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MODELING AND ANALYSIS OF A QUEUEING SYSTEM EXISTING IN A BANK

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Abstract In this paper we consider an interesting scheme of queueing system existing in a bank in Sri Lanka in which the service mechanism consists of three steps performed by two servers with batch service. The analysis is done by a fluid flow model the fitness of which is examined using an appropriate simulation model. The quality of service of the bank in terms of the system time of the customers is improved by proposing a policy to determine the batch size. The study shows that a considerable saving in the system time of the customers can be obtained without any additional cost. The other purpose of this paper is to illustrate the effect of traffic intensity, initial conditions and the service time distribution of the system on the applicability of the fluid flow approximation technique.

I. Introduction

The service organisation under consideration in this paper is a commercial bank in Sri Lanka which has tremendous number of branches located all over the country and is reputed for its wide range of banking services. One of the problems facing this bank is the long customer waiting time in the case of encashment of the cheques. Therefore, there is a pressing need to study the existing procedure in the cheques encashment counter and see if any improvements can be made. The service mechanism in the case of encashment of the cheques is as follows.

The cashier first receives the cheque from the first customer, hands over a token to the customer and enters the particulars such as the cheque number, signature of the cashier, token number, etc. into a scroll book. The cashier repeats this service until he completes a batch. This service is termed as the step 1. The customers whose service at step 1 is over wait in the waiting area until they are called back at step 3.

The cashier then brings the cheques of the first batch along with the scroll book to the ledger officer for authorisation. At the authorisation stage (step 2) the ledger officer checks for the technical mistakes of the cheque such as the correctness of the cheque, date of the cheque, availability of funds and the drawer's signature, etc. and if all the requirements are satisfied he authorises the cheque by putting his signature on the cheque as well as in the scroll book. Officer repeats the same work for each and every cheque. After handing over the cheques to the officer the cashier soon returns to his seat and starts accepting a second batch using another scroll book. When the officer finishes the service at step 2, the cashier brings the scroll book to the ledger officer after completing the step 1's service of the customer currently in service at step 1. When the cashier goes back to his seat he brings with him the previous batch which has already been authorised, and starts the payment of the money (step 3) for the authorised cheques. At step 3 the cashier calls back the relevant token numbers, receives the token back from the customer and makes the payment. After receiving the money the customer leaves the bank. Soon after the payment of the first batch is over, the cashier starts receiving a third batch and continues until the ledger officer finishes the authorisation of the second batch. This process goes on until all the customers are served. The service mechanism can be illustrated more clearly by the following diagram.

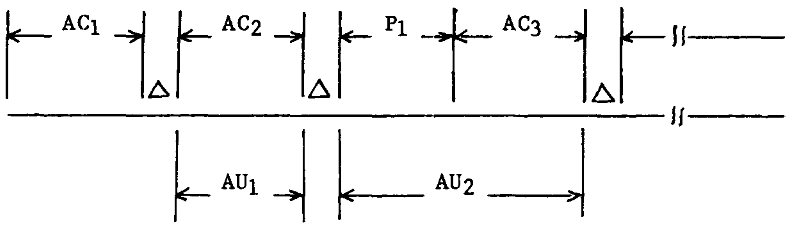

Figure 1. The process

AC_i : the time taken by the cashier to receive the cheques from the customers in the i-th batch

AU_i : the time taken by the ledger officer to authorise the i-th batch

P_i : the time taken by the cashier to pay the money to the customers in the i-th batch

Δ : the setup time which includes the time taken by the cashier in transferring the batches to the officer and the idle time of the officer(if any) until the cashier brings a particular batch. This time is assumed to be a constant.

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2. The Outline of the Study

2.I. The Limitations and the Objectives of the Study

The service procedure of the bank involves two kinds of work:

(i) work where the bank has to adhere to its regulations imposed in order to maintain the security, over which the branch manager has no power to make any alterations.

(ii) work where the branch manager has the power to alter the existing procedure provided that the proposed system is efficient and does not cost any additional money.

Therefore, in this study we focus our attention on the work(ii) and try to propose some modifications to the existing system in order to reduce the system time of the customers.

To analyse this problem we need a suitable model. It is difficult to formulate a stochastic model to this type of complicated queueing problem. Therefore, as a usual practice, we first formulate a simulation model. Then we propose a fluid flow model that gives simple analytical results. It is shown that it fits the simulation model well. Furthermore, it is also shown in this paper that the fluid flow model can be used for the analysis of some detailed properties of the queueing system that cannot be predicted easily by the simulation model. The other purpose of this paper is to illustrate the effect of factors such as traffic intensity, initial conditions and the service time distribution of the system on the applicability of the fluid flow model. The following notations will be used throughout of this paper.

N : number of customers arrived at the bank on a particular day

$\nu = E(N)$

$Q(0)$: queue size at the time $t = 0-$

$\Psi = E[Q(0)]$

λ = arrival rate of the customers

μ_1 = service rate of the cashier at step 1

μ_2 = service rate of the cashier at step 2

μ_3 = service rate of the cashier at step 3

B : size of the initial batch accepted by the cashier

T^* : the time at which the servers complete the service of N customers

$T_e = E[T^*]$

m : number of cycles required to complete service of all the customers

W^* : sum of the system times of N customers

$W = E[W^*]$

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2.2. Simulation Model

In this paper we consider two simulation models namely Simulation model-1 and Simulation model-2 which represent the existing process in the bank and the modified process respectively. In this section we shall describe Simulation model-1 and the other will be appeared in Section 5. In formulating Simulation model-1 we make the following assumptions.

