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THE $M^X/G/1$ QUEUE WITH FINITE WAITING ROOM

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Abstract The $M^X/G/1$ queue with finite waiting room is studied for both the partially rejected model and the totally rejected model. We start by studying the joint distribution of the number of customers at time t and the remaining service time of customer in service. This gives the asymptotic distribution of the number of customers at an arbitrary moment and immediately after a departure. Also the waiting time distribution is easily obtained.

1. Introduction

Several authors have analyzed the single server queueing systems with finite waiting room. Keilson [7] derived several results on the $M/G/1$ and $GI/M/1$ queue. The $M/G/1$ queue was studied by Cohen [4] and he gave some further references. In recent years, Truslove [10] obtained the asymptotic queue length distribution immediately after a departure for the $E_k/G/1$ queue by using a phase technique. Also he [11] studied the busy period for the $E_k/G/1$ queue. Hokstad [5] studied the joint distribution of the number of phases and the remaining service time of a customer in service for the $E_k/G/1$ queue. He obtained not only the queue length but also the waiting time and the mean length of the idle and busy periods. Ohsone [9] studied the $GI/E_k/1$ queue by using the joint distribution of the number of phases and the elapsed time since the last arrival. He obtained the distributions of the number of customers both at an arbitrary moment and immediately after an arrival.

However, there are a few studies on the batch-arrival, single server queueing systems with finite waiting room (Chu [2], Chu et al. [3] and Van Hoorn [12]). Chu [2] studied $M^X/D/1$ queue with finite waiting room. Especially he analyzed the queueing model whose batch size distribution was geometric. This model often appeared in computer communication systems. Van Hoorn [12] gave a stable recursive method to compute state probabilities both at arrival and at arbitrary epochs for a wide class of single server queues with batch arrivals. This class includes finite waiting room models with state dependent Markovian input. But the queueing models which can be computed by his algorithm are restricted to some special service time distributions.

In this paper, we shall find the joint distribution of the number of customers present and the remaining service time of a customer in service for the $M^X/G/1$ queue with finite waiting room in the steady-state. The steady-state distributions of the number of customers both at an arbitrary moment and immediately after a departure are also derived. Also the steady-state waiting time distribution is obtained. Finally we shall show some interesting properties via numerical analysis.

2. Assumptions and Notations

We consider the $M^X/G/1$ queue with finite waiting room. We assume that the number of waiting places is m. Therfore the maximum number of customers allowed in the system is $m+1$.

For the $M^X/G/1$ queue with finite waiting room, it is assumed that the interarrival times in batch are mutually independent and exponentially distributed with common arrival rate λ. Further, consecutive batch sizes are independent and have the common probability function $\{g_i\}_{i=1}^{\infty}$ with mean $g = \sum_{i=1}^{\infty} i g_i$.

We assume that the service times are independent and identically distributed random variables having a distribution function $B(x)$ $(x \geqq 0)$ with a probability density function (p.d.f.) $b(x)$ $(x \geqq 0)$. We can obtain our results without assuming service time to have a density, but this assumption seems to simplify the argument.

At time t, let $N(t)$ denote the number of customers in the system, and let $U(t)$ denote the remaining service time for the customer in service. The state of the system is defined by $(N(t),U(t))$.

We shall analyze this queueing system with respect to two batch acceptance strategies. One of them is a partially rejected model and the other is a totally rejected model. In a partially rejected model, when an arrival batch is larger in size than the number of available free waiting places, it fills the free positions and the remaining customers of the batch are lost. We shall analyze this model in Section 3. On the other hand, in a totally rejected model, when an arriving batch is larger in size than the number of available free waiting places, the whole batch is rejected. We shall analyze this model in Section 4.

