LEASING POLICY UNDER TECHNOLOGICAL ADVANCES
—RISK ANALYSIS FOR LEASE CHARGE DETERMINATION—

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Abstract   Under the situation of the furious competition among the leasing companies, the important problem for a leasing company's management is to determine a lease charge on each lease contract suitably from the management viewpoint. A lease charge is considered to be suitable when the leasing company can get an appropriate profit through the contract. But quite a lot of leasing companies adopt a full payout method for the lease charge determination. It may not be a desirable method in the above mentioned situation because the comparative high lease charge makes its competitive power down. So, it is important for a leasing company to determine suitable lease charges which satisfy both the leasing company's need and its client requirements.

This paper proposes a lease charge determination method for a leasing company based on the risk analysis. The method which we propose in this paper has two eminent characteristics. One of them is that the risk can be treated probabilistically. The other is that the concept of two profit levels (the "necessary profit level" as minimum necessary profit and the "sufficient profit level" as maximum obtainable profit) is introduced as the lease charge determination criterion.

The advantages of this method are as follows: (1) the lease charge which satisfies given risk conditions can be determined by using the disposal price distribution of the lease object, (2) the suitable lease charge can be given in the form of "a certain range" which makes the lease charge negotiation with its client flexible.

In this paper, firstly, we explain the lease charge determination method, secondly, we give the lease charge determination procedures, thirdly, we discuss some characteristics of this method and finally, we show numerical examples.

1. Introduction

In recent years, it is becoming a more and more popular way of thinking to "place a high value on using goods rather than owing them." The stronger this tendency becomes, the more the leasing industry grows. As a result, the competition in this business becomes furious.

Under the situation of the furious competition among the leasing companies, the important problem for a leasing company's management is to determine a lease charge on each lease contract suitably from the management viewpoint. A lease charge is considered to be suitable when the leasing company can get an appropriate profit through the contract. But quite a lot of leasing companies adopt a full payout method for the lease charge determination. This is the method which a lease charge is determined on the basis of the estimation that the disposal price of an object on lease is zero. This method is a leasing company's risk avertion method in the sense that it averts risks by setting a lease charge high. It may not be a desirable method in the above mentioned situation because the comparative high lease charge makes its competitive power down. So, it is important for a leasing company to determine suitable lease charges which satisfy both the leasing company's need and its client requirements (see Fig.1).

The leasing company essentially has the character of "risk business" in the sense that the risk (the profit or loss) of a leasing contract is mainly caused by the disposal price of the object on lease which is changed by the occurrences of technological advances, fluctuation of expenses and changes in economic conditions. So, in order to determine a lease charge suitably, the risk must be considered probabilistically and it must be determined by risk
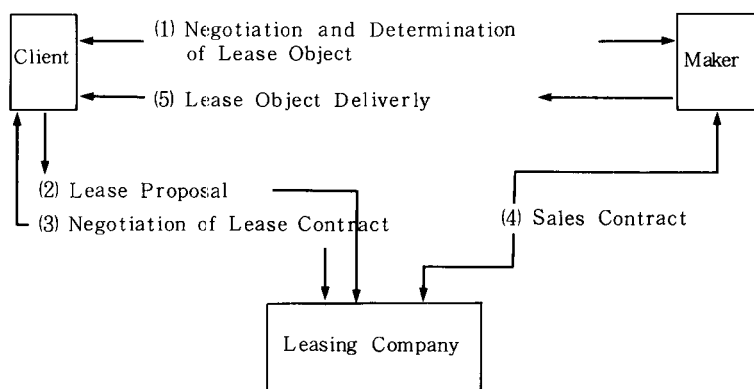
Fig. 1   Lease Contract Business System

analysis based on the risk distribution. Nevertheless, a lease charge has not been determined probabilistically in pertinent methods [1] [3] [5] [6].

Therefore, this paper proposes a lease charge determination method for a leasing company based on the risk analysis. The method which we propose in this paper has two eminent characteristics. One of them is that the risk can be treated probabilistically. The other is that the concept of two profit levels (the "necessary profit level" as minimum necessary profit and the "sufficient profit level" as maximum obtainable profit) is introduced as the lease charge determination criterion.

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In this paper, firstly, we explain the lease charge determination method, secondly, we give the lease charge determination procedures, thirdly, we discuss some characteristics of this method and finally, we show numerical examples.

2. The Lease Charge Determination Method
2.1 The formula of the theoretical lease charge calculation
A formula for calculating a theoretical lease charge is indicated in (2.1).

$$y = (I - S + E)/T$$
$$= (I - S + E_t + E_{is} + E_m + E_c + E_{it})/T, \tag{2.1}$$

where y : theoretical lease charge,
         I : lease object purchase price,
         S : disposal price,
         E : expenses ($E = E_t + E_{is} + E_m + E_c + E_{it}$),
         E_t : taxes,
         E_{is} : insurance,
         E_m : maintenance cost,

E_c : commissions,
E_{it} : interest payment,
and
T : lease period.

