ in (2.2) and exsiting product price $p_1(t)$ are given by the following exponential functions of t:

$$\begin{aligned} b_1(t) &= 50 + 10e^{-0.2t} \\ b_2(t) &= 40 + 60e^{-0.5t} \end{aligned} \tag{5.1}$$

$$\begin{aligned} p_U(t) &= 175 - 50e^{-0.05t} \\ p_L(t) &= 175 - 80e^{-0.05t}(1 + e^{-4t}) \\ \triangle p(t) &\equiv p_U(t) - p_L(t) \\ &= 30e^{-0.05t} + 80e^{-4.05t} \end{aligned} \tag{5.2}$$

$$p_1(t) = 120 + 5e^{-0.1t} \tag{5.3}$$

From (5.1) and (5.3), we have

$$\begin{aligned} \tilde{b}_2(t) &\equiv p_1(t) + b_2(t) - b_1(t) \\ &= 110 + 5e^{-0.1t} + 60e^{-0.5t} - 10e^{-0.2t}. \end{aligned} \tag{5.4}$$

From (5.1) through (5.4), it is easily shown that Assumptions 1 through 3 in Chapter 3 hold. Set price $p^\circ(t)$ and its market share $s(p^\circ(t), t)$ can be determined by using equations defined in Section 2.3. Table 1 represents values of $\tilde{b}_2(t)$, $p_U(t)$, $p_L(t)$, $p_L(t) - \triangle p(t)$, $p^\circ(t)$ and $s(p^\circ(t), t)$ for $t = 1, 2, \cdots, 12$. From the size relation between $\tilde{b}_2(t)$ and $p_U(t)$ and/or $p_L(t) - \triangle p(t)$, it is shown that case (i) occurs at $t = 1$, case (ii) at $t = 2, 3, \cdots, 10$ and case (iii) at $t = 11, 12$, and thus the lemma holds. Furthermore, it can be checked that Propsition 1 holds since $s(p^\circ(t), t)$ is non-decreasing in t.

Next, economic effects under price subsidy policy in Section 4.1 are given in Table 2 for both case (i) at $t = 1$ $(p^\circ(1) = 142.7,\ s(p^\circ(1), 1) = 0)$ and case (ii) at $t = 3$ $(p^\circ(3) = 126.7,\ s(p^\circ(3), 3) = 0.20)$ of the above example. For case (i), market share $s(p^\sharp(1), 1)$ and subsidy amount per unit product SA ($= \{\tilde{b}_2(1) - p^\sharp(1)\}s(p^\sharp(1), 1)$) are shown, when we change new product price $p^\sharp(1)$ under price subsidy policy. For case (ii), market share $s(p^\sharp(3), 3)$, subsidy amount per unit product SA ($= \{p^\circ(3) - \tilde{b}_2(3)\}\{s(p^\sharp(3), 3) - s(p^\circ(3), 3)\}$) and differential subsidy per unit product SA_d ($= \{\tilde{b}_2(3) - p^\sharp(3)\}s(p^\sharp(3), 3) + \{p^\circ(3) - \tilde{b}_2(3)\}s(p^\circ(3), 3)$) are given when we change new product price $p^\sharp(3)$ under price subsidy policy. For case (ii) where price subsidy is executed in the situation of market share being greater than 0, it is shown that an reduction in subsidy amount becomes possible under the differential subsidy.

Lastly, with respect to subsidy policy for technical development, we assume $B_2(t)$ and quantities of $S(p_2, t)$ as follows:

$$B_2(t) = 30 + 50e^{-0.7t} \tag{5.5}$$

$$\begin{aligned} \mathbf{P}_U(t) &= 175 - 40e^{-0.05t} \\ \mathbf{P}_L(t) &= 175 - 70e^{-0.05t}(1 + e^{-5t}) \\ \Delta\mathbf{P}(t) &\equiv \mathbf{P}_U(t) - \mathbf{P}_L(t) \\ &= 30e^{-0.05t} + 70e^{-5.05t} \end{aligned} \tag{5.6}$$

Table 1 Changes of set price and market share with time

case	t	$\tilde{b}_2(t)$	$p_U(t)$	$p_L(t)$	$p_L(t)-\Delta p(t)$	$p^{\circ}(t)$	$s(p^{\circ}(t),t)$
case (i)	1	142.7	127.4	97.5	67.5	142.7	0.00
case (ii)	2	129.4	129.7	102.5	75.4	129.6	0.01
	3	121.6	131.9	106.1	80.3	126.7	0.20
	4	116.9	134.0	109.5	84.9	125.5	0.35
	5	114.2	136.0	112.7	89.3	125.1	0.47
	6	112.7	137.9	115.7	93.5	125.3	0.57
	7	111.8	139.7	118.6	97.4	125.8	0.66
	8	111.3	141.4	121.3	101.2	126.4	0.75
	9	111.0	143.1	123.9	104.8	127.0	0.84
	10	110.8	144.6	126.4	108.2	127.7	0.93
case (iii)	11	110.8	146.1	128.8	111.5	128.8	1.00
	12	110.7	147.5	131.1	114.6	131.1	1.00

