Journal of the Operations Research Society of Japan
Vol. 34, No. 4, December 1991

AN EXACTLY OPTIMAL STRATEGY FOR A SEARCH PROBLEM WITH TRAVELING COST

Kensaku Kikuta
Toyama University

(Received September 3, 1990; Revised February 15, 1991)

Abstract There are n neighboring cells in a straight line. An object is in one of all cells. It is required to determine a strategy that will minimize the expected cost of finding the object. A probability of overlooking the object is equal to zero, when the right cell is searched. Associated with the examination are a traveling cost dependent on the distance from the last cell examined and a fixed examination cost. A procedure for finding an exactly optimal strategy is given.

1. Introduction

This note gives an example of a model treated in [6] (Also see pp. 264–265 of [1], and Section 4 of this note). But a special model in this note still generalizes the model which was treated in [3] and mentioned below (Also see discussions at pp. 1–2 in [5]), in that the examination cost of each cell depends on the location of the cell and distances between cells are different but still additive. We have a procedure which leads to exactly optimal strategies. This procedure applies to the model in [3]. Consequently it is shown that strategies given in [3] as approximately optimal strategies are exactly optimal.

Gluss [3] considered a model in which there are $n+1$ neighboring cells in a straight line, labeled from 0 to n in that order. An object is in one of all cells except for Cell 0, with a priori probabilities $p_1, \cdots, p_n$. At the beginning of the search the searcher is at Cell 0 that is next to Cell 1. It is required to determine a strategy that will minimize the expected cost of finding the object. A probability of overlooking the object is equal to zero, when the right cell is searched. Associated with the examination of Cell $i (1 \leq i \leq n)$ is the examination cost that consists of two parts: (i) a traveling cost $d|i-j|$ $(d > 0)$ of examining Cell i after having examined cell j, and (ii) a fixed examination cost $c > 0$. (i) means that the examination cost varies through the search and is a function of which cell was last examined.

The only difference between his model and the previous one (See [2], p. 90) is that while a traveling cost as well as the fixed examination cost is considered in the former (See [3]), only the fixed examination cost is considered in the latter. On the other hand, a probability of overlooking the object is kept in mind in the latter, while it is equal to zero in the former. The model in [2] is clearly analysed by using a technique of interchanging two adjacent cells in an optimally ordered list (See [2], pp. 67–68). But it does not apply straightforwardly in the former since the cost function depends on the location of the searcher.

Gluss treated two cases: $p_1 \geq \cdots \geq p_n$ and $p_1 \leq \cdots \leq p_n$. He showed that the former case is trivial, that is, the searcher should examine each cell in the order of $1, 2, \cdots, n$, and in the latter case he found approximately optimal strategies when $p_i = 2i/[n(n+1)]$.

In [4], this problem was approached from the game theoretical point of view, assuming a hider instead of the object. He solved a two-person constant-sum game. Another variant of the model of [3] is in [5], where the searcher is at the cell that locates at the center of all cells at the beginning of the search. [7] and [9] are surveys on the search theory. [8] is a text on two-person games. It introduces many interesting problems, some are already solved, others are still open.

In Section 2 the model is stated and the procedure for finding an optimal strategy is given as Theorem 2. Section 3 is spent for the proof of Theorem 1. Finally, a comment on a condition in [6] is added as a remark. In this note, the examination cost of each cell depends on the location of the cell and the distances between cells are different but still additive. On the other hand, in [6], the distances satisfy more general condition, i.e., triangle inequalities.

2. The Model and Results

There are $n+1$ neighboring cells in a straight line, labeled from 0 to n in that order. An object is in one of all cells except for Cell 0, with a priori probabilities such that

$$p_i > 0, \text{ all } i = 1, \cdots, n \text{ and } \sum_{i=1}^{n} p_i = 1. \tag{2.1}$$

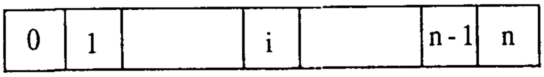

Figure 1

At the beginning of the search the searcher is at Cell 0 that is next to Cell 1. It is required to determine a strategy that will minimize the expected cost of finding the object. A probability of overlooking the object is equal to zero, when the right cell is searched. Associated with the examination of Cell i $(1 \le i \le n)$ is the examination cost that consists of two parts: (i) a traveling cost $d(i,j)$ of examining Cell i after having examined Cell j, and (ii) a fixed examination cost $c(i) > 0$. We assume

$$\begin{aligned} &d(i,j) + d(j,k) = d(i,k) \text{ for all } i,j,k \text{ such that } 1 \le i < j < k \le n, \\ &d(i,j) = d(j,i) \text{ for all } i,j \text{ such that } 1 \le i,j \le n, i \ne j, \text{ and} \\ &d(i,j) > 0 \text{ for all } i,j \text{ such that } 1 \le i,j \le n, i \ne j. \end{aligned} \tag{2.2}$$

For convenience we let $d(i,i) = 0$ for all $i = 1, \cdots, n$. We also assume

$$p_1/c(1) < \cdots < p_n/c(n) \text{ and} \tag{2.3}$$

$$\sum_{r=i+1}^{j} p_r/d(i,j) < \sum_{r=i'+1}^{j'} p_r/d(i',j') \text{ whenever } i < j, i' < j' \text{ and } i < i'. \tag{2.4}$$

While (2.3) means the a priori probability to the fixed examination cost increases, (2.4) means the a priori probability to the unit distance increases, as the distance from Cell 0 becomes large. (2.4) may or may not be relaxed since it is applied only once in the proof of Theorem 1. In the case of [3] both (2.3) and (2.4) reduce to $p_1 < p_2 < \ldots < p_n$.

A (*pure*) *strategy* for the searcher is defined by a permutation on $N \equiv \{1, 2, \cdots, n\}$. The set of all permutations on N is denoted by $\underline{M} \equiv \{\underline{1}, \underline{2}, \cdots, \underline{m}\}$, where $m = n!$. Thus under a strategy $\underline{j}$, he examines Cells $\underline{j}(1), \underline{j}(2), \cdots, \underline{j}(n)$ in this order. This is expressed as $\underline{j} = [\underline{j}(1), \cdots, \underline{j}(n)]$. We set $\underline{j}(n+1) = \underline{j}(0) = 0$ for convenience.