The profit or loss of a lease contract is determined from the values in (2.1). And they vary with many causes. So, these causes can be regarded as risks of a lease contract. The main risks are some technological advances in relation to the lease object, the fluctuation of the interest rate, the variation of the maintenance cost caused by the client's usage pattern of the object and the changes in taxes and insurance.

In spite of the fact that the risks must be taken into account in determining the lease charge for the reasons mentioned above, they have not been considered in the conventional methods. These methods are based on deterministic analysis (see Fig. 2 (a)) [3], [5], [6]. In the deterministic analysis, the profit may be gained as a "result" but it can not be set in advance as a "target". If the "target" of the profit of a lease contract is given in advance, it is convenient for the leasing company to negotiate with its client.

This paper proposes a lease charge determination method which can set the profit as a "target" in advance by introducing the risk evaluation criterion (see Fig. 2(b)). The dotted line indicating environmental risks in Fig. 2(a) differs from the solid line in Fig. 2 (b) from the viewpoint of treating risks.
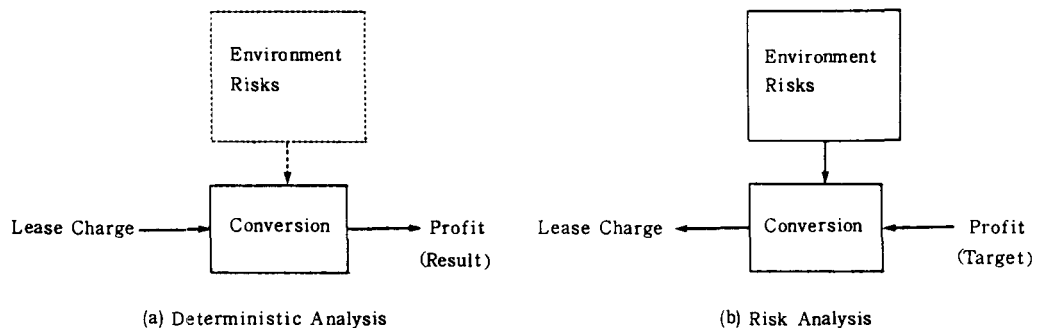
Fig. 2 Comparison between Deterministic and Risk Analyses

2.2 Disposal price distribution

The profit which a leasing company can get from a lease contract largely depends on the disposal price of the lease object. The disposal price of the lease object at the expiration time of the lease contract is determined by the secondhand market conditions of the lease object, which are mainly dominated by what technological advances have occurred.

So, we had better classify the technological advances concerning the impact on the disposal price. If the technological advances of a lease object are not so great, the disposal price is relatively large. Conversely, if the technological advances are great, it is relatively small. Then, we define the former as the gradual technological advances and the latter as the innovative technological advances [2], [7], [8].

It is natural to suppose that the disposal price changes continuously under the gradual technological advances. So, we assume that the probability density function of the disposal price u is given as $g(u) > 0$ for $S_1 < u < S_2$ under the gradual technological advances, where S_1 and S_2 are the lower and upper bounds of the disposal price respectively. The distribution can be estimated by the empirical distribution obtained from the past data. In this case, it is important to construct a database for it. On the other hand, under the innovative technological advances, the disposal price is considered to go down discontinuously. So, we assume that the disposal price is reduced to a very small value, or zero in extreme cases. This is represented by $S_0(S_0 < S_1 < S_2)$.

Furthermore, assuming that the probability of the occurrence of the gradual technological advances and the innovative technological advances are $1 - \alpha$ and α, respectively, the distribution function for the disposal price is given by

$$F(s) = P(S \leq s) = \begin{cases} 0 & s < S_0 \\ \alpha & S_0 < s \leq S_1 \\ \alpha + \int_{S_1}^{s} g(u)\mathrm{d}u & S_1 < s < S_2 \\ 1 & S_2 \leq s, \end{cases} \qquad (2.2)$$

where $g(u)$ is the conditional probability density function of the disposal price under the condition that the gradual technological advances occur but the innovative technological advances don't occur during the lease contract period. This is illustrated in Fig. 3. Here, the case of $\alpha = 1$ is excluded because it is a deterministic case of $S = S_0$. The case of $\alpha = 0$ is a special case where no innovative technological advances occur.
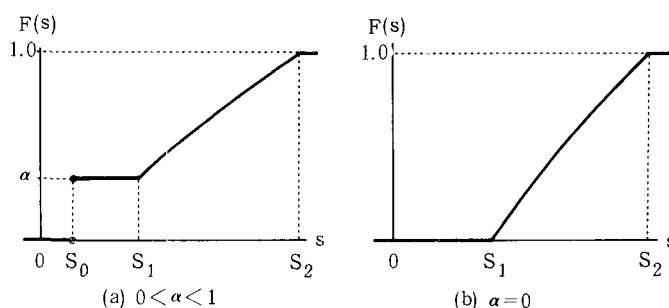

Fig. 3   Distribution Function $F(s)$ of Disposal Price S for a Given α

2.3 A criterion for determining the lease charge
The expected value principle, maximum likelihood principle, stability principle and requirement level principle are generally used as the decision making principles under risk in a theoretical sense [4]. However, considering the decision making principle in a practical sense, the excessive pursuit of a company's short term profit may sometimes result in a decrease of a long term profit. The great profit of a company usually gives rise to furious competition in its business and sometimes strengthening of the governmental control. And the company

decreases its long term profit as a result of these reactions. So, it is desirable to set an upper limit for the profit in order to continue to get a company's suitable profit in a long run.

