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HETEROGENEOUS SUBSCRIBERS AND THE OPTIMAL TWO-PART TARIFF OF TELECOMMUNICATIONS SERVICE

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Abstract In this article, optimal two-part tariffs of a bi-directional telecommunications service are examined. Heterogeneous users are assumed to formulate social demand for the service. Two regimes of supplier are considered: monopoly and non-monopoly. In the latter, the supplier which enjoyed monopoly is faced with the entry of a competitive supplier. Demand externalities which are conspicuous in telecommunications service are introduced into the model. The supplier is assumed to maximize the profit or to be forced to maximize the social benefit derived from the service. A nonlinear optimization method is applied to derive the optimal tariff. It is shown that the tariff diverges from conventional rules for public utility pricing.

1. Introduction

Recent technological advances in microelectronics and other areas have enabled many new electronic communication services to become available. In an "advanced information-oriented society", it is anticipated that tariffs of telecommunications services will less depend upon the distance between caller and callee. For example, communications within a certain range through satellite communications systems are completely independent of the distance. A uniform tariff which are independent of the distance will be applied to telecommunications services.

Most bi-directional telecommunications systems have demand or consumption externalities in that the benefit to a subscriber depends upon the number of subscribers to the system. As the number of subscribers increases, the benefit which a subscriber can obtain from the service also increases.

When a new telecommunications system is to be set up, even an establishment that feels it convenient and is willing to subscribe will not do so alone. It will either subscribe with others simultaneously or agree to subscribe in anticipation of others joining. An establishment is defined as a unit such as an individual, household, office, etc., that is willing to communicate regardless of whether they join the telecommunications system. This characteristic is essential to analyzing the demand and supply of telecommunications services and is discussed repeatedly by several authors.

Squire [9] and Littlechild [5] recognized that the externalities must affect the optimal pricing per call and per phone, and examined corresponding optimal usage and network size. However, they could not make clear in their models the process of how the externalities were generated.

Artle & Averous [1] considered a communication network as a public good, showing that the increase in the value of the service resulting from new subscribers could sustain the continual growth of a network in a stationary population with stationary income.

Rohlfs [8] introduced the notion of an "equilibrium user set", which was a set of users consistent with utility maximization by all individuals (users and nonusers). He examined some general properties of these sets and the conditions for the existence of feasible nonempty equilibrium user sets.

Why basic economic approaches have not been able to deal with demand externalities effectively? It is mainly because they assume that users are anonymous and homogeneous. When an establishment is going to use a telecommunications service, it should subscribe in advance. This means that it is not anonymous. Each establishment has two choices: (i) whether to subscribe or not; and (ii) how much communication should be made through the system. The volume of communication of an establishment depends upon the relationship with others. Each establishment should be heterogeneous.

Oren & Smith [7] assumed that user's demand directly depended upon the number of subscribers and developed an economic model that determined both the minimum user set for startup and the ultimate expansion level of a telecommunications system. Although their demand function dealt with the above problems, their result was theoretically unsatisfactory and did not seem to be useful. Nor did they analyze the case where the supplier was faced with the entry of competitive firms.

This paper develops a new model in which heterogeneous users are assumed to formulate demand for the service and demand externalities are explicitly produced.

2. Heterogeneous Users and Demand Externalities

Let us consider an area in which establishments constitute a complete set with respect to potential inter-establishment communications.

Assume that each establishment is identified with some unique index ξ, and without loss of generality that ξ is within interval [0,1]. Let η be also the index of an establishment in the same set. Any establishment is assumed to have the potential to communicate with others. Assume that establishments distribute in [0,1] continuously and their distribution can be represented with a density function, $g(\eta)$. If the potential of communication from ξ to η can be denoted by a correlation function, $s_0(\xi,\eta)$, the total potential volume of communication of an establishment ξ is defined as:

$$V(\xi) = \int_0^1 s_0(\xi,\eta)g(\eta)d\eta, \tag{2.1}$$

where $s_0(\xi,\eta)g(\eta)d\eta$ is the potential volume of communications sent from an establishment with index ξ to all establishments with indices in the range $[\eta, \eta + d\eta)$.

Each establishment has its own total potential volume, $V(\xi)$. It is desirable to define the labelling index ξ in such a way that establishments' preferences for subscription are monotone with respect to ξ.

Let us arrange the ranking of establishments in the area according to the magnitude of $V(\xi)$. Let the index of establishment which has the minimum total potential volume be $\xi = 1$ and that of the subscriber which has the maximum be 0. The index ξ shows the complete order of all establishments in the area with respect to the total potential volume of communication. We assume that the potential of communication from ξ to η and the distribution of establishments are represented with a correlation function $s(\xi,\eta)$ and a density function $f(\eta)$ respectively after the arrangement. It is plausible to assume that establishments' preferences for joining the system are monotone with respect to the index of subscribers. In this case, if an establishment has subscribed, those who have larger total potential volume of communication, i.e., those who have smaller index, have done so.

All establishments need not subscribe. A subscriber set will be a subset of the full establishments' set. Let the index of subscriber who has the minimum total potential volume be $\xi = y$.

Since the subscriber set is a closed set $[0, y]$, the realizable potential volume of a subscriber with index $0 \leq \xi \leq y$ is defined as

$$S(\xi, y) = \int_0^y s(\xi, \eta) f(\eta) d\eta. \tag{2.2}$$

The realizable social potential volume, Vs, is given by

$$Vs = \int_0^y S(\xi, y) f(\xi) d\xi. \tag{2.3}$$

Note that y is exogenous to any establishment.

Let $D(p, \xi, y)$ be a demand of an establishment ξ at price p for subscriber set y. Note that a demand function denotes consumer's utility maximizing quantity of consumption in microeconomic theory.

Suppose that only the number of subscribers affects the demand of an establishment but it does not care who these subscribers are. This condition is called the "uniform calling pattern" (see Rohlfs[8]).

If we assume that the demand function is separable between price p and the realizable potential volume of communication $S(\xi, y)$ as:

$$v = D(p, \xi, y) = D(p, S(\xi, y)), \tag{2.4}$$

the relationship between the realizable potential and the demand should be written as

$$D(0, \xi, y) = S(\xi, y) \tag{2.5}$$

since $S(\xi, y)$ can be regarded as a potetial demand in case of zero price.