(i) Q(0) is assumed to be Poisson distributed with mean value Ψ

(ii) The customers arrive (Fig.2) in a Poisson process with rate λ

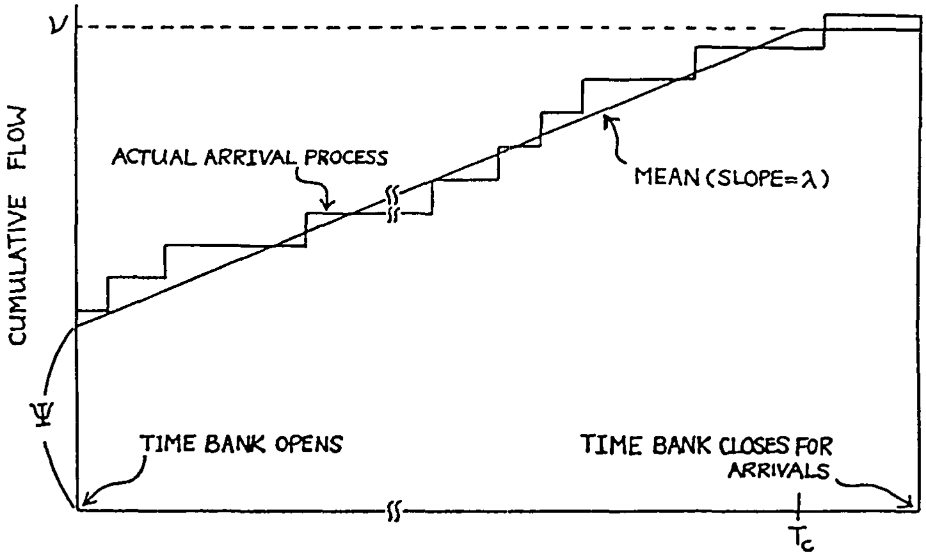

Figure 2. Arrival Process

until time T_c where T_c is determined by $T_c = (\nu - \Psi)/\lambda$. λ is the arrival rate estimated from the actual data.

(iii) In principle, it is assumed that the cashier makes the payment for the previous batch and soon accepts the next batch while the ledger officer authorises the current batch.

(iv) If the officer finishes authorisation of a particular batch when the cashier is serving a customer at step 1, it is assumed that the cashier completes the service at step 1 of that customer and then brings the batch to the officer.

(v) If the officer finishes the authorisation of the current batch before the cashier finishes the payment of the previous batch, then the cashier makes the payment for the current batch soon after the payment of the previous batch and then accepts a new batch of size B. If the number of customers waiting is

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less than B, he accepts only the number available at that time.

(vi) At the time when the cashier becomes free from the payment (step 3) if no customers are waiting, then the next arriving customer and also the customers who arrive during his service time at step 1 are also accepted provided that the total batch is less than or equal to B.

Simulation model-1 is further subdivided as follows depending on the service time distribution at each step.

Simulation model-1(i) : the service time at each step is assumed to be constant at $1/\mu_1$, $1/\mu_2$ and $1/\mu_3$ respectively.

Simulation model-1(ii) : the service time at each step is assumed to be exponentially distributed with means $1/\mu_1$, $1/\mu_2$ and $1/\mu_3$ respectively.

On the basis of the discussion in Section 1 it is natural to assume that the service time at each step is close to a constant value. Therefore, Simulation model-1(i) is considered to be very close to the practical situation. But, there may be a possibility that the service times are random quantities. Simulation model-1(ii) is a representation of such extreme cases.

Since detailed actual data is not available, simulation model is also used for the purpose of estimating some of the system parameters. We now present here a brief discussion on how the system parameters are estimated.

The first author surveyed the queueing situation about four years ago and collected some data at the bank premises on a working day. But, due to the lack of knowledge and experience in the queueing theory field the author had at that time, the data collected was not detailed enough and contains only the informations such as the arrival time, service start time at step 1 and the service end time at step 3 of each customer arrived at the bank. Therefore, the service rates at step 1, step 2 and step 3 cannot be estimated from the actual data. The arrival rate and the average system time could be estimated from the available data and found to be 0.9 customers/minute and 49.4 minutes respectively. The author also observed that the working rates of the cashier at step 1 and step 3 are almost equal but due to the nature of the work at step 2, the service rate of the officer is low compared with that of the cashier at step 1 and step 3. That is, the bottleneck of the system is at step 2. Furthermore, we found from the actual data that, at the time when the bank opens 10 customers have suddenly arrived. Therefore, on the

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basis of these observations and the actual data collected, we use the trial and error method applied on Simulation model-1(i) to estimate the system parameters μ_1, μ_2 and μ_3.

We let B = 1, λ = .9, Ψ = 10 and Δ = .4 in Simulation model-1(i) and the average system time is calculated for different values of μ_1, μ_2 and μ_3. It is found that when μ_1 = 1.6, μ_2 = .7 and μ_3 = 1.6 the simulated average system time(52.6 minutes) is approximately equal to that (49.4 minutes) obtained from the actual data. It can be seen that these estimated values reflect the observed bottleneck property at step 2 too. Therefore, in this paper we proceed our discussion by assuming that μ_1 = 1.6, μ_2 = .7, μ_3 = 1.6, λ = .9, Ψ = 10 and Δ = .4 (case i) represent the practical situation. The sensitivity of Ψ and Δ will be discussed in later sections.

3. Fluid Flow Model

From the discussion in Section 1 we understand that the queueing system under consideration is in heavy traffic conditions. Based on this fact we first put the emphasis on the engineering approach [1,2] by proposing two fluid flow models namely Model 1 and Model 2 to represent the customer behaviour in the existing process and in the modified process respectively. In this section we discuss Model 1 and Model 2 will be discussed in Section 5.

In this model it is assumed that at the time t = 0−, a group of customers of size Ψ suddenly arrives at the bank and after that the customers arrive at the rate of λ until time T_c, where T_c is determined by (ν − Ψ)/ λ . That is the conditions noted in (i) and (ii) of Section 2.2 are considered in terms of means. Furthermore, the essential mechanism noted in (iii) is retained here but the conditions (iv),(v) and (vi) are not necessary to be considered in this model. Thus the fluid flow model becomes simpler than the simulation model.
Model 1 is illustrated in Fig.3.

Before we proceed into the detailed analysis of the model we shall define the following terms that are frequently used in this paper.
i-th cycle : fragment of the service process represented by the time interval starting from the end of payment of the (i − 1)-th batch and lasting until the end of payment of the i-th batch. When i = 1, its atarting time has to be read as 0.

