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EQUIPMENT REPLACEMENT BEHAVIOR UNDER INNOVATIVE TECHNOLOGICAL ADVANCES

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Abstract In considering cost reductions, it is natural to postpone equipment replacement until innovative equipment is made available, if it will appear in the near future. On the other hand, if new equipment with these characteristics is not made available in the near term, existing equipment is upgraded with similar products.

This study discusses equipment replacement where innovative technological advances occur together with gradual technological advances. It aims to clarify theoretically and experimentally the above equipment replacement behavior by using "Control Limit Policy". Next, we give a practical replacement decision method using control limit policy when the appearance time of innovative equipment is forecasted.

This study is significant in the sense of being able to show that control limit policy can propose not only a convenient method in actual replacement decisions, but also an effective method to explain the qualitative aspects of equipment replacement behavior.

1. Introduction

In considering equipment replacement, it is natural to postpone present replacement if innovative equipment with considerable cost reductions will appear in the near future. On the other hand, if new equipment with these characteristics is not made available in the near term, existing equipment is upgraded with similar products. This equipment behavior is brought on by innovative technological advances. This study clarifies theoretical and experimental aspects of this type of equipment replacement behavior by using "Control Limit Policy" as proposed in a previous study by the authors [6]. Finally, a practical replacement decision method is proposed.

In the light of technological advances (TA), management is faced with decisions regarding economical replacement of obsolete equipment. Much research has been conducted in the field of equipment replacement under TA [13, 1, 3, 9, 12, 2, 8, 5, 6, 7]. These studies can be classified into two groups:

(1) research which treats replacement problems for finite planning horizons [9, 3, 5, 6, 7].
(2) research which discusses replacement problems for infinite planning horizons [13, 1, 12, 2, 8].

Concerning TA, we can consider the following two cases: one is the case where present technology is improved partially, as seen in model change cases, and passes almost continuously into another state of technology as an extension of present state of technology (hereafter, we call this gradual technological advances (GTA)). The other case means present technology shifts discontinuously and suddenly to another state which can not be considered as an extension of present state (hereafter, we call this innovative technological advances (ITA)). In economics, there has been a study which grasps a dynamic process of TA through the interaction between imitation and innovation [4]. Here, if the imitation process includes not only the propagation of the same technology but also the appearance of improved technology, the imitation and innovation processes respectively correspond to GTA and ITA. We can also relate the classification [11] of technology development in the firm to these GTA's and ITA's patterns. There has been a study which classifies technology development strategies into the two patterns [10]. Also, concerning a leasing system as a new acquisition method of equipment, there has been a study in determining lease charge taking the ITA's appearance into account [14].

In the above studies, a clear distinction is drawn between GTA and ITA. But, in the conventional research of equipment replacement under TA, these two patterns have not been distinguished clearly or only GTA have been supposed tacitly. So far, the replacement behavior stated at the beginning has been grasped intuitively and has not been explained scientifically and logically. In this sense, this behavior is considered to be a hypothesis. To test this hypothesis, it is necessary to introduce into our model the occurrence structure of ITA as in the case of GTA.

In this problem, any replacement before the appearance time t of innovative equipment is made based on a single sequence of old type equipment, whereas the selection after that time is made from plural replacement alternatives in which two sequences of old and new technology levels exist together. For this reason, replacement behavior is strongly affected by a problem structure with plural replacement alternatives. Therefore, we analyze the problem with plural replacement alternatives in chapter 2 and clarify the replacement behavior under ITA in chapter 3 based on the results of analysis in chapter 2. In chapter 4, we give some considerations on the results obtained in chapter 3 and propose a replacement decision method using control limit policy.

2. Equipment Replacement with Plural Replacement Alternatives

2.1 Problem description

A decision maker will often encounter the following situation in equipment replacement. An equipment purchased at time $x(x = 0, 1, \cdots, n-1)$ is operating at present time $n(n = 1, 2, \cdots, T-1)$. A sequence of new equipment with GTA (called type 1 equipment or equipment 1) is available as replacement alternatives over planning periods $[0, T]$. On the other hand, another sequence of new equipment at technology level higher than type 1 (called type 2 equipment or equipment 2) is also available at any time on planning periods $[0, T]$. It is forecasted that GTA will also occur in type 2 new equipment. GTA bring about a gentle decrease in initial operating cost of new equipment and a gradual change (increase or decrease) in purchase price of new equipment. It will be reasonable to consider that initial operating cost of type 2 new equipment decreases remarkably and its purchase price increases likewise, compared with type 1 new equipment. The decision maker is going to decide the sequence of both replacement time and replaced equipment type so as to minimize the present value at time n of total cost for the remaining planning periods $[n, T]$.

2.2 Notations

In analysis, we use the following notations with respect to type j equipment $(j = 1, 2)$ and discount rate:

$H_j(x, n)$: operating cost at time n of equipment j purchased at time x

$I_j(x)$: purchase price at time x of new equipment j

$V_j(x, n)$: salvage value at time n for equipment j purchased at time x $(V_j(x, n) \leq I_j(n))$

$P_j(x, n)$: present value at time n of total cost for the remaining planning periods $[n, T]$, starting at time n with equipment j purchased at time x and following the optimal policy since time n

d_n : decision at time n, where $d_n = R$ and $d_n = K$ mean taking "replace" and "keep" actions, respectively

α : discount rate per period in discrete compounding interest factor

Generally, initial operating cost $H_j(x, x)$ decreases with respect to x due to GTA, operating cost $H_j(x, n)$ increases with respect to n due to deterioration, and salvage value $V_j(x, n)$ decreases with respect to n due to deterioration and obsolescence. As it is supposed that the technology level of type 2 new equipment is higher than that of type 1, the following is considered to hold:

$$I_1(n) < I_2(n) \ (0 \leq n \leq T-1)$$
$$H_1(x,n) > H_2(x,n) \ (0 \leq x \leq n \leq T-1)$$
$$V_1(x,n) < V_2(x,n) \ (0 \leq x \leq n \leq T-1)$$

It is also considered that $I_j(x) > V_j(x, x)$ holds as a salvage value of new equipment immediately after purchase decreases. These cash flow functions are illustrated in Fig. 1.

Further, we briefly write $g(x) \uparrow x$ or $g(x) \downarrow x$ respectively when $g(x)$ is non-decreasing or non-increasing in terms of x.

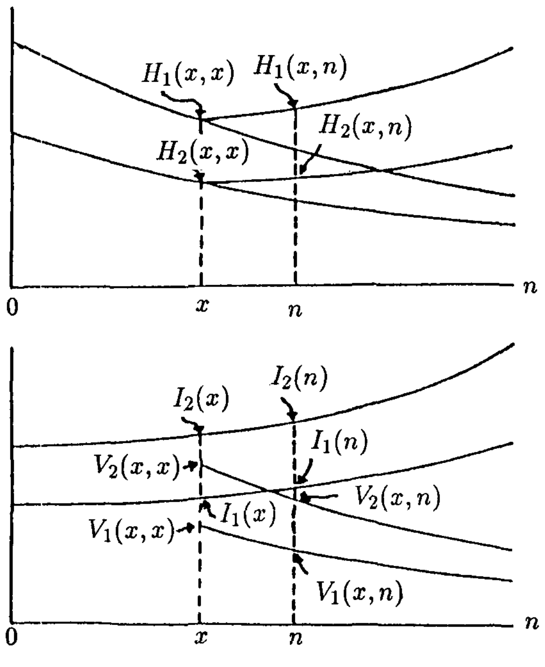


Fig. 1 Cash Flow Functions in the Case with Two Replacement Alternatives

2.3 Formulation

Using dynamic programming (DP) formulation in the same way as [6], the present value at time n of total cost for the remaining planning periods $[n, T]$, starting at time n with equipment j purchased at time x and following the optimal policy from the time on, is given as follows:

$$P_j(x,n) = \min \begin{cases} \min_{a\in\{1,2\}} \{H_a(n,n) + I_a(n) - V_j(x,n) + \alpha P_a(n,n+1)\} & \text{if } d_n = R \\ H_j(x,n) + \alpha P_j(x,n+1) & \text{if } d_n = K \\ \quad \text{for } \forall n = 1,\cdots,T-1;\ \forall x = 0,1,\cdots,n-1;\ \forall j = 1,2 & \end{cases} \tag{2.1}$$

$$P_j(x,T) = -V_j(x,T) \text{ for } \forall x = 0,1,\cdots,T-1;\ \forall j = 1,2 \tag{2.2}$$

Here, let the first and second equations in the right hand side of (2.1) denote respectively the total cost in case of replacing existing equipment j at time $n(d_n = R)$ and the total cost in case of keeping the equipment at time $n(d_n = K)$. The first equation in the right hand side of (2.1) can be rewritten as

$$\begin{aligned} &\min_{a\in\{1,2\}} \{H_a(n,n) + I_a(n) - V_j(x,n) + \alpha P_a(n,n+1)\} \\ &= -V_j(x,n) + \min_{a\in\{1,2\}} \{H_a(n,n) + I_a(n) + \alpha P_a(n,n+1). \end{aligned}$$

This equation shows the salvage value of existing equipment j becomes a sunk cost once the replacement of existing equipment j at time n is decided. Furthermore the optimal sequence of replaced equipment can be determined by considering only the case of starting at time n with new equipment regardless of existing equipment j and following the optimal policy since then. Using this equation, the decision by (2.1) is equivalent to the following:

$$\begin{aligned} &H_j(x,n) + V_j(x,n) + \alpha P_j(x,n+1) \\ &\quad < \qquad\qquad\qquad\qquad \rightarrow d_n = K \\ &\quad \min_{a\in\{1,2\}} \{H_a(n,n) + I_a(n) + \alpha P_a(n,n+1)\} \\ &\quad > \qquad\qquad\qquad\qquad \rightarrow d_n = R \end{aligned} \tag{2.3}$$

Let $j^*(n)$ denote the optimal sequence of replaced equipment satisfying the right hand side of (2.3) when n varies, that is, the sequence of replaced equipment at time n in case of starting at time n with either new equipment 1 or 2 and following the optimal policy. Then, $j^*(n)$ $(n = 0,1,\cdots,T-1)$ can be determined regardless of existing equipment type. It will be shown in the analysis in Chapter 3 and after that $\{j^*(n)\}$ becomes an important equipment sequence characterizing the equipment replacement behavior under ITA.