For a strategy $\underline{j} \in \underline{M}$, let $k = \underline{j}^{-1}(i)$. Assuming that the object is in Cell i, the cost of finding it, written as $f(i, \underline{j})$, is:

$$f(i,\underline{j}) = d(0,\underline{j}(1)) + d(\underline{j}(1),\underline{j}(2)) + \cdots + d(\underline{j}(k-1),\underline{j}(k)) + \sum_{r=1}^{k} c(\underline{j}(r)). \tag{2.5}$$

Thus the expected cost under an a priori probability $p = (p_1, \ldots, p_n)$, written as $f(p, \underline{j})$, is :

$$f(p,\underline{j}) = \sum_{i=1}^{n} p_i f(i,\underline{j}). \tag{2.6}$$

A strategy $\underline{j} \in \underline{M}$ is called *optimal* if it minimizes $f(p, \underline{j})$ subject to $\underline{j} \in \underline{M}$. Our problem is to find optimal strategies.

For any $\underline{j} \in \underline{M}$, define $\rho\underline{j} \in \underline{M}$ by

$$\rho\underline{j} = \underline{j}(n+1-i) \text{ for all } i = 1, \cdots, n. \tag{2.7}$$

$\rho\underline{j}$ reverses the order of examination under $\underline{j}$. Thus, if $\underline{j} = [\underline{j}(1), \underline{j}(2), \cdots, \underline{j}(n)]$, then $\rho\underline{j} = [\underline{j}(n), \cdots, \underline{j}(1)]$. We can assume $\underline{j}$ is as follows:

$$\begin{aligned}
&\underline{j}(0) < \underline{j}(1) < \underline{j}(2) < \cdots < \underline{j}(i_1), \\
&\underline{j}(i_1) > \underline{j}(i_1+1) > \cdots > \underline{j}(i_2), \\
&\underline{j}(i_2) < \underline{j}(i_2+1) < \cdots < \underline{j}(i_3), \\
&\quad \vdots \\
&\underline{j}(i_{2h-1}) > \underline{j}(i_{2h-1}+1) > \cdots > \underline{j}(n) > \underline{j}(n+1).
\end{aligned}$$

Thus, $\underline{j}$ has h peaks and we say $\underline{j}$ is an *h-peaked* strategy. If $\underline{j} \in \underline{M}$ is h-peaked then $\rho\underline{j}$ is also h-peaked. In particular 1-peaked strategies are interesting since less traveling costs are required under them. Thus let $\underline{M}_1$ be the set of all 1-peaked strategies of the searcher.

Theorem 1. If $\underline{j} \in \underline{M}$ is optimal then $\underline{j}$ is 1-peaked.

The proof of this theorem is given in Section 3. From this theorem, we see it suffices to solve:

$$\text{Minimize } f(p,\underline{j}) \text{ subject to } \underline{j} \in \underline{M}_1. \tag{2.8}$$

Hereafter, we consider only strategies in $\underline{M}_1$. For $i = 2, \cdots, n$, let

$$\begin{aligned}
z(i) &= p_{i-1}/[p_i + \ldots + p_n] \text{ and} \\
b(i) &= c(i-1)/[2d(i-1,n) + c(i) + \cdots + c(n)].
\end{aligned} \tag{2.9}$$

Theorem 2. Assume $z(i) \neq b(i)$ for all $i = 2, \cdots, n$. Let

$$\{i-1 : z(i) > b(i)\} = \{i_1, \cdots, i_s\}. \tag{2.10}$$

where $i_1 < i_2 < \cdots < i_s$. Then a unique optimal strategy is a 1-peaked strategy such that it first examines Cells $i_1, \cdots, i_s, n$, in this order, then examines the others.

Proof: Let $\underline{j} \in \underline{M}_1$. Represent this as $\underline{j} = [\underline{j}(1), \cdots, \underline{j}(n)]$. Since j is 1-peaked, $n-1$ is next to n in $\underline{j}$. Suppose $\underline{j}(i) = n-1$ and $\underline{j}(i+1) = n$. Define $\underline{j}' \in \underline{M}_1$ by $\underline{j}' = [\underline{j}(1), \cdots, \underline{j}(i-1), \underline{j}(i+1), \underline{j}(i), \underline{j}(i+2), \cdots, j(n)]$. Then by (2.5) and (2.6),

$$\begin{aligned}
f(p,\underline{j}) - f(p,\underline{j}') =& p_{n-1}f(n-1,\underline{j}) + p_n f(n,\underline{j}) - p_{n-1}f(n-1,\underline{j}') - p_n f(n,\underline{j}') \\
=& p_{n-1}[d(0,n-1) + \sum_{w=1}^{i} c(\underline{j}(w))] + p_n[d(0,n) + \sum_{w=1}^{i+1} c(\underline{j}(w))] \\
& - p_{n-1}[d(0,n) + d(n,n-1) + \sum_{w=1}^{i+1} c(\underline{j}(w))] \\
& - p_n[d(0,n) + \sum_{w=1}^{i-1} c(\underline{j}(w)) + c(\underline{j}(i+1))] \\
=& c(\underline{j}(i))p_n - [2d(n-1,n) + c(\underline{j}(i+1))]p_{n-1} \\
=& [2d(n-1,n) + c(\underline{j}(i+1))]p_n[b(n) - z(n)].
\end{aligned}$$

If $b(n) > z(n)$, then $f(p,\underline{j}) > f(p,\underline{j}')$. $\underline{j}'$ is preferred to $\underline{j}$. If $b(n) < z(n)$, then $\underline{j}$ is preferred. Thus define

$$\begin{aligned}
\underline{M}(p,n) \equiv& \{\underline{j} \in \underline{M}_1 : \underline{j}^{-1}(n-1) < \underline{j}^{-1}(n)\} \text{ if } b(n) < z(n), \text{ and} \\
& \{\underline{j} \in \underline{M}_1 : \underline{j}^{-1}(n-1) > \underline{j}^{-1}(n)\} \text{ if } b(n) > z(n).
\end{aligned}$$

Let $\underline{j} \in \underline{M}(p,n)$. Suppose $\underline{j}(i) = n-2$, and $\{\underline{j}(i+1), \underline{j}(i+2)\} = \{n-1, n\}$. Since $\underline{j}$ is 1-peaked $n-2$ is next to $\{n-1, n\}$ in $\underline{j}$. Define $\underline{j}' \in \underline{M}(p,n)$ by $\underline{j}'(i) = \underline{j}(i+1), \underline{j}'(i+1) = \underline{j}(i+2)$, and $\underline{j}'(i+2) = \underline{j}(i)$, and $\underline{j}'(w) = \underline{j}(w)$ for $w \neq i, i+1, i+2$. Then

$$\begin{aligned}
f(p,\underline{j}) - f(p,\underline{j}') =& p_{n-2}f(n-2,\underline{j}) + p_{n-1}f(n-1,\underline{j}) + p_n f(n,\underline{j}) \\
& - p_{n-2}f(n-2,\underline{j}') - p_{n-1}f(n-1,\underline{j}') - p_n f(n,\underline{j}') \\
=& p_{n-2}[d(0,n-2) + \sum_{w=1}^{i} c(\underline{j}(w))] + c(n-2)[p_{n-1} + p_n] \\
& - p_{n-2}[d(0,n) + d(n,n-2) + \sum_{w=1}^{i+2} c(\underline{j}(w))] \\
=& c(n-2)[p_{n-1} + p_n] - [2d(n-2,n) + c(n-1) + c(n)]p_{n-2} \\
=& [2d(n-2,n) + c(n-1) + c(n)][p_{n-1} + p_n][b(n-1) - z(n-1)].
\end{aligned}$$

