

ILA STOPPING RULES FOR A SEARCH WITH UNCERTAIN SENSOR CAPABILITY

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Abstract Infinitesimal look-ahead stopping rules are discussed when the detection capability of a search sensor as well as the existence of an object in a given area for search is not known with certainty. We attempt to utilize "dummies" to obtain extra information which is expected to give us better estimate of the detection capability of the sensor. Stopping rules are investigated for two different criteria; that is, maximizing the expected net return and minimizing the expected search time subject to the condition that the preassigned detection probability of the object is ensured. We first show a sufficient condition under which an infinitesimal look-ahead stopping rule is optimal in the case of the expected net-value criterion. Second, we show that under the above condition, an optimal stopping rule for the expected search-time criterion has the same structure as the expected net-value case. Finally, we discuss the efficiency of utilizing "dummies" by numerical examples.

1. Introduction

This paper considers a search and stop model taking an uncertainty of a sensor effectiveness into account as well as the existence of an object in a given area.

The detection capability of the search sensor may be easily affected by the circumstances of the search operation. Such a consideration on a search problem was made earlier by Koopman (see p.529 of [1]). Further discussions on an uncertainty of sweep width of a sensor were given by Richardson and Belkin [3]. They investigated the search and stop problem assuming that the prior target-location distribution and the prior sweep width distribution are known, and derived the optimal stopping time for searching from the view-point of trade-off between the search cost and the target value. In this paper, similar search and stop models for two different stopping criteria are dealt with.

An object is supposed to be, but not surely, in a given area. A searcher attempts to find the object using a sensor whose detection capability is not known with certainty. When the object is detected, the search operation comes to the end. If the object is not detected in spite of a certain duration of search, the searcher becomes uneasy to continue his search. He decides at any time before the detection comes whether to stop the search or not according to his criterion.

To estimate the object position and the sweep width of the sensor, Richardson and Belkin used only the information that the object had not been found out up to the present time. Kisi and Tatsuno [2] proposed a new idea for gaining the extra information for the sensor capability to make stopping more effective. The idea is to put "dummies" for search randomly in the search area before the start of the search operation, the detection of which will enable the searcher to estimate the posterior sensor capability. They discussed a stopping rule based on the posterior probability of the object being in the area.

We discuss here optimal stopping rules which utilize the extra information, and (1) maximize the expected net value, i.e., expected recovery value of the object minus expected cumulative cost of search, (2) minimize the expected search operation time subject to the condition that the preassigned detection probability of the object is assured.

2. General Model for a Search and Stop

The following model is considered in this paper: An object is supposed to be, but not surely, in a given area, say A . Let E be the event that the object really exists in the area A . It is assumed that the possible location of the object is equally probable in the area A , and the prior probability of its existence in A , $P\{E\}=p$, is less than unity. The detection capability of the search sensor is constant through the whole operation but not known in advance. It is assumed that the searcher only knows its prior distribution, $F(\alpha)=P\{Z\leq\alpha\}$, $\alpha>0$, where random variable Z denotes the detection capability of the sensor. Finally, we assume a random search, that is to say

$$(2.1) \quad P\{D\leq t|E, Z=\alpha\} = 1 - \exp(-\alpha t),$$

where the random variable D represents the time from the start of the search

to the detection of the object. Under this assumption, α is the reciprocal of the expected detection time when the object is really in A.

Assumption 1. $E\{Z^{-1}\} < \infty$.

3. The ILA Stopping Rule for the Net Value Criterion

Let us define $V(t)$ be the value of the object detected at time t , and $c(t)$ be the cost rate of the search at time t .

Assumption 2. (i) $V(t)$ is a positive, bounded, continuous and nonincreasing function of t . (ii) $c(t)$ is a positive, bounded, continuous and nondecreasing function of t .

Suppose the search is started. If the object is detected at time t , then the operation comes to the end and the searcher receives the net value, $V(t) - \int_0^t c(s)ds$. If a long time elapses without any detection, the search operation should be stopped before his expected net return goes to zero. In this section, we discuss optimal stopping rules which maximize the expected net value.

