Journal of the Operations Research
Society of Japan
Vol. 32, No. 4, December 1989

A COOPERATIVE VARIANT OF DANTZIG-WOLFE DECOMPOSITION METHOD

Byong-Hun Ahn
Korea Advanced Institute of Science and Technology

Seung-Kyu Rhee
Incheon University

(Received October 14, 1988; Revised February 13, 1989)

Abstract Decomposition methods for linear programs are now classical research interests. We revisit this issue, however, in the context of possible adaptation of 'cooperative' rather than 'selfish' division behavior in algorithm development and also of renewed interests in parallel computation. This paper extracts and extends from the Dantzig-Wolfe decomposition framework a new coordination scheme where multiple divisions take turn in playing the role of master unlike many conventional decomposition methods where only single masters are involved and divisions behave 'selfish' (without considering the rest of the system). This new approach can also provide some advantages in analyzing multidivisional organization's information flow as well as in applying in a parallel processing framework.

1. Introduction

The Dantzig-Wolfe (abbreviated as D-W throughout the paper) decomposition principle is usually accepted as a conceptual framework for price-directive coordination of divisional activities by the headquarters of an organization [4,6,14]. If the planning problem of a multi-divisional firm can be formulated as an LP problem (P),

$$
\begin{array}{llll}
(1) & \text{Maximize} & \sum_{j=1}^{n} c_j x_j & \\
 & \text{subject to} & \sum_{j=1}^{n} A_j x_j \leq a, & \\
 & & B_j x_j \leq b_j & \text{for } j = 1, \ldots, n, \\
 & \text{and} & x_j \geq 0 & \text{for } j = 1, \ldots, n,
\end{array}
$$

then the so-called master solves the following approximation problem of (P) at iteration step k, (M^k):

$$
\begin{array}{lllll}
(2) & \text{Maximize} & \sum_{j=1}^{n} \sum_{r=1}^{k} (c_j \bar{x}_j^r) \lambda_{jr} & & \\
 & \text{subject to} & \sum_{j=1}^{n} \sum_{r=1}^{k} (A_j \bar{x}_j^r) \lambda_{jr} \leq a, & & (p^k) \\
 & & \sum_{r=1}^{k} \lambda_{jr} = 1 & \text{for } j = 1, \ldots, n, & (z_j^k) \\
 & \text{and} & \lambda_{jr} \geq 0 & \text{for } r = 1, \ldots, k \text{ and } j = 1, \ldots, n, &
\end{array}
$$

where x_j's are $n_j \times 1$, c_j's are $1 \times n_j$, A_j's are $m \times n_j$, a is $m \times 1$, B_j's are $m_j \times n_j$, b_j's are $m_j \times 1$, and $\bar{x}_j^r$'s are the extreme points of $X_j \equiv \{x_j | B_j x_j \leq b_j,\ x_j \geq 0\}$, assumed to be nonempty and bounded for simplicity. The dual vectors p^k and z_j^k are assumed to have appropriate dimensions. In turn, divisions solve the following subproblem (PS_j^k) *parameterized* by the price $(\bar{p}^k)$ of the corporate resources given by the master problem:

$$(3)\quad \text{Maximize} \quad s_j^k = (c_j - \bar{p}^k A_j)x_j$$
$$\text{subject to} \quad x_j \in X_j.$$

In designing a decentralized resource allocation mechanism, the algorithmic procedure can be interpreted as an information exchange process in a multi-divisional organization [1,6]. Various decomposition methods adopting different iterative communication structures have been developed [2,3,7,11]. These approaches typically assume the existence of artificial headquarters (center) which is solely responsible for balancing the allocation of common resources and the divisions all being 'selfish' in that each division pursues its own goal without paying attention to the rest of the system. In fact, this selfishness, represented by the divisional payoff maximization over local constraints, has been the acronym for the decentralization in the literature.

But many authors in this field have pointed out that these conventional decomposition approaches have such implementational drawbacks as the lack of autonomy or coordinability in the resultant solutions and the large number of required information exchanges *etc.* (see [7,8,19]). What is known as the 'composition' approach is often considered as a symmetric concept to the decomposition. The decomposition approach starts with the mathematical statement of the ideal organizational problem, from which decentralized decision process for a global optimal solution is induced normatively. On the other hand, the composition approach first models the reality of existing decentralized structure and coordination schemes of the organization, and then analyzes and evaluates the derived organizational problem. In fact, the composition approach surpasses the decomposition approach in its implementability, since it is *descriptive*, and even some authors [7,8] have proposed decentralization schemes that utilize the merits of both approaches.

