座標軸成分毎最小化法の収束証明

ABSTRACT

CONVERGENCE PROOF OF COORDINATEWISE MINIMIZATION ALGORITHM FOR CONVEX PROGRAMMING PROBLEM WITH UPPER AND LOWER BOUNDED CONSTRAINTS

Takahumi Oohori Hokkaido Institute of Technology Azuma Ohuchi and Ikuo Kaji Hokkaido University

Q: min f(x) $x \in S$ where $S = \{x \mid 1 \le x \le u\}$ $x, 1, u \in E_n$ $f: E_n \neq E_1$ (differentiable and convex)

In this paper, we give a proof that the coordinatewise minimization (CM) procedure for the above problem converges to the optimal solution.

D'Esopo gave a proof for a special class of Q with the property (A) that f(x) has a unique minimizing argument in each coordinate direction (but need not strictly convex). This paper is the extention of his proof to the more general case that f(x) is differentiable and convex but does not necessarily have the above property.

Problem Q is not only a special class of convex programming problem with only upper and lower bounded constraints, but also a dual of the more general concave programming problem with strictly concave objective function. The Lagrangian dual function with respect to all primal constraints has the above property (A), but the function with respect to only part of the constraints (Lagrange relaxation) does not necessarily have the property. This paper also gives a proof that the CM method for the latter case converges to the optimal solution as well as for the former case.

337