If $b(n-1) > z(n-1)$, then $\underline{j}'$ is preferred to $\underline{j}$. If $b(n-1) < z(n-1)$, then $\underline{j}$ is preferred. Thus define

$$\begin{aligned}
\underline{M}(p,n-1) \equiv& \{\underline{j} \in \underline{M}(p,n) : \underline{j}^{-1}(n-2) < \min\{\underline{j}^{-1}(n-1), \underline{j}^{-1}(n)\}\} \text{ if } b(n-1) < z(n-1), \\
& \{\underline{j} \in \underline{M}(p,n) : \underline{j}^{-1}(n-2) > \max\{\underline{j}^{-1}(n-1), \underline{j}^{-1}(n)\}\} \text{ if } b(n-1) > z(n-1).
\end{aligned}$$

In the same way we can continue and define $\underline{M}(p,n-2), \cdots, \underline{M}(p,2)$ inductively. But by the assumption that $z(i) \neq b(i)$ for all i, we have $\|\underline{M}_1\| = 2^{n-1}, \|\underline{M}(p,n)\| = 2^{n-2}, \cdots, \|\underline{M}(p,3)\| = 2^1$, and $\|\underline{M}(p,2)\| = 2^0 = 1$. Hence finally one element in $\underline{M}_1$ is specified, which is optimal.

Q.E.D.

This theorem gives a procedure that calculates an optimal strategy.

Procedure

1. Calculate $z(i)$'s and $b(i)$'s in advance by means of (2.9) since they depend on only p_i's, $c(i)$'s and $d(i,j)$'s.
2. Determine the set in (2.10). Suppose it is $\{i_1, \ldots, i_s\}$ and $i_1 < \ldots < i_s$.
3. Examine Cells $i_1, \ldots, i_s$ in this order. Then examine Cells $k_1, \ldots, k_{n-s}$ in this order, where $\{k_1, \ldots, k_{n-s}\} = N\backslash\{i_1, \ldots, i_s\}$, and $k_1 > \ldots > k_{n-s}$.

Of course, the assumption in Theorem 2 can be taken away. Then the uniqueness will be lost. But the procedure is still valid. In example 3 the check of this procedure is left to the reader.

A 1-peaked strategy is characterized by a subset of N. Indeed, let $S = \{i_1, \ldots, i_s\} \subset N\backslash\{n\}$, where $i_1 < \ldots < i_s$. Let $N\backslash(S \cup \{n\}) = \{i'_1, \ldots, i'_{n-s-1}\}$, where $i'_1 > \ldots > i'_{n-s-1}$. Define a 1-peaked strategy $\underline{j}$ by $\underline{j}(t) = i_t$ for $t = 1, \cdots, s$, $\underline{j}(s+1) = n, \underline{j}(t) = i'_{t-s-1}$ for $t = s+2, \cdots, n$. Hence write $\underline{j}$ as $\underline{j}_S$. Then $\rho\underline{j} = \underline{j}_{N\backslash(S\cup\{n\})}$. Gluss[3] considered 1-peaked strategies such that $S = \{1, 2, \cdots, s\}$ or $S = \{s+1, \cdots, n-1\}$. Thus define a subclass of 1-peaked strategies, written as $\underline{G}$, by $\underline{G} \equiv \{1\#, \cdots, (n-1)\#, \rho 1\#, \cdots, \rho(n-1)\#\}$ where for $s = 1, \cdots, n-1$, $s\#(i) = i$ if $1 \le i \le s$, and $s\#(i) = n - i + s + 1$ if $s + 1 \le i \le n$. By the definition, $\underline{G} \subset \underline{M}_1$. The next numerical example shows that the minimum is attained by an element in $\underline{M}_1\backslash\underline{G}$.

Example 3. Let $n = 4$. Let $p = (15, 17, 28, 80)/140$. Let $c(1) = c(2) = c(3) = c(4) = 1$, and $d(i,j) = |i - j|$ for all $i, j = 1, 2, 3, 4$. $\underline{G} = \{1\#, 2\#, 3\#, \rho 1\#, \rho 2\#, \rho 3\#\}$. $\underline{M}_1\backslash\underline{G} = \{\underline{j}, \rho\underline{j}\}$, where $\underline{j} = [1, 3, 4, 2]$. By (2.6), we have $f(p, 1\#) = 904/140, f(p, 2\#) = 910/140, f(p, 3\#) = 906/140, f(p, \rho 1\#) = 916/140$, $f(p, \rho 2\#) = 910/140, f(p, \rho 3\#) = 914/140, f(p, \underline{j}) = 900/140$, $f(p, \rho\underline{j}) = 920/140$. From Theorem 1, the minimum is $f(p, \underline{j}) = 900/140$.

This example suggests that the approximation by Gluss[3] does not generalize to the case in this note. But the results in [3] and the next corollary to Theorem 2 give an exact solution to the model in [3].

Corollary 4. Assume $p_i = a + bi$ for $i = 1, \cdots, n$. Here $a = 1/n - b(n+1)/2$, and $0 < b < 2/[n(n-1)]$. Assume $c(i) = c$ for $i = 1, \cdots, n$, and $d(i,j) = |i - j|$ for all $i, j = 1, \cdots, n$. An optimal strategy is in $\underline{G}$.

Proof: $z(i) > b(i)$ becomes $2 - [2a + 2b(n+1)]/(bi + bn + 2a) > c/(2 + c)$ for $i = 2, \cdots, n$. The left hand side of the last inequality is monotone in i. From this and Theorem 2, we have the desired result.

Q.E.D.

3. Proof of Theorem 1

In this section we give the proof of Theorem 1. It must be noted that Lemma 5, Corollary 6, Lemma 7, Lemma 7', and Corollary 8 correspond to Lemma 3, Corollary 1, Lemma 4, Lemma 4', and Corollary 2 of [4] respectively. The assumptions of (2.3) and (2.4) are critical here, while the assumption of $p_1 < p_2 < \cdots < p_n$ was important in [4].