Alternatively, however, the decomposition approach is *normative*, and this feature is what we don't want to miss in many contexts. It can provide a useful reference in the analysis of organizational decentralization [8,19]. And it has various application possibilities in the parallel processing of optimization problems, which is now of the renewed interests with the rapid growth of the distributed information processing systems [10]. This latter is, in fact, one of what revive the otherwise old fashioned research topic of decomposition schemes.

This paper essentially attempts to propose a new decomposition method that is normative yet exhibits higher implementability than conventional decomposition approaches. We do this by modifying the subproblem formulation in the basic D-W decomposition that is a typical example of the price-directive decomposition approaches often characterized by 'lack of autonomy'. In the next section, we justify the assumed particular divisional behavior that is a core building block of the proposed approach. Section 3 provides the mathematical framework that describes our multi-divisional organization problem along with the associated theoretical results. Section 4 elaborates the issue of autonomy embedded in the proposed scheme. In Section 5, computational properties are examined to evaluate the speed of convergence and the required number of iterations which are often of practical concern in the real world information exchanges among divisions. Lastly, it is to be noted that we first establish a desirable yet acceptable divisional behavior and then develop a decomposition scheme based on that, rather than start with rather mechanical algorithmic discussion and then try to associate economic or organizational justification as done by many conventional decomposition methods.

2. Cooperative vs. Myopic Divisions

In the conventional decomposition literature, division models (subproblems) take the form of divisional payoff maximization over local feasible region as in (3) (price-guided subproblems) or maximization of divisional contribution under the corporate resource quota q_j given by the center (resource-guided subproblems):

$$\begin{aligned} (4)\quad & \text{Maximize} && c_j x_j \\ & \text{subject to} && A_j x_j \leq q_j, \\ & \text{and} && x_j \in X_j. \end{aligned}$$

In these divisional problems of conventional decomposition approaches, each division simply maximizes divisional payoff utilizing only the point information on either prices or quotas of common resources most recently announced by the center, while the center utilizes not the point information but the set of accumulated proposals from the divisions. In our approach to be proposed here, we do not assume the existence of the center. In fact, as can be seen later, each division itself plays in turn the role of the center by explicitly carrying the common resources constraints as well as the divisional (local) constraints and also utilizing the set of accumulated information (rather than the point information) on common resources given from other divisions, thus eliminating the need for a center. We call this type of divisions *cooperative divisions*, since resultant divisional objectives are consistent with the global objective, as will be shown in the next section. In this sense, the divisions of conventional decompositions might be called *myopic divisions*.

Now, the question could arise on whether such cooperative divisional behavior can be justified in the real world setting or whether there exists appropriate incentive or motivation schemes that support it. Even though we do not intend to provide a serious theoretical argument for such cooperative behavior, we can at least assert that there have recently appeared numerous examples of strategic business management that stress the consistency of functional strategies (financial, marketing, or manufacturing) with the higher level business or corporate strategy [9,18].

This type of cooperative divisional or functional behavior has often been addressed to in the emerging literature on *manufacturing strategy*. For example, while illustrating the problem of profit centers' managers seeking short-term divisional profits in detail, Hayes and Wheelwright [9, pp. 9-10, 366–367] have emphasized the 'group-consciousness' as Japanese manufacturing's major advantage over U.S.:

> The term *co-destiny* often is used to indicate the close interdependence and the shared expectations that exist between a company and all its affiliated organizations — *the productive confederations.*

In analyzing the causes of conflicts between marketing and manufacturing functions within firms, Powers *et al.* [16] have also warned that the divisional planning based on the poor information on their respective roles in the overall scheme could disturb other divisions' activities. Also Shapiro [17] addressed this conflict issue and suggested that "to lessen the amount of marketing and manufacturing conflict, management can make each function more responsive to other's needs." He also suggested that there could and should exist some appropriate monitoring and rewarding system to motivate such behavior.

Cooperative division behavior in this paper is compatible with the responsiveness (to other divisions) and consistency (with the global objective) mentioned above. To be more specific, in our model framework this cooperative behavior is reflected by explicitly carrying global objective function as well as common resource constraints in each divisional problem. That is, in our assumed division model each division pursues a corporate-wide objective, rather than divisional myopic payoff, and explicitly concerns with others' action possibilities via common resource constraints in deciding its own action plans.

Before moving on to present the cooperative decomposition method, for the sake of generality we categorize the divisions into two: major and minor. That is, we assume that some divisions have the cooperative property as above and other divisions behave just like D-W divisions. Here we denote the cooperative divisions as 'major' divisions. For a business firm, likely candidates are marketing, manufacturing, personnel or finance divisions since these are major inhouse suppliers or consumers of corporatewide resources. And we call the D-W style divisions 'minor' ones such as maintenance, storage management, *etc.* This naming convention is strictly a matter of convenience.