The gross benefit measured by gross consumer surplus is defined as the relevant area under the demand function. Solving (2.4) for p at $p = m$, the gross consumer surplus of an establishment ξ at price $p = m$ can be given by

$$B(m, S(\xi, y)) = \int_0^{v_m} m(v, \xi, y) dv, \tag{2.6}$$

where $v_m = D(m, \xi, y)$.

Gross consumer's surplus, which is measured by the trapezoidal area under the demand function as (2.6), is interpreted as the gross consumer's benefit derived from utility maximizing consumption at a given price.

$B(m, S(\xi, y))$ can also be interpreted as willingness to pay of an establishment ξ when it sends the volume v_m.

Equation (2.6) represents straightforwardly the positive externalities of demand in the use of communications systems. It is possible to prove under general assumptions that the benefit $B(m, S(\xi, y))$ is monotone increasing in the size of subscriber set y and monotone decreasing in the ranking index ξ. The benefit to a subscriber will increase as the number of subscribers increases and those who have greater potential volume can enjoy greater benefit.

Assume that a supplier offers a tariff plan, $C(v)$, which depends on v, the volume sent by a subscriber. Since subscriber's utility maximizing consumption is represented by the unique point on the demand curve corresponding to the unit usage charge m, the maximum net benefit, $\psi(m, \xi, y)$, can be given by subtracting his total payment $C(v)$ from the gross consumer's surplus in (2.6) as:

$$\psi(m, \xi, y) = B(m, S(\xi, y)) - C(v). \tag{2.7}$$

For a subscriber set to be feasible, with respect to the smallest subscriber represented with $\xi = y$ who has the minimum realizable potential volume, the net benefit should be non-negative:

$$\psi(m, y, y) \geq 0. \tag{2.8}$$

If (2.8) is satisfied, those who have greater volume, i.e., those who have smaller index, must have greater non-negative net benefits.

Definition 1 (equilibrium of a subscriber set)
For any $y = y^*$ such that $\psi(m, y^*, y^*) = 0$, the subscriber set $[0, y^*]$ is in equilibrium.

The system expands as far as the net benefit to the smallest subscriber, $\psi(m, y, y)$, has a positive value. It means that the growth of the system is provided by the net benefit to the smallest subscriber. As an illustrative model, let us consider the most straightforward expression of this as follows:

$$\dot{y} \equiv \frac{dy}{dt} = \psi(m, y, y). \tag{2.9}$$

Definition 2 (stability of equilibrium)
An equilibrium set, $[0, y^*]$, of the system (2.9) is asymptotically stable if for any admissible initial point y_0 the solution $\Psi(t; y_0)$ to (2.9) satisfies $\lim_{t\to\infty} \Psi(t; y_0) = y^*$.

Definition 3 (equilibrium subscriber set)
An equilibrium subscriber set is a stable set of subscribers such that no single non-subscriber has the incentive to subscribe to the service (Oren & Smith [7]).

Definition 4 (critical mass)
A critical mass is a minimum unstable subscriber set for the system to be feasible.

Fig. 2.1(a) shows a typical case where the smallest subscribers' benefit function is unimodal for $y \in [0, 1]$. There exist two equilibrium sets, $[0, y_1^*]$ and $[0, y_2^*]$, which satisfy $\psi(m, y^*, y^*) = 0$, for a constant unit usage charge case, $C(v) = mv$.

If (2.9) holds, this figure coincides with the phase diagram of the system. The system will expand or shrink along the path (see Fig. 2.1(b)).

Lemma 1 An equilibrium set $[0, y^*]$ is an "equilibrium subscriber set" if $d\psi(m, y, y,)/dy < 0$ at $y = y^*$.

Lemma 2 An equilibrium set $[0, y^*]$ is a "critical mass" if $d\psi(m, y, y)/dy > 0$ at $y = y^*$.

In Fig. 2.1(b), $[0, y_2^*]$ is an "equilibrium subscriber set", which means the ultimate expansion level of the system. $[0, y_1^*]$ is a "critical mass", which is defined as the minimum user set in order to set up the system. It is obvious that y_0^* such that $y_0^* = 0$ is a stable equilibrium.

If an initial subscriber set is given at $y_0 = y_a$, the system will shrink to zero. However, if $y_0 = y_b$ or $y_0 = y_c$, the system will converge to y_2^*, in the same way as in the case of $y_0 = y_d$.

In some cases, the function is multimodal where multiple equilibria have different levels of critical mass and equilibrium subscriber set.