Lemma 5. Let $\underline{j} \in \underline{M}$ be a 2-peaked strategy such that $\underline{j} = [\underline{j}(1), \cdots, \underline{j}(i_1), \underline{j}(i_1 + 1), \cdots, \underline{j}(i_2), \underline{j}(i_2 + 1), \cdots, \underline{j}(i_2 + s), \cdots, \underline{j}(i_3), \underline{j}(i_3 + 1), \cdots, \underline{j}(n)]$. Let $\underline{j}' \in \underline{M}$ be a 2-

peaked strategy such that $\underline{j}' = [\underline{j}(1), \cdots, \underline{j}(i_1), \underline{j}(i_1+1), \cdots, \underline{j}(i_1+r), \underline{j}(i_2+s), \underline{j}(i_1+r+1), \cdots, \underline{j}(i_2), \underline{j}(i_2+1), \cdots, \underline{j}(i_3), \underline{j}(i_3+1), \cdots, \underline{j}(n)]$ when $\underline{j}(i_1) < \underline{j}(i_3)$, and $\underline{j}' = [\underline{j}(1), \cdots, \underline{j}(i_1), \underline{j}(i_1+1), \cdots, \underline{j}(i_1+r), \underline{j}(i_3-1), \underline{j}(i_1+r+1), \cdots, \underline{j}(i_2), \underline{j}(i_2+1), \cdots, \underline{j}(i_3), \underline{j}(i_3+1), \cdots, \underline{j}(n)]$ when $\underline{j}(i_1) > \underline{j}(i_3)$. Then $f(p, \underline{j}) > f(p, \underline{j}')$.

Proof: Assume $\underline{j}(i_1) < \underline{j}(i_3)$. Suppose $\underline{j}(i_2+s) < \underline{j}(i_1) < \underline{j}(i_2+s+1)$ and $s \geq 1$.

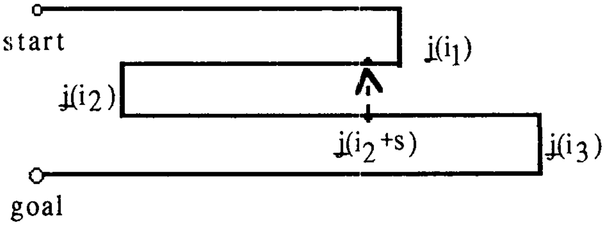

Figure 2

$$
\begin{aligned}
f(p,\underline{j}) = & \cdots + \sum_{t=i_1+1}^{i_2} p_{\underline{j}(t)} \{d(0,\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_1)) + \sum_{w=1}^{t} c(\underline{j}(w))\} \\
& + \sum_{t=i_2+1}^{i_2+s} p_{\underline{j}(t)} \{d(0,\underline{j}(i_1)) + d(\underline{j}(i_2),\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_2)) + \sum_{w=1}^{t} c(\underline{j}(w))\} + \cdots \\
f(p,\underline{j}') = & \cdots + \sum_{t=i_1+1}^{i_1+r} p_{\underline{j}(t)} \{d(0,\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_1)) + \sum_{w=1}^{t} c(\underline{j}(w))\} \\
& + p_{\underline{j}(i_2+s)} \{d(0,\underline{j}(i_1)) + d(\underline{j}(i_2+s),\underline{j}(i_1)) + \sum_{w=1}^{i_1+r} c(\underline{j}(w)) + c(\underline{j}(i_2+s))\} \\
& + \sum_{t=i_1+r+1}^{i_2} p_{\underline{j}(t)} \{d(0,\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_1)) + \sum_{w=1}^{t} c(\underline{j}(w)) + c(\underline{j}(i_2+s))\} \\
& + \sum_{t=i_2+1}^{i_2+s-1} p_{\underline{j}(t)} \{d(0,\underline{j}(i_1)) + d(\underline{j}(i_2),\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_2)) \\
& + \sum_{w=1}^{t} c(\underline{j}(w)) + c(\underline{j}(i_2+s))\} + \cdots
\end{aligned}
$$

Thus,

$$
\begin{aligned}
f(p,\underline{j}) - f(p,\underline{j}') = & - c(\underline{j}(i_2+s)) \sum_{t=i_1+r+1}^{i_2+s-1} p_{\underline{j}(t)} + \{2d(\underline{j}(i_2+s),\underline{j}(i_2)) \\
& + \sum_{w=i_1+r+1}^{i_2+s-1} c(\underline{j}(w))\} p_{\underline{j}(i_2+s)} \\
= & 2d(\underline{j}(i_2+s),\underline{j}(i_2)) p_{\underline{j}(i_2+s)} \\
& + \sum_{t=i_1+r+1}^{i_2+s-1} \{c(\underline{j}(t)) p_{\underline{j}(i_2+s)} - c(\underline{j}(i_2+s)) p_{\underline{j}(t)}\} > 0,
\end{aligned}
$$

by $\underline{j}(i_2+s) \geq \underline{j}(i_2)$, $\underline{j}(i_2+s) \geq \underline{j}(t)$ for all $t : i_1+r+1 \leq t \leq i_2+s-1$, and (2.3).

Assume $\underline{j}(i_3) < \underline{j}(i_1)$.

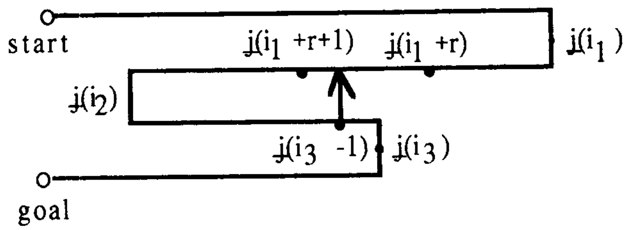

Figure 3

$$f(p,\underline{j}) = \cdots + \sum_{t=i_1+1}^{i_2} p_{\underline{j}(t)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_1)) + \sum_{w=1}^{t} c(\underline{j}(w))\}$$
$$+ \sum_{t=i_2+1}^{i_3-1} p_{\underline{j}(t)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(i_2),\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_2)) + \sum_{w=1}^{t} c(\underline{j}(w))\} + \cdots$$
$$f(p,\underline{j}') = \cdots + \sum_{t=i_1+1}^{i_1+r} p_{\underline{j}(t)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_1)) + \sum_{w=1}^{t} c(\underline{j}(w))\}$$
$$+ p_{\underline{j}(i_3-1)}\{d(0,\underline{j}(i_1+r)) + d(\underline{j}(i_3-1),\underline{j}(i_1+r)) + \sum_{w=1}^{i_1+r} c(\underline{j}(w)) + c(\underline{j}(i_3-1))\}$$
$$+ \sum_{t=i_1+r+1}^{i_2} p_{\underline{j}(t)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_1)) + \sum_{w=1}^{t} c(\underline{j}(w)) + c(\underline{j}(i_3-1))\}$$
$$+ \sum_{t=i_2+1}^{i_3-2} p_{\underline{j}(t)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(i_2),\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_2))$$
$$+ \sum_{w=1}^{t} c(\underline{j}(w)) + c(\underline{j}(i_3-1))\} + \cdots$$