First, we study the rule which uses only the information that the object is not detected up to time t , to evaluate the detection capability of the search sensor. Let X be a random variable which takes value 0 when the object has not been detected and value 1 when the object has been detected. From the assumption of the random search, it is easily seen that $\{Y_1(t)=(t, X), t \geq 0\}$ is a continuous time Markov process with a state space $\Omega_1 = [0, \infty) \times \{0, 1\}$. It is assumed that $P\{Y_1(0)=(0, 0)\} = 1$. Let us define

$$(3.1) \quad \begin{aligned} V(t, x) &= \begin{cases} 0, & \text{when } x=0, \\ V(t_0), & \text{when } x=1, \end{cases} \\ c(t, x) &= c(t) \text{ for all } x, \end{aligned}$$

where $t_0 = \inf\{t: Y_1(t)=(t, 1)\}$. We will call any nonnegative real-valued random variable τ , a stopping time (with respect to $\{Y_1(t)\}$), if for all $t \geq 0$ $\{\tau \leq t\}$ is contained in the sigma field generated by $\{Y_1(s), 0 \leq s \leq t\}$. Then, our objective is to find a stopping time τ_1 which maximizes

$$(3.2) \quad E[V(Y_1(\tau_1)) - \int_0^{\tau_1} c(Y_1(s))ds \mid Y_1(0)=(0, 0)].$$

Let G_1 be the infinitesimal operator of $\{Y_1(t), t \geq 0\}$. The infinitesimal operator associated with a function V is given by

$$(3.3) \quad (G_1 V)(t, x) = \lim_{h \rightarrow 0^+} \frac{E[V(Y_1(t+h)) - V(Y_1(t)) \mid Y_1(t) = (t, x)]}{h}$$

$$= \begin{cases} p_t \alpha_t V(t), & \text{when } x=0, \\ 0, & \text{when } x=1, \end{cases}$$

where

$$(3.4) \quad p_t = P\{E > t\},$$

$$(3.5) \quad \alpha_t = \int_0^\infty \alpha dF(\alpha | t) = \int_0^\infty \alpha dP\{Z \leq \alpha \mid D > t, E\}.$$

From Assumptions 1 and 2, we need only consider stopping rules such that $E[\tau_1] < \infty$, and hence the Dynkin's formula is applicable to our problem (more precisely, see Ross [4]). Thus, (3.2) reduces to

$$(3.6) \quad E\left[\int_0^{\tau_1} \{(G_1 V)(Y_1(s)) - c(Y_1(s))\} ds \mid Y_1(0) = (0, 0)\right].$$

Lemma 1. (i) $(G_1 V)(t, x)$ is nonincreasing in x for any fixed t .

(ii) $(G_1 V)(t, x)$ is nonincreasing in t for any fixed x .

Proof: (i) From (3.3), the assertion is obvious. (ii) When $x=1$, it easily follows from (3.3). When $x=0$, it suffices to prove that $p_t \alpha_t$ is non-increasing in t .

$$p_t \alpha_t = p \int_0^\infty \alpha e^{-\alpha t} dF(\alpha) / \int_0^\infty (1 - p + p e^{-\alpha t}) dF(\alpha).$$

By virtue of Schwarz's inequality, it is easily seen that the right-hand side of the above equation is nonincreasing in t . □

Let us define the set B_1 as follows.

$$(3.7) \quad B_1 = \{(t, x) \in \Omega_1 : (G_1 V)(t, x) - c(t) \leq 0\}.$$

From Lemma 1, it follows that B_1 is closed in the sense that

$$P\{\exists h \geq 0 : Y_1(t+h) \notin B_1 \mid Y_1(t) = (t, x)\} = 0 \text{ for all } (t, x) \in B_1.$$

Thus, it is easily seen that an infinitesimal look-ahead (ILA) stopping rule is optimal. That is to say, the searcher stops the search at state (t, x) if and only if the ILA gain is no greater than the stopping gain. (See, for example, Ross [4]). Using the fact that $\lim_{t \rightarrow \infty} p_t \alpha_t = 0$, we obtain the following theorem.