3. Cooperative Decomposition Method

Let M denote the nonempty index set of the major divisions, *i.e.* a subset of $\{j|j = 1,\dots,n\}$, and the remaining ones are minor divisions ($j \notin M$). The problem facing a major division (CS_j^k) is modeled as one similar to the master problem of D-W method:

$$
\begin{array}{llll}
(5) \quad \text{Maximize} & \pi_j^k = c_j x_j + \sum_{i \neq j} \sum_{r \in R_i} (c_i \bar{x}_i^r)\lambda_{ir} & & \\
\text{subject to} & A_j x_j + \sum_{i \neq j} \sum_{r \in R_i} (A_i \bar{x}_i^r)\lambda_{ir} \leq a, & & (p_j) \\
 & B_j x_j \leq b_j, & & (u_j) \\
 & \sum_{r \in R_i} \lambda_{ir} = 1 & \text{for each } i \neq j, & (z_{ij}^k) \\
\text{and} & x_j \geq 0, \ \lambda_{ir} \geq 0 & \text{for } r \in R_i \text{ and } i \neq j, &
\end{array}
$$

where:

$\bar{x}_i$ is a feasible solution vector of division i ($\bar{x}_i \in X_i$),

p_j is a price vector for the corporate resources evaluated by division j,

z_{ij} is a dual variable for the contribution margin on the objective of division i ($i \neq j$) evaluated by j, and

R_i denotes an index set of proposals from division i available to j at the iteration k.

Note that this formulation is similar to the D-W master problem in that it takes the convex combination of accumulated proposals of $(c_i\bar{x}_i^r)$ and $(A_i\bar{x}_i^r)$ of other divisions. It differs, however, in that Problem (5) explicitly include the local constraints $B_j x_j \leq b_j$ and that $\bar{x}_i$ is not necessarily an extreme point of $X_i = \{x_i|B_i x_i \leq b_i,\ x_i \geq 0\}$. This type of formulation is possible for each major division, and in fact the proposed scheme simply solves in sequence this set of cooperative subproblems. We formalize this scheme as follows:

Algorithm

Step 0 (Initialization)

Set $k := 0$, $j := 1$, $R_i := \emptyset$ for each $i = 1, \ldots, n$, and $phase := 1$. Go to Step 1.

Step 1 (Quantity Adjustment)

If $j \in M$, solve (CS_j^k) and let $(\bar{x}_j^k, \bar{p}_j^k)$ denote an optimal solution pair.

If $\pi_j^k = \pi_j^{k-1}$, then go to Step 3.

If $\pi_j^k - \pi_i^k < \epsilon$ for some $i \in M$, $i \neq j$, and for predefined tolerance $\epsilon > 0$, then go to Step 2.

Otherwise update R_j by adding the index k to R_j, set $j := j + 1$, and repeat Step 1.

If $j \notin M$, solve (PS_j^k) with the most recently computed price $\bar{p}_i^k$ for $i \in M$, and let $\bar{x}_j^k$ denote the optimal solution. Update R_j, set $j := j + 1$, and repeat.

Step 2 (Price Adjustment)

Solve (PS_j^k) with the price $\bar{p}_i^k$ recently generated by $i \in M$, $i \neq j$, and let $\bar{x}_j^k$ denote the resulting optimal solution. Update R_j, set $j := j + 1$, and return to Step 1.

Step 3 (Termination)

If $phase = 1$ and $\pi_j^k > 0$, then the problem is infeasible, stop.

If $phase = 1$ and $\pi_j^k = 0$, then $phase := 2$, $j := j + 1$, and go to Step 1.

If $phase = 2$, then the optimal solution is obtained. Stop.

Note *Throughout the phase 1, the objective function takes the form of* $\sum$*(artificial variables introduced to combining constraints). If* $j > n$ *in setting* $j := j + 1$, *reset* $j := 1$ *and* $k := k + 1$.

The essence of Step 1 is that at kth iteration a major division solves a 'cooperative' subproblem (CS_j^k) to produce $\bar{x}_j^k$ and $\bar{p}_j^k$, and a minor division solves D-W type 'selfish' subproblem (PS_j^k) using the price vector produced from the major division which has been visited most recently. Of course as in D-W method, the minor divisions produce proposals to the next major division. The previously solved (visited) major divisions also provide the activity proposals $\bar{x}_j^k$ to major divisions that follow. The role of Step 2 is to break a 'deadlock' (that is defined in the third 'if' in Step 1) among major divisions, if it ever occurs during the iterations of Step 1. The further discussion of this 'deadlock' follows later. It is also noted that, as in conventional LP algorithms, phase 1 is introduced to find an initial set of feasible activities.

