AN ANALYSIS OF AIRLINE SEAT ALLOCATION

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Abstract   In this paper we consider the airline seat allocation between high and low fares with and without stochastic cancellations. We also analyze the problem of simultaneously determining seat allocation and overbooking levels for two different classes of passengers, which also extends the existing literature in three ways. First, the cost of lost sales, which has been ignored in the existing literature, is explicitly incorporated into the model. Second, the overbooking phenomenon is also explicitly treated. Third, the concept of spill rate is clarified into the passenger spill rate and the flight rate. It is found that the results obtained here are in closer agreement with actual airline practice.

1. Introduction

In an era of increasing pricing freedom, airline companies no longer offer seats for sale at a single fare. Recognizing that different groups of consumers have different willingnesses to pay for the same seat, airline companies offer seats at a wide range of air fares. However, to prevent consumers willing to pay high fares from buying seats at low fares, the airlines attach various restrictions to their tickets such as early time bookings, Saturday night stayovers or reduced service without food to discount fares. (See [10], [11].)

The process of determining fares, associated restrictions and the number of seats to offer at a given fare is referred to as "Airline Revenue Management". (See [6].) Within this area of airline revenue management, the decision process of determining the number of seats to be protected for various classes of passengers is called the airline seat allocation problem. The key idea of the seat allocation problem is to limit the number of discounted seats when there is a strong demand expected from high fare consumers, so as to maximize the expected revenue per flight. Airline revenue management includes (i) determination of the initial allocation of seats between high and low fare classes, (ii) determination of high and low fare levels, (iii) determination of restrictions associated with low fares, and (iv) dynamic monitoring of seats for sale on a given flight and readjusting the allocation of seats between high and low fares so as to maximize expected revenues.

The purpose of this paper is to analyze some of the first element (i), determination of the initial allocation of seats between high and low fares. This static model with one period is of great importance to airlines as the two classes of passengers have very different time patterns of booking. High fare passengers tend to book in the last days or hours before a departure time. Because of this, the initial seat allocation suggested in a static model is critical to airline profitability. (See [10].) On the other hand, the dynamic seat allocation problem treated with the fourth element (iv) would be considered in some where else. Readers interested in airline revenue management should refer to the survey article by Belobaba [2]. Beckmann [1] derived an approximate solution to the problem that maximized expected revenue with oversale penalty and allowed for the possibility of 'stand by' passengers. Rothstein [8] and Shlifer and Vardi [9] proposed various overbooking control methods for a single fare class.

Liberman and Yechiali [7] considers the hotel overbooking problem, which is similar to the airline seat allocation problem with only one class of customers. As far as we know, no work has been done on the problem of simultaneously determining seat allocation and overbooking levels for different classes of passengers.

In this paper we simultaneously consider the seat allocation between high and low fares passengers and the overbooking problem, which also extends the existing literature in three ways. First, the cost of lost sales which has been ignored in the existing literature (see [2], [5], [8]) is explicitly incorporated into the model. Second, the overbooking phenomenon is also explicitly treated. Third, the concept of spill rate is clarified into the passenger spill rate and the flight spill rate. It is found that the results obtained here are in more closer agreement with actual airline practice.

2. A Simple Seat Allocation Model

In this section, we consider a rather simple seat allocation model in which there are two class of passengers, low and high fares passengers. Assume that they do not both cancel their booking reservations. So, in this case the airline company does not have to overbook to hold out against the cancellations of their passengers' bookings. We make three assumptions as follows:

Assumption (A) The low fare and high fare demands are independent to each other.

Assumption (B) Demand for low fares occurs earlier than for high fares, e.g. low fare demand has a minimum advance booking requirement.

Assumption (C) The refused passengers do not pick up other flights of the same carrier (called the total loss of the spilled sales).

Assumption (B) is known as early birds. Assumption (C) excludes the possibility that the denied low fare passengers may then purchase a high fare ticket, which is called "grade up". Define X the random variable of the number of high fare demand with distribution function $F(x)$ and Y the random variable of the number of low fare demand with distribution function $G(y)$, respectively. We use the following notations:

$p_1 = $ the high fare,

$p_2 = $ the low fare,

$C = $ the airplane capacity,

$\pi_1 = $ the cost of goodwill loss per high fare passenger due to the shortage of the capacity,

$L = $ the number of seats allocated to low fare passengers, so at least $(C - L)$ seats are available to high fare passengers,

$a \wedge b = \min(a, b), a^+ = \max(a, 0)$, and $E = $ expectation operator.