Then, define functions $C_j(x,n)$ and $R_j(x,n)$, which play an important role in the analysis, as follows:

$$\begin{aligned} C_j(x,n) &\equiv H_j(x,n) + V_j(x,n) - \alpha V_j(x,n+1) \\ &\qquad \text{for } \forall n = 1,2;\cdots,T-1;\ \forall x = 0,1,\cdots,n;\ \forall j = 1,2 \\ R_j(x,n) &\equiv P_j(x,n) + V_j(x,n) \text{ for } \forall n = 1,2,\cdots,T; \forall x = 0,1,\cdots,n;\ \forall j = 1,2 \end{aligned}$$

Using the above notations, the decision by (2.1) and (2.2) is equivalent to

$$R_j(x,n) = \min \begin{cases} \min_{a\in\{1,2\}} \{C_a(n,n) + \alpha R_a(n,n+1) + I_a(n) - V_a(n,n)\} & \text{if } d_n = R \\ C_j(x,n) + \alpha R_j(x,n+1) & \text{if } d_n = K \end{cases} \tag{2.4}$$

$$\text{for } \forall n = 1,2,\cdots,T-1;\ \forall x = 0,1,\cdots,n-1;\ \forall j = 1,2$$

$$R_j(x,T) = 0 \quad \text{for } \forall x = 0,1,\cdots,T-1;\ \forall j = 1,2. \tag{2.5}$$

The decision by (2.3) is equivalent to

$$C_j(x,n) + \alpha R_j(x,n+1) \begin{array}{c} < \\ \\ > \end{array} \min_{a\in\{1,2\}}\{C_a(n,n) + \alpha R_a(n,n+1) + I_a(n) - V_a(n,n)\} \quad \begin{array}{l} \to d_n = K \\ \\ \to d_n = R. \end{array} \tag{2.6}$$

2.4 Control limit policy

In analyzing the replacement problem with plural alternatives and the replacement behavior under ITA, the control limit policy proposed in [6] plays an important role. In this section we therefore define "Control Limit Policy" and give a sufficient condition for which the optimal policy determined using (2.4) and (2.5) constitutes a control limit policy.

Definition 1: When we adopt a rule of $d_n = R$ if $0 \le x \le x_j(n)$ and $d_n = K$ if $x_j(n) < x \le n$ concerning replacement decision at time n for existing equipment $j(\forall j = 1,2)$ purchased at time $x(\forall x = 0,1,\cdots,n-1)$, we define the threshold value $x_j(n)$ and the sequence $\{x_j(n)\}$ $(n = 1,2,\cdots,T-1)$ control limit at time n for equipment j and control limit policy for equipment j, respectively. If there exist $x_j(n)$ and $\{x_j(n)\}$ under the optimal policy, then we denote them by $x_j^*(n)$ and $\{x_j^*(n)\}$ respectively.

In order to clarify some properties of the optimal policy, we define other control limits minimizing the present value of total cost under the limited alternative sets $\{1\}$ and $\{1,2\}$.

Definition 2: Let $x^*_{(j\cdot1)}(n)$ denote a control limit at time n where we optimally replace existing equipment $j(\forall j = 1,2)$ only for new equipment 1 on or after time n, if it exists. Let $x^*_{(j\cdot12)}(n)$ be a control limit at time n where we optimally replace existing equipment j for either new equipment 1 or 2 on or after time n, if it exists.

From Definitions 1 and 2, we have the relation of $x_j^*(n) = x^*_{(j\cdot12)}(n)$. Next, we give a sufficient condition for which the optimal policy determined using (2.4) and (2.5) constitutes a control limit policy. It can be shown in the same way as discussed in [6] (see pp. 397–398) that the following lemma holds with respect to the left hand side of (2.6).

Lemma 1: If $C_j(x,n) \downarrow x$ on $[0,n]$ for $\forall n = 1,2,\cdots,T-1$; $\forall j = 1,2$, then $C_j(x,n) + \alpha R_j(x,n+1) \downarrow x$ holds (we abbreviate the proof).

Theorem 1: If $C_j(x,n) \downarrow x$ on $[0,n]$ for $\forall n = 1,2,\cdots,T-1$; $\forall j = 1,2$, then there exist the control limit policies in Definitions 1 and 2 (we abbreviate the proof).

We can give sufficient conditions for the existence of the control limit policies from the convexity of $C_j(x,n)$ on x in the same way as [6] (see pp. 398–402) in the case, where $H_j(x,n)$ and $V_j(x,n)$ are given by the bi-nonlinear function forms of equipment purchase time x and use periods $n-x$ such that $H_j(x,n) = h_j(x)\psi_{h_j}(n-x)$, $V_j(n-x) = v_j(x)\psi_{v_j}(n-x)$. Here, note that the exponential case in section 3.4 becomes a special case of bi-nonlinear functions.

2.5 An important factor characterizing equipment replacement

The relation between $x^*_{(1\cdot1)}(n)$ and $x^*_{(1\cdot12)}(n)$ is significant in the replacement problem with two replacement alternatives at every time for planning periods $[0,T]$. We first consider the relation in this section. Generally, in equipment replacement for planning periods $[0,T]$, it is economical to replace existing equipment for type 2 new equipment if the remaining planning periods $[n,T]$ are longer, and to replace for type 1 new equipment if the remaining periods are shorter. Therefore, there is a higher possibility that $j^*(T-1)=1$, that is, $H_1(T-1,T-1)+I_1(T-1)-\alpha V_1(T-1,T) < H_2(T-1,T-1)+I_2(T-1)-\alpha V_2(T-1,T)$ from (2.2) and (2.3). Since new equipment 2 is disqualified as a replacement alternative if $j^*(n)=1$ at every time n on $[0,T]$, hereafter we assume that there exists at least a time n such that $j^*(n)=2$. Therefore, let n_{12} denote ($1 \le n_{12} \le T-1$) the time n such that

$$j^*(n) = \begin{cases} 2 & \text{for } n = n_{12}-1 \\ 1 & \text{for } n = n_{12}, n_{12}+1, \cdots, T-1. \end{cases} \tag{2.7}$$

Time n_{12} represents the minimum time n such that replacement sequence $\{j^*(n)\}$, starting at time n and following the optimal policy after that time, is given only by new equipment 1 on or after time n.

There exist $x^*_{(1\cdot 1)}(n)$ and $x^*_{(1\cdot 12)}(n)$ in Definition 2 under the assumption of $C_j(x,n) \downarrow x$ from Theorem 1. Further, under the assumptions of $C_j(x,n) \downarrow x$ and (2.7), we can easily show

$$x^*_{(1\cdot 12)}(n) > x^*_{(1\cdot 1)}(n) \quad \text{for} \quad n = n_{12}-1$$
$$x^*_{(1\cdot 12)}(n) = x^*_{(1\cdot 1)}(n) \quad \text{for} \quad n = n_{12}, n_{12}+1, \cdots, T-1$$

by reductive absurdum. Then, it is seen that n_{12} becomes the smallest time n such that $x^*_{(1\cdot 1)}(n)$ coincides with $x^*_{(1\cdot 12)}(n)$ from time $T-1$ through time n. From this, a typical relation between $x^*_{(1\cdot 1)}(n)$ and $x^*_{(1\cdot 12)}(n)$ is shown in Fig. 2. Here, let n_1^- and n_1^+ be the smallest and largest values of n satisfying $x^*_{(1\cdot 1)}(n) > 0$, respectively, and let n_{12}^- be the smallest value of n satisfying $x^*_{(1\cdot 12)}(n) > 0$. Though there may be a relation of $n_{12}^- \ge n_1^-$, generally it is considered that the replacement time of existing equipment 1 purchased at time 0 comes earlier if we add new equipment 2 as a alternative. Therefore, hereafter we analyze a problem supposing the case of $n_{12}^- < n_1^-$ as shown in Fig. 2.

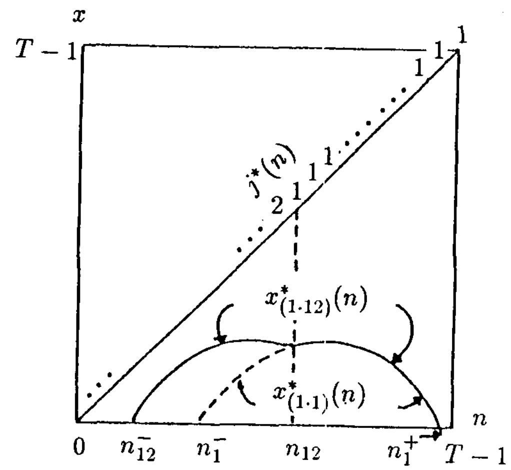


Fig. 2 Typical Relation between $x^*_{(1\cdot 1)}(n)$ and $x^*_{(1\cdot 12)}(n)$

Next, we discuss the relation between $x^*_{(1\cdot 12)}(n)$ and $x^*_{(2\cdot 12)}(n)$. If the technology level of equipment 1 is lower than that of equipment 2, $H_1(x,n) > H_2(x,n)$ in operating cost is considered. Therefore, when we can ignore salvage values of both equipments 1 and 2 (for example, cases (1) where we cannot resale used equipment if it was made for special purpose,

(2) where it is inevitable to destroy existing equipment for replacement as in case of heat treatment furnace, (3) where existing equipment reaches the physical lifetime due to the full operation under an equipment operating policy of earlier capital investment recovery or (4) where economical obsolescence of existing equipment is marked due to rapid technological advances, etc.), or when we can consider the operating cost difference per period $H_1(x,n) - H_2(x,n)$ is greater than the difference in decreases of salvage value per period $\{V_2(x,n) - \alpha V_2(x,n+1)\} - \{V_1(x,n) - \alpha V_1(x,n+1)\}$, the following relation holds:

$$C_1(x,n) > C_2(x,n) \quad \text{for } \forall n = 1,2,\cdots,T-1; \forall x = 0,1,\cdots,n \tag{2.8}$$

Under the conditions of $C_j(x,n) \downarrow x$ and (2.8), we can show

$$x^*_{(1\cdot12)}(n) \geq x^*_{(2\cdot12)}(n) \quad \text{for } \forall n = 1,2,\cdots,T-1 \tag{2.9}$$

by induction from (2.4) and (2.5). The relation (2.9) implies that next optimal replacement time of existing equipment 2 becomes later than that of existing equipment 1 if the purchase times of equipment 1 and 2 are same.