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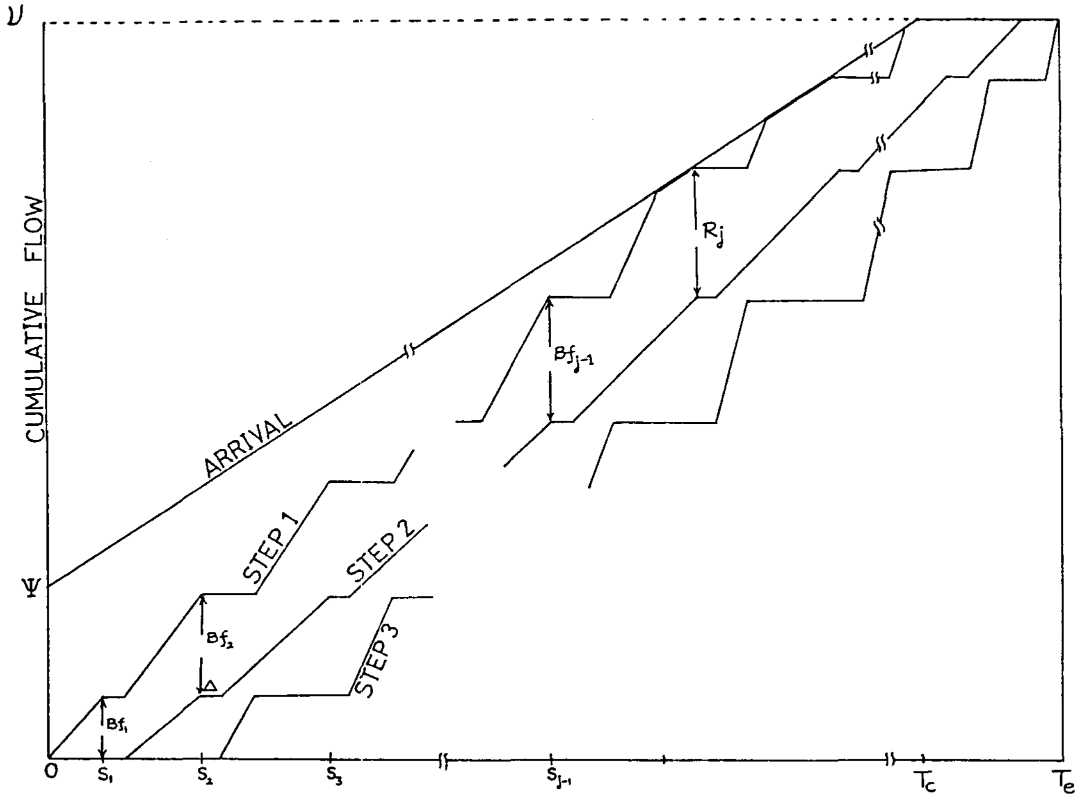

Figure 3. Model I

i-th cycle time : time taken to complete the i-th cycle.

We assume that during the acceptance of the j-th batch the queue becomes zero for the first time. When the initial batch size is B, let the size of the n-th batch be Bf_n, where $f_1 = 1$, $f_2 = \mu_1/\mu_2$ and

(3.1) $f_n = \mu_1(f_{n-1}/\mu_2 - f_{n-2}/\mu_3), \quad j-1 \geq n \geq 3.$

(3.1) is obtained by considering the fact that, while the ledger officer authorises the (n-1)-th batch the cashier pays the (n-2)-th batch and soon accepts the n-th batch. When $\mu_2 \leqq \sqrt{\mu_1\mu_3}/2$, f_n can be expressed explicitly as

(3.2) $f_n = (\Theta_1^n - \Theta_2^n)/(2\Theta_1 - \mu_1/\mu_2), \quad 1 \leqq n \leqq j-1$

where Θ_1 and Θ_2 are the real roots of the quadratic equation

(3.3) $\mu_1^{-1}\Theta^2 - \mu_2^{-1}\Theta + \mu_3^{-1} = 0.$

The validity of (3.2) can be shown as follows.

Letting $f_n = \Theta^n$ in (3.1) we get the quadratic equation (3.3). In order that Θ_1 and Θ_2 be real, the condition $\mu_2 \leqq \sqrt{\mu_1\mu_3}/2$ should be satisfied. The above estimates of the system parameters satisfy this condition. Therefore, f_n can be written as $f_n = C_1\Theta_1^n + C_2\Theta_2^n$,

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where C_1 and C_2 are arbitrary constants. On applying the boundary conditions $f_1 = 1$ and $f_2 = \mu_1/\mu_2$ the constants C_1 and C_2 can be calculated. Thus we can obtain (3.2). It is to be noted that in case(i) the f_n rapidly increases.

Now we define the times $s_1 = Bf_1/\mu_1$, $s_2 = s_1 + \Delta + Bf_1/\mu_2$ and $s_i = s_1 + (i-1)\Delta + B\sum_{n=1}^{i-1} f_n/\mu_2$ for $i \geqq 3$ and the set $S = \{ s_i \mid s_i < T_c \}$.

At first, note that $B\sum_{n=1}^{j} f_n \geqq \Psi + s_j\lambda$ and $B\sum_{n=1}^{i} f_n < \Psi + s_i\lambda$ for $i < j$, where $s_i, s_j \in S$. That is, during the acceptance of the j-th batch the queue becomes zero for the first time. Therefore, the size of the j-th batch R_j can be given by

$$R_j = \Psi + s_j\lambda - B\sum_{n=1}^{j-1} f_n. \tag{3.4}$$

The batch sizes thereafter are assumed to be

$$R_i = (\Delta + R_{i-1}/\mu_2)\lambda \quad , \quad i \geqq j+1. \tag{3.5}$$

And we determine m from the inequality

$$B\sum_{n=1}^{j-1} f_n + \sum_{n=j}^{m} R_n \geqq \nu > B\sum_{n=1}^{j-1} f_n + \sum_{n=j}^{m-1} R_n.$$

Therefore, the size of the last batch (say R_m') can be obtained as

$$R_m' = \nu - B\sum_{n=1}^{j-1} f_n - \sum_{n=j}^{m-1} R_n. \tag{3.6}$$

For convenience hereafter we shall write R_m' as R_m.

The cycle times $T_i (i = 1, 2, \ldots, m)$ can be obtained from Fig.3 as

$$T_1 = 2\Delta + Bf_1(1/\mu_1 + 1/\mu_2 + 1/\mu_3)$$

$$T_i = \Delta + Bf_i/\mu_2 - Bf_{i-1}/\mu_3 + Bf_i/\mu_3, \quad 2 \leqq i \leqq j-1$$

$$T_j = \Delta + R_j/\mu_2 - Bf_{j-1}/\mu_3 + R_j/\mu_3$$

$$T_i = \Delta + R_i/\mu_2 - R_{i-1}/\mu_3 + R_i/\mu_3, \quad j+1 \leqq i \leqq m-1$$

$$T_m = \begin{cases} \Delta + R_m/\mu_2 - R_{m-1}/\mu_3 + R_m/\mu_3 & \text{if } R_m/\mu_2 > R_{m-1}/\mu_3 \\ \Delta + R_m/\mu_3 & \text{if } R_m/\mu_2 \leqq R_{m-1}/\mu_3. \end{cases} \tag{3.7}$$