3. Partially Rejected $M^X/G/1$ Queue with Finite Waiting Room

Let us define

$$(3.1)\qquad p_0(t) = P(N(t)=0),\quad p_k(u,t)du = P((N(t)=k)\ \&\ (u<U(t)\leq u+du))$$
$$u \geq 0,\quad k = 1,2,\ldots,m+1.$$

Relating the states of the system at time t and $t + dt$, we obtain by an ordinary argument that under certain regularity assumptions

$$(3.2)\qquad \frac{\partial}{\partial t}p_0(t) = -\lambda p_0(t) + p_1(0,t)$$

$$(3.3)\qquad \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial u}\right)p_1(u,t) = -\lambda p_1(u,t) + p_0(t)\lambda g_1 b(u) + p_2(0,t)b(u)$$

$$(3.4)\qquad \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial u}\right)p_k(u,t) = -\lambda p_k(u,t) + p_0(t)\lambda g_k b(u) + \sum_{i=1}^{k-1} p_i(u,t)\lambda g_{k-i}$$
$$+ p_{k+1}(0,t)b(u),\qquad k = 2,\ldots,m$$

$$(3.5)\qquad \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial u}\right)p_{m+1}(u,t) = p_0(t)\sum_{i=m+1}^{\infty}\lambda g_i b(u) + \sum_{i=1}^{m} p_i(u,t)\sum_{j=m-i+1}^{\infty}\lambda g_j\ .$$

As we will restrict ourselves now to study the steady-state distribution, we let $t \to \infty$, and thus derivatives with respect to t tend to zero in (3.2)-(3.5). Let

$$(3.6)\qquad p_0 = \lim_{t\to\infty} p_0(t),\qquad p_k(u) = \lim_{t\to\infty} p_k(u,t),\qquad k = 1,2,\ldots,m+1,$$

which gives the steady-state distribution of $(N(t),U(t))$. Also define

$$(3.7)\qquad B^*(s) = \int_0^\infty e^{-su}b(u)du = \int_0^\infty e^{-su}dB(u)$$

$$(3.8)\qquad P_k^*(s) = \int_0^\infty e^{-su}p_k(u)du,\qquad k = 1,2,\ldots,m+1.$$

From (3.2)-(3.5) it follows that

$$(3.9)\qquad \lambda p_0 = p_1(0)$$

$$(3.10)\qquad (\lambda - s)P_1^*(s) = p_0\lambda g_1 B^*(s) + p_2(0)B^*(s) - p_1(0)$$

$$(3.11)\qquad (\lambda - s)P_k^*(s) = p_0\lambda g_k B^*(s) + \sum_{i=1}^{k-1} P_i^*(s) + p_{k+1}(0)B^*(s) - p_k(0),$$
$$k = 2,\ldots,m$$

$$(3.12)\qquad -sP_{m+1}^*(s) = p_0\sum_{i=m+1}^{\infty}\lambda g_i B^*(s) + \sum_{i=1}^{m} P_i^*(s)\sum_{j=m-i+1}^{\infty}\lambda g_j - p_{m+1}(0).$$

Inserting $s = 0$ into (3.10), we have

$$(3.13)\qquad \lambda P_1^*(0) = \lambda g_1 p_0 + p_2(0) - p_1(0).$$

Next insert $s = \lambda$ into (3.10). It follows that

$$(3.14)\qquad 0 = \lambda g_1 p_0 B^*(\lambda) + p_2(0)B^*(\lambda) - p_1(0).$$

From (3.9), (3.13) and (3.14), we obtain

$$(3.15)\qquad p_2(0) = \frac{[1 - g_1 B^*(\lambda)]\lambda p_0}{B^*(\lambda)}$$

$$(3.16)\qquad P_1^*(0) = \frac{[1 - B^*(\lambda)]p_0}{B^*(\lambda)} .$$

Next inserting $s = \lambda$ into (3.11), we have

$$(3.17)\qquad p_{k+1}(0) = \frac{p_k(0) - \lambda g_k B^*(\lambda)p_0 - \sum_{i=1}^{k-1} P_i^*(\lambda)\lambda g_{k-i}}{B^*(\lambda)}, \qquad k = 2,\dots,m.$$

Thus we need $P_i^*(\lambda)$, $(i = 1,\dots,k-1)$ in order to get the explicit expressions for $p_k(0)$.