All variables are treated as continuous.

Defining $ER(L)$ the expected total revenue per flight when L seats are allocated to the low fare demand, $ER(L)$ can be written as follows:

$$ER(L) = E_Y[p_2(Y \wedge L)] + E_Y E_{X|Y}[p_1(X \wedge (C - Y \wedge L)) - \pi_1(X - C + Y \wedge L)^+] \quad (1)$$

which can be rewritten as

$$ER(L) = p_2[\int_0^L y dG(y) + L\overline{G}(L)]$$
$$+ \overline{G}(L)\{p_1 \int_0^{C-L} x dF(x) + \int_{C-L}^{\infty} [p_1(C - L) - \pi_1(x - C + L)]dF(x)\}$$

where $\overline{F}(\cdot) = 1 - F(\cdot)$ and $\overline{G}(\cdot) = 1 - G(\cdot)$. The problem is of choosing L to maximize $ER(L)$ given by (1) (or (1)') subject to $0 \leq L \leq C$. Let f and g be density functions of X and Y, respectively. $f(\cdot)$ and $g(\cdot)$ are assumed to be positive on $(0, C)$.

Proposition 1 If $g(L)/\overline{G}(L) \leq f(C - L)/\overline{F}(C - L)$ for all L, then an optimal number of seats to allocate to low fare demand L^* is given by

$$
L^* = \begin{cases} 0 & \text{if } p_2 < (p_1 + \pi_1)\overline{F}(C), \\ C - \overline{F}^{-1}(\frac{p_2}{p_1 + \pi_1}) & \text{if } (p_1 + \pi_1)\overline{F}(C) \leq p_2 \leq (p_1 + \pi_1)\overline{F}(0), \\ C & \text{, otherwise.} \end{cases} \tag{2}
$$

Proof. $ER(L)$ is differentiable. We shall show that $ER(L)$ is strictly concave with respect to L under the condition that $g(L)/\overline{G}(L) \leq f(C - L)/\overline{F}(C - L)$. Hence, if it possesses a maximum, it is a unique one. Differentiating $ER(L)$ results in

$$
\frac{dER(L)}{dL} = \overline{G}(L)[p_2 - (p_1 + \pi_1)\overline{F}(C - L)]. \tag{3}
$$

Differntiating $ER(L)$ twice, we have

$$
\frac{d^2 ER(L)}{dL^2} = -g(L)p_2 + (p_1 + \pi_1)[g(L)\overline{F}(C - L) - f(C - L)\overline{G}(L)] < 0.
$$

Note that $\overline{F}(C - L)$ is monoton increasing in L and then the inverse of $\overline{F}(\cdot)$ exists. Since $g(\cdot) > 0$ implies $\overline{G}(\cdot) > 0$, letting $dER(L)/dL = 0$ with condition $0 \leq L \leq C$ yields equation (2).

Remarks: Note that the optimal allocation to low fare demand L^* is independent of the distribution of low fare demand, provided $\overline{G}(\cdot) > 0$. L^* is decreasing as π_1 increases, and depends only on the relative low fare $p_2/(p_1 + \pi_1)$.

It is difficult to provide an economic interpretation on the condition of proposition 1, $g(L)/\overline{G}(L) \leq f(C - L)/\overline{F}(C - L)$, which looks like a failure rate often appearing in reliability theory. Instead of doing that, we consider a special case where there is strong demand from low fare passengers, that is, $\overline{G}(C) = 1$. For instance, the peak season may have the demand distribution satisfying $\overline{G}(C) = 1$.