Using (3.7) and Fig.3 we can calculate X, the area covered by the

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departure process and the time axis. It is given as follows.

$$(3.8)\qquad X = [B^2 \sum_{i=1}^{j-1} f_i^2]/(2\mu_3) + [\sum_{i=j}^{m} R_i^2]/(2\mu_3)$$

$$+ Bf_1T_2 + B(f_1 + f_2)T_3 + \ldots + B(\sum_{n=1}^{j-1} f_n)T_j$$

$$+ [B\sum_{i=1}^{j-1} f_i + R_j]T_{j+1} + [B\sum_{i=1}^{j-1} f_i + R_j + R_{j+1}]T_{j+2} + \ldots$$

$$+ [B\sum_{i=1}^{j-1} f_i + \sum_{i=j}^{m-1} R_i]T_m$$

Next, if there does not exist $j < m$ defined above, then the queue becomes zero for the first time during the acceptance of the last batch. Therefore, the size of the last batch R_m can be given by

$$(3.9)\qquad R_m = \nu - B\sum_{i=1}^{m-1} f_i$$

where $m = |S| + 1$.

In this case, we have

$$(3.10)\qquad T_m = \begin{cases} \Delta + R_m/\mu_2 - Bf_{m-1}/\mu_3 + R_m/\mu_3 & \text{if } R_m/\mu_2 > Bf_{m-1}/\mu_3 \\ \Delta + R_m/\mu_3 & \text{if } R_m/\mu_2 \leqq Bf_{m-1}/\mu_3 \end{cases}$$

and

$$(3.11)\qquad X = [B^2 \sum_{i=1}^{m-1} f_i^2]/(2\mu_3) + R_m^2/(2\mu_3)$$

$$+ Bf_1T_2 + B(f_1 + f_2)T_3 + \ldots + B(\sum_{i=1}^{m-1} f_i)T_m.$$

In Model 1 the cumulative system time of the customers W is represented by the area between the arrival process and the departure process. Then, since the expected service completion time of all the customers is $T_e = \sum_{i=1}^{m} T_i$, it can be given by

$$(3.12)\qquad W = \begin{pmatrix}\text{area covered by the arrival}\\ \text{process and the time axis}\end{pmatrix} - \begin{pmatrix}\text{area covered by the departure}\\ \text{process and the time axis}\end{pmatrix}$$

$$= \Psi T_c + T_c(\nu - \Psi)/2 + (T_e - T_c)\nu - X$$

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$= \nu T_e - T_c(\nu - \Psi)/2 - X,$

where X is given by (3.8) or (3.11). The average system time corresponding to the model is equal to W/ν. In addition, when $j = 1$ or 2 a slight modification to the above analysis of the model is required.

In order to examine the fitness of the fluid flow model, the approximate values calculated from (3.12) when $\nu = 100$, $\Psi = 10$, $\Delta = .4$, $\mu_1 = 1.6$, $\mu_2 = .7$ and $\mu_3 = 1.6$ are plotted in Fig.4 along

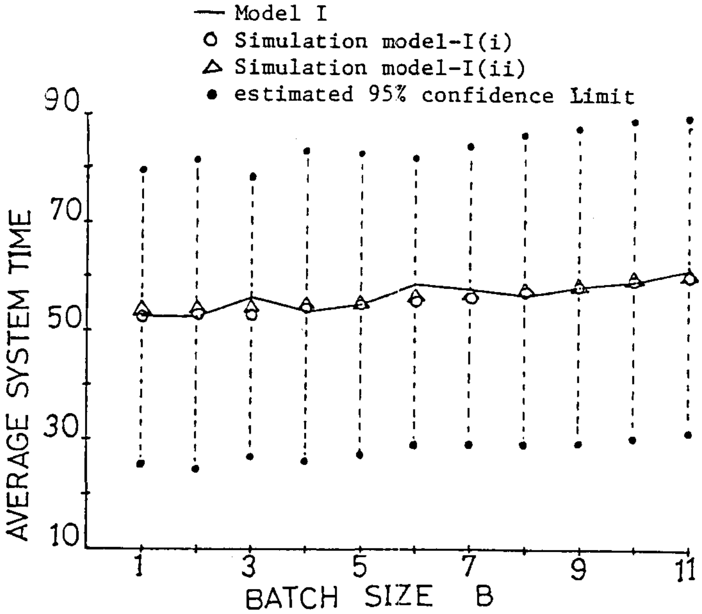

Figure 4. The average system time of the customers ($\mu_1 = \mu_3 = 1.6$, $\mu_2 = .7$, $\lambda = .9$, $\Delta = .4$)

with the simulated results obtained from Simulation model-1(i) under the same parameter values. The simulated results corresponding to the Simulation model-1(ii) are also plotted in Fig.4 to check whether the fluid flow model is susceptible to the extreme case where service time in each step is exponential. The average system time in case of simulation means that W^* divided by the number of customers N on each run. It is to be noted that in the simulation models the same set of random numbers is used for each value of batch size and the simulated values appear in Fig.4 are averages of 2000 runs. These values are almost same as the values corresponding to 1000 runs. In the case of Simulation model-1(i) the estimated standard deviations when B=1,6 and 11 are 13.6, 13.2 and 14.5 respectively and we get similar values for other B. The 95% confidence limits of the average system time in case of the Simulation model-1(i) are also shown in Fig.4.

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From Fig.4 it can be seen that the proposed fluid flow model gives good approximate results even when the service times are exponentially distributed.

It is found that in some cases f_n gradually decreases for $n \geqq n_0$ for some n_0. In such cases Model 1 can be analysed in a similar way. A numerical example is given in Section 5.

4. A Consideration on the " Fluctuation Property " of the Average System Time when the Batch Size changes

In Section 3 we examined the accuracy of the proposed model by comparing the approximate values with the simulated ones(Fig.4). It can be seen in Fig.4 that the average system time of Model 1 fluctuates considerably whereas this fluctuation is nearly negligible in the case of simulation. This discrepancy of the fluctuation property between the simulated and the approximated values leads us to further consideration of Model 1. In this section we focus our attention on the detailed analysis of the " fluctuation property " of the average system time when the batch size changes.

In order to strengthen this phenomena we illustrate in Fig.5 the calculated values from Model 1 when $\nu = 100$, $\Psi = 10$, $\mu_1 = .7$, $\mu_2 = .1$, $\mu_3 = .4$, $\Delta = .4$ and $\lambda = .9$(case ii). This situation may not exist in the real case because the average system time is very high. Fig.5 shows that the average system time varies in a parabolic shape within a certain range of B values. This parabolic shaped " fluctuation property " of the average system time can be verified as follows.