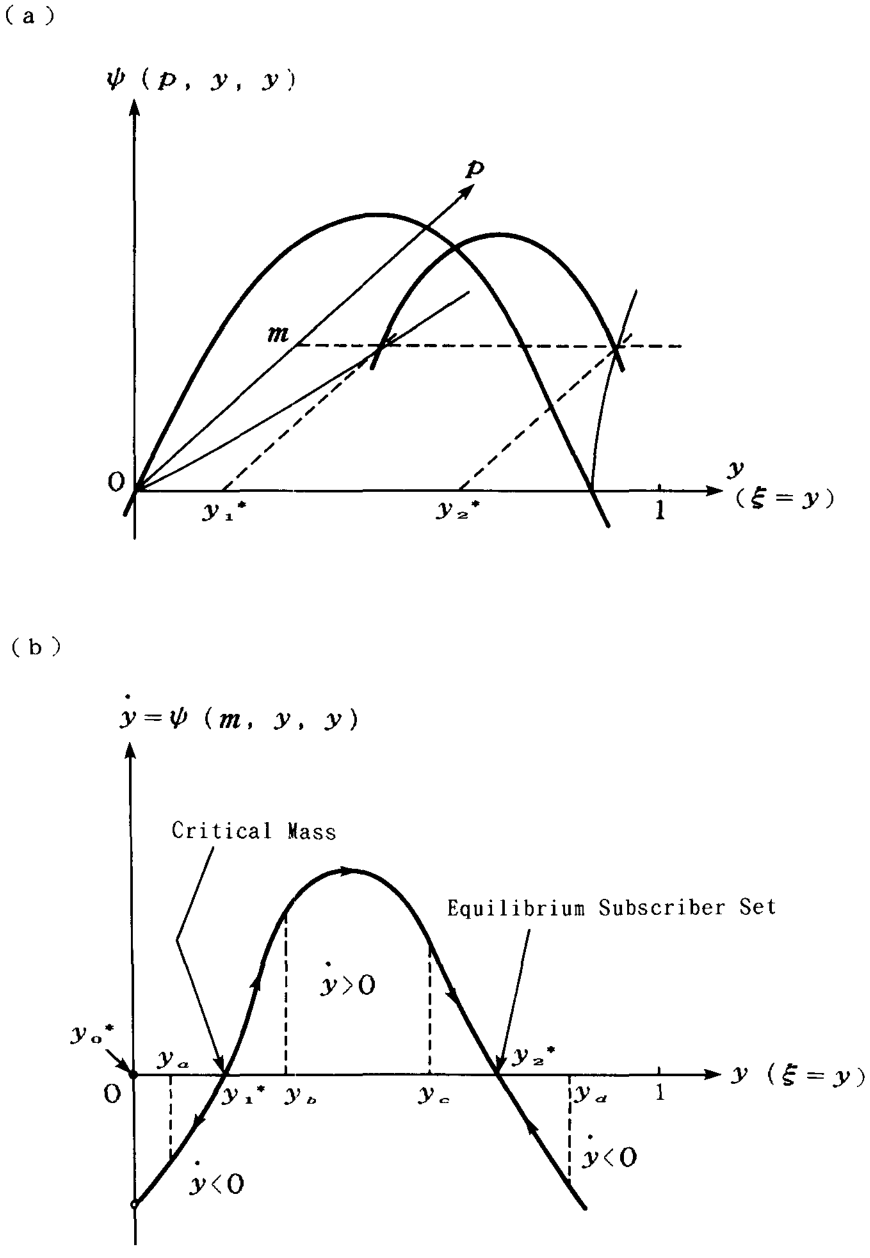

Fig. 2.1 Critical Mass and Equilibrium Subscriber Set in the Case of a Unimodal Smallest Subscriber's Benefit Function

3. Optimal Two-Part Tariff in the Case of Monopoly

3.1 Model Formulation

3.1.1 Assumptions

Assumption 1. A two-part tariff is applied to the pricing of telecommunications service:

$$C(v) = C_0 + mv \tag{3.1}$$

where m : unit usage charge
C_0 : fixed charge
v : volume of communication

Assumption 2. The benefit derived from receiving call is ignored.

This external effect arises when a call confers a benefit on the recipient as well as the originator. In theory, efficient pricing should take account of this externality. It would argue for further divergence of the price of a call discussed below. However, it is sometimes criticized to involve it (see Brown & Sibley [3], for example). One reason is that it only involves two people and can probably "internalized". For example, two frequent callers could arrange to share the cost of calling. Another reason is that not all benefits are positive. Since no one can distinguish between positive and negative externalities, it is probably not useful to incorporate them into pricing formulas.

Assumption 3. Each establishment has its ranking index ξ according to the potential volume of communication. The index of an establishment ξ distributes uniformly and continuously in [0,1]. The establishment, $\xi = 0$, has the largest potential volume and $\xi = 1$ has the smallest.

In the previous section, the distribution of establishments is assumed to be represented with a density function. However, here we assume for simplicity that their distribution is uniform. Although pricing formulas must be affected by the distribution, to introduce it explicitly makes their derivation complex and may not provide effective case for price alternations.

Assumption 4. The preference order of joining the system is monotone decreasing in ξ.

Assumption 5. The demand function $D(m, \xi, y)$ is continuous for $0 \leq y \leq 1, 0 \leq \xi \leq y$, and $m \geq 0$.

Assumption 6. The demand function satisfies the following inequalities:

$$\begin{aligned} &\partial D(m, \xi, y)/\partial m \leq 0, \\ &\partial D(m, \xi, y)/\partial \xi \leq 0, \\ &\partial D(m, \xi, y)/\partial y \geq 0. \end{aligned} \tag{3.2}$$

Assumption 7. The supplier bears fixed cost per subscriber, k, and constant marginal cost per unit of volume transmitted, r.

3.1.2 The Structure of Telecommunications Market

A monopoly supplier provides one kind of telecommunications service. It gains profits $m - r$ per volume of communication and $C_0 - k$ per subscriber. Subscribers are offered a two-part tariff from the supplier. They will determine their usage level so as to maximize their net benefit. Even if they do not make any calls, they have to pay fixed charge. They can forgo the service if their net benefit is negative. Non-subscribers can subscribe if their net benefit is expected to be non-negative.

The sum of the profit of the supplier and the total net benefit to subscribers is interpreted as the social benefit derived from the service.

Subtracting subscriber's total usage charge payment which is proportional to his volume of consumption from his gross consumer's surplus yields (net) consumer's surplus. It is measured by the triangular area between the demand curve and the price line. It is obvious that the subscriber's net benefit corresponds with consumer's surplus if there exist no fixed charge payment.

If fixed charge payment should be taken into account, the net benefit is defined as the consumer's surplus minus the fixed charge payment.

Let us define $\varphi(m, \xi, y)$ as the consumer's surplus of an establishment ξ for subscriber set y at unit usage charge m:

$$\varphi(m, \xi, y) = \int_m^\infty D(p, \xi, y)dp. \tag{3.3}$$

The total consumers' surplus of all subscribers for subscriber set y is:

$$\Phi(m, y) = \int_0^y \varphi(m, \xi, y)d\xi. \tag{3.4}$$

The total demand for the service is equal to

$$D^T(m, y) = \int_0^y D(m, \xi, y)d\xi. \tag{3.5}$$

From the above definitions, following equations hold:

$$\partial\varphi(m, \xi, y)/\partial m = -D(m, \xi, y), \tag{3.6}$$

$$\partial\Phi(m, y)/\partial m = -D^T(m, y). \tag{3.7}$$

The total profit of the supplier, R, is the sum of total net revenues from subscribers and from the supply of communications service:

$$R = (C_0 - k)y + (m - r)\int_0^y D(m, \xi, y)d\xi. \tag{3.8}$$

Since each subscriber must pay fixed charge, C_0, regardless of his volume of communication, the net benefit left to an establishment ξ and all subscribers are

$$\varphi(m, \xi, y) - C_0, \tag{3.9}$$

$$\Phi(m, y) - yC_0, \tag{3.10}$$

respectively.