Theorem 1. Let

$$(3.8) \quad \tau_1^* = \begin{cases} \min\{t : Y_1(t) \in B_1\}, & \text{if } pE[Z] > c(0)/V(0), \\ 0, & \text{if } pE[Z] \leq c(0)/V(0). \end{cases}$$

Then the rule characterized by τ_1^* is an ILA stopping rule which maximizes

the expected net value. (We call this rule Net-value Rule I hereafter.) \square

Corollary. Let s_0 be the unique root of the equation:

$$(3.9) \quad \int_0^\infty (1-p+p(1-\alpha \frac{c(t)}{V(t)})e^{-\alpha t})dF(\alpha) = 0.$$

Then, Net-value Rule I is of the form $\tau_1^* = \min\{s_0, D\}$ if $pE[Z] > c(0)/V(0)$. \square

Note that if $p=1$, then the theorem is equivalent to Theorem 3 in [3].

In carrying out the search operation effectively in such an uncertain environment, it is desirable to obtain the information as much as possible about the detection capability of the sensor during the search operation. In the following, we employ the idea discussed in [2] for gaining the information for the sensor capability to make stopping more effective.

Before initiating the search operation the searcher randomly scatters n false-objects (we call them "dummies" hereafter) having the same signal characteristics as the true object in the area A . Then the searcher starts the search. If a dummy is detected at some time, the searcher counts one, and then restores the dummy into A . Two things are assumed about the restoration of the detected dummy: First, the restoration takes a nonnegative constant cost C_r . Second, the searcher loses the position of the restored dummy immediately, and so the distribution of n dummies is always uniform in A . Let $N(t)$ be the total number of counts up to time t . Then, from the second assumption of the above together with the assumption of a random search, $\{N(t), t \geq 0\}$ is a Poisson Process.

$$(3.10) \quad P\{N(t)=k|Z=\alpha\} = (\alpha t)^k \exp(-\alpha t)/k!, \quad k=0,1,\dots$$

We assume that random variables $N(t)$ and D are independent if conditioned on Z .

Let us define a continuous time Markov process $\{Y_2(t)=[t, N(t), X], t \geq 0\}$ with state space $\Omega_2 = [0, \infty) \times \{0, 1, \dots\} \times \{0, 1\}$. We assume that $P\{Y_2(0)=[0, 0, 0]\} = 1$. In this case, the value resulted from the detection of the object and the search cost rate are to be

$$V(t, k, x) = \begin{cases} -kC_r, & \text{when } x=0, \\ V(t_0) - kC_r, & \text{when } x=1, \end{cases}$$

for any k , and $c(t, k, x) = c(t)$ for all k and x , where $t_0 = \inf\{t: Y_2(t)=[t, \cdot, 1]\}$. Let G_2 be an infinitesimal operator of $\{Y_2(t), t \geq 0\}$. The infinitesimal operator associated with a function V is given by

$$(3.11) \quad (G_2 V)(t, k, x) = \begin{cases} p_t, k^\alpha t, k^{V(t) - \alpha t}, k C_r, & \text{when } x=0, \\ 0, & \text{when } x=1, \end{cases}$$

where

$$(3.12) \quad p_{t,k} = P\{E|D>t, N(t)=k\} = \frac{\int_0^\infty \alpha^k e^{-(n+1)\alpha t} dF(\alpha)}{\int_0^\infty (1-p+pe^{-\alpha t}) \alpha^k e^{-n\alpha t} dF(\alpha)},$$

$$(3.13) \quad \alpha_{t,k} = \int_0^\infty \alpha dP\{Z \leq \alpha | D > t, N(t) = k, E\} = \frac{\int_0^\infty \alpha^{k+1} e^{-(n+1)\alpha t} dF(\alpha)}{\int_0^\infty \alpha^k e^{-(n+1)\alpha t} dF(\alpha)}.$$

Then our problem is again to find a stopping rule τ_2 which maximizes

$$(3.14) \quad E\left[\int_0^{\tau_2} \{(G_2V)(Y_2(s)) - c(Y_2(s))\} ds \mid Y_2(0) = [0, 0, 0]\right].$$

Let $B_2 = \{[t, k, x] \in \Omega_2 : (G_2V)(t, k, x) - c(t) \leq 0\}$. If B_2 is closed, then an ILA stopping rule characterized by

$$(3.15) \quad \tau_2^* = \begin{cases} \inf\{t : Y_2(t) \in B_2\}, & \text{when } pV(0) - nC_r > E[Z]^{-1}c(0), \\ 0, & \text{when } pV(0) - nC_r \leq E[Z]^{-1}c(0), \end{cases}$$

is optimal. Unfortunately, B_2 is not always closed because of the non-monotonicity of $(G_2V)(t, k, 0)$ in k for any fixed t . In the following, we give a sufficient condition under which τ_2^* given by (3.15) is optimal. For each $k = 0, 1, \dots$, let

$$(3.16) \quad t_k = \begin{cases} \inf\{t \geq 0 : (G_2V)(t, k, 0) - c(t) \leq 0\}, & \text{when } pV(0) - nC_r > E[Z]^{-1}c(0), \\ 0, & \text{when } pV(0) - nC_r \leq E[Z]^{-1}c(0). \end{cases}$$