Table 2 Economic effects of price subsidy policy

case	t	$p^{\#}(t)$	$s(p^{\#}(t),t)$	SA	SA_d
case (i)					
$p^{\circ}(1)=142.7$		97.5	1.00	45.2	—
$p_L(1)=97.5$	1	100.0	0.91	39.1	—
$p_U(1)=127.4$		110.0	0.58	19.0	—
$s(p^{\circ}(1),1)=0.00$		120.0	0.24	5.6	—
case (ii)					
$p^{\circ}(3)=126.7$		106.1	1.00	20.6	16.5
$p_L(3)=106.1$	3	110.0	0.85	14.2	10.9
$p_U(3)=131.9$		115.0	0.65	7.7	5.3
$s(p^{\circ}(3),3)=0.20$		120.0	0.46	3.1	1.7

From (5.1), (5.3) and (5.5), we have

$$\begin{aligned}\tilde{B}_2(t) &\equiv p_1(t)+B_2(t)-b_1(t)\\ &=100+5e^{-0.1t}+50e^{-0.7t}-10e^{-0.2t}.\end{aligned} \tag{5.7}$$

From (5.1) through (5.7), it is easily checked that Assumptions 5 through 7 hold. Table 3 compares market shares with and without subsidy for technical development, which are represented by $s(p^\circ(t),t)$ and $S(\mathbf{P}^\circ(t),t)$, respectively. It is checked that Proposition 2 holds since market share $S(P^\circ(t),t)$ is greater than or equal to $s(p^\circ(t),t)$ for $t=1,2,\cdots,12$.

Table 3 Comparison between policies with and without technical development subsidy

t	policy without subsidy		policy with subsidy	
	$p^\circ(t)$	$s(p^\circ(t),t)$	$\mathbf{P}^\circ(t)$	$S(\mathbf{P}^\circ(t),t)$
1	142.7	0.00	129.0	0.27
2	129.6	0.01	124.2	0.54
3	126.7	0.20	122.4	0.70
4	125.5	0.35	122.0	0.82
5	125.1	0.47	122.3	0.92
6	125.3	0.57	123.1	1.00
7	125.8	0.66	125.6	1.00
8	126.4	0.75	128.0	1.00
9	127.0	0.84	130.3	1.00
10	127.7	0.93	132.5	1.00
11	128.8	1.00	134.6	1.00
12	131.1	1.00	136.5	1.00

6. Some considerations on the case where a demand peculiar to new product is created

In the preceding chapters, we have considered the premise 3 that the whole demand for products consists of conventional and substitutional products and the pricing policy for new product determines only the market share of new product. In this chapter, we discuss a case where, in addition to substitutional demand for existing product, a potential demand peculiar to new product is created by improving remarkably the performance of new product or adding new function not provided with existing one, and it is realized by pricing policy for new product. Concretely, we discuss how the set price and market share, where we consider the demand peculiar to realized new product, change compared with the case where we don't consider the demand. Here we discuss the case where market share $s(p_2,t)$ for substitutional demand $x(t)$ is non-linear illustrated in Fig. 3. Letting the total demand be $z(p_2,t)$ in case where newly created demand $y(p_2,t)$ exists for new product, we obtain

$$z(p_2,t) = x(t) + y(p_2,t). \tag{6.1}$$

Let set price of new product be $p_z^\circ(t)$ in this case. In this chapter, we discuss the case where market share $s(p_2,t)$ for substitutional demand $x(t)$ generally has a non-linear function as illustrated in Fig. 3. With respect to $y(p_2,t)$ changes in p_2, it will be valid to assume a similar