From Fig.5 we may determine a range of B within which the number of cycles m becomes the same. We shall denote this range as $r(i) \leqq B < r(i-1)$ $(i = 1,2,..)$. It can be seen that $r(2) = 13$, $r(3) = 2$, $r(4) = 1$ from Fig.5. That is, the number of cycles is 4 for $1 \leqq B < 2$ and 3 for $2 \leqq B < 13$. Now, we shall assume that B is in $[r(i), r(i-1))$. Then from the equation (3.9) we get that

$$R_i = \nu - B \sum_{n=1}^{i-1} f_n$$

$$= \nu - BF(i-1)$$

where

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$$F(i) = f_1 + f_2 + \dots + f_i$$
$$= \theta_1(1 - \theta_1^i)/(r(1 - \theta_1)) - \theta_2(1 - \theta_2^i)/(r(1 - \theta_2))$$

and $r = 2\theta_1 - \mu_1/\mu_2$. Therefore, the equation (3.12) can be written as

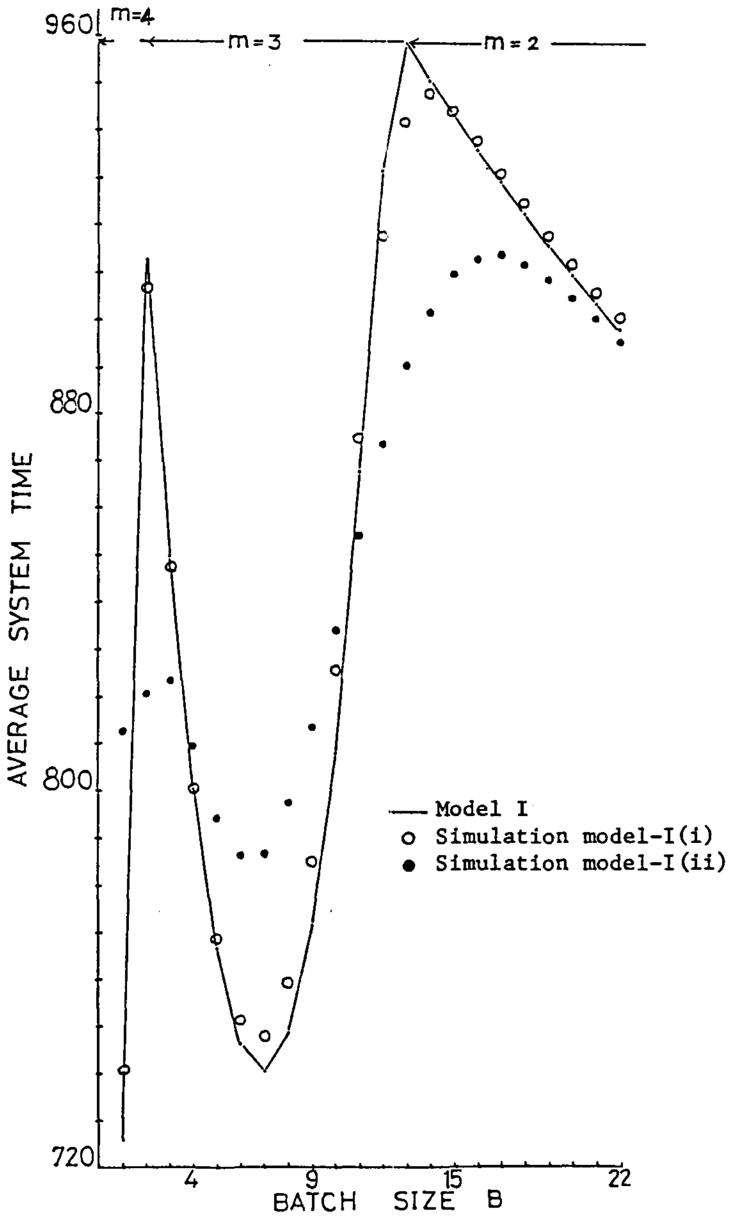

Figure 5. The average system time of the customers
(μ_1=.7, μ_2=.1, μ_3=.4, λ =.9, Δ =.4)

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(4.1) $W = B^2\xi_1 + B\xi_2 + \xi_3$

when $R_i/\mu_2 > Bf_{i-1}/\mu_3$ and X is expressed by (3.11). Here,

(4.2) $\xi_1 = [1/(2\mu_3) + 1/(2\mu_2)][F(i-1)]^2 + [3/(2\mu_3) + 1/(2\mu_2)]G(i-1)$

(4.3) $\xi_2 = \nu/\mu_1 - \Delta H(i-1) - \nu F(i-1)(1/\mu_2 + 1/\mu_3)$

(4.4) $\xi_3 = \nu^2[1/\mu_2 + 1/(2\mu_3)] + \nu\Delta(i+1) - T_c(\nu - \Psi)/2$

are constants . The G(i) and H(i) are given below.

(4.5) $$G(i) = \sum_{n=1}^{i} f_n^2 = \Theta_1^2(1-\Theta_1^{2i})/(r^2(1-\Theta_1^2)) + \Theta_2^2(1-\Theta_2^{2i})/(r^2(1-\Theta_2^2)) - 2\Theta_1\Theta_2(1-\Theta_1^i\Theta_2^i)/(r^2(1-\Theta_1\Theta_2))$$

(4.6) $$H(i) = \sum_{p=1}^{i}\sum_{n=1}^{p} f_n = i(\Theta_1-\Theta_2)/(r(1-\Theta_1)(1-\Theta_2)) + \Theta_2^2(1-\Theta_2^i)/(r(1-\Theta_2)^2) - \Theta_1^2(1-\Theta_1^i)/(r(1-\Theta_1)^2)$$

Thus, (4.1) shows the parabolic shaped "fluctuation property" of W. We can show the same property in the case when X is expressed by (3.8).

Fig.5 illustrates that Model 1 behaves closely with Simulation model-1(i). It can also be seen that the behavioural pattern of Simulation model-1(ii) resembles to Model 1.

5. An Improvement of the Queueing Situation

It can be noticed from Fig.4 that the proposed model gives over-estimated values for some of B. But, usually the fluid flow approximation technique gives under-estimated results. This unusual behaviour of the proposed model is due to the following reason. When f_n rapidly increases with n, the number of customers enters into a particular batch becomes large and as a result of this the customers in that batch wait a long time until the payment. This contributes to the rapid increment of W.