From (3.10) and (3.11), we have

$$(3.18)\qquad P_1^{*(n)}(\lambda) = -\frac{\lambda p_0 B^{*(n+1)}(\lambda)}{(n+1)B^*(\lambda)}, \qquad n = 0,1,\dots,m-2$$

$$(3.19)\qquad P_k^{*(n)}(\lambda) = -\frac{1}{n+1}[\lambda p_0 g_k B^{*(n+1)}(\lambda) + \sum_{i=1}^{k-1} P_i^{*(n+1)}(\lambda)\lambda g_{k-i}$$

$$+p_{k+1}(0)B^{*(n+1)}(\lambda)], \qquad k = 2,\dots,m-1;\ n=0,1,\dots,m-1-k.$$

Here and in the following we let

$$(3.20)\qquad B^{*(n)}(\lambda) = [\frac{d^n}{ds^n}B^*(s)]_{s=\lambda} .$$

$P_k^{*(n)}(\lambda)$ is defined in the same way.

From (3.17)-(3.19), $p_k(0)$, $k = 3,\dots,m,m+1$ are calculated by using p_0.

Inserting $s = 0$ into (3.11) and the derivative of (3.12), we have

$$(3.21)\qquad \lambda P_k^*(0) = \lambda g_k p_0 + \sum_{i=1}^{k-1} P_i^*(0)\lambda g_{k-i} + p_{k+1}(0) - p_k(0), \qquad k = 2,\dots,m$$

$$(3.22)\qquad P_{m+1}^*(0) = -p_0 \sum_{i=m+1}^{\infty} \lambda g_i B^{*(1)}(0) - \sum_{i=1}^{m} P_i^{*(1)}(0) \sum_{j=m-i+1}^{\infty} \lambda g_j .$$

Further differentiate (3.10) and (3.11) and insert $s = 0$. It follows that

$$(3.23)\qquad P_1^{*(1)}(0) = \frac{1}{\lambda}P_1^*(0) + p_0\lambda g_1 B^{*(1)}(0) + p_2(0)B^{*(1)}(0)$$

$$(3.24)\qquad P_k^{*(1)}(0) = \frac{1}{\lambda}P_k^*(0) + p_0\lambda g_k B^{*(1)}(0) + \sum_{i=1}^{k-1} P_i^{*(1)}(0)\lambda g_{k-i}$$

$$+p_{k+1}(0)B^{*(1)}(0), \qquad k = 2,\dots,m.$$

The right hand side of (3.22) will be calculated by (3.23) and (3.24). Since the probability of i $(1 \leqq i \leqq m+1)$ customers present at an arbitrary moment is $P_i^*(0)$ and the probability the system is empty at an arbitrary moment is P_0, we can completely determine the steady-state probabilities at an arbitrary moment with the normalizing condition

$$(3.25) \qquad P_0 + \sum_{i=1}^{m+1} P_i^*(0) = 1.$$

Furthermore the probability that i customers remain in the system immediately after a departure is given by

$$(3.26) \qquad q_i = \frac{p_{i+1}(0)}{\sum_{k=1}^{m+1} p_k(0)}, \qquad i = 0,\ldots,m.$$

This quantity is determined by using (3.9), (3.15) and (3.17).

We can also obtain the Laplace transform of the waiting time distribution and the loss probability. Let $W_F^*(s)$ denote the Laplace transform of the waiting time distribution of the first customer of an actual arrived batch. $W_F^*(s)$ is represented by

$$(3.27) \qquad W_F^*(s) = \frac{P_0 + \sum_{i=1}^{m} P_i^*(s)[B^*(s)]^{i-1}}{1 - P_{m+1}^*(0)}.$$

And the loss probability of the first customer of an arriving batch is $P_{m+1}^*(0)$.

Let r_n $(n = 1,2,\ldots)$ be that the probability of an arbitrary customer being in the n-th position of the batch. Burke [1] showed, using a result in the renewal theory, that it is given by

$$(3.28) \qquad r_n = \frac{1}{g}\sum_{i=n}^{\infty} g_i .$$

Let $W_A^*(s)$ denote the Laplace transform of the waiting time distribution of an arbitrary customer of an actual arrived batch. Using r_n, $W_A^*(s)$ is represented by

$$(3.29) \qquad W_A^*(s) = \frac{P_0 \sum_{i=1}^{m} r_i [B^*(s)]^{i-1} + \sum_{i=1}^{m} P_i^*(s) \sum_{j=1}^{m+1-i} r_j [B^*(s)]^{i+j-2}}{P_0 \sum_{i=1}^{m} r_i + \sum_{i=1}^{m} P_i^*(0) \sum_{j=1}^{m+1-i} r_j}$$