3. Equipment Replacement under the Appearance of Innovative Equipment

3.1 Problem description

We suppose, in stead of considering the problem in section 2.1, that new equipments 1 with GTA are available for planning periods $[0,T]$ and it is forecasted that new equipments 2 at much higher level of technology (hereafter called innovative equipments) appear since some time t on $[0,T]$. It is found that GTA also occur for equipments 2. Then, if we can forecast the appearance time t of innovative equipment, we can make a replacement decision scientifically and rationally. Even if we can not forecast time t, we will be able to obtain some effective informations by analysis, supposing some values of time t. Therefore, we assume a case of the appearance time t of innovative equipment being known.

Now, we denote function $f(\cdot)$ by $f(\cdot;t)$ in order to show that the function is affected by the appearance time t of innovative equipment. Thus, $H_j(x,n), I_j(n), V_j(x,n), P_j(x,n), C_j(x,n)$ and $R_j(x,n)$ are represented by $H_j(x,n;t), I_j(n;t), V_j(x,n;t), P_j(x,n;t), C_j(x,n;t)$ and $R_j(x,n;t)$, respectively. Here, operating cost of equipment 1 should be noted by $H_1(x,n)$ since it is not affected by time t and remains $H_1(x,n)$. However, we express it by $H_1(x,n;t)$ for the description convenience. Control limits $x^*_j(n)$ in Definition 1, and, $x^*_{(j\cdot1)}(n)$ and $x^*_{(j\cdot12)}(n)$ in Definition 2 are expressed by $x^*_j(n;t)$, and, $x^*_{(j\cdot1)}(n;t)$ and $x^*_{(j\cdot12)}(n;t)$, respectively. It is evident that any replacement before time t is made by new equipment 1 and any replacement on or after time t is made from plural replacement alternatives of new equipments 1 and 2. From the fact, we define a replacement alternative set $A(n;t)$ available at arbitrary time n as follows:

$$A(n;t) \equiv \begin{cases} \{1\} & \text{if } n < t \\ \{1,2\} & \text{if } n \geq t \end{cases}$$

Let $j^*(n;t)$ be the optimal type of replaced equipment starting with new equipment at time n and following the optimal policy when innovative equipment appears at time t ($j^*(n;t)$ is given by the equipment type which attains the minimum value of the first equation in the right hand side of (3.2)). Using $j^*(n)$ defined with respect to (2.3), it is clear that $j^*(n;t)$ can be expressed as follows:

$$j^*(n;t) = \begin{cases} 1 & \text{if } n < t \\ j^*(n) & \text{if } n \geq t \end{cases}$$

GTA bring about a gentle decrease in initial operating cost of new equipments and a gradual change (increase or decrease) in purchase price of new equipments for each type. On the other hand, with respect to cash flow functions of innovative equipment appearing at time t, initial operating cost decreases radically, and purchase price increases rapidly, compared with new equipment 1. As purchase price of new equipment 1 is considered to decrease at time t radically, we suppose a case where this price curve shifts to another curve preforecasted since time t (see an example of $I_1(x;t)$ curve in Fig. 3). In this chapter, we assume that (1) operating cost of equipment 1 is not influenced by the appearance time t of innovative equipment, (2) purchase price of new equipment 1 is not affected by a change of time t so long as time n lies before or after time t, (3) purchase price of new equipment 1 at time n decreases when innovative equipment appears at time n, and (4) operating cost and purchase price of new equipment 2 are not affected by a change of time t so long as time n lies after time t. These assumptions are expressed as follows:

$$\begin{aligned}
&\text{(i) } H_1(x,n;t'-1) = H_1(x,n;t') \text{ for } \forall n \geq 1; \forall x = 0,1,\cdots,n \\
&\text{(ii) } H_2(x,n;t'-1) = H_2(x,n;t') \text{ for } \forall n \geq t'; \forall x = t',t'+1,\cdots,n \\
&\text{(iii) } I_1(n;t'-1) = I_1(n;t') \text{ for } \forall n \leq t'-2 \text{ or } \forall n \geq t' \\
&\text{(iv) } I_1(n;t'-1) < I_1(n;t') \text{ for } n = t'-1 \\
&\text{(v) } I_2(n;t'-1) = I_2(n;t') \text{ for } \forall n \geq t'
\end{aligned} \tag{3.1}$$

A typical pattern of cash flow functions is illustrated in Fig. 3.

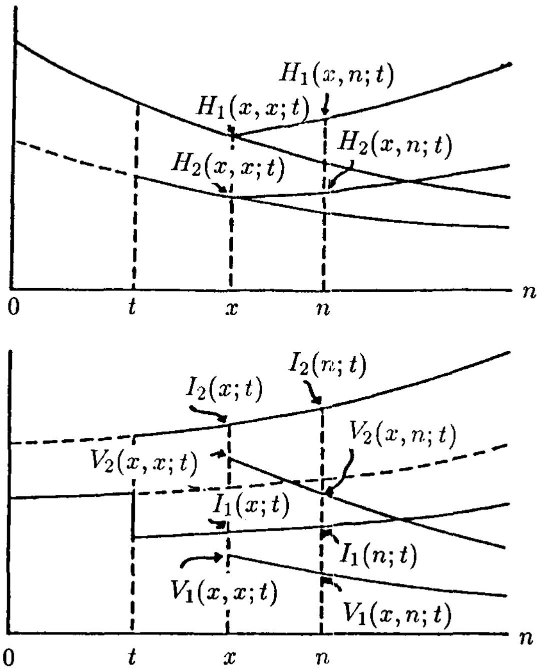


Fig. 3 Cash Flow Functions in the Case of Innovative Equipment Appearing at Time t

Under the above circumstances, the decision maker is going to decide a sequence of both replaced equipment type and replacement time so as to minimize the present value of total cost for the remaining planning periods $[n,T]$.

3.2 Formulation

Using the notations described in section 3.1 and rewriting (2.1)–(2.6), we can formulate a replacement problem where innovative equipment appears at time t. Here, we restrict the descriptions to those of $C_j(x,n;t), R_j(x,n;t)$ and (2.4)–(2.6) which will be often used in the later analysis.

Those are rewritten as follows:

$C_j(x,n;t) \equiv H_j(x,n;t) + V_j(x,n;t) - \alpha V_j(x,n+1;t)$
for $\forall n = 1,2,\cdots,T-1; \forall x = 0,1,\cdots,n; \forall j \in A(x;t)$
$R_j(x,n;t) \equiv P_j(x,n;t) + V_j(x,n;t)$
for $\forall n = 1,2,\cdots,T; \forall x = 0,1,\cdots,n; \forall j \in A(x;t)$

$$R_j(x,n;t) = \min \begin{cases} \min_{a\in A(n;t)} \{C_a(n,n;t) + \alpha R_a(n,n+1;t) + I_a(n;t) - V_a(n,n;t)\} & \text{if } d_n = R \\ C_j(x,n;t) + \alpha R_j(x,n+1;t) & \text{if } d_n = K \end{cases} \tag{3.2}$$

for $\forall n = 1,2,\cdots,T-1; \forall x = 0,1,\cdots,n-1; \forall j \in A(x;t)$

$$R_j(x,T;t) = 0 \quad \text{for } \forall x = 0,1,\cdots,T-1; \forall j \in A(x,t) \tag{3.3}$$

$$\begin{array}{ll} C_j(x,n;t) + \alpha R_j(x,n+1;t) & \\ \quad < & \to d_n = K \\ \quad \min_{a\in A(n;t)} \{C_a(n,n;t) + \alpha R_a(n,n+1;t) + I_a(n;t) - V_a(n,n;t)\} & \\ \quad > & \to d_n = R \end{array} \tag{3.4}$$

3.3 Control limits $x_1^*(n;t)$ and $x_2^*(n;t)$

We can show that Lemma 2 and Theorem 2 hold respectively corresponding to Lemma 1 and Theorem 1:

Lemma 2: (i) If $C_1(x,n;t) \downarrow x$ on $[0,n]$ for $\forall n = 1,2,\cdots,T-1$, then $C_1(x,n;t)+\alpha R_1(x,n+1;t) \downarrow x$ holds (we abbreviate the proof).
(ii) If $C_2(x,n;t) \downarrow x$ on $[t,n]$ for $\forall n = t+1,t+2\cdots,T-1$, then $C_2(x,n;t)+\alpha R_2(x,n+1;t) \downarrow x$ holds (we abbreviate the proof).

Theorem 2: (i) If $C_1(x,n;t) \downarrow x$ on $[0,n]$ for $\forall n = 1,2,\cdots,T-1$, then there exists control limit policy $\{x_1^*(n;t)\}$ and $\{x_{(1\cdot 1)}^*(n;t)\}$ (we abbreviate the proof).
(ii) If $C_2(x,n;t) \downarrow x$ on $[t,n]$ for $\forall n = t+1,t+2\cdots,T-1$, then there exists control limit policy $\{x_2^*(n;t)\}$ (we abbreviate the proof).

Control limits $x_{(1\cdot 1)}^*(n)$ and $x_{(1\cdot 12)}^*(n)$ are functions of n, not depending on the appearance time t of innovative equipment. On the other hand, control limit $x_{(1\cdot 1)}^*(n;t)$ $(x_{(1\cdot 1)}^*(n;0) = x_{(1\cdot 1)}^*(n))$ is a function of n, depending on time t. In this section, we shall theoretically examine how control limits $x_1^*(n;t)$ and $x_2^*(n;t)$ are concerned with $x_{(1\cdot 1)}^*(n), x_{(1\cdot 12)}^*(n)$ and $x_{(1\cdot 1)}^*(n;t)$, and $x_{(2\cdot 12)}^*(n)$, respectively. We clarify some characteristics of the equipment replacement behavior under ITA when we let time t change.