From Equations (3.8) and (3.10) the social benefit derived from this communications service, W, can be represented as

(3.11)
$$W = \{\Phi(m,y) - yC_0\} + R = \int_0^y \varphi(m,\xi,y)d\xi - yk + (m-r)\int_0^y D(m,\xi,y)d\xi.$$

3.1.3 Equilibrium Condition

From Definition 1, for $[0,y]$ to be an equilibrium subscriber set, the gross benefit to the smallest subscriber, $B(m,y,y)$, must be equal to his total charge paid:

(3.12)
$$B(m,y,y) = C_0 + mD(m,y,y).$$

$B(m,y,y)$ can be represented as sum of the consumer's surplus, $\varphi(m,y,y)$, and his total usage charge paid, $mD(m,y,y)$, so that Equation (3.12) can be rewritten as:

(3.13)
$$\varphi(m,y,y) = C_0.$$

Supplier's or policy maker's instruments are unit usage charge, m, and fixed charge, C_0. Any size of subscriber set can be attained by combining them appropriately. The equilibrium condition (3.13) shows that C_0, m and y are not independent one another. If two of them are optimized, the rest is in effect optimized. We firstly maximize the objective function with respect to m and y. The optimal C_0 can be derived uniquely from these values.

3.2 Optimal Two-Part Tariff

3.2.1 Profit Maximization

[Supplier's profit maximization problem]

Maximize

(3.8)
$$R = (C_0 - k)y + (m-r)\int_0^y D(m,\xi,y)d\xi$$

subject to

(3.13)
$$\varphi(m,y,y) = C_0$$

with respect to

$$m \text{ and } y.$$

Substituting (3.13) for C_0 in (3.8), the problem can be rewritten as the unconstrained maximization:

(3.14)
$$\max_{\{m,y\}} R = \max_{\{m,y\}} \{(\varphi(m,y,y) - k)y + (m-r)D^T(m,y)\}.$$

Assuming that the second order conditions are satisfied, from the first order condition with respect to m, we have

(3.15)
$$(m-r)/m = (1 - yD(m,y,y)/D^T)/e_m$$

or

$$m = re_m/(e_m - 1 + yD(m,y,y)/D^T) \quad (3.15')$$

where e_m is the price elasticity of total demand, D^T, with respect to unit usage charge, m:

$$e_m = -(\partial D^T/\partial m)(m/D^T) \geq 0.$$

Since the average demand, $D^T(m,y)/y$, is greater than the demand of the smallest subscriber, $D(m,y,y)$, the inequality, $m \geq r$, must hold. The marginal charge should be proportional to and greater than the marginal cost.

From the first order condition with respect to y, we have also

$$C_0 = \{k - (m-r)\partial D^T/\partial y\}/(1-e_y), \quad (3.16)$$

where e_y is the elasticity of the benefit to the smallest subscriber with respect to the size of subscriber set:

$$e_y = -\frac{\partial \varphi(m,y,y)}{\partial y}\frac{y}{\varphi(m,y,y)} \geq 0.$$

$\partial D^T/\partial y$ is the increase in demand of all subscribers derived from the marginal subscriber's joining. The fixed charge, C_0, diverges from the fixed cost, k, by the two factors. It will be determined by the relative strength of these opposing factors whether the fixed charge C_0 is actually greater or lower than the fixed cost, k.

3.2.2 Social Benefit Maximization

[Social benefit maximization problem]

Maximize

$$W = \int_0^y \varphi(m,\xi,y)d\xi - yk + (m-r)\int_0^y D(m,\xi,y)d\xi \quad (3.11)$$

subject to

$$\varphi(m,y,y) = C_0 \quad (3.13)$$

with respect to

m and y

Since the social benefit does not depend on the fixed charge, C_0, the maximization problem can be solved by differentiating directly Equation (3.11). Assuming that the second order conditions are satisfied, the first order condition with respect to m is reduced to

$$m = r. \quad (3.17)$$

The unit usage charge, m, should be set equal to the constant marginal cost, r. We have also from the first order condition with respect to y:

$$\partial\Phi/\partial y - k = 0. \quad (3.18)$$

Since $\partial\Phi/\partial y$ is not equal to the benefit to the smallest subscriber, $C_0 = k$ does not hold. If interdependencies between subscribers are not assumed, the fixed charge should be equal to k obviously.

Applying the mean value theorem for integrals to the total consumers' surplus function, there exists $\omega \in [0, y]$ such that

$$\int_0^y \varphi(m,\xi,y)d\xi = \varphi(m,\omega,y)(y-0). \tag{3.19}$$

Thus, we have approximately

$$\partial\Phi/\partial y = \varphi(m,\omega,y) + C_0 - \varphi(m,y,y) + y\partial\varphi(m,\omega,y)/\partial y.$$

Substituting this for Equation (3.18) yields

$$C_0 = k - [\varphi(m,\omega,y) - \varphi(m,y,y)] - y\partial\varphi(m,\omega,y)/\partial y. \tag{3.20}$$

Let us define ω, which satisfies Equation (3.19), as an "average subscriber". The second term of the right hand side of Equation (3.20) (in brackets) is the difference of the consumer's surplus between the average subscriber and the smallest subscriber. The last term is the total incremental benefit to the existing subscribers induced by marginal subscriber's joining the system, which is measured at the average subscriber. Both of them have nonnegative values from the definition. Therefore, in order to maximize the social benefit, the fixed charge, C_0, must be set lower than the fixed cost, k, by the amount of the above two terms.

The optimal pricing for social benefit maximization causes the deficit per subscriber

$$[\varphi(m,\omega,y) - \varphi(m,y,y)] + y\partial\varphi(m,\omega,y)/\partial y$$

on the supplier. It should be made up by subsidies and so on, in the same way as in the case of marginal cost pricing of public utility which has a decreasing average cost. Tables 3.1~3.2 summarize the above results.

Table 3.1: The Optimal Two-Part Tariff in the Case of Monopoly
--- Profit Maximization ---

Optimal Two-Part Tariff	Comparison with Costs r and k
Unit Usage Charge $m = r e_m / (e_m - 1 + v_y/\overline{v})$ where $v_y = D(m, y, y)$, $\overline{v} = D^T(m, y)/y$ $e_m = -(\partial D^T/\partial m)(m/D^T)$	$m \geqq r$
Fixed Charge $C_0 = \{k - (m-r)\partial D^T/\partial y\}/(1-e_y)$ where $e_y = -\dfrac{\partial\varphi(m, y, y)}{\partial y}\dfrac{y}{\varphi(m, y, y)}$	$C_0 \gtreqless k$

Table 3.2: The Optimal Two-Part Tariff in the Case of Monopoly
--- Social Benefit Maximization ---

Optimal Two-Part Tariff	Comparison with Costs r and k
Unit Usage Charge $m=r$	$m=r$
Fixed Charge $C_0=k-[\varphi(m,\omega,y)-\varphi(m,y,y)]$ $-y\partial\varphi(m,\omega,y)/\partial y$ where $\omega\in[0,y]$	$C_0<k$

4. Optimal Two-Part Tariff after the Entry of a Market Contestant

4.1 Model Formulation

4.1.1 Additional Assumptions and Definitions

In most advanced countries, telecommunications market has been developed with monopolistic supply by a public enterprise. Recently, however, it was deregulated and a competitive supplier entered. Although a monopoly was broken by the entry, it is often argued that the old supplier still gains an enormous profit. This section develops a model for examining the tariff of old supplier's service.