The denominator of (3.29) is the probability that an arbitrary customer of an arriving batch can enter the system. Thus, (3.29) is the Laplace transform of the conditional waiting time distribution given that an arbitrary customer of an arriving batch entered the system. Moreover, the loss probability of an arbitrary customer of an arriving batch is

(3.30) $$p_0\sum_{i=m+2}^{\infty} r_i + \sum_{i=1}^{m+1} P_i^*(0) \sum_{j=m+2-i}^{\infty} r_j .$$

4. Totally Rejected $M^X/G/1$ Queue with Finite Waiting Room

Similarly to Section 3, we can construct the partial differential equations in the case of the totally rejected model as follows.

(4.1) $$\frac{\partial}{\partial t} p_0(t) = -\sum_{i=1}^{m+1} \lambda g_i p_0(t) + p_1(0,t)$$

(4.2) $$(\frac{\partial}{\partial t} - \frac{\partial}{\partial u})p_1(u,t) = -\sum_{i=1}^{m} \lambda g_i p_1(u,t) + p_0(t)\lambda g_1 b(u) + p_2(0,t)b(u)$$

(4.3) $$(\frac{\partial}{\partial t} - \frac{\partial}{\partial u})p_k(u,t) = -\sum_{i=1}^{m+1-k} \lambda g_i p_k(u,t) + p_0(t)\lambda g_k b(u)$$
$$+ \sum_{i=1}^{k-1} p_i(u,t)\lambda g_{k-i} + p_{k+1}(0,t)b(u), \quad k = 2,\dots,m,$$

(4.4) $$(\frac{\partial}{\partial t} - \frac{\partial}{\partial u})p_{m+1}(u,t) = p_0(t)\lambda g_{m+1} b(u) + \sum_{i=1}^{m} p_i(u,t)\lambda g_{m+1-i} .$$

We also assume (3.6) to study the steady-state distribution. From (4.1)-(4.4), we have

(4.5) $$\sum_{i=1}^{m+1} \lambda g_i p_0 = P_1(0)$$

(4.6) $$(\sum_{i=1}^{m} \lambda g_i - s)P_1^*(s) = p_0 \lambda g_1 B^*(s) + P_2(0)B^*(s) - P_1(0)$$

(4.7) $$(\sum_{i=1}^{m+1-k} \lambda g_i - s)P_k^*(s) = p_0 \lambda g_k B^*(s) + \sum_{i=1}^{k-1} P_i^*(s)\lambda g_{k-i} + P_{k+1}(0)B^*(s)$$
$$- P_k(0), \quad k = 2,\dots,m,$$

(4.8) $$-sP_{m+1}^*(s) = p_0 \lambda g_{m+1} B^*(s) + \sum_{i=1}^{m} P_i^*(s)\lambda g_{m+1-i} - P_{m+1}(0).$$

In addition, we assume that $g_i \neq 0$ $(i = 1,\dots,m+1)$ to be able to analyze this model. In the case that the batch size distribution is geometric, this assumption is satisfied. Inserting $s = \sum_{i=1}^{m} \lambda g_i$ and $s = 0$ into (4.6) and using (4.5), we have

(4.9) $$P_2(0) = \frac{\lambda p_0[\sum_{i=1}^{m+1} g_i - g_1 B^*(\sum_{j=1}^{m} \lambda g_j)]}{B^*(\sum_{j=1}^{m} \lambda g_j)}$$

$$(4.10)\qquad P_1^*(0) = \frac{\sum_{i=1}^{m+1} g_i p_0[1 - B^*(\sum_{j=1}^{m} \lambda g_j)]}{\sum_{i=1}^{m} g_i B^*(\sum_{j=1}^{m} \lambda g_j)}$$

Next inserting $s = \sum_{i=1}^{m+1-k} \lambda g_i$ into (4.7), we have

$$(4.11)\qquad p_{k+1}(0) = \frac{p_k(0) - \lambda g_k B^*(\sum_{j=1}^{m+1-k} \lambda g_j) - \sum_{i=1}^{k-1} P_i^*(\sum_{j=1}^{m+1-k} \lambda g_j)\lambda g_{k-i}}{B^*(\sum_{j=1}^{m+1-k} \lambda g_j)}, \qquad k = 2,\ldots,m.$$