5.I. A " new Policy " to decide the Batch Size

The fact discussed above suggests the phenomena that the average system time can be reduced by preventing the rapid increment of the size of a particular batch. One way of approaching this goal is by limiting

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the batch sizes in each cycle to a constant value. i.e.,we let the cashier accept batches of constant size k, irrespective of the time taken by the officer to authorise the previous batch. We call this policy as " new policy ". It may be possible to think other dynamical policies as well but the new policy is considered for its simplicity and easiness of implementation.

When the batch size is constant, the fluid flow model can be analysed in a similar way as mentioned in Section 3. Let this model be denoted by Model 2. It is to be noted that in Model 2, the expression obtained for the cumulative system time of the customers slightly varies depending on whether $1/\mu_2 \geqq 1/\mu_1 + 1/\mu_3$ (the case of officer bottleneck) or $1/\mu_2 < 1/\mu_1 + 1/\mu_3$ (case of cashier bottleneck).

5.2. Simulation Model-2

In order to examine whether the fluid flow model still fits if the existing process is modified under the new policy , we propose another simulation model namely Simulation model-2. Simulation model-2 is similar to Simulation model-1 except the fact that in each cycle the cashier accepts batches of size equal to a constant value k, irrespective of the time taken by the officer to authorise the previous batch.

Simulation model-2 is further subdivided into Simulation model-2(i) ~ (v) depending on the service time distribution and in this section we consider only the following model and the rest will be appeared in Section 6.

Simulation model-2(i) : the service time at each step is constant at $1/\mu_1$, $1/\mu_2$ and $1/\mu_3$ respectively.

The simulated results obtained from Simulation model-2(i) and the approximate values obtained from Model 2 for two sets of parameter values namely $\nu = 100$, $\Psi = 10$, $\mu_1 = .7$, $\mu_2 = .1$, $\mu_3 = .4$ and $\nu = 100$, $\Psi = 10$, $\mu_1 = 1.6$, $\mu_2 = .7$, $\mu_3 = 1.6$ when $\lambda = .9$ are plotted in Fig.6 and Fig.7.

Fig.6 and Fig.7 show that the proposed fluid flow model gives very good approximate results under the new policy . i.e., the fluid flow model well fits the situation even if the existing process is modified under the new policy . Furthermore, the optimal constant batch size indicated by Model 2 is k = 3(when $\Delta = .4$) and the corresponding minimum average system time is 38.6 minutes. If we let the cashier accept the first batch of size 3 in the existing process, we can see from Fig.4 that Model 1 indicates an average system time of 56.3 minutes. i.e., the

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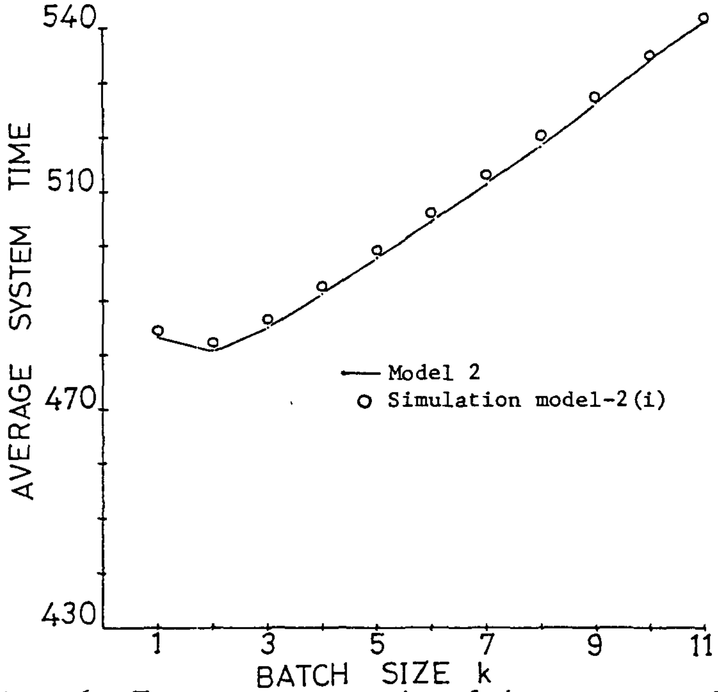

Figure 6. The average system time of the customers under new policy(μ_1=.7, μ_2=.1, μ_3=.4, λ =.9, Δ =.4)

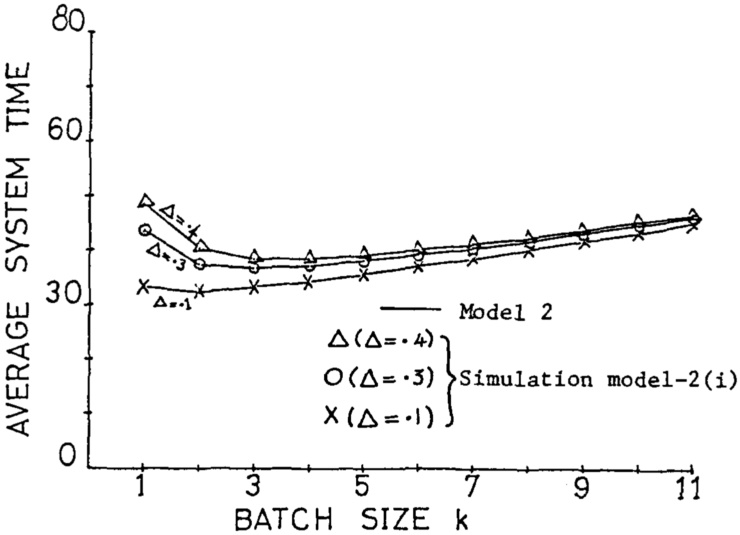

Figure 7. The average system time of the customers under new policy(μ_1= μ_3=1.6, μ_2=.7, λ =.9)

average system time decreases when the existing process is modified under new policy . This minimum value corresponding to the modified process

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is less than even the minimum value 52.9(when B = 1 in Fig.4), that can be expected from the existing process. The simulated results also reflect the same property. Therefore, we can conclude that if the cashier accepts batches of constant size 3 in each cycle, the queueing situation can be improved considerably. It can also be seen in Fig.7 that when Δ decreases, the average system time also decreases considerably for values $k \leq 3$. Moreover, Fig.6 and Fig.7 show that the fluid flow model gives under-estimated results. This fact supports the explanation given at the beginning of Section 5.