Accordingly, in evaluating the lease charge, it is important to have a viewpoint that a leasing company must acquire profits above the "necessary profit level" Z_1 (minimum profit which the management gives) as well as limit the profits below the "sufficient profit level" Z_2 (maximum profit which the management also gives). However, there is always a possibility that a profit might be less than Z_1 or more than Z_2, so that it is necessary to adopt the "decision making principle under risk" that determines acceptance levels on the basis of the probability.

Let $Z(y)$ be the present value of the profit gained from the lease contract when the lease charge y is given. Let ε_1 and ε_2 respectively denote the degrees of risk bearing (hereafter called risk levels) predetermined with respect to Z_1 and Z_2. Then, our problem is formulated as follows:

$$\text{select } y, \text{subject to} \tag{2.3}$$

$$P[Z(y) \leq Z_1] \leq \varepsilon_1 \tag{2.4}$$

$$P[Z(y) \geq Z_2] \leq \varepsilon_2 \tag{2.5}$$

Here, $Z(y)$ is expressed by

$$Z(y) = y \sum_{k=1}^{T} \delta_k - (I\nu_T - S)\delta_T - \sum_{k=1}^{T} E_k \delta_k$$
$$= \lambda_1 y + \lambda_2 S - \lambda_3, \tag{2.6}$$

where T:lease contract period,

I:lease object purchase price,

S:disposal price at the expiration time of a lease contract,

$E_k (k = 1, \cdots, T)$:expenses at the end of the k-th period (taxes+insurance+maintenance cost),

δ_k:present worth factor used to discount the cash flow at the end of the k-th period to the beginning of the first period,

ν_T:factor which converts the lease object purchase price at the beginning of the first period into the value at the end of the T-th period,

i_j:discount factor at the end of the j-th period,

r_j:interest rate at the j-th period.

Here $\delta_k, \nu_k, \lambda_1, \lambda_2$ and λ_3 are described as follows:

$$\delta_k = \prod_{j=1}^{k}(1+i_j)^{-1}, \nu_k = \prod_{j=1}^{k}(1+r_j), \lambda_1 = \sum_{k=1}^{T}\delta_k, \lambda_2 = \delta_T, \lambda_3 = I\nu_T\delta_T + \sum_{k=1}^{T}E_k\delta_k.$$

3. Lease Charge Determination Procedure Considering the Disposal Price as a Risk

3.1 A method for determining the lease charge when the disposal price distributes.

Risk factors for $Z(y)$ are technological advances, expenses E_k, present worth factor δ_k and conversion factor δ_T. Among them, technological advances are usually considered to be

the most influential factor. Because the technological advances concerning the lease object often make its disposal price S down drastically. Neither the factors δ_k and ν_T nor E_k are considered as main risk factors when the economic situations are stable during the lease contract period and when there is little possibility of fluctuation in expenses such as taxes, insurance, etc. The technological advances may mainly influence the profit of the lease business through the price distribution of S.

Therefore, in this section, the disposal price S is assumed to have a probability density function.

Using (2.6), (2.4) and (2.5) can be written as follows:

$$P(Z(y) \leq Z_1) = P(\lambda_1 y + \lambda_2 S - \lambda_3 \leq Z_1) = P(S \leq (Z_1 - \lambda_1 y + \lambda_3)/\lambda_2) \leq \varepsilon_1 \qquad (3.1)$$

$$P(Z(y) \geq Z_2) = P(\lambda_1 y + \lambda_2 S - \lambda_3 \geq Z_2) = P(S \geq (Z_2 - \lambda_1 y + \lambda_3)/\lambda_2) \leq \varepsilon_2 \qquad (3.2)$$

It is obvious that (3.1) is equivalent to the inequality of the form $y \geq y_1$ or $y > y_1$. Similarly (3.2) is equivalent to the inequality $y \leq y_2$ or $y < y_2$.

Determining y which satisfies (2.4)

The relation between α (the probability of the occurrence of the innovative technological advances) and ε_1 (the risk level at the necessary profit level Z_1) can be classified into two cases: (i) $\alpha \leq \varepsilon_1$ and (ii) $\alpha > \varepsilon_1$.