(4.11) shows that we need $P_i^*(\sum_{j=1}^{m+1-k} \lambda g_j)$ $(i = 1,\ldots,k-1)$ in order to obtain $p_{k+1}(0)$ from $p_k(0)$. By the following two formulae which are obtained from (4.6) and (4.7),

$$(4.12)\qquad P_1^*(\sum_{j=1}^{m+1-k} \lambda g_j) = \frac{p_0 \lambda g_1 B^*(\sum_{j=1}^{m+1-k} \lambda g_j) + p_2(0) B^*(\sum_{j=1}^{m+1-k} \lambda g_j) - p_1(0)}{\sum_{j=m+2-k}^{m} \lambda g_j}$$

$$(4.13)\qquad P_i^*(\sum_{j=1}^{m+1-k} \lambda g_j) = \frac{1}{\sum_{j=m+2-k}^{m+1-i} \lambda g_j}[p_0 \lambda g_i B^*(\sum_{j=1}^{m+1-k} \lambda g_j) + \sum_{\ell=1}^{i-1} P_\ell^*(\sum_{j=1}^{m+1-k} \lambda g_j)\lambda g_{i-\ell}$$

$$+ p_{i+1}(0) B^*(\sum_{j=1}^{m+1-k} \lambda g_j) - p_i(0)], \qquad i = 2,\ldots,k-1,$$

we can calculate $P_i^*(\sum_{j=1}^{m+1-k} \lambda g_j)$ $(i = 1,\ldots,k-1)$ recursively.

Inserting $s = 0$ into (4.7), we have

$$(4.14)\qquad P_k^*(0) = \frac{p_0 \lambda g_k + \sum_{i=1}^{k-1} P_i^*(0)\lambda g_{k-i} + p_{k+1}(0) - p_k(0)}{\sum_{i=1}^{m+1-k} \lambda g_i}, \qquad k = 2,\ldots,m.$$

Differentiating (4.8) with respect to s and inserting $s = 0$, we have

$$(4.15)\qquad P_{m+1}^*(0) = -p_0 \lambda g_{m+1} B^{*(1)}(0) + \sum_{i=1}^{m} P_i^{*(1)}(0)\lambda g_{m-i+1}.$$

Differentiating (4.6) and (4.7) with respect to s, and inserting $s = 0$, we have

$$(4.16)\qquad P_1^{*(1)}(0) = \frac{p_0 \lambda g_1 B^{*(1)}(0) + p_2(0) B^{*(1)}(0) + P_1^*(0)}{\sum_{i=1}^{m} \lambda g_i}$$

(4.17) $$p_k^{*(1)}(0) = \frac{1}{\sum_{i=1}^{m+1-k} \lambda g_i} [p_0 \lambda g_k B^{*(1)}(0) + \sum_{i=1}^{k-1} P_i^{*(1)}(0) \lambda g_{k-i} + p_{k+1}(0) B^{*(1)}(0) + P_k^{*}(0)], \quad k = 2,\ldots,m.$$

Inserting (4.16) and (4.17) into (4.15), we obtain $P_{m+1}^{*}(0)$. Finally, by (4.10), (4.14), (4.15) and the normalizing condition,

(4.18) $$p_0 + \sum_{i=1}^{m+1} P_i^{*}(0) = 1 ,$$

we can determine the steady-state probabilities at an arbitrary moment.

Analogously to Section 3, the probability of i $(0 \leq i \leq m)$ customers in the system immediately after a departure is determined by using (4.5), (4.9) and (4.11).