If the set of major division indices M contains only one element, the above algorithm becomes the typical D-W decomposition. On the other extreme occasion, that is, with $M = \{j | j = 1, \ldots, n\}$, the algorithm might be seen as Gauss-Seidal implementation of

D-W decomposition. Which and how many major divisions are in the system is closely related to the issue of communication structure of an organization. Even though our formulation assumes M is given apriori, it would be quite meaningful to find an optimal M associated with the given cost and technical data of an organization.

Now in order is convergence discussion. Consider the sequence of the objective values of major divisions $\{\pi^1_{j_1}, \pi^1_{j_2}, \ldots, \pi^1_{j_l}, \pi^2_{j_1}, \pi^2_{j_2}, \ldots\}$ obtained in Step 1, where l is the number of major divisions and $j_1, j_2, \ldots, j_l \in M$.

Proposition 1 *A sequence of* Step 1 *given above monotonically converges to a limit in a finite number of iterations.*

Proof. First we want to show that the sequence converges. Consider a subsequent pair of two major divisions j_1 and j_2. Note that the optimal solution $(\bar{x}^k_{j_1}, \tilde{\lambda})$ of j_1 is also a feasible solution for the latter division j_2 via $(\bar{x}^k_{j_2}, \hat{\lambda})$, where $\bar{x}^k_{j_2} = \sum_{r \in R_{j_2}} \bar{x}^r_{j_2} \tilde{\lambda}_{j_2 r}$, $\hat{\lambda}_{j_1 k} = 1$, and $\hat{\lambda}_{jr} = \tilde{\lambda}_{jr}$ for other j's. This naturally leads to $\pi^k_{j_2} \geq \pi^k_{j_1}$. Hence the sequence is monotonically increasing. The boundedness of the sequence comes directly from the fact that (CS^k_j) is a restrictive approximation of (P). Since the sequence is monotonically increasing and bounded, it converges to some limit, say $\bar{\pi}$.

Now we show that a limit point can be obtained in a finite number of iteration steps. Suppose on the contrary that an infinite sequence needs to be generated to converge to $\bar{\pi}$. Then we can take sufficiently large iteration counter K such that $0 < \pi^K_{j_2} - \pi^K_{j_1} < \epsilon$ where ϵ is sufficiently small positive constant. Note that it can be assumed R_j's $(j \notin M)$ are not different for j_1 and j_2 because K is so large and the number of extreme points in X_j's $(j \notin M)$ are finite. Assume that j_2 starts with an equivalent solution to that of j_1's optimal one as in above. Then there exists at least one element of x_{j_2} which can be a candidate to enter the basis. This implies that an adjacent extreme point of the j_2's feasible region (denoted as $\bar{X}_{j_2}$ which is a subset of X_{j_2} and has a fixed number of bases) can have a better objective value. Note that the bases of $\bar{X}_{j_2}$ can be classified into two categories; those defined by common constraints' slack variables, the others belonging to the bases of original X_{j_2}. But the former group of bases are projected from the global feasible region and is equivalent to those of j_1's. Since K is large enough, the latter group of extreme points can be assumed to be known to j_1. This contradicts to the assumption $\pi^k_{j_2} > \pi^k_{j_1}$. This completes the proof. □

Hence, this sequence converges monotonically to some limit. If this limit happens to be equal to the true global optimal objective value and the material balances of common resources are met, we are done. Unfortunately, this limit is not guaranteed to be true optimal. This premature cycling or 'deadlock' has interesting implications. First, if this iteration scheme is applied to decentralized decision process of an organization, this 'deadlock' would pose no problem, since what matters here is the convergence rate during the first several iterations (see [3,4]). As shown in the computational experiments later, our cooperative scheme typically performs well during the first several iterations. Secondly, this 'dead lock' phenomena arises possibly because the major divisions are mutually cooperative rather than are 'selfish' individually. Thirdly, if such 'deadlock' needs to be detected and broken for the sake of completeness as a solution finding algorithm, we may well introduce the 'selfish' step of D-W method. That is, if the objective value sequence does not improve as much as a specified threshold, each division is temporarily allowed to be 'selfish' by solving D-W type subproblem (PS) rather than 'cooperative' subproblem (CS). This step essentially generates additional extreme points of the major divisions' feasible regions X_j's. This is exactly what Step 2 (price adjustment step) does in our algorithm. Since Step 2 is invoked only a finite number of times (if ever needed), Proposition 1 leads to the following.

