## ABSTRACT

# An Analysis of a Stochastic Resource Allocation Model with a Nonnegative Production Function

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In this paper, we deal with a stochastic resource allocation model. Suppose that we have resource $X_n$ at time $n$, and allocate $A_{n+1}X_n$ out of $X_n$ for production and $(1 - A_{n+1})X_n$ for consumption at time $n$, where $A_{n+1}$ is the proportion of the resource that is allocated for production at time $n$. Then utility $U_{n+1}f((1 - A_{n+1})X_n)$ is obtained and resource $X_{n+1} = R_{n+1}g(A_{n+1}X_n)$ is occurred at time $n + 1$, where $U_{n+1}$ and $R_{n+1}$ are random parameters, $f$ is the utility function, and $g$ is the production function that is an extension of $g(x) = x$ in [7]. We are interested in the way of allocation that maximizes the sum of the expected utilities.

We show necessary and sufficient conditions for an allocation to be optimal in this model. The sufficient condition is obtained via the technique of dynamic programming in the same way as one in [7]. Although the necessary condition is shown by means of the similar way used in [9], the sigma additive set function is substituted for the measure as the utility function that we treat in this paper takes more generally a real value. The necessary and sufficient conditions are represented via a martingale: the supremum of the conditional expected utilities forms a supermartingale if we arbitrarily allocate the resource, however it is necessary and sufficient for an allocation to be optimal that the supremum is a martingale.

Further, we have obtained an optimal allocation in the resource allocation model with a logarithmic utility function and a nonlinear production function $g(x) = x^p (p > 0)$, utilizing the sufficient condition for an allocation to be optimal obtained above. The optimal allocation includes one obtained in [7].