The assumptions and definitions in the previous section are valid. We add some assumptions and definitions concerned with a new supplier and market competition.

Assumption 8. The new supplier provides new telecommunications service through its own trunk circuits and terminal circuits leased from the old supplier.

Assumption 9. Old and new services are homogeneous.

Assumption 10. Only subscribers to old service can utilize new service. Establishments which are going to utilize new service must be subscribers to old service in advance.

Assumption 11. Users of new service have to pay usage charge, n, to the new supplier, and fixed charge, C_0, to the old supplier even if they do not utilize old service.

Assumption 12. The unit usage charge of new service, n, must not exceed that of old service: $n \leq m$.

Assumption 13. Users of new service must pay fixed charge, A, for the leased terminal circuits to the old supplier.

Assumption 14. The old supplier bears fixed cost per subscriber, k, and constant marginal cost per unit of volume, r, while the new supplier does constant marginal cost per unit of volume, r_2.

As regulators continue to open the telecommunications market, it is expected that the new supplier will specialize in providing service for the utility's largest subscribers (see Einhorn [4]). Consequently, the old utility supplier will concentrate on establishments above some index, e.

Assumption 15. The establishment with the index, $e, (0 \leq e \leq 1)$, is indifferent between old and new services.

If the new supplier specializes in providing service for the largest volume customers, total demands for old and new services can be written as:

$$D_1^T = \int_e^y D(m,\xi,y)d\xi, \tag{4.1}$$

$$D_2^T = \int_0^e D(n,\xi,y)d\xi. \tag{4.2}$$

The net benefit to a subscriber with index $e \leq \xi \leq y$ is given by the consumer's surplus minus fixed charge:

$$\varphi(m,\xi,y) - C_0. \tag{4.3}$$

The total net benefit to subscribers only to the old service is:

$$W_1 = \int_e^y \varphi(m,\xi,y)d\xi - C_0(y-e). \tag{4.4}$$

The net benefit to a subscriber with index $0 \leq \xi \leq e$ is defined as:

$$\varphi(n,\xi,y) - (C_0 + A). \tag{4.5}$$

The total net benefit to subscribers to the new service is:

$$W_2 = \int_0^e \varphi(n,\xi,y)d\xi - (C_0 + A)e. \tag{4.6}$$

The profits of the old and new suppliers are:

$$R_1 = [C_0 - k]y + (m-r)\int_e^y D(m,\xi,y)d\xi + Ae, \tag{4.7}$$

$$R_2 = (n-r_2)D_2^T = (n-r_2)\int_0^e D(n,\xi,y)d\xi. \tag{4.8}$$

The total subscribers' benefit derived from these telecommunications services is given by the sum of W_1 and W_2:

$$\begin{aligned} W &= W_1 + W_2 \\ &= \int_e^y \varphi(m,\xi,y)d\xi + \int_0^e \varphi(n,\xi,y)d\xi - C_0 y - Ae. \end{aligned} \tag{4.9}$$

Adding R_1 to R_2 yields the total suppliers' profit:

$$\begin{aligned} R &= R_1 + R_2 \\ &= [C_0 - k]y + Ae + (m-r)\int_e^y D(m,\xi,y)d\xi \\ &\quad + (n-r_2)\int_0^e D(n,\xi,y)d\xi. \end{aligned} \tag{4.10}$$

The social benefit derived from these telecommunications services is

$$(4.11)\quad S = W + R = \int_e^y \varphi(m,\xi,y)d\xi + \int_0^e \varphi(n,\xi,y)d\xi + (m-r)\int_e^y D(m,\xi,y)d\xi + (n-r_2)\int_0^e D(n,\xi,y)d\xi - ky.$$

4.1.2 Equilibrium and Feasibility Conditions

For $[0,y]$ to be an equilibrium subscriber set, the net benefit to the smallest subscriber must be equal to zero:

$$(4.12)\quad \varphi(m,y,y) - C_0 = 0.$$

The establishment with the index e is indifferent between old and new services if and only if

$$(4.13)\quad \varphi(m,e,y) = \varphi(n,e,y) - A.$$

The net benefit to any subscriber in $[e,y]$ obtained from old supplier's service must exceed the benefit obtained if it subscribes to the new service. If not, some subscribers in $[e,y]$ would be able to enjoy more benefits from subscription to the new service than from remaining customers of the old supplier. Thus, we have the feasibility condition:

$$(4.14)\quad \varphi(m,\xi,y) \geq \varphi(n,\xi,y) - A \quad \text{for} \quad \xi \in [e,y].$$

Proposition

If there exists a subscriber who has the index $\xi = e$ $(0 \leq e \leq y)$ which is indifferent between old and new services, those with indices $e \leq \xi \leq y$ subscribe to the old service only and those with indices $0 \leq \xi < e$ subscribe to the new service.

Proof

From Assumption 3, 5 and 6, the benefit $\varphi(p,\xi,y)$ is monotone decreasing in ξ. For $\xi \in [0,y]$ and $n \leq m$, we have from (3.6):

$$\left.\frac{\partial\varphi(p,\xi,y)}{\partial p}\right|_{p=n} \leq \left.\frac{\partial\varphi(p,\xi,y)}{\partial p}\right|_{p=m}$$

and

$$\varphi(n,\xi,y) \geq \varphi(m,\xi,y) \text{ for } \xi \in [0,1].$$

Since $\varphi(n,\xi,y)$ is steeper than $\varphi(m,\xi,y)$ for $n \leq m$ with respect to $\xi \in [0,y]$ and $\varphi(n,\xi,y)$ is greater than $\varphi(m,\xi,y)$ for all $\xi \in [0,1], \varphi(n,\xi,y) - (C_0 + A)$ intersects $\varphi(m,\xi,y) - C_0$ from above. See Fig. 4.1. If the intersection, $\xi = e$, satisfies $0 \leq e \leq y$, subscribers with indices $0 \leq \xi < e$ utilize new service and the rest with indices $e \leq \xi \leq y$ utilize old service only.