Proposition 2 *The algorithm terminates with an optimal solution of (P) within a finite number of iterations.*

Proof. The objective values of (CS_j^k)'s, which are local approximation problems for (P), increase monotonically as the iteration proceeds. When the algorithm terminates, it is assured that all the π_j^k's for $j \in M$ are equal and (PS_j^k) is solved for all $j = 1, \ldots, n$. Recall that the price coefficients are consistent in solving (PS_j^k)'s. This implies that the global optimality is obtained at the first major subproblem followed by a series of price adjustments, just as in master problem of D-W method. The finite convergence can be also guaranteed by the finiteness of X_j's as in D-W method. The desired result follows. □

Several computational strategies can be devised for the algorithm, due to the embedded flexibility in selection of the computation (visitation) sequence among major divisions. Parallel implementation of divisional calculation is possible in network environment as in Ho [10]. Intermediate cycling over a subset of divisions may well work. Some major divisions might be skipped temporarily during visitation depending on the computational contribution in the previous iterations. All these options and others not explicitly discussed here can be interesting topics of computational research.

4. Divisional Autonomy and Cooperative Decomposition

Many authors [7,8,11] have argued that the lack of 'autonomy' is one of the major drawbacks in conventional decomposition approaches in light of the decentralized decision processes. A subproblem's optimal activity pattern, given a set of optimal information on common resources from the master, does not in general guarantee the systemwide material balances. This phenomenon is also referred to as the problem of *coordinability* [13].

Especially in LP setting, the lack of coordinability in price-guided decomposition methods has been considered as inevitable (see [7,12,13,15]). The lack of autonomy is known (see Eto [7]) to arise from the fact that, while each division desires to take local extreme point solution, it is forced at optimum to accept an interior point solution (a proper convex combination of local extreme points) due to the existence of common resource constraints. Alternatively interpreting, if one division adheres to a local optimum, it will restrict other divisions' activities due to common resource constraints. From the very definition of our cooperative divisions, this type of autonomy problem is lessened, since each division explicitly carry the common resource constraints. In an attempt to solve this lack of autonomy problem, Eto [7] suggested to bring the *systems bureau* into the existing multi-divisional organization, yet assumed the conventional myopic divisional behavior. In this framework, the center is given an additional role of the systems bureau along with conventional price-setting role. In other words, to induce myopic divisions to accommodate full autonomy, the center is imposed an increased role. In this context, our approach addresses this issue from the opposite direction, namely, reducing the role of the center and letting individual divisions to share its role.

Jennergren [12] has analyzed this in detail and proposed a price-schedules approach that utilizes a quadratic price function to enforce each division to use the corporate resources at a globally optimal level determined by the center. Geometrically speaking, this quadratic subproblem defined on the same local feasible region as in (PS) has a unique optimal solution which may not be an extreme point at all.

We have presented an alternative approach that improves the divisional autonomy among divisions. Here we show that the cooperative major divisions are all autonomous while the selfish divisions are not. That is, once the iteration satisfies the termination criteria, we obtain at this final iteration the local optimal solutions of the major divisions that are consistent with the global optimal solution.

Proposition 3 *If every major division $j \in M$ sets $\lambda_{ik} = 1$ for $i \in M$ and $i < j$ at the final round of the iteration, the resulting solution $\bar{x}_j^*$ will coincide with a global optimal solution of (P).*

Proof. It is obvious from the fact that $\pi_{j_1}^{k-1} = \pi_{j_2}^{k-1} =, \ldots, = \pi_{j_1}^{k}$ at final iteration k. In other words, at final iteration k, the cooperative subproblem of major division j (CS_j^k) attains its optimal activities $\bar{x}_j^*$ with all the preceding major divisions' activities set at $\bar{x}_i^*$ $(i < j)$. Hence, after the final round completes visiting all major divisions in sequence, the last major division ends up with a global optimal solution that includes all major divisions' autonomous decisions. □

It has been shown that, if some major divisions are cooperative in the sense that their objectives are compatible with the systemwide goal and they respect other major divisions' local decisions, the global optimum can be attained through the interactions only among these divisions without relying on the center's (master's) coordination. This can be interpreted as the center's role being shared by major divisions. Of course, the minor divisions remain selfish as in D-W method.

Gijsbrechts [8] has extensively surveyed the implementability issues of the various decomposition and composition methods such as convergence assurance, rate of convergence, monotonic improvement of the objective value, computational burden, real system fidelity of models, adaptability to various organizational structures, and consistency with the real spirit of decentralization. The decomposition scheme presented here, though discussed within the LP format, provides the flexible framework of decentralization, and allows many of Gijsbrechts' implementability issues to be accommodated (recall that the definition of M is restricted only to be nonempty). Some of these issues are illustrated in the computational experiments.