We can also obtain the Laplace transform of the waiting time distribution and the loss probability. Let $W_B^{*}(s)$ denote the Laplace transform of the waiting time distribution of the first customer of an actual arrived batch. $W_B^{*}(s)$ is represented by

(4.19) $$W_B^{*}(s) = \frac{p_0 \sum_{i=1}^{m+1} g_i + \sum_{i=1}^{m} P_i^{*}(s)[B^{*}(s)]^{i-1} \sum_{j=1}^{m+1-i} g_j}{p_0 \sum_{i=1}^{m+1} g_i + \sum_{i=1}^{m+1} P_i^{*}(0) \sum_{j=1}^{m+1-i} g_j}.$$

The loss probability of an arriving batch is

(4.20) $$p_0 \sum_{i=m+2}^{\infty} g_j + \sum_{i=1}^{m+1} P_i^{*}(0) \sum_{j=m+2-i}^{\infty} g_j.$$

5. Numerical Results

(1) Partially Rejected Model

Table 1 gives numerical values of some measures for the constant and geometric batch sizes for $g = 5$, and $m = 10$.

Table 1. Various measures of the partially rejected $M^X/G/1$ with finite waiting room

PLF : The loss probability of the first customer of the batch

EWF : The mean waiting time of the first customer of an actual arrived batch (mean service time = 1)

PLA : The loss probability of an arbitrary customer

EWA : The mean waiting time of an arbitrary actual arrived customer (mean service time = 1)

ρ : Traffic intensity ($\rho=\lambda g/\mu$, where $1/\mu$ is mean service time)

Constant batch size $M^X/M/1$ with finite waiting room g = 5, m = 10

ρ	PLF	EWF	PLA	EWA
0.2	0.00095	0.71380	0.00815	2.64758
0.4	0.00715	1.60806	0.03717	3.34977
0.6	0.02169	2.58282	0.08760	4.05147
0.8	0.04451	3.53834	0.15310	4.71268
1.0	0.07335	4.40711	0.22502	5.31070
1.2	0.10538	5.15911	0.29616	5.83700
1.4	0.13827	5.79132	0.36220	6.29236
1.6	0.17049	6.31499	0.42120	6.68261
1.8	0.20119	6.74650	0.47310	7.01566
2.0	0.23001	7.10242	0.51812	7.29976

Constant batch size $M^X/D/1$ with finite waiting room g = 5, m = 10

ρ	PLF	EWF	PLA	EWA
0.2	0.00040	0.60260	0.00628	2.55150
0.4	0.00324	1.39989	0.02989	3.18206
0.6	0.01057	2.33234	0.07387	3.84935
0.8	0.02315	3.30569	0.13488	4.51132
1.0	0.04028	4.23109	0.20547	5.13469
1.2	0.06046	5.05191	0.27790	5.69877
1.4	0.08214	5.74688	0.34660	6.19465
1.6	0.10413	6.31934	0.40871	6.62212
1.8	0.12570	6.78467	0.46334	6.98625
2.0	0.14646	7.16166	0.51072	7.29455

Constant batch size $M^X/H_2/1$ with finite waiting room g = 5, m = 10
(H_2 represents a hyperexponential distribution with balanced mean whose coefficient of variation is 2)

ρ	PLF	EWF	PLA	EWA
0.2	0.00360	1.01953	0.01354	2.90774
0.4	0.02142	2.10545	0.05546	3.75347
0.6	0.05361	3.11523	0.11776	4.49270
0.8	0.09465	4.00040	0.18913	5.12434
1.0	0.13899	4.75541	0.26097	5.66042
1.2	0.18289	5.39170	0.32839	6.11545
1.4	0.22431	5.92577	0.38927	6.50288
1.6	0.26239	6.37425	0.44315	6.83422
1.8	0.29693	6.75198	0.49035	7.11901
2.0	0.32809	7.07156	0.53156	7.36508

Geometric batch size $M^X/M/1$ with finite waiting room $g = 5$, $m = 10$

ρ	PLF	EWF	PLA	EWA
0.2	0.00577	0.70620	0.12109	3.35559
0.4	0.01482	1.45284	0.16303	3.76224
0.6	0.02743	2.21237	0.21028	4.17774
0.8	0.04347	2.95788	0.26082	4.59316
1.0	0.06250	3.66667	0.31250	5.00000
1.2	0.08385	4.32260	0.36336	5.39099
1.4	0.10679	4.91650	0.41190	5.76050
1.6	0.13061	5.44562	0.45713	6.10475
1.8	0.15473	5.91120	0.49857	6.42168
2.0	0.17869	6.31770	0.53607	6.71071