Thus,

$$f(p,\underline{j}) - f(p,\underline{j}') = -\,c(\underline{j}(i_3-1)) \sum_{t=i_1+r+1}^{i_3-2} p_{\underline{j}(t)} + \{2d(\underline{j}(i_3-1),\underline{j}(i_2))$$
$$+ \sum_{w=i_1+r+1}^{i_3-2} c(\underline{j}(w))\}p_{\underline{j}(i_3-1)}$$
$$=2d(\underline{j}(i_3-1),\underline{j}(i_2))p_{\underline{j}(i_3-1)} + \sum_{t=i_1+r+1}^{i_3-2} \{c(\underline{j}(t))p_{\underline{j}(i_3-1)} - c(\underline{j}(i_3-1))p_{\underline{j}(t)}\}$$
$$>0,$$

since $\underline{j}(i_3-1) \geq \underline{j}(i_2)$, $\underline{j}(i_3-1) \geq \underline{j}(t)$ for all $t : i_1+r+1 \leq t \leq i_3-2$, and (2.3).

Q.E.D.

Corollary 6. Let $\underline{j} \in \underline{M}$ be a 2-peaked strategy such that $\underline{j} = [\underline{j}(1),\cdots,\underline{j}(i_1),\underline{j}(i_1+1),\cdots,\underline{j}(i_2),\underline{j}(i_2+1),\cdots,\underline{j}(i_2+s),\cdots,\underline{j}(i_3),\underline{j}(i_3+1),\cdots,\underline{j}(n)]$. Let $\underline{j}'$ be a 2-peaked strategy such that $\underline{j}' = [\underline{j}(1), \cdots, \underline{j}(i_1), \underline{j}'(i_1+1), \cdots, \underline{j}'(i_2-i_1+s-1), \underline{j}(i_2), \underline{j}(i_2+s+1), \cdots, \underline{j}(i_3), \underline{j}(i_3+1), \cdots, \underline{j}(n)]$ where $\{\underline{j}'(i_1+1), \cdots, \underline{j}'(i_2-i_1+s-1)\} = \{\underline{j}(i_1+1), \cdots, \underline{j}(i_2-1), \underline{j}(i_2+1), \cdots, \underline{j}(i_2+s)\}$ and $\underline{j}'(i_1+1) > \cdots > \underline{j}'(i_2-i_1+s-1)$ when $\underline{j}(i_1) < \underline{j}(i_3)$, and let $\underline{j}' = [\underline{j}(1), \cdots, \underline{j}(i_1), \cdots, \underline{j}(i_1+r), \underline{j}'(i_1+r+1), \cdots, \underline{j}'(i_3-2), \underline{j}(i_2), \underline{j}(i_3), \underline{j}(i_3+1), \cdots, \underline{j}(n)]$, where $\{\underline{j}'(i_1+r+1), \cdots, \underline{j}'(i_3-2)\} = \{\underline{j}(i_1+r+1), \cdots, \underline{j}(i_2-1), \underline{j}(i_2+1), \cdots, \underline{j}(i_3-1)\}$ and $\underline{j}'(i_1+r+1) > \cdots > \underline{j}'(i_3-2)$ when $\underline{j}(i_1) < \underline{j}(i_3)$. Then $f(p, \underline{j}) > f(p, \underline{j}')$.

Proof: Assume $\underline{j}(i_1) < \underline{j}(i_3)$. Suppose $\underline{j}(i_2+s) < \underline{j}(i_1) < \underline{j}(i_2+s+1)$ and $s \geq 1$.

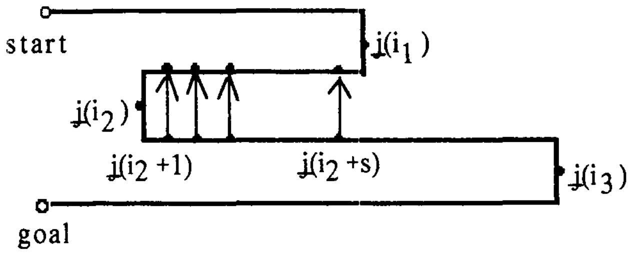

Figure 4.

Apply the first half of Lemma 5 s times, starting with $\underline{j}(i_2+s)$, then $\underline{j}(i_2+s-1), \cdots$.

Next assume $\underline{j}(i_1) > \underline{j}(i_3)$. Apply the second half of Lemma 5, $(i_3 - i_2 - 1)$ times, starting with $\underline{j}(i_3-1)$, then $\underline{j}(i_3-2), \cdots$.

Q.E.D.

Perhaps Corollary 6 and the following lemma can be merged and shortened. But the proof will be complicate in notation if we merge. Thus we do not. Further Corollary 6 in itself says a property of a strategy for the searcher.

Lemma 7. Let $\underline{j} \in \underline{M}$ be a 2-peaked strategy such that $\underline{j} = [\underline{j}(1), \cdots, \underline{j}(i_1), \cdots, \underline{j}(i_2), \underline{j}(i_2+1), \cdots, \underline{j}(i_3), \cdots, \underline{j}(i_3+s), \cdots, \underline{j}(n)]$, where $\underline{j}(i_1) < \underline{j}(i_2+1)$ and $\underline{j}(i_3+s) > \underline{j}(i_1) > \underline{j}(i_3+s+1)$. Let $\underline{j}' = [\underline{j}(1), \cdots, \underline{j}(i_1), \underline{j}(i_2+1), \cdots, \underline{j}(i_3), \cdots, \underline{j}(i_3+s), \underline{j}'(i_3+s-i_2+i_1+1), \cdots, \underline{j}'(n)]$ where $\{\underline{j}'(i_3+s-i_2+i_1+1), \cdots, \underline{j}'(n)\} = \{\underline{j}(i_1+1), \cdots, \underline{j}(i_2), \underline{j}(i_3+s+1), \cdots, \underline{j}(n)\}$ and $\underline{j}'(i_3+s-i_2+i_1+1) > \cdots > \underline{j}'(n)$. Then $f(p, \underline{j}) > f(p, \underline{j}')$.