Control limit $x_1^*(n;t)$ determining the replacement of existing equipment 1

Definition 3: The region of t, where $x_1^*(n;t)$ $(1 \leq n \leq T-1)$ does not change when we let time t change, is called a "non-influence zone". On the contrary, if $x_1^*(n;t)$ changes, the t region is called an "influence zone".

Hereafter, we classify time t region into the following three cases:

(i) $0 \leq t < n_{12}^-$
(ii) $n_{12}^- \leq t < n_{12}$
(iii) $n_{12} \leq t \leq T$

(i) the case of $0 \leq t < n_{12}^-$

Because of the relation $A(n;0) = \{1,2\}$ at any time n when $t = 0, x_1^*(n;0) = x_{(1\cdot12)}^*(n)$ holds. Next we consider the case of $0 < t < n_{12}^-$. In this case, we further classify the relation between time n and time t into the cases of $n \geq t$ and $n < t$, and analyze them.

the case of $n \geq t$

Since we obtain $A(n;t) = \{1,2\}$ at any time n such that $n \geq t$, $x_1^*(n;t)$ on or after time n is given by the value of x equalizing the left and right hand sides of (3.4). Taking the calculation process by (3.2) and (3.3) into account, this implies that $x_1^*(n;t)$ coincides with $x_{(1\cdot12)}^*(n)$ at any time n such that $n \geq t$.

the case of $n < t$

For any n such that $n < t < n_{12}^-$, we have $A(n;t) = 1$. Since there hold $x_{(1\cdot12)}^*(n) = 0$ from $n < n_{12}^-$ and $x_{(1\cdot1)}^*(n) = 0$ from $n < n_{12}^- < n_1^-$, we obtain $x_1^*(n;t) = x_{(1\cdot12)}^*(n) = 0$.

Therefore, considering the result of $x_1^*(n;t) = x_{(1\cdot12)}^*(n)$ for $n \geq t$ and $n < t$, and the calculation process by (3.2) and (3.3), $x_1^*(n;t) = x_{(1\cdot12)}^*(n)$ holds for $0 < t < n_{12}^-$. Thus, the results in the cases $t = 0$ and $0 < t < n_{12}^-$ yield the following property.

Property 1: If $0 \leq t < n_{12}^-$, then $x_1^*(n;t) = x_{(1\cdot12)}^*(n)$ $(1 \leq n \leq T-1)$ holds as shown in Fig. 4 (i). Thus, the t region such that $0 \leq t < n_{12}^-$ becomes a non-influence zone in Definition 3.

Property 1 implies the following: Replacement is made considering plural replacement alternatives in case where time t is fairly near at time 0. This case is not affected by any change of t within this region.

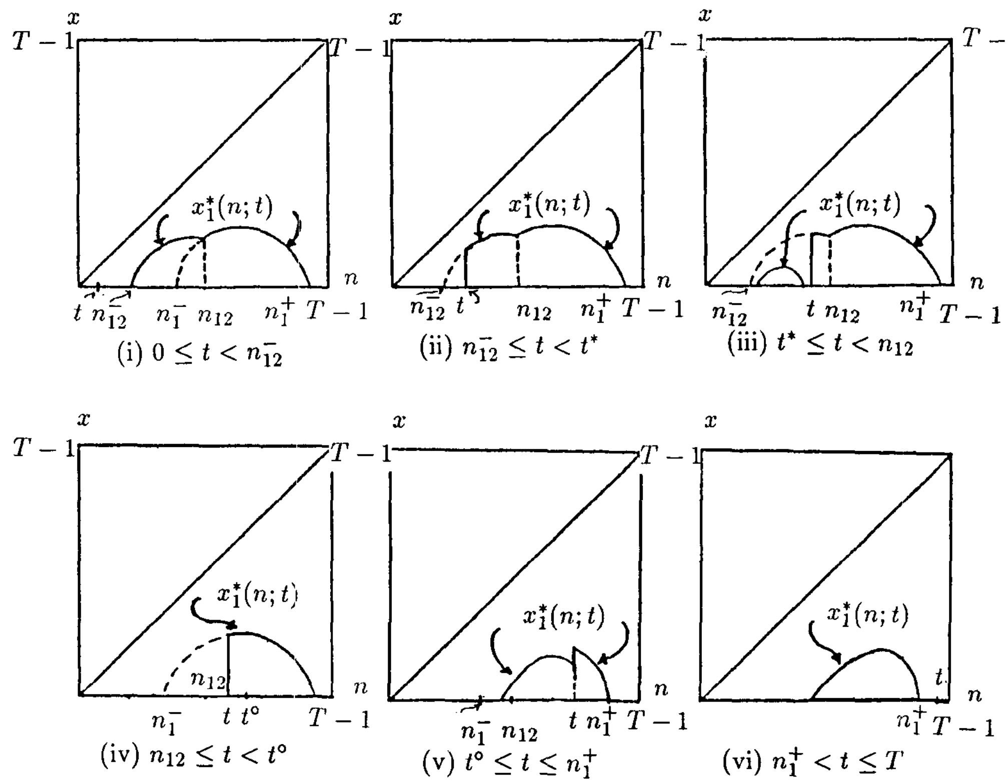


Fig. 4 Control Limit Patterns $x_1^*(n;t)$ for Time t's Regions

(ii) the case of $n_{12}^- \le t < n_{12}$

Control limit $x_1^*(n;t)$ at any time n such that $n_{12}^- \le n < t$ does not coincide with $x_{(1\cdot 1)}^*(n)$ even if $A(n;t) = \{1\}$ holds for such n, since $x_1^*(n;t)$ at any time n such that $t \le n < n_{12}$ does not correspond to $x_{(1\cdot 1)}^*(n)$ but to $x_{(1\cdot 12)}^*(n)$. The results of numerical experiments expressed later (see Fig. 9 (ii)-(vii)) show that patterns Fig. 4 (ii), (iii) and (v) appear when time t becomes larger (here, t^* represents the minimum value of t such that $x_1^*(n;t) > 0$ appears at some time n before time t). However, when we compare the size relation between $C_1(x,n;t')$ and $C_1(x,n;t'-1)$ in order to examine the change of $x_1^*(n;t)$ with time t, we have possibilities of $C_1(x,t'-1;t') > C_1(x,t'-1;t'-1)$ at $n = t'-1$ and $C_1(x,t'-2;t') < C_1(x,t'-2;t'-1)$ at $n = t'-2$. For this reason, it is difficult to prove the change of $x_1^*(n;t)$ with time t theoretically and completely from the recurrence equation by DP. Therefore, in this case, we shall analyze a property with respect to Fig. 4 (ii) by adding conditions of salvage value for existing equipment being independent of the appearance time t of innovative equipment (for example, case where salvage value of equipment can be ignored) to the conditions of (2.7), (3.1) and $C_j(x,n;t) \downarrow x (j = 1,2)$. These added conditions are respectively expressed by

$$\begin{aligned} &\text{(i)} V_1(x,n;t'-1) = V_1(x,n;t') \quad \text{for } \forall n \ge 1; \forall x = 0,1,\cdots,n \\ &\text{(ii)} V_2(x,n;t'-1) = V_2(x,n;t') \quad \text{for } \forall n \ge t'; \forall x = t',t'+1,\cdots,n. \end{aligned} \tag{3.5}$$

Then we have the following property.

Property 2: Assume that the conditions of $C_j(x,n;t) \downarrow x (j=1,2)$, (2.7), (3.1) and (3.5) hold. If there exists a time t' such that $x_1^*(n;t')=0$ for any time n with $n<t'$, then there holds $x_1^*(n;t'-1)=0$ for any time n with $n<t'-1$ (see Fig. 4 (i) and (ii)).

Proof: From (3.1) (i) and (ii), and (3.5) (i) and (ii), we have

$$C_j(x,n;t'-1)=C_j(x,n;t') \text{ for } \forall n=1,2,\cdots,T;\forall x=0,1,\cdots,n;\forall j\in A(x;t') \tag{3.6}$$

Therefore, for $n\geq t'$, we can show, using (3.1) (iii) and (v), (3.5) (i) and (ii), and (3.6),

$$R_j(x,n;t'-1)=R_j(x,n;t') \text{ for } \forall n=t',t'+1,\cdots,T-1;\forall x=0,1,\cdots,n;\forall j=1,2 \tag{3.7}$$

by induction from (3.2) and (3.3). Next, we consider the relation between $R_1(x,n;t'-1)$ and $R_1(x,n;t')$ at $n=t'-1$. From (3.2), $R_1(x,n;t'-1)$ is given by

$$R_1(x,n;t'-1)=\min\begin{cases}\min\limits_{a\in\{1,2\}}\{C_a(n,n;t'-1)+\alpha R_a(n,n+1;t'-1) \\ \qquad +I_a(n;t'-1)-V_a(n,n;t'-1)\} & \text{if } d_n=R\\ C_1(x,n;t'-1)+\alpha R_1(x,n+1;t'-1) & \text{if } d_n=K\\ \quad \text{for } n=t'-1;\forall x=0,1,\cdots,n-1\end{cases} \tag{3.8}$$

and $R_1(x,n;t')$ is given by

$$R_1(x,n;t')=\min\begin{cases}C_1(n,n;t')+\alpha R_1(n,n+1;t')+I_1(n;t')-V_1(n,n;t') & \text{if } d_n=R\\ C_1(x,n;t')+\alpha R_1(x,n+1;t') & \text{if } d_n=K\end{cases} \tag{3.9}$$

$$\text{for } n=t'-1;\forall x=0,1,\cdots,n-1,$$

because of $A(n;t')=\{1\}$ at $n=t'-1$. From (3.6) and (3.7), with respect to the second equations in the right hand sides of (3.8) and (3.9), we have

$$C_1(x,n;t'-1)+\alpha R_1(x,n+1;t'-1)=C_1(x,n;t')+\alpha R_1(x,n+1;t'). \tag{3.10}$$

From (3.1) (iv), (3.5) (i), (3.6) and (3.7), with respect to the first equations in the right hand sides of (3.8) and (3.9), we have

$$\min_{a\in\{1,2\}}\{C_a(n,n;t'-1)+\alpha R_a(n,n+1;t'-1)+I_a(n;t'-1)-V_a(n,n;t'-1)\} \tag{3.11}$$

$$< C_1(n,n;t')+\alpha R_1(n,n+1;t')+I_1(n;t')-V_1(n,n;t').$$

Noticing $C_1(x,n;t)+\alpha R_1(x,n+1;t)\downarrow x$ for any t from Lemma 2, with respect to the left hand sides of (3.8) and (3.9), we have

$$\begin{aligned}&R_1(x,n;t'-1)\downarrow x;R_1(x,n;t')\downarrow x\\ &R_1(x,n;t')-R_1(x,n;t'-1)\geq 0;R_1(x,n;t')-R_1(x,n;t'-1)\downarrow x\end{aligned} \tag{3.12}$$

from (3.10) through (3.11), as illustrated in Fig. 5.