In the case of (i), the distribution function of the disposal price $F(s)$ is illustrated in Fig. 4. Let $S_*(\alpha)$ denote the value of s which satisfies the equation $F(s) = \varepsilon_1$, that is, $F[S_*(\alpha)] = P[S \leq S_*(\alpha)] = \varepsilon_1$. Here, $S_*(\alpha)$ is a decreasing function of α in the range $0 \leq \alpha \leq \varepsilon_1$ and the maximum value of $S_*(\alpha)$ is $S_*(0)$. The value y which satisfies (2.4) is determined in the following way from (3.1).
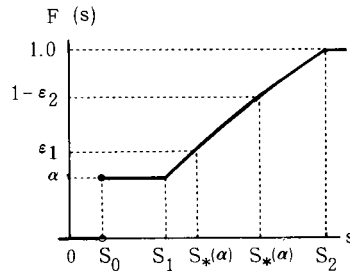

Fig. 4 $S_*(\alpha)$ and $S^*(\alpha)$

Let the y which satisfies the equation $(Z_1 - \lambda_1 y + \lambda_3)/\lambda_2 = S_*(\alpha)$ be denoted by $y_1^*(\alpha)$. Then, from the equation $P[S \leq S_*(\alpha)] = \varepsilon_1$ and (3.1), we can derive

$$P[S \leq (Z_1 - \lambda_1 y_1^*(\alpha) + \lambda_3)/\lambda_2)] = P[S \leq S_*(\alpha)] = \varepsilon_1 \qquad (3.3)$$

Each $Z(y)$ at $S = S_0, S_1, S_*(\alpha)$ and $S_2(S_0, S_1, S_*(\alpha)$ and S_2 are shown in Fig. 4) is a linear function of y as shown in Fig. 5. The values of y which satisfy $Z(y) = Z_1$ at $S = S_*(\alpha), S_1$ and S_0 are represented by $y_1^*(\alpha), y_{11}$ and y_{10}, respectively. Since $S_*(\alpha)$ is decreasing in

$\alpha, y_1^*(\alpha)$ becomes increasing in α. Therefore, the maximum value y_{11} of $y_1^*(\alpha)$ is equal to $y_1^*(\varepsilon_1)$.
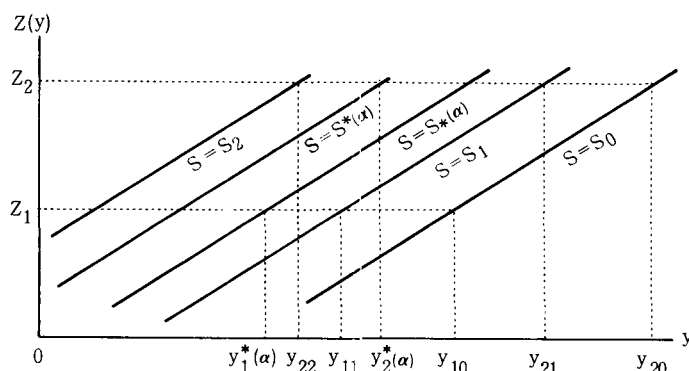

Fig. 5 $Z(y)$ at Lease Charge y. Given Each Disposal Price S

From (3.1) and (3.3), it is easily shown that y which satisfies (2.4) is given by

$$y \geq y_1^*(\alpha), \qquad (3.4),$$

where $y_1^*(0) \leq y_1^*(\alpha) \leq y_{11} = y_1^*(\varepsilon_1)$.

In the case of (ii) $\varepsilon_1 < \alpha < 1$, it is easily understood that y which satisfies (2.4), is given by

$$y > y_{10}. \qquad (3.5)$$

Determining y which satisfies (2.5)

The relation between α and $1 - \varepsilon_2$ can be classified into two cases: (i) $0 \leq \alpha \leq 1 - \varepsilon_2$; (ii) $1 - \varepsilon_2 < \alpha < 1$.

In the case of (i), let $S^*(\alpha)$ denote the value of s which satisfies the equation $F(s) = 1 - \varepsilon_2$, as shown in Fig. 4, that is, $F[S^*(\alpha)] = P[S \leq S^*(\alpha)] = 1 - \varepsilon_2$ (let $S_1 = S^*(1 - \varepsilon_2)$). Here, $S^*(\alpha)$ is decreasing in α in the interval $0 \leq \alpha \leq 1 - \varepsilon_2$, so that the maximum value of $S^*(\alpha)$ is given by $S^*(0)$. The value y which satisfies (2.5) is determined in the following way from (3.2).

Let the y which satisfies the equation $(Z_2 - \lambda_1 y + \lambda_3)/\lambda_2 = S^*(\alpha)$ in (3.2) be denoted by $y_2^*(\alpha)$. Then, from the equation $P[S \geq S^*(\alpha)] = \varepsilon_2$ and (3.2), we can derive

$$P[S \geq (Z_2 - \lambda_1 y_2^*(\alpha) + \lambda_3)/\lambda_2] = P[S \geq S^*(\alpha)] = \varepsilon_2 \qquad (3.6)$$

As shown in Fig. 5, let $y_{22}, y_2^*(\alpha), y_{21}$ and y_{20} denote the values which satisfy $Z(y) = Z_2$ at $S = S_2, S^*(\alpha), S_1$ and S_0, respectively. Since $S^*(\alpha)$ is a decreasing function of $\alpha, y_2^*(\alpha)$ is increasing in α and the maximum value y_{21} of $y_2^*(\alpha)$ is $y_{21} = y_2^*(1 - \varepsilon_2)$.