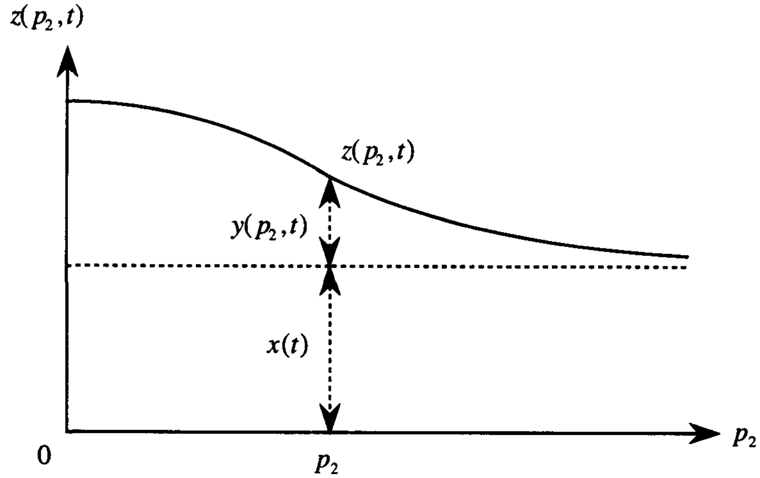


Figure 10 Relation between p_2 and $z(p_2,t)$

non-linear curve to $s(p_2,t)$ in Fig. 3 from the same reasoning of $s(p_2,t)$. These relations are illustrated in Fig. 10. Letting the total profit of industry at time t be $r_z(p_2,t)$ we have

$$\begin{aligned} r_z(p_2,t) &= m_1(t)x(t) + \{p_2 - \tilde{b}_2(t)\}s(p_2,t)x(t) - \sum_{j=1}^{2} a_j(t) + \{p_2 - b_2(t)\}y(p_2,t) \\ &= r(p_2,t) + r_y(p_2,t) \\ r_y(p_2,t) &\equiv \{p_2 - b_2(t)\}y(p_2,t) \end{aligned} \tag{6.2}$$

from (2.5) and (6.1). Here, we denote $r_y(p_2,t)$ as the total amount of gross profit obtained from newly created demand peculiar to new product. And let p_2 values maximizing $r(p_2,t)$, $r_y(p_2,t)$ and $r_z(p_2,t)$ be $p^\circ(t)$, $p_y^\circ(t)$, and $p_z^\circ(t)$, respectively. Noticing the relations of $\tilde{b}_2(t) > b_2(t)$ and

$$\begin{aligned} r_z(p_2,t) &< m_1(t)x(t) - \sum_{j=1}^{2} a_j(t) \qquad \text{for } p_2 < b_2(t) \\ r_z(p_2,t) &\geq m_1(t)x(t) - \sum_{j=1}^{2} a_j(t) \qquad \text{for } p_2 \geq \tilde{b}_2(t), \end{aligned}$$

it is seen that $p_z^\circ(t)$, that is, the p_2 value which maximizes $r_z(p_2,t)$, can't be achieved in the p_2 region of $p_2 < b_2(t)$. Therefore, the relation $p_z^\circ(t) \geq b_2(t)$ holds.

Partially differentiating with respect to p_2 in order to maximize $r_z(p_2,t)$, we have

$$\begin{aligned} \frac{\partial r_z(p_2,t)}{\partial p_2} &= \frac{\partial r(p_2,t)}{\partial p_2} + \frac{\partial r_y(p_2,t)}{\partial p_2} \\ \frac{\partial r_y(p_2,t)}{\partial p_2} &= y(p_2,t) + \{p_2 - b_2(t)\}\frac{\partial y(p_2,t)}{\partial p_2}. \end{aligned} \tag{6.3}$$

Under the demand curve in Fig. 10, a typical relation between p_2 and $\dfrac{\partial r_y(p_2,t)}{\partial p_2}$ is illustrated in Fig. 11.

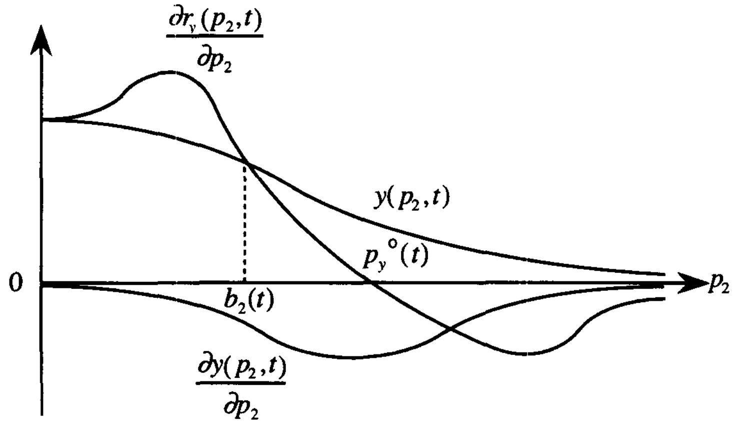


Figure 11 Typical relation between p_2 and $\partial y(p_2,t)/\partial p_2, \partial r_y(p_2,t)/\partial p_2$

Here, in order to simplify the following discussion, assume that there exist only one p_2 values satisfying $\partial r(p_2,t)/\partial p_2 = 0$, $\partial r_y(p_2,t)/\partial p_2 = 0$ and $\partial r_z(p_2,t)/\partial p_2 = 0$, and these values are represented by $p^\circ(t)$, $p_y^\circ(t)$ and $p_z^\circ(t)$, respectively.