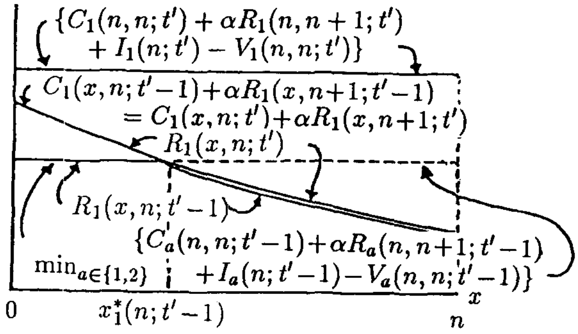


Fig. 5 Typical Relation at Time $n(=t'-1)$ between $R_1(x,n;t'-1)$ and $R_1(x,n;t')$

Next, we consider the relation between $R_1(x,n-1;t'-1)$ and $R_1(x,n-1;t')$ at time $n-1(=t'-2)$. From the assumption $C_1(x,n;t)\downarrow x$ in Property 2, (3.6) and (3.12), we obtain the relations of

$$\begin{aligned}&C_1(x,n-1;t'-1)+\alpha R_1(x,n;t'-1)\downarrow x; C_1(x,n-1;t')+\alpha R_1(x,n;t')\downarrow x\\&C_1(x,n-1;t')+\alpha R_1(x,n;t')\geq C_1(x,n-1;t'-1)+\alpha R_1(x,n;t'-1)\\&\{C_1(x,n-1;t')+\alpha R_1(x,n;t')\}-\{C_1(x,n-1;t'-1)+\alpha R_1(x,n;t'-1)\}\downarrow x.\end{aligned}\tag{3.13}$$

From (3.1) (iii) and (3.5) (i), we have

$$I_1(n-1;t'-1)-V_1(n-1,n-1;t'-1)=I_1(n-1;t')-V_1(n-1,n-1;t')\tag{3.14}$$

at $n-1(=t'-2)$. Since $x_1^*(n;t')$ is determined as the value of x equalizing the first and second equations at the right hand sides in (3.9), the assumption of $x_1^*(n;t')=0$ in Property 2 means that the following relation holds with respect to the first and second equations in the right hand side of (3.9):

$$\begin{aligned}&C_1(x,n;t')+\alpha R_1(x,n+1;t')\\&\quad\leq C_1(n,n;t')+\alpha R_1(n,n+1;t')+I_1(n;t')-V_1(n,n;t')\text{ for }\forall n<t'.\end{aligned}\tag{3.15}$$

Therefore, the assumption $x_1^*(n-1;t')=0$ in Property 2 implies

$$\begin{aligned}&C_1(x,n-1;t')+\alpha R_1(x,n;t')\leq C_1(n-1,n-1;t')+\alpha R_1(n-1,n;t')+\\&\quad I_1(n-1;t')-V_1(n-1,n-1;t')\text{ for }\forall x=0,1,\cdots,n-1.\end{aligned}\tag{3.16}$$

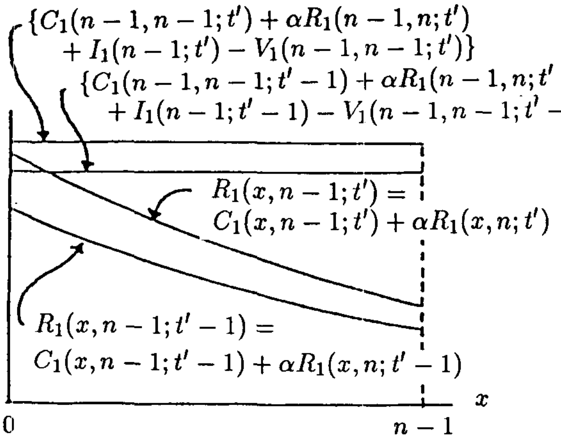


Fig. 6 Typical Relation at Time $n(=t'-2)$ between $R_1(x,n-1;t'-1)$ and $R_1(x,n-1;t')$

With respect to the right and left hand sides of (3.16) where we replace t' for $t'-1$, we can show, as illustrated in Fig. 6, the relation

$$C_1(x,n-1;t'-1) + \alpha R_1(x,n;t'-1) \leq C_1(n-1,n-1;t'-1) + \alpha R_1(n-1,n;t'-1) + I_1(n-1;t'-1) - V_1(n-1,n-1;t'-1) \quad \text{for } \forall x = 0,1,\cdots,n-1 \tag{3.17}$$

from (3.13) through (3.15). From this, it is clear that

$$x_1^*(n-1;t'-1) = 0 \tag{3.18}$$

holds and the left hand sides of (3.16) and (3.17) respectively become $R_1(x,n-1;t')$ and $R_1(x,n-1;t'-1)$. Therefore, we obtain the relations of

$$\begin{aligned} &R_1(x,n-1;t'-1) \downarrow x; R_1(x,n-1;t') \downarrow x \\ &R_1(x,n-1;t') - R_1(x,n-1;t'-1) \geq 0; R_1(x,n-1;t') - R_1(x,n-1;t'-1) \downarrow x. \end{aligned} \tag{3.19}$$

Generally, we can show by induction that the relations obtained by replacing $n-1$ for k ($k = 1,2,\cdots,t'-2$) in (3.18) and (3.19) hold. Thus, the proof is complete. □

Though it is difficult to prove theoretically the results of numerical experiments which represent the case of Fig. 4 (iii) such that $x_1^*(n;t) > 0$ at some time n before time t, we can infer the reason as follows: At time $n = t-1$ just before the appearance time t of innovative equipment, it is considered that the difference between the first and second equations in the right hand side of (3.2) is larger than that at time $n = t$, because a replacement alternative at time $n = t-1$ is limited to new equipment 1 and $I_1(n;t) - V_1(n,n;t)$ at $n = t-1$ tends to be larger than that at time $n = t$ from (3.1) (iv) and (3.5) (i). Thus the case of $x_1^*(n;t) = 0$ occurs when $n = t-1$. Then, we obtain the relation of $R_1(x,n;t) = C_1(x,n;t) + \alpha R_1(x,n+1;t)$. The relation $x_1^*(n;t) = 0$ is maintained for a while even if present time n becomes successively smaller than t. But, the decreasing function $C_1(x,n;t)$ of x is added to $R_1(x,n+1;t)$ at time $n+1$ in the second equation of the right hand side, the second equation becomes more rapidly larger until the size relation between the first and second equations in the right hand side of (3.2) is reversed at last. Thus, the case of $x_1^*(n;t) > 0$ appears at some time $n < t$.

(iii) the case of $n_{12} \leq t \leq T$

Noticing the relation of $n < n_{12} \leq t$ at any time n before time n_{12}, replacement alternative at time n is limited to new equipment 1. On the other hand, we have obtained $x_1^*(n;t) = x_{(1\cdot 1)}^*(n;t)$ from the definition of time n_{12} at any time n such that $n \geq n_{12}$. From the facts, we obtain $x_1^*(n;t) = x_{(1\cdot 1)}^*(n;t)$ $(1 \leq n \leq T-1)$ taking the calculation process by (3.2) and (3.3) into account. Thus, the following property holds.

Property 3: If $n_{12} \leq t \leq T$ holds, then $x_1^*(n;t)$ $(1 \leq n \leq T-1)$ coincides with $x_{(1\cdot 1)}^*(n;t)$ as shown in Fig. 4 (iv), (v) and (vi).

Property 3 indicates the importance of investigating the behavior of $x_{(1\cdot 1)}^*(n;t)$ for time t with $n_{12} \leq t \leq T$. This behavior can be sufficiently clarified by examining the change pattern of $x_{(1\cdot 1)}^*(n;t)$ with $0 \leq t \leq T$, which involves the basic pattern of $x_{(1\cdot 1)}^*(n;0)$ at time $t = 0$. Therefore, we notice $x_{(1\cdot 1)}^*(n;0)$.

Control limit $x_{(1\cdot 1)}^*(n;0)$ is illustrated by Fig. 7 (0). Let n_1^- and n_1^+ be the smallest and largest values satisfying $x_{(1\cdot 1)}^*(n) > 0$, respectively. (Note that n_1^- and n_1^+ become identical values described in Fig. 2 since $x_{(1\cdot 1)}^*(n) = x_{(1\cdot 1)}^*(n;0)$ holds.) Next, letting time t increase from time 0 to time n_1^- $(0 \leq t < n_1^-)$, it is seen that Fig. 7 (i) pattern does not change by the same reason as the case of Fig. 4 (i). When we let time t increase from time n^- to $t^o - 1$ $(n_1^- \leq t < t^o)$, Fig. 7 (ii) pattern appears. (Here, t^o is the smallest value of time t such that the pattern of $x_{(1\cdot 1)}^*(n;t) > 0$ appears at some time n before time t.) With respect to this pattern, we can easily show that the same phenomenon as Property 2 appears. When we let time t increase from time t^o to n_1^+ $(t^o \leq t \leq n_1^+)$, the pattern such that $x_{(1\cdot 1)}^*(n;t) > 0$ at some time n before time t, as illustrated in Fig. 7 (iii), appears. Though it is also difficult to prove this case theoretically, we can check up on the matter by numerical experiments as described later. When we let time t increase from time $n_1^+ + 1$ to time $T(n_1^+ < t \leq T)$, Fig. 7 (iv) pattern appears. Fig. 7 (iv) implies that replacements are not affected by any change of time t within $(n_1^+, T]$. Thus, the region $(n_1^+, T]$ becomes a non-influence zone.