If $\{t \geq 0 : (G_2V)(t, k, 0) - c(t) \leq 0\} = \emptyset$, then t_k is defined to be ∞ .

Lemma 2. If $t_0 \geq t_1 \geq \dots$, the set B_2 is closed.

Proof: If $(G_2V)(t, k, 0)$ is nonincreasing in t for any fixed k , and if $t_0 \geq t_1 \geq \dots$, then it is obvious that

$$P\{\exists h \geq 0 : Y_2(t+h) \notin B_2 \mid Y_2(t) = [t, k, x]\} = 0 \text{ for all } [t, k, x] \in B_2.$$

Thus, considering Assumption 2, it suffices to prove that $p_{t,k}$ and $\alpha_{t,k}$ are both nonincreasing in t for any fixed k . Since $p_{t,k}$ was proved to be nonincreasing in [2], we only investigate the monotonicity of $\alpha_{t,k}$ here. Denoting $\int_0^\infty \alpha^k \exp(-n\alpha t) dF(\alpha)$ by $h(t, k)$ for $k = 0, 1, \dots$,

$$\alpha_{t,k} = h(t, k+1)/h(t, k),$$

$$h'(t, k) \equiv \frac{\partial}{\partial t} h(t, k) = -(n+1)h(t, k+1).$$

Then

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_{t,k} &= \{h'(t,k+1)h(t,k) - h(t,k+1)h'(t,k)\} / h^2(t,k) \\ &= (n+1)\{h^2(t,k+1) - h(t,k+2)h(t,k)\} / h^2(t,k), \end{aligned}$$

for $k=0,1,\dots$. Using the Schwarz's inequality, $\partial \alpha_{t,k} / \partial t \leq 0$ for all k . This concludes the proof. □

It should be noted that if $pV(0) - nC_r > E[Z]^{-1}c(0)$ and $\{t \geq 0: (G_2V)(t,k) - c(t) \leq 0\} \neq \emptyset$, then t_k is obtained as the unique root of the equation: $\alpha_{t,k} \{p_{t,k} \times V(t) - nC_r\} = c(t)$, i.e.,

$$\begin{aligned} (3.17) \quad \int_0^\infty \{p[1 - \alpha V(t) / c(t)] e^{-\alpha t} + 1 - p\} \alpha^k e^{-n\alpha t} dF(\alpha) \\ + n \frac{C_r}{c(t)} \alpha_{t,k} \int_0^\infty (1 - p + p e^{-\alpha t}) \alpha^k e^{-n\alpha t} dF(\alpha) = 0, \quad k=0,1,\dots \end{aligned}$$

Let us call the stopping rule characterized by τ_2^* as Net-value Rule II, and $\{t_0 \geq t_1 \geq \dots\}$ as the ILA condition. The next theorem which gives the structure of Net-value Rule II follows easily from Lemma 2.

Theorem 2. Under the ILA condition, Net-value Rule II is optimal. This rule indicates that if the searcher cannot find out the object up to time t , then his decision at time t is

" Continue the search if $t < t_{N(t)}$ ",
 " Stop it if $t \geq t_{N(t)}$ ". □

Remark. (i) If the prior of the detection capability Z is a gamma distribution and the density function $f(\alpha)$ is given by

$$(3.18) \quad f(\alpha) = \begin{cases} \frac{\beta^\lambda}{(\lambda-1)!} \alpha^{\lambda-1} e^{-\beta\alpha}, & \alpha > 0, \\ 0, & \alpha \leq 0, \end{cases}$$

where $\lambda > 1$ is an integer, and if $V(t) \equiv V$, $c(t) \equiv c$, then the ILA condition is satisfied when $(G_2V)(u_0, 0, 0) > c$, where u_0 is the unique root of the equation:

$$(G_2V)(u_0, 0, 0) = (G_2V)(u_0, 1, 0).$$

Fig. 1 shows an example of the ILA condition in the case where $C_r = 0$ and $n=1$. When the point (λ, p) lies to the right of the curve corresponding to the given value of $\beta c / V$, Net-value Rule II is optimal.