Assume that we can sell as many low fare seats as we desire up to the capacity C of the plane. Let $R(L, X)$ be the revenue obtained if L seats are sold to low fare demand and $C - L$ to high fare demand and a demand for high fare X is realized. Then, for $0 \leq L \leq C$ we have

$$
R(L, X) = p_2 L + p_1 (C - L) \wedge X - \pi_1 (X - C + L)^+. \tag{4}
$$

Note that for each $X, R(L, X)$ is concave in L without any condition. So, the expected revenue $ER(L) \equiv E[R(L, X)]$ is also concave in L. Because of the concavity, the optimal number of low fares L^* to maximize the expected revenue can be determined by looking at the incremental expected revenue from selling an additional low fare and stop selling when this becomes negative. The following corollary immediately follows from the fact that the assumption of proposition 1 is satisfied if $\overline{G}(C) = 1$ for all $C > 0$.

Corollary If there is an unlimited low fare demand, one should sell only L^* low fares and protect $C - L^*$ seats for high fares, where L^* is given by

$$
L^* = \min\{L \geq 0 : \frac{p_2}{p_1 + \pi_1} \leq \overline{F}(C - L)\}.
$$

3. Optimal Seat Allocation with Overbooking

In this section we treat with an optimal seat allocation model allowing overbooking and cancellations. Assume that high fare passengers may cancel their reservations but low fare ones can not. Hence, only high fare demands are overbooked. We must determine both the maximum level to accept the reservations from low fare passengers and the overbooking ratio for high fare passengers. A sequence of operations is as follows; (i) choose the number of low fare seats to reserve, (ii) observe the number of realized booking progress of low fare passengers, and then (iii) determine the number of high fare seats to protect.

In addition to the notations listed in section 2, we use the following:

Z = the number of cancellations of high fare reservations, $Z \leq X$,

π_2 = the cost per denied boarding due to overbooking,

$K(y)$ = the number of seats allowed to book for high fare passengers when $Y = y$, which is assumed to satisfy

$$K(y) = (C - y \wedge L)(1 + \alpha), \; 0 \leq \alpha < 1.$$

Remembering L as the seat allocation for low fares, a pair of (L, α) is called a booking strategy and set $B = (L, \alpha)$. We may possibly have $L + K(y) \geq C$ but $L \leq C$.

Remarks; (i) If high fare passengers do not cancel their reservations, so that the airline does not necessarily overbook, then the booking strategy can be written as $B = (L, 0)$, which is reduced to $K(y) = C - y \wedge L$ as same as in section 2. (ii) α can be interpreted as an overbooking ratio for high fare passengers because

$$\frac{y \wedge L + (C - y \wedge L)(1 + \alpha) - C}{C - y \wedge L} = \frac{\alpha(C - y \wedge L)}{C - y \wedge L} = \alpha.$$

So, $\alpha(C - y \wedge L)$ seats are overbooked for high fares. $\alpha = 0$ corresponds to no overbooking with which cases are treated in section 2.

Assumption (D) High fare passengers may cancel their reservations independently with the equal probability $(1 - \theta)$.

It is well known under assumption (D) that the probability distribution of the number of cancellations z, given the number of reservations x is binomial with the mean $x(1 - \theta)$, that is, $H(z \mid x) = P\{Z \leq z \mid X = x\} = \sum_{k=0}^{z} \binom{x}{k}(1 - \theta)^k \theta^{x-k}, z = 0, 1, \cdots, x$. So, a passenger may board on with probability θ. Put $\beta \equiv (1 + \alpha)$ and $B = (L, \beta)$ in stead of (L, α). Define $ER(L, \beta)$ the expected profit obtained from a flight when a booking strategy $B = (L, \beta)$ is used. Then, we obtain