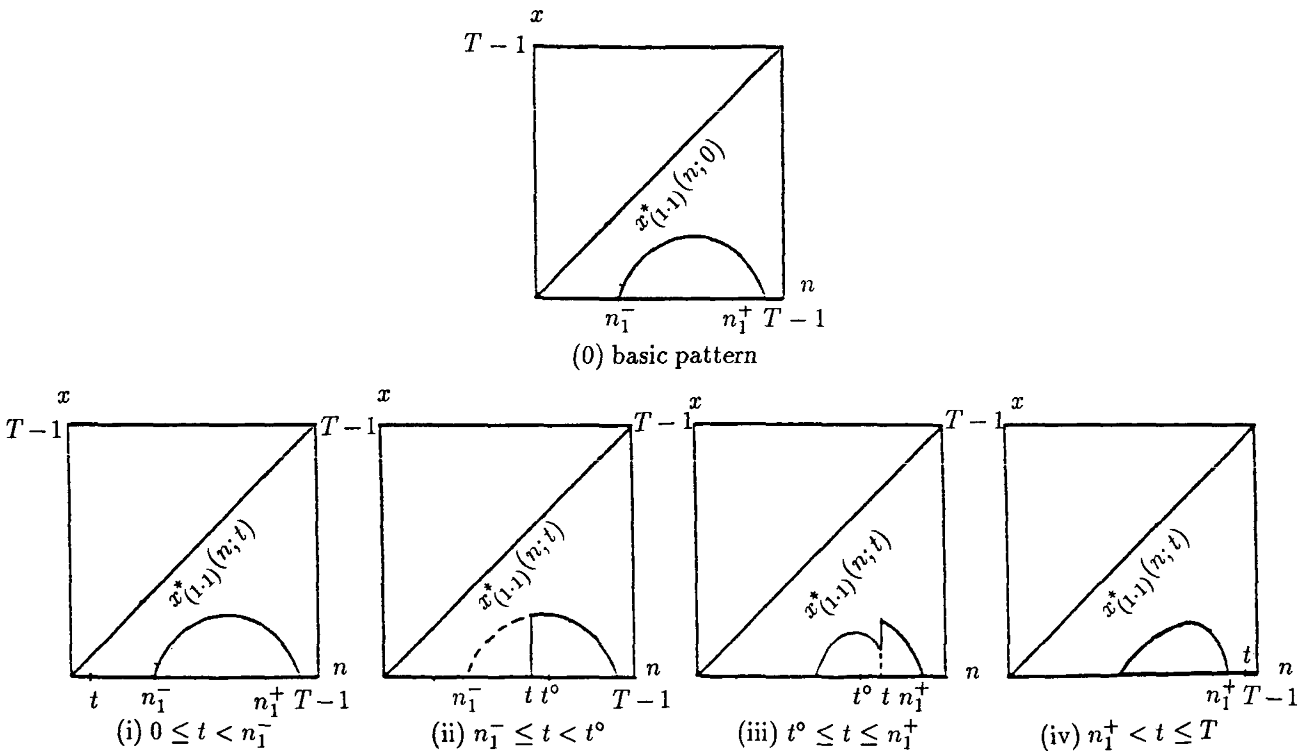


Fig. 7 Control Limit Patterns $x^*_{(1\cdot1)}(n;t)$ for Time t's Regions

Based on the preparation described above, we shall consider the behavior of $x^*_{(1\cdot 1)}(n;t)$ for time t with $n_{12} \le t \le T$. Fig. 7 (i) pattern can not appear since $n_1^- < n_{12} \le t \le T$ holds. For time t with $n_1^+ < t \le T$, Fig. 7 (iv) pattern appears. For time t with $n_{12} \le t \le n_1^+$, either of patterns Fig. 7 (ii) and (iii) or pattern Fig. 7 (iii) appears corresponding to the size relation between n_{12} and t^o. Thus, $x_1^*(n;t)$ for time t with $n_{12} \le t \le T$ is illustrated in Fig. 4 (iv), (v) and (vi) corresponding to Fig. 7 (ii), (iii) and (iv), respectively. Note that either of patterns Fig. 4 (iv), (v) and (vi) or patterns Fig. 4 (v) and (vi) appears.

Control limit $x_2^*(n;t)$ determining the replacement of innovative equipment 2

We discuss how control limit $x_2^*(n;t)$, determining the replacement of existing innovative equipment 2 introduced at time $x(x \ge t)$, varies when we let time t change from time 0 through time T. We shall treat the case where $\{j^*(n)\}$ is determined by the relation of (2.7) as in the case of $x_1^*(n;t)$ and consider the change of $x_2^*(n;t)$ in relation to $x^*_{(2\cdot 12)}(n)$.

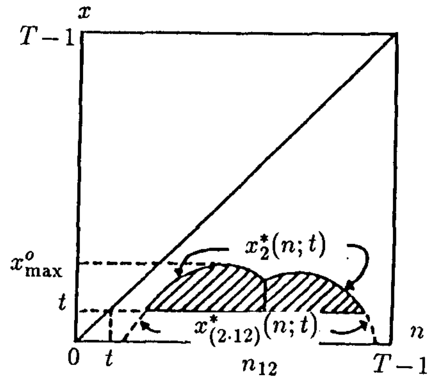


Fig. 8 Control Limit $x_2^*(n;t)$ and the Region of $d_n = R$

A graph of $x^*_{(2\cdot 12)}(n)$ is illustrated in Fig. 8. Here, since n_{12} is, as explained in (2.7), determined by the optimal policy starting with new equipment regardless of existing equipment type, it has the same meaning in the case of Fig. 2. That is, replacement at time $n = n_{12} - 1$ is made by new equipment 2 and replacement at any time n such that $n \ge n_{12}$ is made only by new equipment 1. Now we denote $x^o_{\max}$ maximum value of $x^*_{(2\cdot 12)}(n)$ for $1 \le n \le T-1$ (if there does not exist $x^*_{(2\cdot 12)}(n)$ for $1 \le n \le T-1$, then we let $x^o_{\max} \equiv 0$). Noticing that the relation of $n \ge x \ge t$ holds in case where we replace existing equipment 2 purchased at time x for new equipment at time n. It is understood that the following property holds:

Property 4: We suppose the case of $x^o_{\max} > 0$ under the conditions of $C_j(x,n;t) \downarrow x(j = 1,2)$ and (2.7). With respect to the replacement of innovative equipment, the necessary and sufficient condition for the existence of a region such that $d_n = R$ is given by $t \le x^o_{\max}$, and the region is given by the shadowed portion enclosed by $x^*_{(2\cdot 12)}(n)$ and $x = t$ in Fig. 8.

Property 4 shows that $x_2^*(n;t)$ is fundamentally determined by the structure of $x^*_{(2\cdot 12)}(n)$.

3.4 Numerical example

Because it is difficult to theoretically clarify the change of $x_1^*(n;t)$ when time t varies for $t^* \le t < n_{12}$ in case of Fig. 4 (iii) and $t^o \le t \le n_1^+$ in case of Fig. 4 (v), numerical experiments are needed. Therefore, in this section, we show a typical numerical example of control limits $x_1^*(n;t)$ and $x_2^*(n;t)$. In the example, cash flow functions $H_j(x,n), I_j(x)$ and $V_j(x,n)$ of equipment $j (j = 1,2)$ for the case with plural replacement alternatives in chapter 2 are given as follows:

$$\begin{aligned} &H_j(x,n) = h_j\tau_j^x\rho_j^{n-x} \ (h_j \equiv H_j(0,0) > 0, h_1 > h_2, \rho_j \ge 1, 0 < \tau_j < 1) \\ &I_j(x) = v_j\delta_j^x \ (v_j \equiv I(0), v_1 < v_2, \delta_j > 0) \\ &V_j(x,n) = \beta_j I_j(x)\phi_j^{n-x} \ (0 \le \beta_j < 1, 0 < \phi_j < 1) \end{aligned} \tag{3.20}$$

Here, the parameters τ_j, ρ_j, δ_j and ϕ_j for equipment j represent a decreasing rate of initial operating cost due to GTA, an increasing rate of operating cost due to deterioration, a changing rate of purchase price and a decreasing rate of salvage value, respectively. Also, β_j represents a falling rate of salvage value immediately after its purchase. Then the cost function per period $C_j(x,n)$ is given by

$$C_j(x,n) = h_j\rho_j(\tau_j/\rho_j)^x + \beta_j v_j(1-\phi_j\alpha)\phi_j^n(\delta_j/\phi_j)^x \tag{3.21}$$

and becomes convex with respect to x. Thus, we can concretely describe the condition $C_j(x,n) \downarrow x$ in Theorem 1 in the same way as [6]. Especially in case where we can ignore salvage values (case of $\beta_j = 0$), it is obvious that $C_j(x,n) \downarrow x$ holds from (3.20) and (3.21). Furthermore, for the convenience of numerical analysis, we suppose the case of

$$\tau_1 = \tau_2 = \tau, \rho_1 = \rho_2 = \rho, \delta_1 = \delta_2 = \delta, \phi_1 = \phi_2 = \phi. \tag{3.22}$$

The appearance of innovative equipment is considered to result in (1) sudden decreases in both purchase price of new equipment 1 and salvage value of existing equipment 1, (2) an increase in purchase price of innovative equipment compared with new equipment 1 and (3) an decrease in operating cost for innovative equipment compared with equipment 1. Here, we refer to the case where purchase prices $I_1(n;t)$ and $I_2(n;t)$ rapidly decreases and rises, respectively, when innovative equipment appears and operating cost $H_2(x,n;t)$ of innovative equipment decreases abruptly. That is, we assume that the ITA's effect can be represented by

$$\begin{aligned} I_1(n;t) &= \mu I_1(n) && t \le n \le T-1, 0 < \mu < 1 \\ I_2(n;t) &= \nu I_1(n) && t \le n \le T-1, 1 < \nu \\ H_2(x,n;t) &= \lambda H_1(x,n) && t \le x \le n \le T-1, 0 < \lambda < 1. \end{aligned} \tag{3.23}$$

Parameters values used in the example are shown in Table 1.