(ii) When $C_r = 0$ and $n=0$ ($k=0$), (3.17) is reduced to (3.9); that is, t_0 coincides with s_0 in this case.

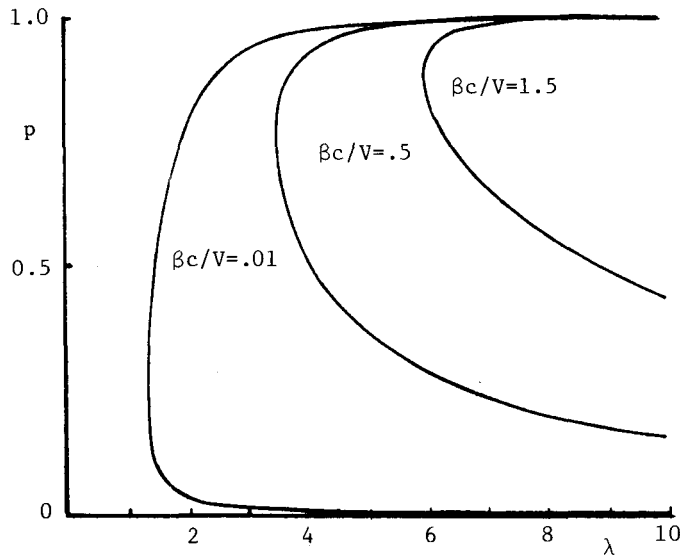


Fig. 1. Example of the region in which ILA rule is optimal
($n=1, C_r=0$)

4. ILA Stopping Rule for the Operation Time Criterion

In this section, we investigate another stopping rule which utilizes n dummies in the same way as the case of Net-value Rule II. The searcher is supposed to adopt the following stopping rule: The search is stopped when the search operation time exceeds a prescribed time T_k ($k=0,1,\dots$), if the object cannot be found out, and if the detections of dummies count k times.

Assumption 3. $\{T_k\}$ is a nonincreasing sequence, and $\lim_{k \rightarrow \infty} T_k = 0$.

Let \mathcal{T} be the class of all the stopping rules $t = \{T_0, T_1, \dots\}$ which satisfy Assumption 3.

Our objective in this section is to find a stopping rule $\{T_k, k=0,1,\dots\} \in \mathcal{T}$ which minimizes the expected search operation time subject to the condition that the preassigned detection probability of the object is assured. Let $D(t)$ be the event that the object is detected under a rule $t = \{T_0, T_1, \dots\}$. Let $S(t)$ be the time at which the search operation terminates under rule t ; that is, the time of detecting the object or stopping the search, whichever comes first. Then our problem is described as follows: Under Assumption 3,

$$(4.1) \quad \begin{aligned} & \text{minimize}_{t \in \mathcal{T}} \int_0^\infty E[S(t) | Z=\alpha] dF(\alpha), \\ & \text{subject to } \int_0^\infty P\{D(t) | Z=\alpha\} dF(\alpha) = \gamma. \end{aligned}$$

4.1. Necessary condition for an optimal stopping rule

It is assumed that the searcher adopts a stopping rule $t \in T$. Then the probabilities conditioned on Z with which the search operation terminates by detecting the object/stopping the search are given in the following. (We neglect the higher order terms of Δt .)

(i) Detecting the object in $(t, t+\Delta t) \subset (T_{k+1}, T_k)$.

$$\Delta P_D^k(t|\alpha) = p \sum_{j=0}^k e^{-\alpha t} \frac{(n\alpha t)^j}{j!} e^{-n\alpha t} \alpha \Delta t.$$

(ii) Stopping the search in $(t, t+\Delta t) \subset (T_{k+1}, T_k)$ by finding a dummy.

$$\begin{aligned} \Delta P_\tau^k(t|\alpha) &= p e^{-\alpha t} \frac{(n\alpha t)^k}{k!} e^{-n\alpha t} n\alpha \Delta t \\ &+ (1-p) \frac{(n\alpha t)^k}{k!} e^{-n\alpha t} n\alpha \Delta t. \end{aligned}$$