Q.E.D.

The equilibrium condition (4.13) can be substituted for the feasibility condition (4.14) if the proposition holds.

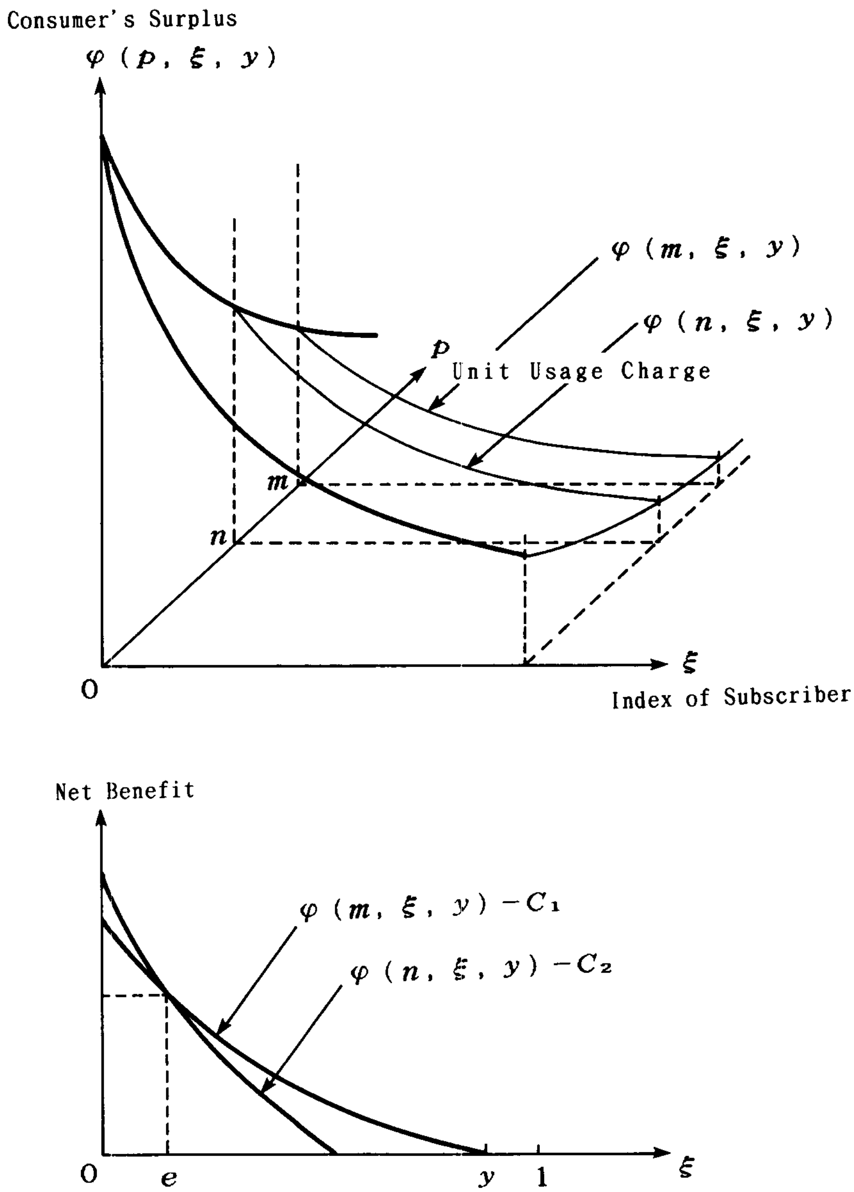

Fig. 4.1 : Consumer's Surplus Function $\varphi(p, \xi, y)$

and Net Benefit

The new supplier will not be feasible if the price is set below the associated cost. Since the old supplier has a great influence on pricing, it is possible for the old supplier to set the price low enough to let the new supplier withdraw from telecommunications. The feasibility condition of the new supplier is:

$$n \geq r_2. \tag{4.15}$$

Equality holds if the new supplier breaks even.

4.2 Optimal Two-Part Tariff

We assume that the new supplier set its unit usage charge at marginal cost: $n = r_2$, which means the unit usage charge is the lowest.

4.2.1 Profit Maximization

The profit maximization problem is formulated as the maximization of (4.7) subject to (4.12) and (4.13). Substituting (4.12) for C_0, and (4.13) for A, the problem can be rewritten as the unconstrained maximization:

$$\begin{aligned}(4.16)\qquad &\max_{\{m,y\}} R_1\\ &= \max_{\{m,y\}}\{[\varphi(m,y,y)-k]y+(m-r)\int_e^y D(m,\xi,y)d\xi\\ &\quad +[\varphi(n,e,y)-\varphi(m,e,y)]e\}.\end{aligned}$$

Assuming that the second order conditions are satisfied, from the first order necessary condition with respect to m, we have:

$$\begin{aligned}(4.17)\qquad (m-r)/m =&[1-yD(m,y,y)/D_1^T\\ &+eD(m,e,y)/D_1^T]/\epsilon_m\end{aligned}$$

or

$$\begin{aligned}(4.18)\qquad m =&r\epsilon_m/[\epsilon_m-1+yD(m,y,y)/D_1^T\\ &-eD(m,e,y)/D_1^T]\end{aligned}$$

where

$$\epsilon_m = -(\partial D_1^T/\partial m)/(D_1^T/m) \geq 0.$$

It is clear that

$$yD(m,y,y)/D_1^T - eD(m,e,y)/D_1^T \leq 1.$$

Thus, inequality

$$m \geq r \tag{4.19}$$

must hold in the same way as in monopoly. Although the unit usage charge is reduced by $eD(m,e,y)/D_1^T$, it is still greater than the constant marginal cost.

From the first order condition with respect to y, we have also

$$\begin{aligned}(4.20)\qquad C_0 =&\{k-(m-r)\frac{\partial D_1^T}{\partial y}\\ &-[\frac{\partial\varphi(n,e,y)}{\partial y}-\frac{\partial\varphi(m,e,y)}{\partial y}]e\}/(1-\epsilon_y),\end{aligned}$$

where $\epsilon_y = -\dfrac{\partial\varphi(m,y,y)}{\partial y}\dfrac{y}{\varphi(m,y,y)} \geq 0.$

The term

$$[\frac{\partial \varphi(n,e,y)}{\partial y} - \frac{\partial \varphi(m,e,y)}{\partial y}]e$$

is added to the fixed charge. The sign of this term is unknown. However, if we assume the scale effect of the subscriber set would be more crucial for the subscriber $\xi = e$ faced with a lower price, it is non-negative. Its willingness to call would increase more as the number of subscribers increases at a lower price.