Proof:

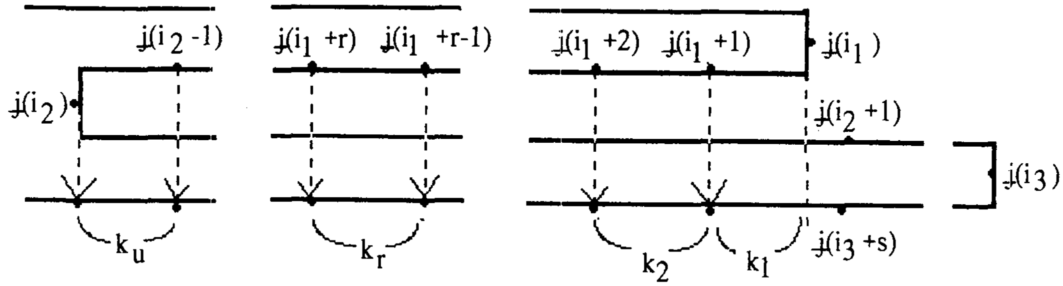

Figure 5.

Let $u = i_2 - i_1$. Observing that for t with $t \geq i_3 + 1$,

$$\begin{aligned}&d(0,\underline{j}(i_1)) + d(\underline{j}(i_2),\underline{j}(i_1)) + d(\underline{j}(i_3),\underline{j}(i_2)) + d(\underline{j}(t),\underline{j}(i_3))\\ &= 2d(\underline{j}(i_1),\underline{j}(i_2)) + d(0,n) + d(n,\underline{j}(t)),\end{aligned}$$

$$\begin{aligned}f(p,\underline{j}) = \cdots &+ \sum_{t=i_1+1}^{i_2} p_{\underline{j}(t)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_1)) + \sum_{w=1}^{t} c(\underline{j}(w))\}\\ &+ \sum_{t=i_2+1}^{i_3} p_{\underline{j}(t)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(i_2),\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_2)) + \sum_{w=1}^{t} c(\underline{j}(w))\}\\ &+ \sum_{t=i_3+1}^{i_3+s} p_{\underline{j}(t)}\{2d(\underline{j}(i_1),\underline{j}(i_2)) + d(0,n) + d(n,\underline{j}(t)) + \sum_{w=1}^{t} c(\underline{j}(w))\}\\ &+ \sum_{r=1}^{u} \sum_{t=i_3+s+k_1+\ldots+k_{r-1}+1}^{i_3+s+k_1+\ldots+k_r} p_{\underline{j}(t)}\{2d(\underline{j}(i_1),\underline{j}(i_2)) + d(0,n) + d(n,\underline{j}(t))\\ &+ \sum_{w=1}^{t} c(\underline{j}(w))\} + \cdots.\end{aligned}$$

On the other hand, seeing that $\underline{j}'$ is 1-peaked,

$$\begin{aligned}f(p,\underline{j}') = \cdots &+ \sum_{t=i_2+1}^{i_3} p_{\underline{j}(t)}\{d(0,\underline{j}(t)) + \sum_{w=1}^{i_1} c(\underline{j}(w)) + \sum_{w=i_2+1}^{t} (\underline{j}(w))\}\\ &+ \sum_{t=i_3+1}^{i_3+s} p_{\underline{j}(t)}\{d(0,n) + d(n,\underline{j}(t)) + \sum_{w=1}^{i_1} c(\underline{j}(w)) + \sum_{w=i_2+1}^{t} c(\underline{j}(w))\}\\ &+ \sum_{r=1}^{u} \sum_{t=i_3+s+k_1+\ldots+k_{r-1}+1}^{i_3+s+k_1+\ldots+k_r} p_{\underline{j}(t)}\{d(0,n) + d(n,\underline{j}(t)) + \sum_{w=1}^{i_1} c(\underline{j}(w))\\ &+ \sum_{w=i_2+1}^{t} c(\underline{j}(w)) + \sum_{w=i_1+1}^{i_1+r-1} c(\underline{j}(w))\} + \sum_{r=1}^{u} p_{\underline{j}(i_1+r)}\{d(0,n) + d(n,\underline{j}(i_1+r))\\ &+ \sum_{w=1}^{i_1} c(\underline{j}(w)) + \sum_{w=i_2+1}^{i_3+s+k_1+\ldots+k_r} c(\underline{j}(w)) + \sum_{w=i_1+1}^{i_1+r} c(\underline{j}(w))\}\\ &\vdots\end{aligned}$$

Hence,

$$f(p,\underline{j}) - f(p,\underline{j}') = A + B,$$

where

$$
\begin{aligned}
A \equiv & 2 \sum_{t=i_2+1}^{i_3} p_{\underline{j}(t)} d(\underline{j}(i_1), \underline{j}(i_2)) + 2 \sum_{t=i_3+1}^{i_3+s} p_{\underline{j}(t)} d(\underline{j}(i_1), \underline{j}(i_2)) \\
& + 2 \sum_{r=1}^{u} \sum_{t=i_3+s+k_1+\ldots+k_{r-1}+1}^{i_3+s+k_1+\ldots+k_r} p_{\underline{j}(t)} d(\underline{j}(i_1), \underline{j}(i_2)) - 2 \sum_{r=1}^{u} p_{\underline{j}(i_1+r)} d(\underline{j}(i_1), n) \\
= & 2d(\underline{j}(i_1), \underline{j}(i_2)) \sum_{t=i_2+1}^{i_3+s} p_{\underline{j}(t)} - 2d(\underline{j}(i_1), n) \sum_{r=1}^{u} p_{\underline{j}(i_1+r)} \\
& + 2 \sum_{r=1}^{u} \sum_{t=i_3+s+k_1+\ldots+k_{r-1}+1}^{i_3+s+k_1+\ldots+k_r} p_{\underline{j}(t)} d(\underline{j}(i_1), \underline{j}(i_2)) \\
\geq & 2 \sum_{r=1}^{u} \sum_{t=i_3+s+k_1+\ldots+k_{r-1}+1}^{i_3+s+k_1+\ldots+k_r} p_{\underline{j}(t)} d(\underline{j}(i_1), \underline{j}(i_2)) > 0,
\end{aligned}
$$