5. Computational Properties of the Cooperative Decomposition

Here we summarize the results from limited computational experiments focusing on the comparison of some decomposition parameters. We have randomly generated groups of examples based on the number of total divisions (n), the number of major divisions, the number of common resources (m), and the subproblem sizes $(m_j \times n_j)$. The D-W method and our cooperative decomposition method were coded in FORTRAN 77 (Microsoft FORTRAN Version 4.0) and run on Compaq Deskpro 386 PC, and the LP solution subroutines were borrowed from MINOS. In coding the algorithms, advanced techniques for reducing computation time have not been utilized, since the purpose of the test is to compare the

Table 1: Computational Results for $n = 2$

	Dantzig-Wolfe Method			Cooperative Method					
				$M = \{1\}$			$M = \{1,2\}$		
	I_1	I_2	R_3	I_1	I_2	R_3	I_1	I_2	R_3
$m = 5$	2.04	7.52	99.23%	2.34	7.44	99.64%	2.00	5.37	99.97%
(27)	0.43	1.99	1.34%	0.48	2.45	0.47%	0.00	1.79	0.20%
10	2.00	9.41	94.70%	2.59	8.12	99.05%	2.00	6.88	98.61%
(17)	0.00	1.54	8.41%	0.49	2.27	1.49%	0.00	2.40	2.90%
15	2.00	10.1	96.10%	2.55	8.82	98.26%	2.00	6.91	99.03%
(11)	0.00	2.47	2.51%	0.50	2.12	2.05%	0.00	1.78	1.36%
$m_j \times n_j$									
5×10	2.03	8.66	96.15%	2.48	7.72	99.04%	2.00	6.69	99.30%
(29)	0.32	2.19	6.85%	0.50	2.02	1.53%	0.00	2.18	2.03%
10×15	2.00	8.58	98.38%	2.46	8.15	99.34%	2.00	5.54	99.42%
(26)	0.28	2.34	2.15%	0.50	2.74	1.17%	0.00	1.91	1.59%
Total									
	2.02	8.62	97.21%	2.47	7.93	99.18%	2.00	6.15	99.34%
(55)	0.30	2.26	5.31%	0.50	2.40	1.38%	0.00	2.14	1.83%

Table 2: Computational Results for $n = 3$

	Dantzig-Wolfe Method			Cooperative Method								
				$M = \{1\}$			$M = \{1,2\}$			$M = \{1,2,3\}$		
	I_1	I_2	R_3	I_1	I_2	R_3	I_1	I_2	R_3	I_1	I_2	R_3
$m = 5$	2.00	8.39	99.63%	2.57	7.74	99.79%	2.22	6.91	99.75%	1.96	6.96	99.89%
(23)	0.00	1.28	0.97%	0.50	1.42	0.32%	0.41	1.72	0.62%	0.21	2.76	0.20%
10	2.00	9.73	97.85%	2.64	8.18	99.10%	2.36	7.82	99.06%	2.00	8.00	99.08%
(11)	0.00	1.48	1.96%	0.48	2.04	1.13%	0.48	2.52	1.81%	0.00	3.02	1.89%
15	2.00	11.3	97.56%	2.86	10.3	99.17%	2.29	10.1	99.37%	2.00	10.6	99.83%
(7)	0.00	1.83	1.31%	0.35	1.98	0.43%	0.45	2.75	0.80%	0.00	1.84	0.11%
$m_j \times n_j$												
5×10	2.00	9.04	98.66%	2.78	8.39	99.37%	2.48	7.87	99.36%	2.00	8.22	99.45%
(23)	0.00	1.00	1.75%	0.41	1.47	0.86%	0.50	1.73	1.41%	0.00	3.02	1.36%
10×15	2.00	9.50	98.84%	2.44	8.17	99.66%	2.00	7.50	99.67%	1.94	7.39	99.93%
(18)	0.00	2.46	1.44%	0.50	2.41	0.50%	0.00	3.13	0.59%	0.23	2.91	0.12%
Total												
	2.00	9.24	98.74%	2.63	8.29	99.50%	2.27	7.71	99.50%	1.98	7.85	99.66%
(41)	0.00	1.80	1.62%	0.48	1.94	0.74%	0.44	2.45	1.14%	0.15	3.00	1.05%