If there holds $p_y^\circ(t) \le p^\circ(t)$, we have

$$\begin{aligned}\left[\frac{\partial r_z(p_2,t)}{\partial p_2}\right]_{p_2=p^\circ(t)} &= \left[\frac{\partial r(p_2,t)}{\partial p_2} + \frac{\partial r_y(p_2,t)}{\partial p_2}\right]_{p_2=p^\circ(t)} \\ &= \left[\frac{\partial r_y(p_2,t)}{\partial p_2}\right]_{p_2=p^\circ(t)} \le 0.\end{aligned}$$

From this, we have the relation of $p_z^\circ(t) \le p^\circ(t)$. Conversely, since the relation $p_z^\circ(t) \le p^\circ(t)$ implies the relation $\left[\frac{\partial r_y(p_2,t)}{\partial p_2}\right]_{p_2=p^\circ(t)} \le 0$, we have the relation $\left[\frac{\partial r_y(p_2,t)}{\partial p_2}\right]_{p_2=p^\circ(t)} \le 0$. This means that there holds the relation $p_y^\circ(t) \le p^\circ(t)$. From $s(p_2,t) \downarrow p_2$, the relation $p_z^\circ(t) \le p^\circ(t)$ is equivalent to the relation $s(p_z^\circ(t),t) \ge S(p^\circ(t),t)$.

Therefore, under the demand curve in Fig. 10 and the assumption that there exist only one $p^\circ(t)$, $p_y^\circ(t)$ and $p_z^\circ(t)$ and $p^\circ(t)$, and between $s(p_z^\circ(t),t)$ and $s(p^\circ(t),t)$, we have

$$p_y^\circ(t) \le p^\circ(t) \Leftrightarrow p_z^\circ(t) \le p^\circ(t) \Leftrightarrow s(p_z^\circ(t),t) \ge s(p^\circ(t),t). \tag{6.4}$$

The relation (6.4) implies that the following three relations are all equivalent: (1) set price $p_y^\circ(t)$ for newly created demand $y(p_2,t)$ being less than or equal to set price $p^\circ(t)$ for substitutional demand $x(t)$, (2) set price $p_z^\circ(t)$ for total demand $z(p_2,t)$ being less than or equal to set price $p^\circ(t)$, and (3) market share $s(p_z^\circ(t),t)$ for total demand being greater than or equal to $s(p^\circ(t),t)$ for substitutional demand. If $p_y^\circ(t) < p^\circ(t)$ holds, set price of new product for total demand decreases from $p^\circ(t)$ to $p_z^\circ(t)$, because the profit obtained from newly created demand for new product is so great that the price may be reduced. Then, the existence of newly created demand acts in the direction of increasing market share for substitutional demand. On the contrary, if $p_y^\circ(t) > p^\circ(t)$ holds, its existence acts in the direction of increasing set price of new product and decreasing market share. In case where higher market share for new product is needed from a social viewpoint, it is desirable that there exists a situation where higher profit for newly created demand can be attained at lower set price.

7. Conclusion

In this study, a linear model has been constructed and analyzed to grasp a trend of new product diffusion, considering two characteristics of production and market under TA. The model supposes a situation where the industry sets the price of new product so as to maximize the total profit obtained from mixed production of existing and new products. The results of analyses are as follows: (1) changes in set price of new product and its trend of diffusion are determined by the relation between the reduction in equivalent variable cost and changes with time in upper limit price for new product, that is, under some mild assumptions, changes in the price of new product can be determined by the size relation of the above relation, and the market share increases monotonously. Next, economic effects of subsidies for price and technical development on new product have been discussed. We have pointed out that (2) in executing a subsidy policy, it is important to identify newly generated profit, follow the concept of differential subsidy and construct a system returning flexibly the profit obtained from the subsidy policy to the desired end of new product diffusion, and (3) with regard to the subsidy policy for technical development, the market share of new product under the subsidy policy exceeds one without the subsidy under some mild conditions. Finally, this model has been extended to a model in which there is a potential demand peculiar to new product and market share is given by non-linear function. Here, it has been discussed how the set price differs compared with the case of non-existence of newly created demand peculiar to new product. The result shows that (4) the size relation between set prices with and without newly created demand of new product is corresponding to the size relation between market shares at each set price in both cases.

In the light of today's growing necessity of spreading the technology desirable from the social point of view such as mitigating environmental impacts, the viewpoints presented in this study are considered to be more and more valuable in future.

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