by (2.4). Furthermore,

$$
\begin{aligned}
B \equiv & \sum_{t=i_2+1}^{i_3} p_{\underline{j}(t)} \sum_{w=1}^{u} c(\underline{j}(i_1+w)) + \sum_{t=i_3+1}^{i_3+s} p_{\underline{j}(t)} \sum_{w=1}^{u} c(\underline{j}(i_1+w)) \\
& + \sum_{r=1}^{u} \sum_{t=i_3+s+k_1+\ldots+k_{r-1}+1}^{i_3+s+k_1+\ldots+k_r} p_{\underline{j}(t)} \sum_{w=i_1+r}^{i_2} c(\underline{j}(w)) - \sum_{r=1}^{u} p_{\underline{j}(i_1+r)} \sum_{w=i_2+1}^{i_3+s+k_1+\ldots+k_r} c(\underline{j}(w)) \\
= & \sum_{w=1}^{u} c(\underline{j}(i_1+w)) \sum_{t=i_2+1}^{i_3+s} p_{\underline{j}(t)} + \sum_{r=1}^{u} \sum_{w=i_1+r}^{i_2} c(\underline{j}(w)) \sum_{t=1}^{k_r} p_{\underline{j}(i_3+s+k_1+\ldots+k_{r-1}+t)} \\
& - \sum_{w=i_2+1}^{i_3+s} c(\underline{j}(w)) \sum_{r=1}^{u} p_{\underline{j}(i_1+r)} - \sum_{r=1}^{u} p_{\underline{j}(i_1+r)} \sum_{t=1}^{r} \sum_{w=i_3+s+k_1+\ldots+k_{t-1}+1}^{i_3+s+k_1+\ldots+k_t} c(\underline{j}(w)) \\
= & \sum_{w=1}^{u} c(\underline{j}(i_1+w)) \sum_{t=i_2+1}^{i_3+s} p_{\underline{j}(t)} - \sum_{w=i_2+1}^{i_3+s} c(\underline{j}(w)) \sum_{r=1}^{u} p_{\underline{j}(i_1+r)} \\
& + \sum_{r=1}^{u} \sum_{w=r}^{u} c(\underline{j}(w+i_1)) \sum_{t=1}^{k_r} p_{\underline{j}(i_3+s+k_1+\ldots+k_{r-1}+t)} \\
& - \sum_{r=1}^{u} \sum_{w=r}^{u} p_{\underline{j}(i_1+w)} \sum_{t=1}^{k_r} c(\underline{j}(i_3+s+k_1+\ldots+k_{r-1}+t)) \\
> & 0,
\end{aligned}
$$

by (2.3). Q.E.D.

Lemma 7'. Let $\underline{j} \in \underline{M}$ be a 2-peaked strategy such that $\underline{j} = [\underline{j}(1), \cdots, \underline{j}(i_1), \cdots, \underline{j}(i_2), \underline{j}(i_3), \cdots, \underline{j}(n)]$, where $\underline{j}(i_1) > \underline{j}(i_3)$ and $\underline{j}(i_1+s) > \underline{j}(i_3) > \underline{j}(i_1+s+1)$. Let $\underline{j}' = [\underline{j}(1), \cdots, \underline{j}(i_1), \cdots, \underline{j}(i_1+s), \underline{j}(i_3), \underline{j}'(i_1+s+2), \cdots, \underline{j}'(n)]$, where $\{\underline{j}'(i_1+s+2), \cdots, \underline{j}'(n)\} = \{\underline{j}(i_1+s+1), \cdots, \underline{j}(i_2), \underline{j}(i_3+1), \cdots, \underline{j}(n)\}$ and $\underline{j}(i_3) > \underline{j}'(i_3+s+2) > \cdots > \underline{j}'(n)$. Then $f(p, \underline{j}) > f(p, \underline{j}')$.

Proof: Let $u = i_2 - i_1$. Noting that $i_3 = i_2 + 1$,

$$\begin{aligned} f(p,\underline{j}) = \cdots &+ \sum_{t=i_1+s+1}^{i_2} p_{\underline{j}(t)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_1)) + \sum_{w=1}^{t} c(\underline{j}(w))\} \\ &+ p_{\underline{j}(i_2+1)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(i_2),\underline{j}(i_1)) + d(\underline{j}(i_2+1),\underline{j}(i_2)) + \sum_{w=1}^{i_3} c(\underline{j}(w))\} \\ &+ \sum_{r=0}^{u-s-1} \sum_{t=i_2+1+k_0+\ldots+k_{r-1}+1}^{i_2+1+k_0+\ldots+k_r} p_{\underline{j}(t)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(i_2),\underline{j}(i_1)) \\ &+ d(\underline{j}(i_2+1),\underline{j}(i_2)) + d(\underline{j}(t),\underline{j}(i_2+1)) + \sum_{w=1}^{t} c(\underline{j}(w))\} + \cdots. \end{aligned}$$

On the other hand, seeing that $\underline{j}'$ is 1-peaked,

$$\begin{aligned} f(p,\underline{j}') = \cdots &+ p_{\underline{j}(i_2+1)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(i_2+1),\underline{j}(i_1)) + \sum_{w=1}^{i_1+s} c(\underline{j}(w)) + c(\underline{j}(i_2+1))\} \\ &+ \sum_{r=0}^{u-s-1} \sum_{t=i_2+1+k_0+\ldots+k_{r-1}+1}^{i_2+1+k_0+\ldots+k_r} p_{\underline{j}(t)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(t),\underline{j}(i_1)) \\ &+ \sum_{w=1}^{t} c(\underline{j}(w)) - \sum_{w=i_1+s+r+1}^{i_2} c(\underline{j}(w))\} \\ &+ \sum_{r=1}^{u-s} p_{\underline{j}(i_1+s+r)}\{d(0,\underline{j}(i_1)) + d(\underline{j}(i_1+s+r),\underline{j}(i_1)) \\ &+ \sum_{w=1}^{i_2+1+k_0+\ldots+k_{r-1}} c(\underline{j}(w)) - \sum_{w=i_1+s+r+1}^{i_2} c(\underline{j}(w))\} + \cdots \\ &\vdots \end{aligned}$$

Hence,

$$f(p,\underline{j}) - f(p,\underline{j}') = A + B,$$

where

$$\begin{aligned} A \equiv & p_{\underline{j}(i_2+1)} 2d(\underline{j}(i_2+1),\underline{j}(i_2)) \\ &+ \sum_{r=0}^{u-s-1} \sum_{t=i_2+1+k_0+\ldots+k_{r-1}+1}^{i_2+1+k_0+\ldots+k_r} p_{\underline{j}(t)} 2d(\underline{j}(i_2+1),\underline{j}(i_2)) > 0. \end{aligned}$$