Therefore, from (3.2) and (3.6), it is easily shown that y which satisfies (2.5) is given by

$$y \leq y_2^*(\alpha), \qquad (3.7)$$

where $y_2^*(0) \leq y_2^*(\alpha) \leq y_{21} = y_2^*(1-\varepsilon)$. In the case of (ii) $\alpha > 1 - \varepsilon_2, y$ which satisfies (2.5) is given by

$$y < y_{20}. \tag{3.8}$$

Setting the lease charge y suitable for negotiating with the client

Eq. (2.5) is equivalent to $P[Z(y) < Z_2] \geq 1 - \varepsilon_2$. Therefore, from the relations of (2.4), (2.5) and $Z_1 < Z_2$, it can be easily understood that the following relation holds:

$$\varepsilon_1 + \varepsilon_2 < 1, \text{ that is, } \varepsilon_1 < 1 - \varepsilon_2. \tag{3.9}$$

Then, from (3.4), (3.5), (3.7) and (3.8), the following relations can be derived concerning the "contractable lease charge" y for a given probability α in the case of $0 \leq \alpha < 1$: Here the "contractable lease charge" is denoted as a lease charge range suitable in the meaning of satisfying (2.4) and (2.5) simultaneously.

  (i) A "contractable lease charge" exists for a given probability α in the case of $0 \leq \alpha \leq \varepsilon_1 < 1 - \varepsilon_2$, if and only if $y_1^*(\alpha) \leq y_2^*(\alpha)$. Then, the range of y is $[y_1^*(\alpha), y_2^*(\alpha)]$.

  (ii) A "contractable lease charge" exists for a given probability α in the case of   (3.10) $\varepsilon_1 < \alpha \leq 1 - \varepsilon_2$, if and only if $y_{10} < y_2^*(\alpha)$. Then, the range of y is $(y_{10}, y_2^*(\alpha)]$.

  (iii) A "contractable lease charge" exists for any probability α in the case of $\varepsilon_1 < 1 - \varepsilon_2 < \alpha$. Then, the range of y is (y_{10}, y_{20}).

Here, note that y is given in the form of a range whenever a "contractable lease charge" exists. Especially, when

$$y_{10} < y_{22} \tag{3.11}$$

holds, y which satisfies (2.4) and (2.5) at the same time is determined in the form of a range for any α with $0 \leq \alpha < 1$. From Fig. 5 and (2.6), it is easily shown that (3.11) is equivalent to

$$Z_2 - Z_1 > \lambda_2(S_2 - S_0). \tag{3.12}$$

Eq. (3.12) implies that the sufficient condition for the existence of a range for any α with $0 \leq \alpha < 1$ can be given only by the relation between the difference $Z_2 - Z_1$ and the difference $S_2 - S_0$.

3.2 Modification of the necessary profit level Z_1 or sufficient profit level Z_2

If a "contractable lease charge" does not exist, it is necessary to modify Z_1, Z_2, ε_1 and ε_2 in order to make a "contractable lease charge" exist. In this study, we set ε_1 and ε_2 as fixed values and we modify Z_1 and Z_2 values. Since they are determined from the company policy, the modification means a policy change.

The following describes the method for finding a "contractable lease charge" by setting Z_1 as a constant and Z_2 as a variable. It can easily be found that the reverse case can be derived by almost the same method.

From (3.10), a "contractable lease charge" does not exist when $y_1^*(\alpha) > y_2^*(\alpha)$ for a certain α such that $0 \leq \alpha \leq \varepsilon_1 < 1 - \varepsilon_2$. In this case, as shown in Fig. 6 (a), let $Z_{21}^*(\alpha)$ be the profit at $S = S^*(\alpha)$ when $y_1^*(\alpha)$ is set as a lease charge determined by $Z(y) = Z_1$ at $S = S_*(\alpha)$. Then, it is possible to find a "contractable lease charge" by selecting Z_2 such that

$$Z_2 \geq Z_{21}^*(\alpha). \tag{3.13}$$

For an α such that $\varepsilon_1 < \alpha \leq 1 - \varepsilon_2$, a "contractable lease charge" doesn't exist when $y_{10} \geq y_2^*(\alpha)$. In this case, as shown in Fig. 6 (b), let $Z_{22}^*(\alpha)$ be the profit at $S = S^*(\alpha)$ when
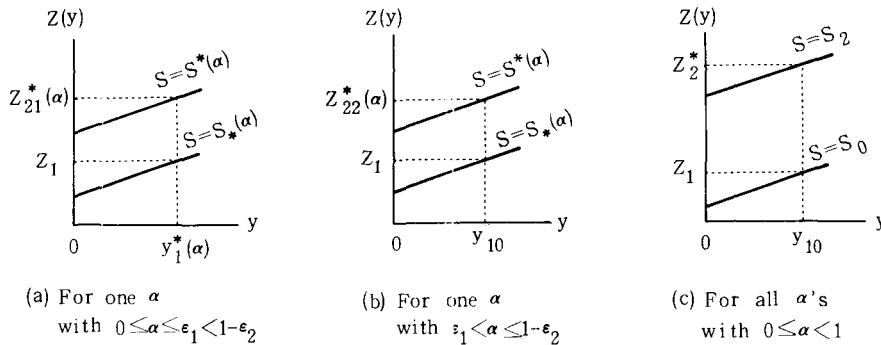
(a) For one α
with $0 \le \alpha \le \varepsilon_1 < 1-\varepsilon_2$