4.2.2 Social Benefit Maximization Subject to a Breakeven Constraint

When social benefit is to be maximized, competition is of no use because the old supplier has no incentive to maximize its profit and is rather willing to bear deficits. There is no need to get a market share. The result is the same as in social benefit maximization in the case of monopoly.

However, if a breakeven constraint is imposed, the old supplier must change its supply schedule. From the assumption that the new supplier breaks even, only the budget constraint on the old supplier should be taken into account. The maximization problem is revised to the maximization of (4.11) subject to $R_1 \geq 0$ in (4.7). The Lagrangean can be formulated as:

$$\begin{aligned}(4.21) \qquad L = & \int_e^y \varphi(m,\xi,y)d\xi + \int_0^e \varphi(n,\xi,y)d\xi \\ & + (m-r)\int_e^y D(m,\xi,y)d\xi + (n-r_2)\int_0^e D(n,\xi,y)d\xi - ky \\ & + \alpha\{[\varphi(m,y,y)-k]y + (m-r)\int_e^y D(m,\xi,y)d\xi \\ & + [\varphi(n,e,y) - \varphi(m,e,y)]e\},\end{aligned}$$

where $\alpha \geq 0$ is a Lagrangean multiplier. Assuming an interior solution, we have (4.22) or (4.23) from the first order condition with respect to m:

$$(4.22) \qquad (m-r)/m = \frac{\alpha}{1+\alpha}[1 - yD(m,y,y)/D_1^T + eD(m,e,y)/D_1^T]/\epsilon_m,$$

$$(4.23) \qquad m = r\epsilon_m/[\epsilon_m - \frac{\alpha}{1+\alpha}\{1 - yD(m,y,y)/D_1^T + eD(m,e,y)/D_1^T\}].$$

No more it holds that unit usage charge is set at constant marginal cost. The profit margin should be positive according to the difficulty to satisfy the breakeven constraint. However, it cannot attain the level in the case of profit maximization. If the breakeven constraint is not binding, $\alpha = 0$ and it coincides with the case of social benefit maximization in monopoly.

On the other hand, from the first condition with respect to y, the fixed charge should be set lower:

$$\begin{aligned}(4.24) \qquad C_0 = & \{\frac{1+\alpha}{\alpha}k + \frac{1}{\alpha}(\partial\Phi_1/\partial y + \partial\Phi_2/\partial y) - \frac{1+\alpha}{\alpha}(m-r)\frac{\partial D_1^T}{\partial y} \\ & - [\frac{\partial \varphi(n,e,y)}{\partial y} - \frac{\partial \varphi(m,e,y)}{\partial y}]e\}/(1-\epsilon_y), \\ & \text{where } \Phi_1 = \int_e^y \varphi(m,\xi,y)d\xi, \quad \Phi_2 = \int_0^e \varphi(n,\xi,y)d\xi.\end{aligned}$$

It means that the fixed charge is set lower for more subscribers to join the system, which will raise the social benefit. However, the deficit from this per capita pricing must be covered by the higher usage pricing.

It should be noted that the two-part tariff does not take the most convenient and traditional way to satisfy a breakeven constraint that fixed charge and marginal usage charge are set equal to fixed cost and constant marginal cost, respectively.

Table 4.1: The Optimal Two-part Tariff in the Case of Non-Monopoly
--- Profit Maximization ---

Optimal Two-Part Tariff	Comparison with Costs r and k
Unit Usage Charge $m = r\varepsilon_m / [\varepsilon_m - 1 + yD(m, y, y)/D_1^T - eD(m, e, y)/D_1^T]$ where $\varepsilon_m = -(\partial D_1^T / \partial m)(m / D_1^T)$	$m \geqq r$
Fixed Charge $C_0 = \{k - (m - r)\frac{\partial D_1^T}{\partial y} - [\frac{\partial \varphi(n, e, y)}{\partial y} - \frac{\partial \varphi(m, e, y)}{\partial y}] e\} / (1 - \varepsilon_y)$ where $\varepsilon_y = -\frac{\partial \varphi(m, y, y)}{\partial y} \frac{y}{\varphi(m, y, y)}$	$C_0 \gtreqless k$

Table 4.2: The Optimal Two-Part Tariff in the Case of Non-Monopoly
--- Social Benefit Max. Sub. to a Breakeven Constraint ---

Optimal Two-Part Tariff	Comparison with Costs r and k
Unit Usage Charge $m = r\varepsilon_m / [\varepsilon_m - \frac{\alpha}{1+\alpha}\{1 - yD(m, y, y)/D_1^T + eD(m, e, y)/D_1^T\}]$	$m \geqq r$
Fixed Charge $C_0 = \{\frac{1+\alpha}{\alpha} k + \frac{1}{\alpha}(\partial \Phi_1 / \partial y + \partial \Phi_2 / \partial y) - \frac{1+\alpha}{\alpha}(m - r)\frac{\partial D_1^T}{\partial y} - [\frac{\partial \varphi(n, e, y)}{\partial y} - \frac{\partial \varphi(m, e, y)}{\partial y}] e\} / (1 - \varepsilon_y)$	$C_0 \leqq k$

5. Numerical Example

5.1 Formulation

Assume that establishments distribute uniformly. Let a realizable potential volume and a linear demand function be

(5.1) $$S(\xi, y) = \alpha(1-\xi)(2-y)y/2,$$

(5.2) $$D(m, \xi, y) = (1-m)S(\xi, y) = \alpha(1-m)(1-\xi)(2-y)y/2.$$

In the case of monopoly, total demand is given by

(5.3) $$D^T(m, y) = \alpha(1-m)(2-y)^2 y^2/4.$$

Consumer's surplus of a subscriber ξ and total consumers' surplus at price m are represented as

(5.4) $$\varphi(m, \xi, y) = \alpha(1-m)^2(1-\xi)(2-y)y/4,$$

(5.5) $$\Phi(m, y) = \alpha(1-m)^2(2-y)^2 y^2/8,$$

respectively. The fixed charge is

(5.6) $$C_0 = \varphi(m, y, y) = \alpha(1-m)^2(1-y)y(2-y)/4.$$

In the case of non-monopoly, the total demands of subscribers to the old and new systems are

