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ABSTRACT

STRUCTURE OF OPTIMAL SOLUTIONS OF A KNAPSACK PROBLEM SUBJECT TO A GIVEN TOTAL NUMBER OF VARIABLES USED

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This paper is concerned with solving a knapsack problem subject to the constraint that total number of variables used in the solution is bounded above by a given number. The approach taken in [6,7] for solving the (equalty-constraint) knapsack problem is extended to this case. Like the case of the equality-constraint knapsack problem [6,7], the feasibility of the problem is not trivial and examined first. A necessary and sufficient condition for the feasibility is obtained through a special structure of the problem. All dual feasible bases of the linear programming relaxation problem of the problem are obtained explicitly so that the cone spanned by all right hand side vectors for which the problem is feasible is decomposed into the cones generated by the columns of the dual feasible bases. Associated with each dual feasible basis is a group relaxation problem of the problem. The group minimization problem is written down explicitly and studied. Unlike the case of the single constraint integer linear programming problems [6,7], there may still remain an infinite number of right hand side vectors for which the problem is not solved optimally through its group minimization problem. To resolve this difficulty (the infinity of the number of unsolved right hand side vector), a new relaxation problem is introduced in which the nonnegativity of only one of the two variables corresponding to a dual feasible basis is relaxed. It is determined for what right hand side vectors the original problem is solved or is not solved by solving the new relaxation problem thus introduced. By solving all the new relaxation problems, one finds that only a finite number of problems remain unsolved. After solving the problem for all right hand side vectors, a tree structure is exploited to represent all the optimal solutions to the problem for which its right hand side vectors are changed all over a dual feasible basis.