Further,

$$\begin{aligned}
B \equiv & p_{\underline{j}(i_2+1)} \sum_{w=i_1+s+1}^{i_2} c(\underline{j}(w)) + \sum_{r=0}^{u-s-1} \sum_{t=i_2+1+k_0+\ldots+k_{r-1}+1}^{i_2+1+k_0+\ldots+k_r} p_{\underline{j}(t)} \sum_{w=i_1+s+r+1}^{i_2} c(\underline{j}(w)) \\
& - \sum_{r=1}^{u-s} p_{\underline{j}(i_1+s+r)} \sum_{w=i_3}^{i_3+k_0+\ldots+k_{r-1}} c(\underline{j}(w)) \\
= & p_{\underline{j}(i_2+1)} \sum_{w=i_1+s+1}^{i_2} c(\underline{j}(w)) - \sum_{r=1}^{u-s} p_{\underline{j}(i_1+s+r)} c(\underline{j}(i_3)) \\
& + \sum_{r=1}^{u-s} \sum_{t=i_3+k_0+\ldots+k_{r-2}+1}^{i_3+k_0+\ldots+k_{r-1}} p_{\underline{j}(t)} \sum_{w=i_1+s+r}^{i_2} c(\underline{j}(w)) - \sum_{r=1}^{u-s} p_{\underline{j}(i_1+s+r)} \sum_{w=i_3+1}^{i_3+k_0+\ldots+k_{r-1}} c(\underline{j}(w)) \\
= & p_{\underline{j}(i_2+1)} \sum_{w=i_1+s+1}^{i_2} c(\underline{j}(w)) - \sum_{r=1}^{u-s} p_{\underline{j}(i_1+s+r)} c(\underline{j}(i_3)) \\
& + \sum_{r=1}^{u-s} \sum_{t=1}^{k_r-1} p_{\underline{j}(i_3+k_0+\ldots+k_{r-2}+t)} \sum_{w=r}^{u-s} c(\underline{j}(i_1+s+w)) \\
& - \sum_{r=1}^{u-s} p_{\underline{j}(i_1+s+r)} \sum_{t=1}^{r} \sum_{w=1}^{k_t-1} c(\underline{j}(i_3+k_0+\ldots+k_{t-2}+w)) \\
= & p_{\underline{j}(i_2+1)} \sum_{w=i_1+s+1}^{i_2} c(\underline{j}(w)) - \sum_{r=1}^{u-s} p_{\underline{j}(i_1+s+r)} c(\underline{j}(i_3)) \\
& + \sum_{r=1}^{u-s} \sum_{t=1}^{k_r-1} p_{\underline{j}(i_3+k_0+\ldots+k_{r-2}+t)} \sum_{w=r}^{u-s} c(\underline{j}(i_1+s+w)) \\
& - \sum_{t=1}^{u-s} \sum_{r=t}^{u-s} p_{\underline{j}(i_1+s+r)} \sum_{w=1}^{k_{t-1}} c(\underline{j}(i_3+k_0+\ldots+k_{t-2}+w)) \\
= & \sum_{r=1}^{u-s} \{ p_{\underline{j}(i_3)} c(\underline{j}(i_1+s+r) - p_{\underline{j}(i_1+s+r)} c(\underline{j}(i_3)) \} \\
& + \sum_{r=1}^{u-s} \sum_{t=1}^{k_{r-1}} \sum_{w=r}^{u-s} \{ p_{\underline{j}(i_3+k_0+\ldots+k_{r-2}+t)} c(\underline{j}(i_1+s+w)) \\
& - p_{\underline{j}(i_1+s+w)} c(\underline{j}(i_3+k_0+\ldots+k_{t-2}+t)) \} \\
& > 0,
\end{aligned}$$

by (2.3). Q.E.D.

Corollary 8. For any $\underline{j} \in \underline{M} \backslash \underline{M}_1$, there is $\underline{j}' \in \underline{M}_1$ such that $f(p, \underline{j}) > f(p, \underline{j}')$.

Proof: Suppose $\underline{j} \in \underline{M}$ is h-peaked$(h \geq 2)$. Let $\underline{j}' \in \underline{M}$ be a 1-peaked strategy which is transferred from $\underline{j}$ by repeated operations indicated in Corollary 6, Lemma 7 and Lemma 7'. Then $f(p, \underline{j}) \geq f(p, \underline{j}')$.

Q.E.D.

This corollary implies Theorem 1.

4. A Remark

It is interesting to compare the condition $z(i) > b(i)$ in (2.10) with the condition $p_i > v(i)$ in Lemma 5.3 of [6]. Let $w(m,i) \equiv mc(i) + 2\max\{d(0,i), d(i,n)\}$ for $i = 1,\cdots,n$ and $m = 1,2,\cdots$. Let $c^i_{\min} \equiv \min\{c(i') : i' \neq i\}$ for $i = 1,\cdots,n$. Define

$$v_m(i) \equiv w(m,i)/[w(m,i) + c^i_{\min}] \text{ and}$$
$$v(i) \equiv \min\{v_m(i) : m = 1,2,\cdots\}.$$

Since $v_m(i)$ is increasing in m, we have $v(i) = v_1(i)$. Assume $d(i,j) = |j-i|$ for all i,j, and $c(i) = c$ for all i. Then, if $n \geq 3$, the condition $p_1 > v(1)$ becomes $p_1 > [c+2(n-1)]/[2c+2(n-1)]$. On the other hand, $z(2) > b(2)$ becomes $p_1 > c/[nc+2(n-1)]$. Thus two conditions are different. (2.1) and (2.3) imply $p_1 < 1/n$ since $c(1) = \cdots = c(n)$. We have $[c+2(n-1)]/[2c+2(n-1)] > 1/n$. Thus Lemma 5.3 in [6] may not apply at least for $i = 1$. In [6] conditions are discussed in a more general setting. Indeed, overlooking probabilities are kept in mind. This seems to make the analysis more difficult.

Acknowledgement The author wish to thank the referees for their helpful comments and suggestions.

References

[1] R. Ahlswede and I. Wegener: *Search Problems*, John Wiley & Sons, Chichester, 1987.
[2] R. Bellman: *Dynamic Programming*, Princeton University Press, Princeton, New Jersey, 1957.
[3] B. Gluss: Approximately Optimal One-Dimensional Search Policies in Which Search Costs Vary through Time, *Nav. Res. Log. Quart.*, Vol. 8 (1961), 277–283.
[4] K. Kikuta: A Hide and Seek Game with Traveling Cost, *Journal of the Operations Research Society of Japan*, Vol. 33 (1990), 168–187.
[5] K. Kikuta: A One-Dimensional Search with Traveling Cost, *Journal of the Operations Research Society of Japan*, Vol. 33 (1990). 262–276.
[6] U. Lössner and I. Wegener: Discrete Sequential Search with Positive Switch Cost, *Math. Oper. Res.*, Vol. 7 (1982), 426–440.
[7] T. Nakai: *Tansaku Riron Tenbo* (In Japanese), mimeo, 1986.
[8] W.H. Ruckle: *Geometric Games and Their Applications*, Pitman, Mass., 1983.
[9] M. Sakaguchi: Tansaku Riron (In Japanese), *BASIC Sugaku*, Vol. 14 (1981), 61–67.

Kensaku Kikuta
Faculty of Economics
Toyama University
Gofuku 3190, Toyama 930
Japan