(5.7) $$D_1^T = \alpha(1-m)(2-y)y[(2-y)y-(2-e)e]/4$$

(5.8) $$D_2^T = \alpha(1-n)(2-y)y(2-e)e/4.$$

The access charge can be formulated as

(5.9) $$A = \alpha(1-e)y(2-y)[(1-n)^2-(1-m)^2]/4.$$

Table 5.1 Parameters

Constant marginal cost of the (old) supplier (r)	0.2
Constant marginal cost of the new supplier ($r_2 = n$)	0.18
Fixed Cost per Subscriber (k)	
Monopoly case:	5.0
Non-monopoly case:	1.6
Parameter (α)	100.0

The total consumers' surplus of subscribers to the old and new systems are

$$(5.10)\qquad \Phi_1 = \alpha(1-m)^2[y^2(2-y)^2 - e(2-e)y(2-y)]/8$$

$$(5.11)\qquad \Phi_2 = \alpha(1-n)^2 e(2-e)y(2-y)/8,$$

respectively. An example of the value of parameters is given in Table 5.1.

5.2 The Result

5.2.1 Monopoly

The indices derived are shown in Table 5.2. 60 and 85 per cent of total establishments join the system in profit maximization and social benefit maximization, respectively. The other solution, y_c, in [0,1] is the "critical mass". 9.2 per cent of subscription level is required in profit maximization and 8.4 per cent in social benefit maximization. It is evident from these values that more subscribers are necessary for the system to be set up and the final subscription level is lower in profit maximization.

The supplier can obtain the profit 0.24 per volume of communication in profit maximization while fixed charge per subscriber must be set lower than fixed cost. As we have shown, fixed charge diverges from fixed cost by two factors. In this example, the increase in revenue by the increase in total demand induced by the subscription of the marginal establishment overcomes the inverse of his own subscription benefit.

In the case of social benefit maximization, although the supplier has to bear the deficit 2.13, total subscribers' benefit and social benefit derived are 2.8 and 1.6 times as large as in profit maximization.

For the supplier to adopt the criterion of social benefit maximization, it will be enough to make sure of the profit 1.20 which will be attained in profit maximization. If we assume that a lump sum tax system which does not break marginal conditions can be utilized and if the loss (=3.33) which the supplier will suffer from social benefit maximization can be compensated, the net subscribers' benefit left is greater than in profit maximization ($2.94 < 8.13 - 3.33$).

Table 5. 2 Numerical Result in the Case of Monopoly

	Profit maximization	Social benefit maximization
Equilibrium subscriber set (y^*)	0.60	0.85
Critical mass (y_c)	0.092	0.084
Marginal usage charge (m)	0.44	0.2
Fixed charge per subscriber (C_0)	2.80	2.49
Total demand (D^T)	10.50	20.31
Profit (R)	1.20	-2.13
Total consumers' surplus (Φ)	2.94	8.13
Social benefit (W)	2.46	3.88

5.2.2 Non-Monopoly

The indices computed are represented in Table 5.3. 76 and 92 per cent of establishments are subscribers to the old supplier's system in the cases of profit maximization and social benefit maximization subject to a breakeven constraint. 10 and 7 per cent are subscribers to the new system in the respective cases.

In each case, fixed charge per subscriber, C_0, is lower than fixed cost. The supplier would set C_0 low enough to facilitate small establishments to join the system. However, the deficit is covered by unit usage charge.

The old supplier obtains profit 3.44 in profit maximization. On the other hand, it is evidently equal to zero and the subscribers' benefit and social benefit obtained are equally 6.40 in social benefit maximization subject to a breakeven constraint.

This example shows the case where it is worth considering to adopt the criterion of social benefit maximization through the intervention by authority in order to achieve the efficiency in supplying a communication service.

Table 5.3 Numerical Result in the Case of Non-Monopoly

	Profit max.	Social benefit max. sub. to breakeven
Equilibrium subscriber set (y^*)	0.76	0.92
Indifferent subscriber (e)	0.10	0.07
Marginal usage charge (m)	0.54	0.22
Fixed charge per subscriber (C_0)	1.18	1.02
Profit of the old supplier (R)	3.44	0.0
The volume of communication		
provided by the old supplier (D_1^T)	8.08	16.63
the new supplier (D_2^T)	3.67	2.75
Total net subscribers' benefit (W)	1.46	6.40
Social benefit (S)	4.91	6.40

6. Conclusion

The preliminary results of this research suggest that there is an intricate linkage between tariff, the volume of communication and the size of subscriber set. This paper is designed to address these relationships and develop a model in which demand externalities are defined as a distinct feature.

Introduction of a competition policy is now considered more suitable for securing a certain grade of communications services. We have tried to show that an optimization method can be usefully applied to policy issues in telecommunications pricing.

We have analyzed both the cases of monopoly and non-monopoly. Two objectives of pricing are considered: (a) supplier's profit maximization; and (b) social benefit maximization in each case. The pricing strategies derived are as follows:

(i) Monopoly

(a) Profit maximization — Unit usage charge is set greater than constant marginal cost while fixed charge per subscriber diverges from fixed cost according to the two opponent factors resulting from the externality effects.

(b) Social benefit maximization — Unit usage charge is set at marginal cost. Fixed charge must be set lower than fixed cost. Supplier's deficit must be made up in some way.

(ii) Non-monopoly

(a) Profit maximization — Unit usage charge is set lower than in the monopoly case, but still greater than constant marginal cost. Fixed charge diverges from fixed cost according to the three factors.

(b) Social benefit maximization subject to a breakeven constraint — This is an intermediate case between profit maximization and social benefit maximization. The pricing (i)-(b) is slightly altered. Fixed charge is still lower than fixed cost while unit usage charge is set greater than constant marginal cost so as to satisfy the breakeven constraint.

One problem confronting us is how to identify and estimate the telecommunications demand function. This may be crucial in applying the model. Past studies in telecommunications economics paid no attention to this problem. In the numerical example, we have considered the simplest case where the demand function is linear. However, it seems more plausible to assume that it is represented, for example, by an exponential function in which the distance between users impedes communications. This is also closely related to the problem of incorporating the distribution of subscribers. A solution of these problems will be sought in future study.

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