$$
\begin{aligned}
ER(L, \beta) &= p_1 E_Y E_{X|Y} E_{Z|X}[X \wedge ((C - Y \wedge L)\beta - Z] + p_2 E[Y \wedge L] \\
&\quad - \pi_1 E_Y E_{X|Y}[X - (C - Y \wedge L)\beta]^+ \\
&\quad - (p_1 + \pi_2) E_Y E_{X|Y} E_{Z|X}[Y \wedge L + X \wedge ((C - Y \wedge L)\beta) - Z - C]^+ \\
&= p_1\{E[X - Z \mid X \leq (C - Y)\beta]P[X \leq (C - Y)\beta] \\
&\quad + E[(C - Y)\beta - Z \mid X > (C - Y)\beta]P[X > (C - Y)\beta]\}P[Y \leq L] \\
&\quad + p_1\{E[X - Z \mid X \leq (C - L)\beta]P[X \leq (C - L)\beta] \\
&\quad + E[(C - L)\beta - Z \mid X > (C - L)\beta]P[X > (C - L)\beta]\}P[Y > L] \\
&\quad + p_2\{E[Y \mid Y \leq L]P[Y \leq L] + E[L \mid Y > L]P[Y > L]\} \\
&\quad - \pi_1\{E[X - (C - Y)\beta \mid Y \leq L]^+ P[Y \leq L] \\
&\quad + E[X - (C - L)\beta \mid Y > L]^+ P[Y > L]\} \\
&\quad - (p_1 + \pi_2)\{E[Y + X - Z - C \mid X \leq (C - Y)\beta]P[X \leq (C - Y)\beta] \\
&\quad + E[Y + (C - Y)\beta - Z - C \mid X > (C - Y)\beta]P[X > (C - Y)\beta]\}P[Y \leq L]
\end{aligned}
$$

$$- (p_1 + \pi_2)\{E[L + X - Z - C \mid X \le (C - L)\beta]P[X \le (C - L)\beta]$$
$$+ E[L + (C - L)\beta - Z - C \mid X > (C - L)\beta]P[X > (C - L)\beta]\}P[Y > L]$$
$$= p_1\theta\{\overline{G}(L)[\int_0^{(C-L)\beta} x dF(x) + \overline{F}((C - L)\beta)(C - L)\beta]$$
$$+ \int_0^L [\int_0^{(C-y)\beta} x dF(x) + \overline{F}((C - y)\beta)(C - y)\beta]dG(y)\}$$
$$+ p_2 \int_0^L y dG(y) + p_2 L\overline{G}(L)$$
$$- \pi_1\{\overline{G}(L)\int_{(C-L)\beta}^\infty (x - (C - L)\beta)dF(x)$$
$$+ \int_0^L \int_{(C-y)\beta}^\infty (x - (C - y)\beta)dF(x)dG(y)\}$$
$$- (p_1 + \pi_2)\{\overline{G}(L)[\int_{C-L}^{(C-L)\beta} (L + \theta x - C)dF(x)$$
$$+ \overline{F}((C - L)\beta)(L - C)(1 - \beta\theta)]$$
$$+ \int_0^L [\int_{C-y}^{(C-y)\beta} (y + \theta x - C)dF(x) + \overline{F}((C - y)\beta)(y - C)(1 - \beta\theta)]dG(x)\}. \quad (5)$$

The first two terms are the revenues for the high and low fares, respectively. The third one is the cost of lost sales due to the booking limit. The last one is the cost of denied booking which occurs whenever the sum of the numbers of low fares and of confirmed high fares is larger than the capacity of the plane. Since the number of cancelled bookings is binomially distributed $E[X - Z] = E_Z E_X[X - E(Z \mid X) \mid Z] = E[X - X(1 - \theta) = \theta E[X]$. The problem is to find an optimal booking strategy (L^*, β^*) so as to maximize $ER(L, \beta)$. After taking and rearranging the partial derivatives with respect to L and β, we have*

$$p_2 - \pi_2 H * F((C - L^*)\beta^*) = \overline{F}((C - L^*)\beta^*)[\beta^*(p_1\theta + \pi_1) + \alpha\pi_2 H(\alpha(C - L) \mid (C - L)\beta)], \quad (6)$$

$$\overline{G}(L^*)\overline{F}((C - L^*)\beta^*)Q(L^*, \beta^*) = \int_0^{L^*} \overline{F}((C - y)\beta^*)Q(y, \beta^*)dG(y), \quad (7)$$

where

$$Q(L, \beta) = (C - L)[p_1\theta + \pi_1 - \pi_2 H(\alpha(C - L) \mid (C - L)\beta)], \quad (8)$$

$$Q(y, \beta) = (C - y)[p_1\theta + \pi_1 - \pi_2 H(\alpha(C - y) \mid (C - y)\beta)], \quad (9)$$

and

$$H * F(u) = \int_0^u H(L + x - C \mid x)dF(x). \quad (10)$$

An optimal booking strategy, seat allocation for low fares and overbooking ratio for high fares, must jointly satisfy equations (6) and (7). Note that such an optimal booking strategy is no longer independent of $G(\cdot)$, the probability distribution of low fare demands. Equations (8) and (9) are the net profits obtained from high fare passengers when $Z \le (C - y \wedge L)\beta$. Equation (7) can, therefore, be interpreted as follows; under the optimal booking strategy the expected profit from high fare demands when $Y \ge L$ should be equal to the one when $Y < L$. It seems to us that finding a closed form of an optimal booking strategy jointly satisfying equations (6) and (7) is almost impossible. For this reason we consider a special case of overbooking problems where there is only one class of fares, say high fares.