In order to examine the efficiency of the new policy when the bottleneck is at the cashier, the minimum average system times are calculated from Model 1 and 2 for some cases and are given in Table.1 along with case(i) and (ii) appeared previously.

Table 1. Minimum average system times for some cases

case	μ_1	μ_2	μ_3	bottleneck	Model 1	Model 2	f_n
i	1.6	.7	1.6	officer	52.9	38.6	rapidly increases
ii	.7	.1	.4	officer	725.5	481.0	rapidly increases
iii	.9	.8	3.1	cashier	42.5	39.0	decreases
iv	3.1	.8	.9	cashier	41.8	37.6	increases
v	1.4	.8	1.4	cashier	37.7	38.4	decreases

It is to be noted that for case(iii), (iv) and (v) $\lambda = .9$, $\Delta = .4$, $\Psi = 10$ and $\nu = 100$ and μ_1, μ_2, μ_3 for case(iii) and (iv) are selected such that the quantity $\lambda(1/\mu_1 + 1/\mu_3)$ calculated for case(iii) and (iv) are equal to the quantity $\lambda(1/\mu_2)$ calculated for case(i).

Table 1 shows that the new policy is effctive when the bottleneck of the system is at the officer. But, when the bottleneck is at the cashier the improvement due to the new policy is not attractive. It is to be noted that from the practical point of view the cashier bottleneck situation is hardly possible to occur due to the nature of work of the officer.

6. A Discussion on the Applicability of the Fluid Flow Model

So far in our analysis we made conclusions on the basis of the assumption that in the practical situation the service time at each step is a constant value. Even though this assumption is reasonable in this

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case, it is of our interest to examine the applicability of the fluid flow model in extreme cases where the service time is expressed by a suitable probability distribution such as exponential distribution. Therefore, in this section we discuss this concept in details. We also analyse in this section, how far the applicability of the fluid flow model depends on the initial conditions and the traffic intensity of the system.

In order to examine the effect of the service time distribution on the fitness of the fluid flow model, we first draw Fig.8 in which the

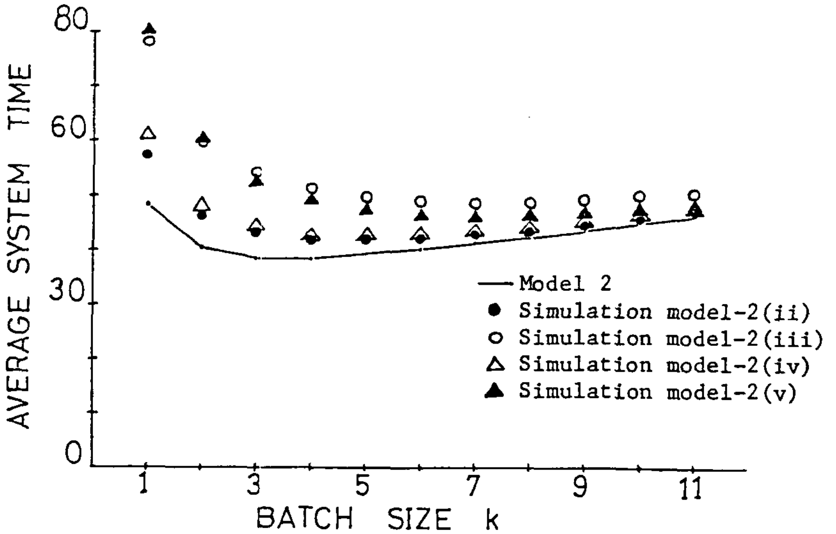

Figure 8. The average system time of the customers ($\mu_1 = \mu_3 = 1.6$, $\mu_2 = .7$, $\lambda = .9$, $\Delta = .4$)

simulated results are obtained from the following simulation models.

Simulation model-2(ii) : service time in step 1 is exponentially distributed with mean $1/\mu_1$ and in step 2 and step 3 it is constant at $1/\mu_2$ and $1/\mu_3$ respectively.

Simulation model-2(iii) : service time in step 2 is exponentially distributed with mean $1/\mu_2$ and in step 1 and step 3 it is constant at $1/\mu_1$ and $1/\mu_3$ respectively.

Simulation model-2(iv) : service time in step 3 is exponentially distributed with mean $1/\mu_3$ and in step 1 and step 2 it is constant at $1/\mu_1$ and $1/\mu_2$ respectively.

Simulation model-2(v) : service time in each step is exponentially distributed with means $1/\mu_1$, $1/\mu_2$ and $1/\mu_3$ respectively.

It can be seen from Fig.8 that the fluid flow model shows a reasonable correspondence with Simulation model-2(ii) and (iv) when $B \geqq 4$ (with

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discrepancy $\Omega < 10\%$. Ω is mentioned below). Furthermore, Simulation model-2(ii) and (iv) indicate a minimum average system time of 42 minutes which is still less than the minimum(52.9 minutes) that can be expected from the existing situation. That is, the new policy is effective in this case too. Moreover, the fluid flow model can be used to determine the optimum batch size and the corresponding minimum average system time approximately since the discrepancy is less arround the optimum value of B.

But, the fluid flow model does not show a good correspondence with Simulation model-2(iii) and (v) and the effect of the new policy is negligible in this case. This implies that the fitness of the fluid flow model seriously depends on the service time distribution specially at the bottleneck step(step 2) of the system.

In order to perform a quantitative analysis on how far the fitness of the fluid flow model depends on the initial conditions and the traffic intensity of the system, we measure the applicability of the model in terms of relative percentage discrepancy Ω, defined as follows.

(6.1) $\Omega = (t_2 - t_1)\text{x}100/t_1$,

where

t_1 = minimum average system time corresponding to Model 2

t_2 = minimum average system time corresponding to simulation

We shall define the traffic intensity of the system as

(6.2) $\rho = \lambda\ (1/\mu_2 + \Delta/k)$

depending on batch size k. The system parameters were set at $\mu_1 = 1.6$, $\mu_2 = .7$, $\mu_3 = 1.6$ and $\Delta = .4$ and Ω is obtained for different values of λ and Ψ. When calculating ρ it is found that in some cases the optimum batch size corresponding to the simulation slightly differs from that calculated from Model 2. Therefore, ρ in (6.2) is calculated using the optimum batch size k corresponding to Model 2. When calculating Ω, t_2 in (6.1) is taken as the simulated value corresponding to the optimum batch size indicated by Model 2. It is to be noted that the simulated value corresponding to the optimum batch size indicated by Model 2 is almost same as the minimum of simulated average system times.