Table 3: Computational Results for $n = 5$

	Dantzig-Wolfe Method			Cooperative Method											
				$M = \{1\}$			$M = \{1,3\}$			$M = \{1,2,3\}$			$M = \{1,2,3,4,5\}$		
	I_1	I_2	R_3	I_1	I_2	R_3	I_1	I_2	R_3	I_1	I_2	R_3	I_1	I_2	R_3
$m = 5$	2.00	7.50	99.84%	2.50	8.00	99.81%	2.00	7.00	99.93%	2.00	5.50	99.95%	2.00	5.33	99.99%
(6)	0.00	1.61	0.16%	0.50	1.53	0.27%	0.00	2.24	0.10%	0.00	1.38	0.09%	0.00	1.11	0.02%
10	2.00	9.00	98.99%	2.50	9.00	99.72%	2.50	8.00	99.95%	2.00	6.00	99.79%	2.00	10.0	99.68%
(2)	0.00	2.00	1.01%	0.50	2.00	0.28%	0.50	1.00	0.05%	0.00	3.00	0.21%	0.00	6.00	0.32%
15	2.00	9.43	98.79%	2.57	9.29	99.12%	2.00	8.00	99.85%	2.00	7.57	99.96%	2.00	8.57	99.92%
(7)	0.00	0.73	0.99%	0.50	0.70	0.92%	0.00	1.31	0.19%	0.00	1.05	0.07%	0.00	2.26	0.09%
$m_j \times n_j$															
5×10	2.00	7.67	99.59%	2.67	8.00	99.02%	2.00	6.00	99.97%	2.00	6.00	99.92%	2.00	7.00	99.92%
(3)	0.00	1.70	0.55%	0.47	0.82	1.36%	0.00	0.82	0.04%	0.00	0.82	0.12%	0.00	1.63	0.08%
10×15	2.00	8.83	99.15%	2.50	8.92	99.51%	2.08	8.00	99.88%	2.00	6.67	99.94%	2.00	7.58	99.92%
(12)	0.00	1.52	0.98%	0.50	1.50	0.40%	0.28	1.73	0.16%	0.00	2.01	0.12%	0.00	3.59	0.18%
Total															
	2.00	8.60	99.24%	2.53	8.73	99.48%	2.07	7.60	99.90%	2.00	6.53	99.93%	2.00	7.47	99.92%
(15)	0.00	1.63	0.92%	0.50	1.44	0.74%	0.25	1.78	0.15%	0.00	1.86	0.12%	0.00	3.30	0.17%

1. I_1 : # of iterations in *phase* 1
2. I_2 : # of total iterations
3. R_3 : the ratio of the objective value of the $(I_1 + 3)th$ iteration to the optimum
4. The results from each problem set are categorized in terms of m and $(m_j \times n_j)$.
5. The numbers in leftmost parentheses denote the # of problems solved in the category.
6. For each category, the upper line lists the sample means and the lower line the standard deviations.

number of iterations required to find a feasible and/or optimal solution and the initial rate of convergence. Our method, however, has not been meant to provide faster computation. It is meant to provide a flexible framework for decomposition. Summarized in Tables 1, 2 and 3 are the following three measures; the number of iterations in *phase* 1 (I_1), the number of iterations required to get an optimal solution (I_2), and the ratio (R_3) of the objective value of the $(I_1 + 3)th$ iteration to the optimum. I_1 and I_2 are usual measures and R_3 is to check the initial rate of convergence. R_3 is the typical indicator of how fast (in terms of the number of iterations) a given information flow structure (specified by M) approaches to the global optimum during initial iterations in the light of limits in information exchange in a multi-divisional organization (see for example [4,5]).

Examining the tables, first we note that our cooperative decomposition has revealed notable superiority over D-W's as far as the number of iterations and initial rate of convergence are concerned. The likely reason that cooperative scheme converges more rapidly in terms of the number of required iterations is that cooperative divisions utilize their action possibility sets adapted to the expected responses of other divisions. Second, as the size of M (indicating the degree of decentralization) grows, I_2 decreases and R_3 increases. But in Table 3, the extreme case of $M = \{1,2,3,4,5\}$ shows worse result than those of $M = \{1,3\}$ and $M = \{1,2,4\}$. This phenomena consistently appears in most cases. This may indicate that there is an optimal degree of decentralization in terms of which and how many divisions are cooperative.

6. Summary and Further Researches

We have presented a cooperative scheme where several major divisions act like the D-W's master. These major divisions are explicitly concerned with the material balances of *all common resources*. While D-W method may not exploit divisional feasible regions effectively due to its price only control mechanism, major divisions with local approximation of systemwide objective and constraints can generate efficient proposals adapting to other divisions' feasible activity proposals. This kind of major division may be seen as a price-directive master for minor divisions and a quantity adjusting division among major divisions. One possible extension of our cooperative scheme would be to associate each subset of common resources to respective major divisions: for example, the finance division is concerned only with corporate's financial resources, the marketing division with product availability and demands, the personnel division with the availability and recruiting of manpower, and the like. The decomposition method in this line may benefit from the 'cross decomposition' ideas of van Roy [20].