(b) For one α
with $\varepsilon_1 < \alpha \le 1-\varepsilon_2$

(c) For all α's
with $0 \le \alpha < 1$

Fig. 6   Sufficient Profit Level Z_2 $(Z_{21}^*(\alpha), Z_{22}^*(\alpha), Z_2^*)$
for Given Necessary Profit Level Z_1

setting y_{10} as a lease charge determined by $Z(y) = Z_1$ at $S = S_*(\alpha)$. Then, it is possible to find a "contractable lease charge" by selecting Z_2 such that

$$Z_2 > Z_{22}^*(\alpha). \tag{3.14}$$

For any α with $1 - \varepsilon_2 < \alpha$, a "contractable lease charge" always exists. From (3.11), if $y_{10} < y_{22}$, a range exists for any α in a case of $0 \le \alpha < 1$. However, if $y_{10} \ge y_{22}$, let Z_2^* be the profit at $S = S_2$ when we set y_{10} as a lease charge determined by $Z(y) = Z_1$ at $S = S_0$ (see Fig. 6 (c)). Then, it is possible to make a "contractable lease charge" exist for any α with $0 \le \alpha < 1$ by selecting Z_2 such that

$$Z_2 > Z_2^*. \tag{3.15}$$

3.3 Some considerations about a "contractable lease charge"

Here, we discuss the case where a "contractable lease charge" exists for any value α.

The relation between α and a "contractable lease charge" y

An examination is conducted on how the "contractable lease charge" varies according to the change of α. As indicated in Section 3.1, the relation between α and the "contractable lease charge" y is given by Fig. 7 from (3.10). In Fig. 7, if the probability of the occurrence of the innovative technological advances α satisfies $1 - \varepsilon_2 < \alpha < 1$, the range of y is (y_{10}, y_{20}). The lower and upper bounds of y, viz., y_{10} and y_{20}, are determined from $Z(y) = Z_1$ and $Z(y) = Z_2$ at $S = S_0$ in Fig. 5. This means that the lease charge is set by the full payout system. If α lies in the range $\varepsilon_1 < \alpha \le 1 - \varepsilon_2$, the range of y is $(y_{10}, y_2^*(\alpha)]$. When α satisfies $0 \le \alpha \le \varepsilon_1$, the range of y is $[y_1^*(\alpha), y_2^*(\alpha)]$. Both the upper and lower bounds of y, viz., $y_1^*(\alpha)$ and $y_2^*(\alpha)$, decrease as α becomes smaller. This indicates that the smaller the probability of the innovative technological advances is, the greater the possibility of setting a low lease charge is. If α is at or near ε_1 of $1 - \varepsilon_2$, the lower or upper bound for y changes discontinuously. Therefore, we must take care when α is forecasted to be at or near ε_1 or $1 - \varepsilon_2$.
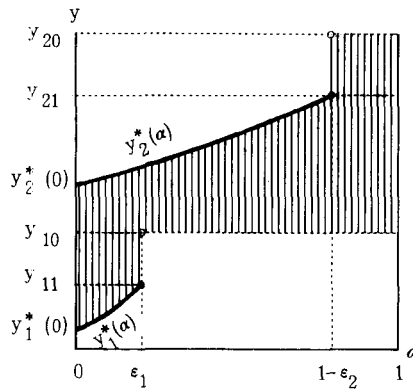
Fig. 7   Contractable Lease Charge Range for Each α

Some characteristics of a "contractable lease charge"

As the graph of $Z(y)$ is a straight line with the slope $(\sum_{k=1}^{T} \delta_k)$, the range of $y(\Delta y)$ is basically determined by the slope and the difference (ΔS) between S_2 and S_0 (see Fig. 8). This indicates that the smaller $\sum_{k=1}^{T} \delta_k$ is and the greater ΔS is, the greater Δy is. Now, $\sum_{k=1}^{T} \delta_k$ is determined from the conditions of the financial environment and ΔS is determined by the characteristics of technological advances and various economic conditions during the lease contract period.
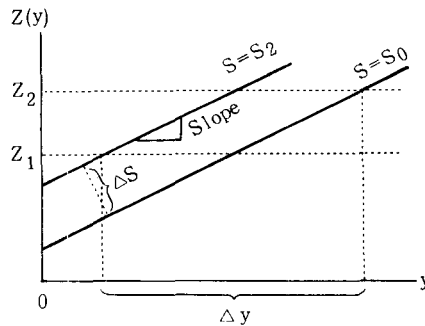
Fig. 8 $Z(y)$ and y at a Given S

4. Lease Charge Determination Procedure Considering the Disposal Price, the Expenses, etc., as Risks

Now we are concerned with the case where expenses at every period $E_k(k = 1, 2, \cdots, T)$ are

considered to vary at random as risks. Let

$$P = (S\delta_T - \sum_{k=1}^{T} E_k \delta_k)/\delta_T. \tag{4.1}$$

Then, (2.6) can be rewritten

$$Z(y) = y \sum_{k=1}^{T} \delta_k - (I\nu_T - P)\delta_T. \tag{4.2}$$

The P in (4.1) is a random variable because S and E_k are random variables. Then, (4.2) has the same form as (2.6) by replacing S by P and setting E_k to zero. Therefore, all the methods discussed in Section 3.3 are applicable to this case. As seen in (4.2), S is conceptually dependent on E_k.