The Ω values calculated are plotted in Fig.9. If we set the applicability range as 5%, then it can be seen from Fig.9 that the fluid flow model is applicable for $\rho \geqq 1.06$ when $\Psi \geqq 8$. This shows the fact that the simple model (fluid flow model) is sufficiently effective for large ρ and Ψ.

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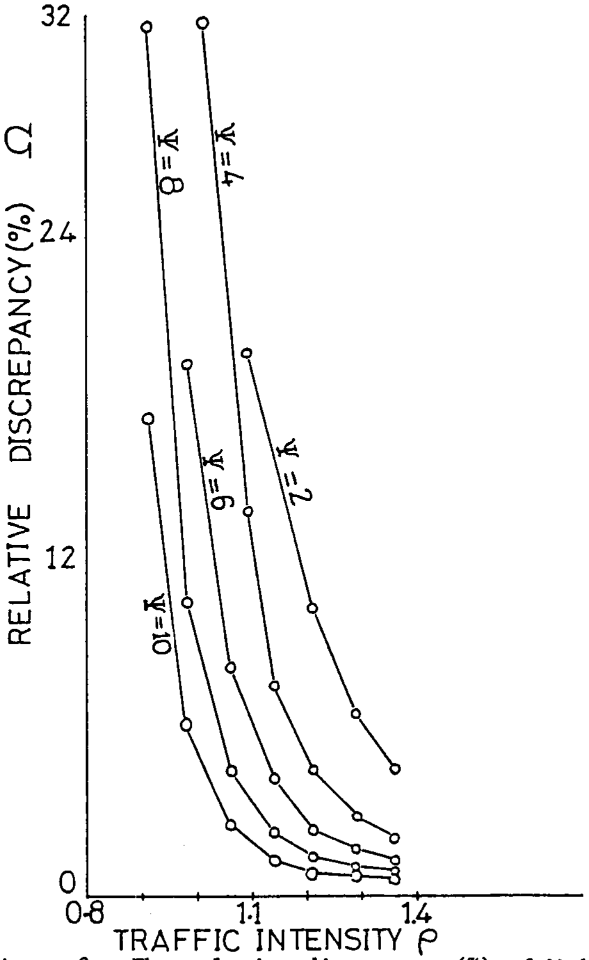

Figure 9. The relative discrepancy(%) of Model 2

7. A Remark on Further Improvement

Further improvement of the queueing situation is possible by increasing the service rates of the servers. The average system times are calculated from Model 2 for different values of k when μ_1, μ_2 and μ_3 take larger values, keeping $\lambda = .9$, $\nu = 100$, $\Psi = 10$, $\Delta = .4$ and the minimum average system times corresponding to each situation are plotted in Fig.10. From Fig.10 it can be calculated that if the officer increases his working rate by 12%[(1/.7 - 1/.8)x100x.7] keeping the cashier's working rate the same (i.e., $\mu_1 = \mu_3 = 1.6$), then the average system time can further be decreased by 24%[(38.6-29.3)x100/38.6]. Further increment of the officer's working rate (i.e.,for $\mu_2 > 0.8$) keeping

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$\mu_1 = \mu_3 = 1.6$, does not reduce the average system time significantly

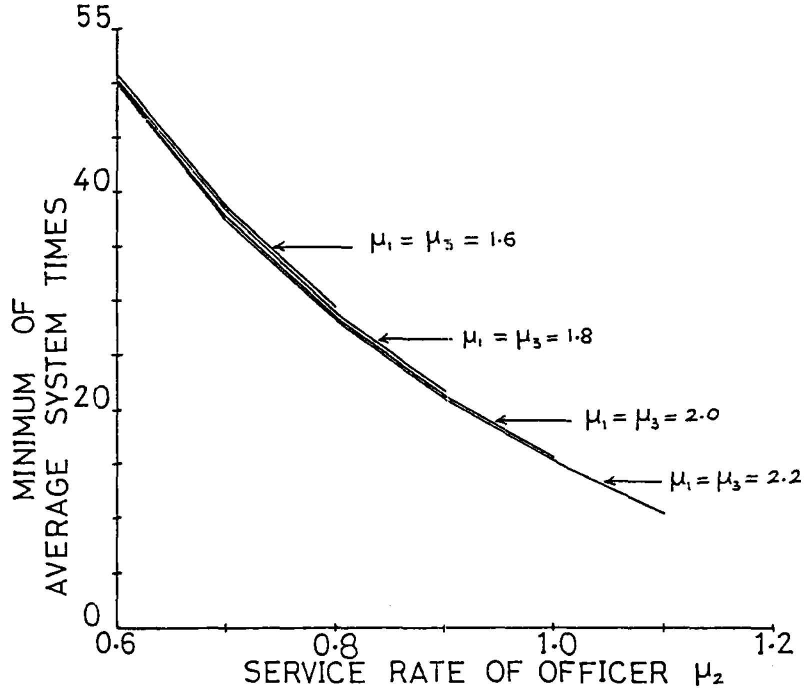

Figure 10. The minimum of average system times

because in this case the bottleneck of the system is at the cashier. Moreover, if the cashier's working rate at step 1 and step 3 are increased by 20%[(1/1.6 - 1/2)x1.6x100] and the working rate of the officer is increased by 24%[(1/.7 - 1/.9)x.7x100], then the average system time can be reduced further by 50% [(38.6-19.3)x100/38.6].

8. Conclusion

In this paper we introduce a new scheme of queueing system in which a single counter is used twice for two kinds of services. The service here is done in batch in three steps and the size of the batch is determined mainly on the service time at step 2. Unlike the ordinary queueing systems, this queueing system is complicated in structure and therefore the probabilistic approach seems to be difficult. Therefore, we first propose a simulation model which can be considered to be close to the behaviour of the existing system. Then a fluid flow model which gives simple analytical results is proposed and it is found that the fluid flow model well fits(Fig.4 and Fig.7) the simulated results. It is also shown that the fluid flow model can be used to analyse(Fig.5) some detailed properties

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of the system. Moreover, the fluid flow model is used to propose a new policy that reduces(Fig.7) the average system time of the customers, in which the batch size is controlled to be a constant. Finally, on the basis of the fluid flow model, we illustrate (Fig.8 and Fig.9) how far the applicability of the fluid flow technique depends on factors such as traffic intensity, initial conditions and the service time distribution of the system. It is found that for low values of Ψ and ρ, the efficiency of the fluid flow model rapidly decreases. The applicability of the fluid flow model also depends on the service time distribution specially at the bottleneck step of the system.

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