Table 1 Parameter Values

$T = 40$ periods (1 period= 3 months), $h_0 = 155$, $v_0 = 190$
$\tau = 0.989846$/period (0.96/year), $\rho = 1.00496$/period (1.02/year)
$\delta = 1.00496$/period (1.02/year), $\phi = 0.927842$/period (0.05/10 years)
$\beta = 0$, $\lambda = 0.8$, $\mu = 0.9$, $\nu = 2.0$, $\alpha = 0.974004$/period (0.90/year)

Fig. 9, Fig. 10 and Fig. 11 show control limits $x_1^*(n;t), x_{(1\cdot 1)}^*(n;t)$ and $x_2^*(n;t)$ in the cases of starting with equipments 1, 1 and 2 respectively. The asterisk "$*$" in Fig. 9, Fig. 10 and Fig. 11 denotes $d_n = R$ at time n for equipment purchased at time x and the values of $j^*(n;t)$ in Fig. 9 and Fig. 11 are noted on the straight line of $x = n$.

In Fig. 9, patterns (i) and (viii) show control limits $x_1^*(n;0)(= x_{(1\cdot 12)}^*(n))$ and $x_1^*(n;38)$ $(= x_{(1\cdot 1)}^*(n;40))$, respectively. Pattern (i) satisfies the relation (2.7). Patterns from (ii) through (vii) represent control limits $x_1^*(n;t)$ at $t = 14, 19, 20, 27, 28$ and 31, respectively. It is seen that each case of Fig. 4 except Fig. 4 (iv) appears by letting $n_{12}^- = 8, n_1^- = 12, t^* = 20, n_{12} = 29$ and $n_1^+ = 37$ for all the patterns from (i) through (viii). In the pattern change from (iv) through (vi), we may expect $x_1^*(n;t) \uparrow t$ for $n < t$. However, there exist counter examples that it does not hold (for example, see the pattern change at times $n = 10, 11$ and 12 from (iv) to (v)).

It is shown that $x_{(1\cdot 1)}^*(n;31)$ in Fig. 10 (v) and $x_{(1\cdot 1)}^*(n;38)$ in Fig. 10 (vi) correspond to $x_1^*(n;31)$ in Fig. 9 (vii) and $x_1^*(n;38)$ in Fig. 9 (viii), respectively, and thus Property 3 holds, that is, patterns Fig. 4 (v) and (vi) appear.

Control limit $x_2^*(n;t)$ determining the replacement of existing equipment 2 is given by Fig. 11. It is shown that Property 4 holds.

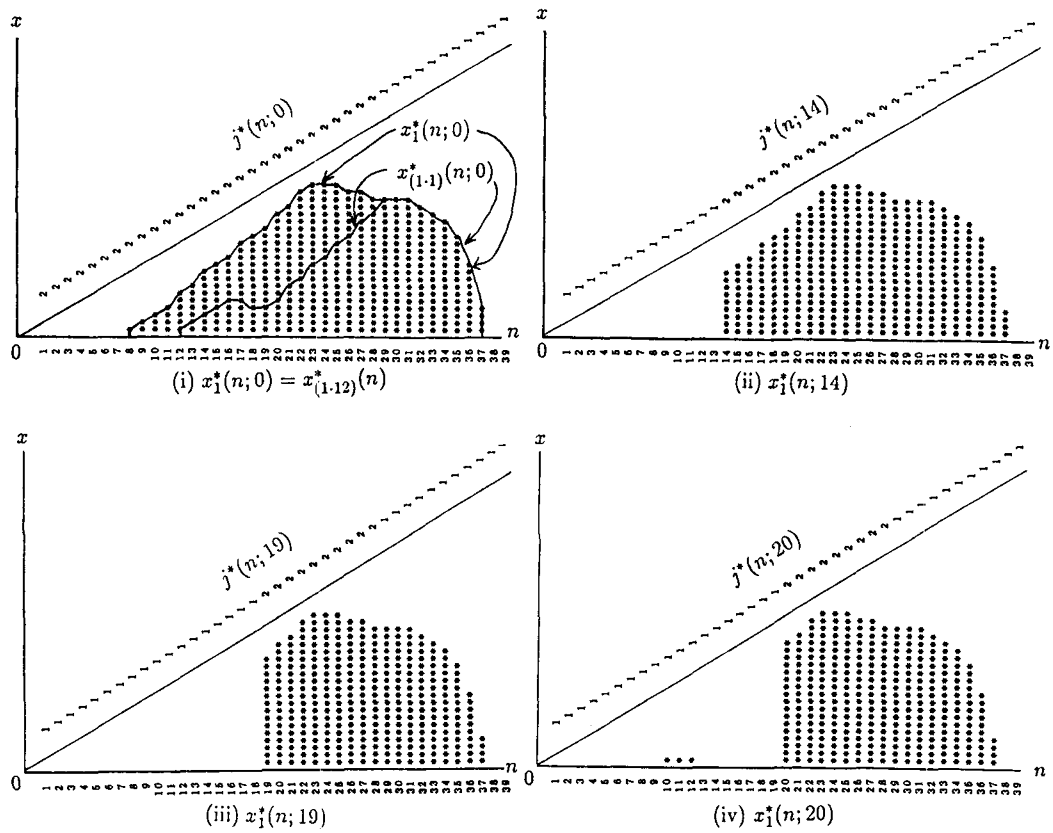


Fig. 9-1 Typical Relation between Time t and Control Limit $x_1^*(n;t)$
$(n_{12}^- = 8, n_1^- = 12, t^* = 20, n_{12} = 29, n_1^+ = 37)$

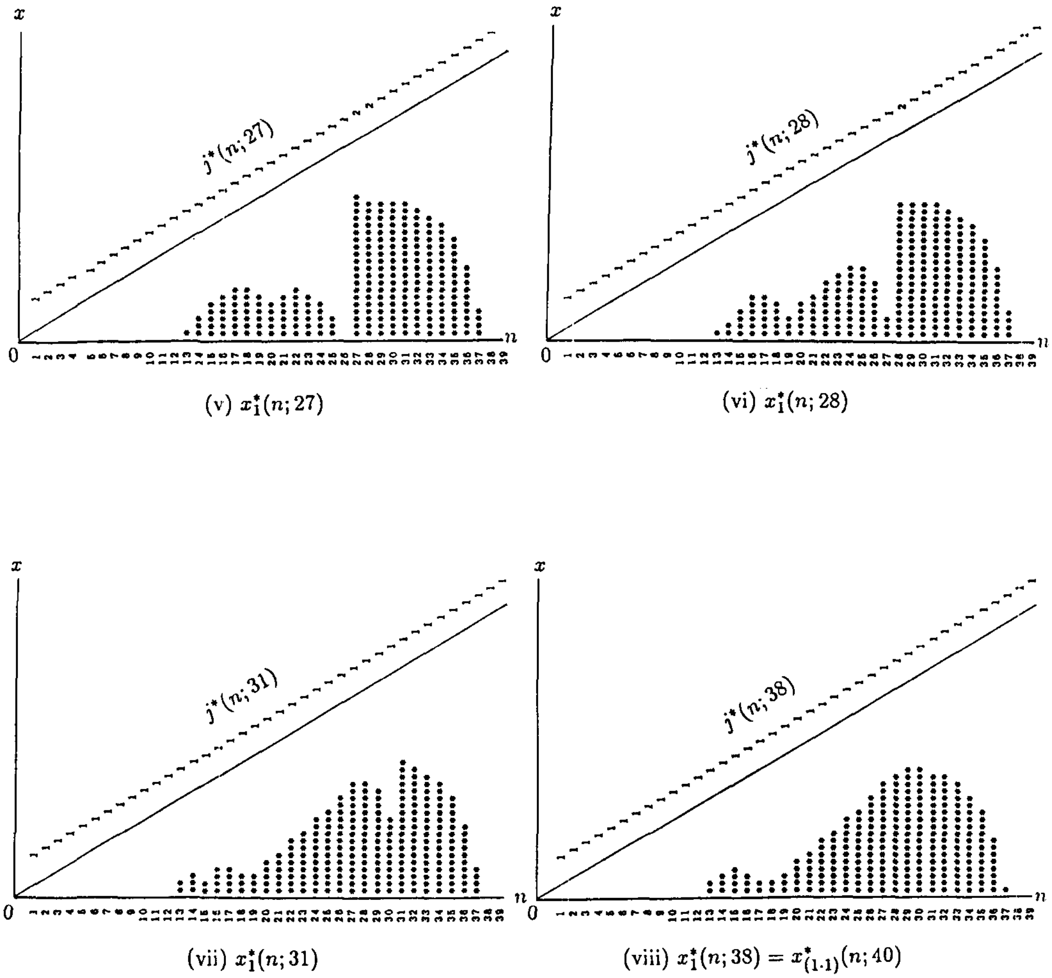


Fig. 9-2 Typical Relation between Time t and Control Limit $x_1^*(n;t)$
$(n_{12}^- = 8, n_1^- = 12, t^* = 20, n_{12} = 29, n_1^+ = 37)$