For example, if the maintenance cost is high, the lease object disposal prices S is comparatively high. In practice, however, S and E_k can be regarded as independent. The reason is that the distribution of S is mostly determined, not by E_k but by technological advances or other factors such as style, color, etc. Therefore, the empirical distribution of P can be generated by using each empirical distribution of S and E_k. From the above discussion , the same method proposed in the previous section can be used to determine the lease charge when the expenses E_k are considered to be risks as well as S.

Furthermore, this method can be used for the case where δ_k and ν_k are considered to be risks as well as S and E_k. The main risks of the discount factor i_j and interest rate $r_j(j = 1, 2, \cdots, T)$ are the estimation errors caused by making wrong forecasts of them. And it is usually difficult to estimate their values accurately. Then, from the practical viewpoint, it is better to use the upper bound value S, the middle value M, and the lower bound value L of the discount factor and the interest rate than to use their estimates directly for the analysis. So, we will show how to use this method in the following examples.

5. Numerical examples

For the simplicity of the analysis, we assume that the disposal price has a uniform distribution under the gradual technological advances and the values of i_j, r_j and E_j don't change at each period. So, we represent them by i, r and E respectively. Then, δ_k and ν_k are given by

$$\begin{aligned} \delta_k &= 1/(1+i)^k \\ \nu_k &= (1+r)^k \end{aligned} \tag{5.1}$$

The parameter values used are given in Table 1. Fig. 9 shows the relation between α and the "contractable lease charge" y. We can point out several results of a "contractable lease charge" as follows. (1) Both the lower and upper bounds for y change drastically near $\alpha = 0.1 = \varepsilon_1$ and $\alpha = 0.9 = 1 - \varepsilon_2$. (2) The monthly lease charge y is between about 3.2 and 3.6 million yen. (3) When α is estimated between 0.9 and 1, the lease charge must be determined by using the full payout method.

Comparing (a) with (b) of Fig. 10, the followings are concluded: (1) The changes of the ranges of the "contractalbe lease charge" at and near $\alpha = 0.1 = \varepsilon_1$ or $\alpha = 0.9 = 1 - \varepsilon_2$ in the case of $S_0 = 0$ is much larger than the changes in the case of $S_0 = 1000$. (2) The ranges

Table 1   Parameter Values in Numerical Examples

$I = 10000$	$Z_1 = 4000$	$Z_2 = 6500$
$S_0 = 1000$	$S_1 = 1500$	$S_2 = 2000$
$E = 10/\text{month}$	$\varepsilon_1 = 0.1$	$\varepsilon_2 = 0.1$
$T = 48\text{months}$	$i = 0.06/\text{year}$	$r = 0.062/\text{year}$

Monetary unit represents ten thousand yen.
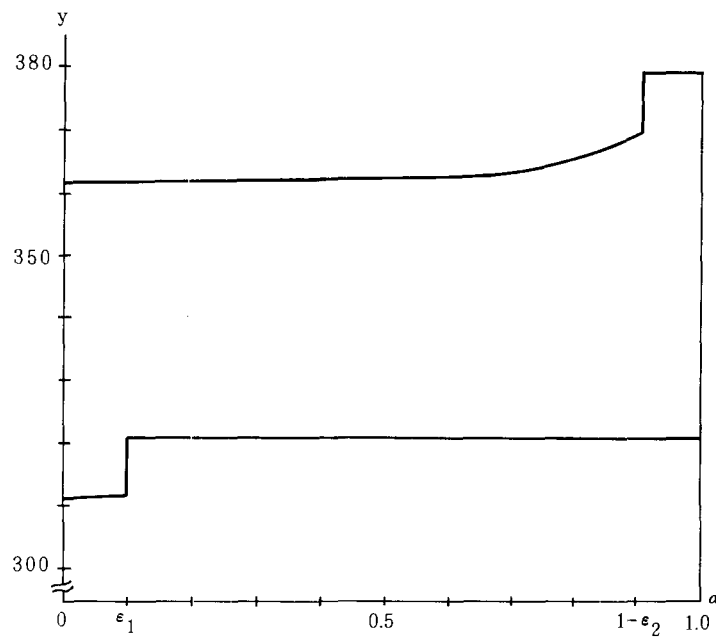
Fig. 9   Contractable Lease Charge Range for Each α

of the "contractable lease charge" in the case of $S_0 = 0$ are narrower than the ranges in the case of $S_0 = 1000$ in $0.1 \leq \alpha \leq 0.9$. In other words, if the disposal price is low, the range of y is narrow.