Another topic of interest would be to investigate in a rigorous manner the relationship between the extent of decentralization and the efficiency of information flow. It would be also interesting to investigate the effect of the 'visitation' sequences among major divisions upon the efficiency of organization's information flow as well as the computational burden. It has been pointed out that during the iterations of the proposed cooperative scheme the situation of cycling or 'deadlock' could be encountered. The implications of this phenomena and the associated 'deadlock' breaking scheme needs further study within actual decentralized decision making environments.

Lastly, the results presented so far have been in an LP representation of the multi-divisional resource allocation. Our results need to be extended to more general problem settings such as nonlinear program representation. This is a topic of our subsequent ongoing work.

References

[1] W.J. Baumol and T. Fabian, "Decomposition, Pricing for Decentralization and External Economies," *Management Science*, Vol.11, No.1, pp.1-32, 1964

[2] R.M. Burton, W.W. Damon, and D.W. Loughridge, "The Economics of Decomposition: Resource Allocation vs. Transfer Pricing," *Decision Sciences*, Vol.5, No.3, pp.297-310, 1974

[3] R.M. Burton and B. Obel, "The Multilevel Approach to Organizational Issues of the Firm: A Critical Review," *OMEGA*, Vol.5, pp.395-414, 1977

[4] R.M. Burton and B. Obel, "The Efficiency of the Price, Budget, and Mixed Approaches under Varying A Priori Information Levels for Decentralized Planning," *Management Science*, Vol.26, No.4, pp.401-417, 1980

[5] J. Christensen and B. Obel, "Simulation of Decentralized Planning in Two Danish Organizations Using Linear Programming Decomposition," *Management Science*, Vol.24, No.15, pp.1658-1667, 1978

[6] G.B. Dantzig, *Linear Programming and Extensions*, Princeton University Press, Princeton, N.J., 1963

[7] H. Eto, "Effectiveness of Nonhierarchical Decentralized Corporate System," *Large Scale Systems*, Vol.4, pp.189-197, 1983

[8] E. Gijsbrechts, "Implementability of Hierarchical Approaches to Large Scale Organizational Problems," *Large Scale Systems*, Vol.8, pp.87-104, 1985

[9] R.H. Hayes and S.C. Wheelwright, *Restoring Our Competitive Edge: Competing through Manufacturing*, John Wiley and Sons, New York, 1984

[10] J.K. Ho, "Recent Advances in the Decomposition Approach to Linear Programming," *Mathematical Programming Study*, Vol.31, pp.119-127, 1987

[11] L.P. Jennergren, *Studies in the Mathematical Theory of Decentralized Resource Allocation*, Ph.D. Dissertation, Stanford University, 1971

[12] L.P Jennergren, "A Price Schedules Decomposition Algorithm for Linear Programming Problems," *Econometrica*, Vol.41, No.5, pp.965-980, 1973

[13] L.P. Jennergren, "On the Concept of Coordinability in Hierarchical Systems Theory," *International Journal on Systems Science*, Vol.5, No.5, pp.493-497, 1974

[14] L.S. Lasdon, *Optimization Theory for Large Systems*, McMillan, London, 1970

[15] C. Van de Panne, "Decentralization for Structured Linear Programming Models," *Working Paper*, The University of Calgary, 1984

[16] T.L. Powers, J.U. Sterling, and J.F. Wolter, "Marketing and Manufacturing Conflict: Sources and Resolution," *Production and Inventory Management Journal*, pp.56-60, First Quarter, 1988

[17] B.P. Shapiro, "Can Marketing and Manufacturing Coexist?" *Harvard Business Review*, pp.104-114, Sep.-Oct., 1977

[18] W. Skinner, *Manufacturing The Formidable Competetive Weapon*, John wiley and Sons, New York, 1985

[19] D.J. Sweeney, E.P. Winkofsky, P. Roy, and N. Baker, "Composition vs. Decomposition: Two Approaches to Modeling Organizational Decision Processes," *Management Science*, Vol.24, No.14, pp.1491-1499, 1978

[20] T.J. Van Roy, "Cross Decomposition for Mixed Integer Programming," *Mathematical Programming*, Vol.25, pp.46-63, 1983

Byong-Hun AHN:
Department of Management Science,
Korea Advanced Institute of Science and Technology,
P.O. Box 150, Cheongryang, Seoul 131-650, KOREA

Seung-Kyu RHEE:
Department of Business Administration,
Incheon University,
177 Dohwa-Dong, Nam-Ku, Incheon 402-749, KOREA