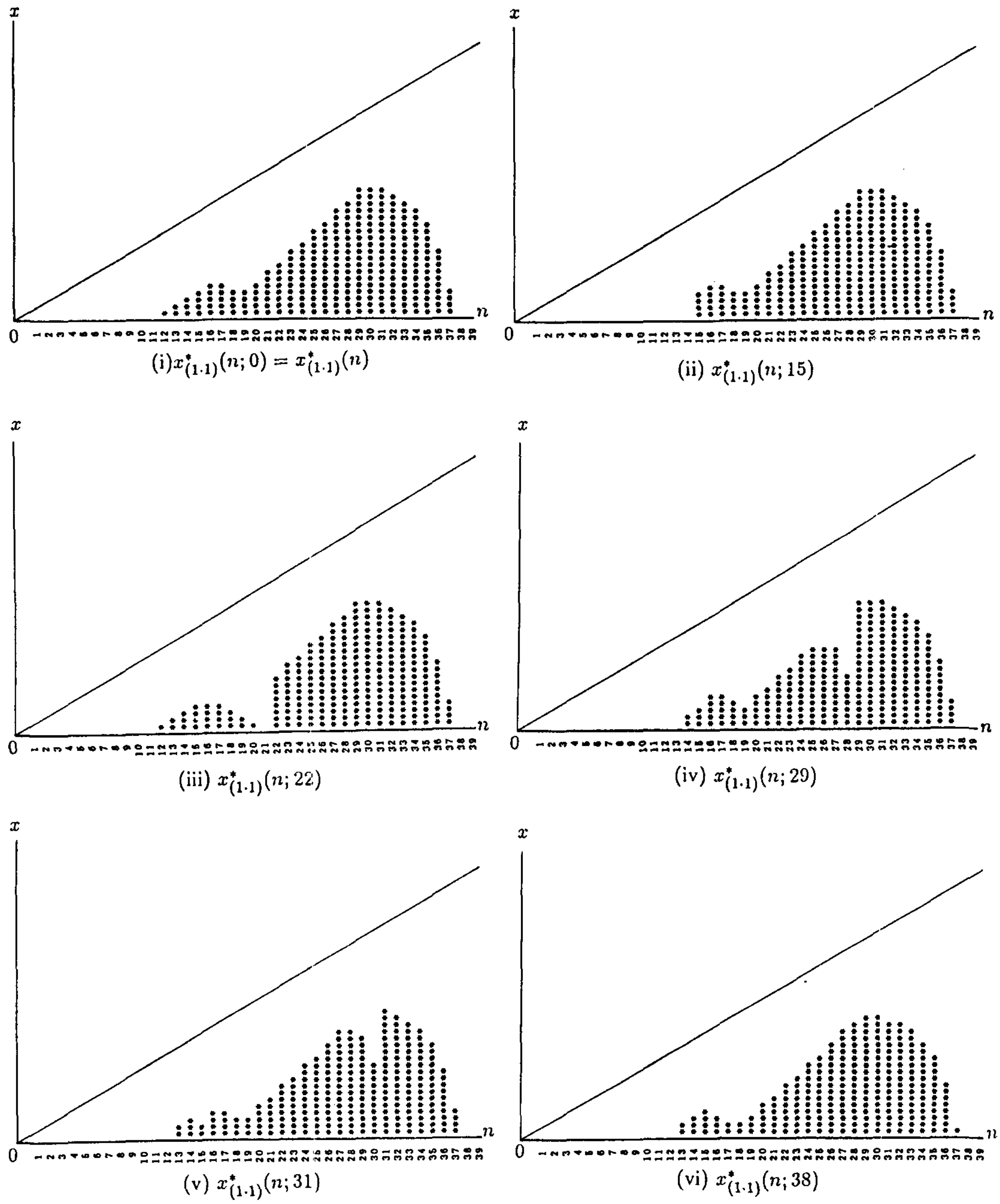
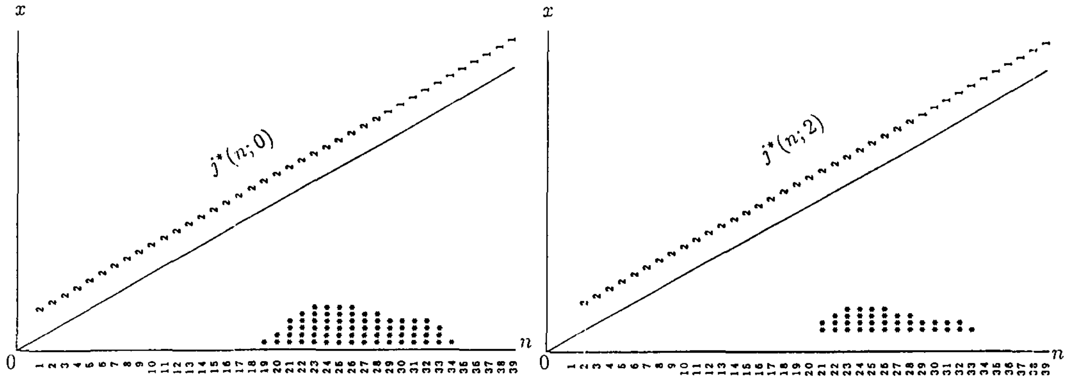


Fig. 10 Typical Relation between Time t and Control Limit $x^*_{(1\cdot1)}(n;t)$
$(n_1^- = 12, n^o = 17, n_1^+ = 37)$



(i) $x_2^*(n;0)$ (ii) $x_2^*(n;2)$

Fig. 11 Typical Relation between Time t and Control Limit $x_2^*(n;t)$

4. Some Considerations

First, we shall consider what is the meaning of control limits $x_1^*(n;t)$ and $x_2^*(n;t)$ given, as time t changes, by respective patterns of Properties 1-3 (Fig. 4), and Property 4 (Fig. 8), as a characteristic of equipment replacement behavior under ITA.

Properties 1-3 (Fig. 4) mean the followings: Non-influence zones exist in cases of (i) $0 \leq t < n_{12}^-$ in Fig. 4 (that is, when we expect ITA in the very near future from time 0 on planning periods $[0, T]$) and (vi) $n_1^+ < t \leq T$ in Fig. 4 (that is, when we expect them in the very near future from time T). The replacements in cases of Fig. 4 (i) and (vi) can be made based on $\{x_{(1\cdot12)}^*(n)\}$ and $\{x_{(1\cdot1)}^*(n;40)\}$, respectively. The case of (ii) $n_{12}^- \leq t < t^*$ in Fig. 4 (that is, we expect ITA in relatively near future from time 0) means that we postpone the replacement of existing equipment 1 until innovative equipment will appear. The case of (iii) $t^* \leq t < n_{12}$ in Fig. 4 (that is, innovative equipment will appear in the relatively remote future from time 0) means that we replace existing equipment 1 with the same type of new one before the appearance time t of innovative equipment, if the introduced time x of existing equipment is old. The case also means that we postpone the replacement of existing equipment until innovative equipment will appear, if the existing one is new. These indicate that the hypothesis of the replacement behavior as stated so far has been supported by theoretical and numerical analyses based on the recurrence equation by DP. Even in the cases of (iv) $n_{12} \leq t < t^o$ and (v) $t^o \leq t \leq n_1^+$ in Fig. 4, where we replace existing type 1 equipment for the same type of new one, it is easily found that the same hypothesis concerning replacement behavior as cases Fig. 4 (ii) and (iii) respectively holds from Fig. 7.

Property 4 (Fig. 8) shows, with respect to control limit $x_2^*(n;t)$ determining replacement of existing equipment 2, that it is economical to replace existing equipment 2 with innovative equipment if it appears earlier on $[0, T]$ and to replace existing equipment 2 for new equipment 1 or keep existing equipment 2 if innovative equipment appears later on $[0, T]$.

Lastly, we show an optimal replacement method by control limit policy starting with new equipment 1 at time 0, given the appearance time t of innovative equipment (see Fig. 12). Here, we assume the conditions of $C_j(x, n; t) \downarrow x$, (2.7) and (3.1). If $t > x_{\max}^o$, no control limit policy $\{x_2^*(n;t)\}$ exists from Property 4 and the replacement of existing equipment 2 is not made from time n. Therefore, we assume the relation $t < x_{\max}^o$. We first draw graphs

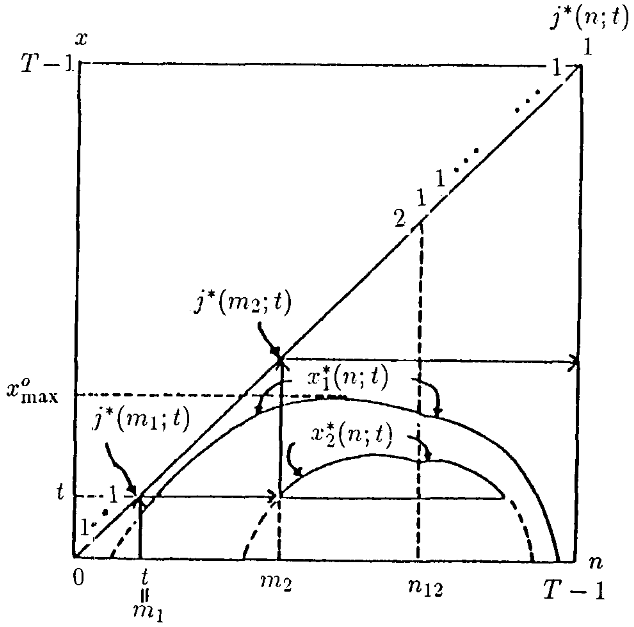


Fig. 12 An Example of Replacement Sequence Determined by $x_1^*(n;t)$ and $x_2^*(n;t)$

of optimal control limit policies $\{x_1^*(n;t)\}$ and $\{x_2^*(n;t)\}$. On the straight line $x = n$ with a slope of 45°, we write optimal equipment sequence $\{j^*(n;t)\}$ $(n = 1, 2, \cdots, T-1)$ defined in section 3.1. The first replacement time under the optimal policy is given by the n-abscissa m_1 of an intersection of $x = 0$ and $x_1^*(n;t)$. Then the type of replaced equipment becomes $j^*(m_1;t)$ on an intersection of a vertical line through m_1 and $x = n$. We draw a line through this intersection parallel to n-axis, let m_2 the n-abscissa of the intersection of the line and $x_1^*(n;t)$ if $j^*(m_1;t) = 1$, or that of the line and $x_2^*(n;t)$ if $j^*(m_1;t) = 2$, and so on. Thus, we can determine optimal replacement sequence of both replacement time and replaced equipment type.

5. Conclusion

This study introduces the concept of innovative technological advances into a model, and clarifies, theoretically and experimentally, equipment replacement behavior under innovative technological advances by control limit policy based on DP formulation. Next we give a practical replacement method by control limit policy when the innovative equipment appearance is forecasted. This is also a practical model involving replacement with plural alternatives as a special case.

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