For a given necessary profit level Z_1 and a sufficient profit level Z_2, we often encounter
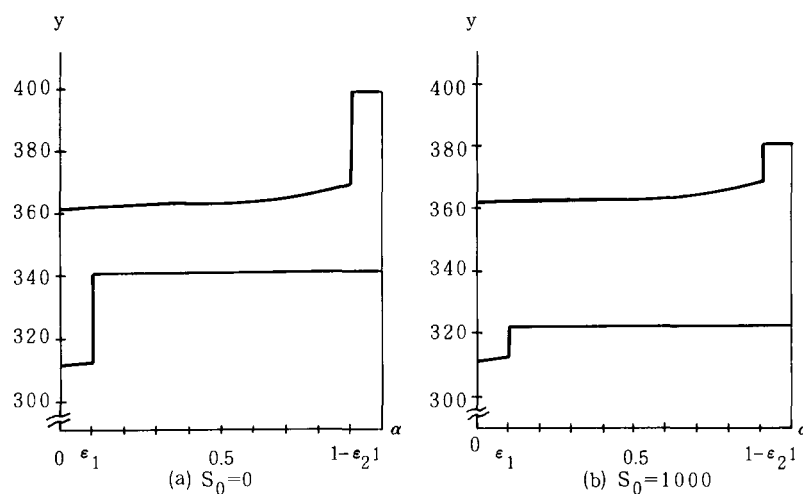
Fig. 10 Contractable Lease Charge at $S_0 = 0$ and $S_0 = 1000$, Given α

Table 2 Contractable Lease Charge and Its Existing Condition
with Respect to Z_1 or Z_2 at Given α

$(\varepsilon_1 = \varepsilon_2 = 0.1, \ Z_1 = 4000, \ Z_2 = 4500)$

α	Z_1 or Z_2 Range for Each α	α	Z_1 or Z_2 Range for Each α
0	$*(310.8 \leq y \leq 315.0)$	0.5	$Z_1 < 3784.7, \ Z_2 > 4714.0$
0.1	$*(311.8 \leq y \leq 315.2)$	0.6	$Z_1 < 3806.5, \ Z_2 > 4694.0$
0.2	$Z_1 < 3758.7, \ Z_2 > 4743.5$	0.7	$Z_1 < 3841.2, \ Z_2 > 4660.0$
0.3	$Z_1 < 3763.0, \ Z_2 > 4736.5$	0.8	$Z_1 < 3906.4, \ Z_2 > 4595.0$
0.4	$Z_1 < 3776.0, \ Z_2 > 4726.5$	0.9	$*(321.1 < y \leq 323.4)$

Asterisk "*" means that y constitutes an Range for $Z_1 = 4000$ and $Z_2 = 4500$.

(a) $i_L = 0.5$, $i_M = 0.6$ and $i_H = 0.7$ (b) $i_L = 0.4$, $i_M = 0.6$ and $i_H = 0.8$
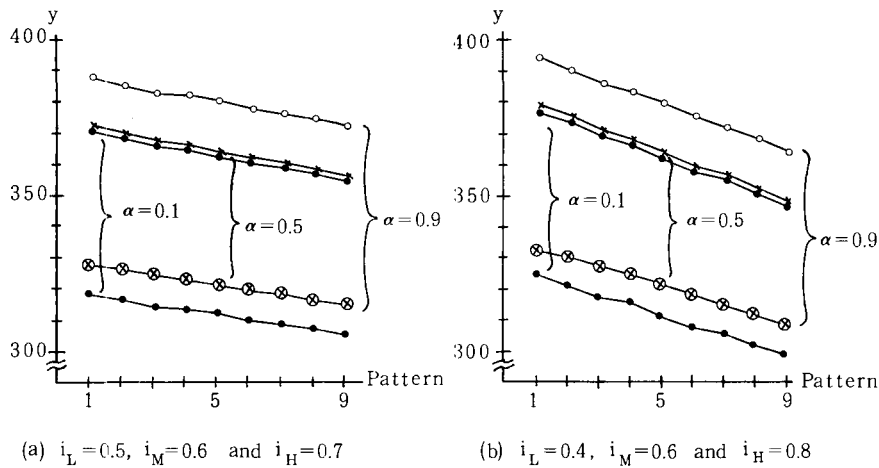
Fig. 12   Contractable Lease Charge, Given Patterns for Discount Factor and Interest Rate

6. Conclusion

This paper proposes a lease charge evaluation method under risks. This method has two characteristics. One is that it evaluates the risks probabilistically in determining the lease charge. The other is that by using this method, the lease charge can be determined to satisfy the conditions which the management policy of the leasing company gives. The risks in the paper are technological advances, fluctuations in expenses and changes of economic conditions, etc. By using this method, we can find the "contractable lease charge" which satisfies the given conditions of the leasing company.

In this paper, firstly, the necessary and sufficient condition for the existence of the "contractable lease cahrge" is derived in the case where technological advances are considered to be the risks of the lease contract, and when the "contractable lease charge" doesn't exist in this case, the criterion modification method is proposed. Secondly, we give the lease charge determination method in the case where the fluctuation of the expenses and changes in economic conditions in addition to technological advances are considered to be the risks. Finally, we give the numerical examples to show the practicality of the method.

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