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* A detailed but messy derivation of equations is omitted here but is available from the author by request.

A special case of overbooking problems.

Suppose that there is only one class of fares, say a high fare class. With each fare booked we associate a random variable

$$D_i = \begin{cases} 1 & \text{if the } i\text{th fare booked confirms,} \\ 0 & \text{if the } i\text{th fare cancels.} \end{cases}$$

Assume that $\{D_1, D_2, \cdots\}$ are independent and identically distributed with mean $ED_i = \theta$. If B seats are booked, then $N(B) = \sum_{i=1}^{B} D_i$ seats are confirmed and the distribution of $N(B)$ is binomial with mean $B\theta$. The revenue as a function of the number of seats available, L, and the number of fares booked, B, is assumed to be

$$R(L, B) = p_1 N(B) - (p_1 + \pi_2)[N(B) - L]^+ - \pi_1 (X - B)^+.$$

Since $E[N(B)] = \theta B$, the expected revenue function is

$$ER(L, B) = p_1 \theta B - (p_1 + \pi_2) E[N(B) - L]^+ - \pi_1 E[X - B]^+.$$

We first compute the incremental revenue from an additional booking

$$\begin{aligned} \Delta R(L, B) &= R(L, B+1) - R(L, B) \\ &= p_1[N(B+1) - N(B)] - (p_1 + \pi_1)[(N(B+1) - L)^+ - (N(B) - L^+] \\ &\quad - \pi_1[(X - (B+1))^+ - (X - B)^+] \\ &= p_1 D_{B+1} - (p_1 + \pi_1) Z_B + \pi_1 Z_B' \end{aligned}$$

where

$$Z_B = \begin{cases} D_{B+1} & \text{if } N(B) > L, \\ 0 & \text{if } N(B) \leq L, \end{cases}$$

and

$$Z_B' = \begin{cases} 1 & \text{if } X > B, \\ 0 & \text{if } X \leq B. \end{cases}$$

Now, the incremental expected revenue can be obtained.

$$\begin{aligned} \Delta ER(L, B) &\equiv ER(L, B+1) - ER(L, B) = E\Delta R(L, B) \\ &= p_1 E(D_{B+1}) - (p_1 + \pi_1) EZ_B - \pi_1 EZ_B' \\ &= p_1 \theta - (p_1 + \pi_1)\theta P\{N(B) > L\} + \pi_1 P\{X > B\}. \end{aligned}$$

Since $N(B)$ is increasing in B, so is $P\{N(B) > L\}$. $P\{X > B\}$ is also decreasing in B. Therefore, $\Delta ER(L, B)$ is decreasing in B, which implies that $ER(L, B)$ is concave in B. So, we should book fares so long as $ER(L, B+1) - ER(L, B)$ is positive. Hence, we arrive at following theorem.

Theorem. If we have L seats protected for high fares, it is optimal to book up to $B^*(L)$ where

$$B^*(L) = \min\{B \geq L; p_1 \theta \leq (p_1 + \pi_2)\theta P\{N(B) > L\} + \pi_1 \overline{F}(B)\}. \tag{11}$$

Since $P\{N(B) > L\}$ can be evaluated from a table of binomial distributions and $\overline{F}(B)$ is given, $B^*(L)$ can easily be calculated. Note that $P\{N(B) > L\}$ and $\overline{F}(B)$ in equation (11) is strictly decreasing in B. So, there exists a unique solution satisfying (11).