Geometric batch size $M^X/D/1$ with finite waiting room $g = 5$, $m = 10$

ρ	PLF	EWF	PLA	EWA
0.2	0.00465	0.62041	0.12000	3.28518
0.4	0.01030	1.29574	0.15917	3.63211
0.6	0.01731	2.00824	0.20298	4.00274
0.8	0.02597	2.73550	0.25013	4.39077
1.0	0.03639	3.45357	0.29939	4.78832
1.2	0.04853	4.14047	0.34906	5.18668
1.4	0.06219	4.77896	0.39759	5.57716
1.6	0.07700	5.35798	0.44373	5.95193
1.8	0.09288	5.87261	0.48664	6.30473
2.0	0.10927	6.32298	0.52587	6.63119

Geometric batch size $M^X/H_2/1$ with finite waiting room $g = 5$, $m = 10$

ρ	PLF	EWF	PLA	EWA
0.2	0.00906	0.94680	0.12416	3.55011
0.4	0.02742	1.85642	0.17300	4.09129
0.6	0.05323	2.69322	0.22762	4.58405
0.8	0.08391	3.44409	0.28378	5.02883
1.0	0.11705	4.10859	0.33848	5.42883
1.2	0.15082	4.69243	0.38993	5.78816
1.4	0.18397	5.20373	0.43725	6.11107
1.6	0.21575	5.65114	0.48016	6.40154
1.8	0.24579	6.04288	0.51877	6.66320
2.0	0.27391	6.38643	0.55334	6.89923

From Table 1, for both constant and geometric batch cases, it is noted that the larger the coefficient of variation of the service time distribution, the larger both PLA and EWA.

(2) Totally Rejected Model

Table 2 gives numerical values of some measures for the geometric batch size for $g = 5$ and $m = 10$.

Table 2. Various measures of the totally rejected $M^X/G/1$ with finite waiting room

PBR : The loss probability of an arbitrary batch

EWB : The mean waiting time of the first customer of an actual arrived batch

ρ : Traffic intensity ($\rho=\lambda g/\mu$, where $1/\mu$ is the mean service time)

Geometric batch size $M^X/G/1$ with finite waiting room $g = 5$, $m = 10$

	$M^X/M/1$		$M^X/D/1$		$M^X/H_2/1$	
ρ	PBR	EWB	PBR	EWB	PBR	EWB
0.2	0.10608	0.37929	0.10559	0.31769	0.10749	0.55584
0.4	0.12822	0.76778	0.12641	0.64374	0.13324	1.09523
0.6	0.15195	1.16859	0.14023	0.98646	0.16176	1.60854
0.8	0.17681	1.55293	0.17084	1.33220	0.19178	2.09076
1.0	0.20236	1.94035	0.19401	1.68140	0.22226	2.53999
1.2	0.22816	2.31895	0.21750	2.03070	0.25244	2.95625
1.4	0.25383	2.68550	0.24105	2.37710	0.28176	3.34073
1.6	0.27905	3.03740	0.26443	2.71727	0.30990	3.69523
1.8	0.30356	3.37307	0.28741	3.04054	0.33664	4.02184
2.0	0.32720	3.69114	0.30981	3.36855	0.36191	4.32274

From Table 2, it is noted that the larger the coefficient of variation of the service time distribution, the larger both PBR and EWB.

6. Conclusion

Recently Manfield et al. [8] analyzed $M^X/M/s$ queue with finite waiting room. In [8] they also investigated $M^X/G/1$ queue with finite waiting room by means of simulation. They said in [8] that the results were clear in showing that all the batch queue performance characteristics were virtually *unaffected* by the service time distribution, provided that the mean batch size is not too small and the most important factor affecting the system performance was the batch size statistics of the arrival process, not only the mean batch size, but also the batch size distribution. (They simulated this model with respect to two service time distributions E_2 and D.)

But the numerical results in Section 5 in this paper show that when the coefficient of variation of the service time distribution is large (e.g. H_2), the effect of the service time distribution cannot be neglected.

Finally, the comment is made that the model analyzed in this paper that the service time distribution is *general* is worthy and applicable to a wider class of problems involving batch arrival process.

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