4. Spill Rates and Overbooking

If passengers may cancel their reservations without any penalties, airline companies tend to overbook. However, such overbookings cause them compensation costs. If a small number of seats are allocated to each fare class to prevent them from overbooking, they also lose refused boarding passengers resulting in a cost of goodwill lost, which is called spilling passengers of the airline. Hence, a booking strategy must be balanced between overbooking and spilling passengers. In this section we discuss the concepts of spill rates and the expected number of overbookings when you must obtain a certain number of "confirmed" seats, say the number of seats equal to the airplane capacity.

There are two possible interpretations of the term "spill rate" in the airline context. The first is that the spill rate is the expected proportion of flights on which some high fare reservations must be refused because of prior low fare bookings, which is often used in "Airline Yield Management" articles. (For Example, see [6], [10].) The second is that the spill rate is the expected proportion of high fare reservations that must be refused out of the total number of such reservations, which seems to be more meaningful since it relates more closely to the amount of high fare revenues lost. We provide formulae for calculating the spill rate under either interpretation and consider such spill rates associated with use of the revenue maximizing seat allocation rule.

The proportion of *flights* refusing high-fare reservations, called the *flight spill rate*, can be expressed as

$$R_1 = P\{X + L \wedge Y > C\} \tag{12}$$
$$= \overline{G}(L)\overline{F}(C - L) + \int_0^L \overline{F}(C - y)dG(y).$$

It is easy to show that when S seats are available for high-fare passengers the expected proportion of reservations refused will be $\overline{F}(S)E[X - S \mid X > S]/E[X]$. Thus we have for the expected proportion of high fare reservations refused R_2 called the *passenger spill rate*;

$$R_2 = \frac{1}{E[X]}\{\overline{G}(L)\overline{F}(S)u_{C-L} + \int_0^{C-S} \overline{F}(C - y)u_{C-y}dG(y)\} \tag{13}$$

where

$$u_s = [(1/\overline{F}(S))\int_S^\infty xdF(x)] - S.$$

Consider the simple seat allocation model discussed in section 2 where the revenue maximizing rule is used to determine L^*, the number of low fare seats to protect. Let S^* be the number of high fare seats to allocate. In this case we have $S^* = C - L^* = \overline{F}^{-1}(p_2/(p_1 + \pi_1))$ from equation (2). $\overline{F}(S^*) = p_2/(p_1 + \pi_1)$. In the extreme case that low fare demand always exceeds the allocation of low fare seats, we have $\overline{G}(L^*) = 1$, and the above formulae become:

$$R_1 = p_2/(p_1 + \pi_1), \text{ and}$$
$$R_2 = [p_2/(p_1 + \pi_1)](u_{S^*}/E[X]).$$

Now, further assume that $p_2/(p_1 + \pi_1) = 0.4$ (a typical ratio) and that high fare demand is approximately normally distributed with mean 100 seats and standard deviation 20 seats. This will yield a spill rate (expressed as a percentage) of 40%, if the first interpretation (R_1) is used. It seems to us that this figure is abnormally too high. However, if the second interpretation is used, we get $u_{S^*} \approx 32$ and then $R_2 \approx (0.4)(32)/100 = 0.128$. A spill rate of 12.8% seems in closer agreement with actual airline practice.

Given the probability distribution for high fare demands, the problem of determining the seat allocation for each fare class is also of determining either spill rate. The more seats are protected, the smaller spill rate we have. However, the more bookings we accept, the higher probability of overbookings we have, while at that time the spill rate is close to zero. So, the seat allocation problem is of trade off between a low spill rate and a high overbooking ratio. Let us consider the following probability. What is the probability that we must accept a certain number of bookings, say B, in order to obtain the number of confirmed seats which is naturally equal to the capacity of airline seats $C, B \geq C$. Let $q(B; C)$ be such probabilities. It is easy to see that such a random variable follows a negative binomial distribution, that is,

$$q(B;C) = \binom{B-1}{C-1} \theta^C (1-\theta)^{B-C}. \tag{14}$$

with mean C/θ and variance $C(1-\theta)/\theta^2$. Note that variance rapidly increases as $\theta \to 0$. This suggests that airline companies should make effort of reducing the cancellation probability. For example, the restriction on tickets or on booking procedures must be imposed. Equation (14) can be expressed in terms of binomial distribution $b(n, c)$, that is,

$$q(B;C) = \theta b(B-1, C-1).$$

which can be evaluated from a table of binomial distributions.

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