(iii) Stopping the search at T_k .

$$P_\tau^k(T_k|\alpha) = p e^{-\alpha T_k} \frac{(n\alpha T_k)^k}{k!} e^{-n\alpha T_k} + (1-p) \frac{(n\alpha T_k)^k}{k!} e^{-n\alpha T_k}.$$

Thus, the conditional probability of detecting the object under rule t is given by

$$(4.2) \quad P\{D(t) | Z=\alpha\} = \sum_{k=0}^{\infty} \int_{T_{k+1}}^{T_k} dP_D^k(t|\alpha) = p\alpha \sum_{k=0}^{\infty} \int_0^{T_k} \frac{(n\alpha t)^k}{k!} e^{-(n+1)\alpha t} dt.$$

Then the expected operation time is given by

$$\begin{aligned} (4.3) \quad E[S(t) | Z=\alpha] &= \sum_{k=0}^{\infty} \int_{T_{k+1}}^{T_k} t \{dP_D^k(t|\alpha) + dP_\tau^k(t|\alpha)\} + \sum_{k=0}^{\infty} T_k P_\tau^k(T_k|\alpha) \\ &= p \sum_{k=0}^{\infty} \int_0^{T_k} \frac{(n\alpha t)^k}{k!} e^{-(n+1)\alpha t} dt + (1-p) \sum_{k=0}^{\infty} \int_0^{T_k} \frac{(n\alpha t)^k}{k!} e^{-n\alpha t} dt. \end{aligned}$$

Let μ denote a Lagrange multiplier. Then, for $k=0,1,\dots$, a necessary condition for optimal T_k is

$$(4.4) \quad \frac{\partial}{\partial T_k} \int_0^{\infty} [E[S(t) | Z=\alpha] - \mu P\{D(t) | Z=\alpha\}] dF(\alpha) = 0.$$

From (4.2) and (4.3), the integrand of (4.4) is given by

$$\sum_{k=0}^{\infty} \int_0^{T_k} \frac{(n\alpha t)^k}{k!} e^{-n\alpha t} \{p(1-\alpha\mu)e^{-\alpha t} + 1 - p\} dt.$$

Differentiating the above by T_k , (4.4) reduces to

$$(4.5) \quad \int_0^\infty \alpha^k e^{-n\alpha T_k} \{p(1-\alpha\mu)e^{-\alpha T_k} + 1 - p\} dF(\alpha) = 0, \quad k=0,1,\dots$$

Then T_k is obtained as a unique root, if it exists, of the above equation. If otherwise, T_k is defined as ∞ . The Lagrange multiplier μ is determined by the condition:

$$\int_0^\infty \left\{ \sum_{k=0}^\infty \int_{T_{k+1}}^{T_k} dF_D^k(t|\alpha) \right\} dF(\alpha) = \gamma,$$

where T_k is given by (4.5) as a function of μ .

Remark. If the Lagrange multiplier μ is replaced by $V(t)/c(t)$ and if $C_r=0$, then (4.5) coincides with (3.17). Thus, it is said that Assumption 3 implies the ILA condition discussed in Sec. 3.

We note that if the searcher utilizes only information that the object has not been detected and if he wishes to minimize the expected operation time subject to the condition that the detection probability γ is assured, then the necessary condition for optimal $\bar{\tau}_1$ is given by

$$\frac{\partial}{\partial \bar{\tau}_1} \int_0^\infty [p\{\alpha \int_0^{\bar{\tau}_1} s e^{-\alpha s} ds + \bar{\tau}_1 e^{-\alpha \bar{\tau}_1}\} + (1-p)\bar{\tau}_1 - \mu p(1 - e^{-\alpha \bar{\tau}_1})] dF(\alpha) = 0.$$

The above is easily reduced to

$$(4.6) \quad \int_0^\infty \{p(1-\mu\alpha)e^{-\alpha \bar{\tau}_1} + 1 - p\} dF(\alpha) = 0.$$

If we set $\mu=V(\bar{\tau}_1)/c(\bar{\tau}_1)$ in (4.6), then we have $\bar{\tau}_1=s_0$. Moreover, if we substitute $n=0$ (consequently $k=0$), then (4.5) reduces to (4.6).

For convenience, we call the stopping rule characterized by $\bar{\tau}_1$ as Operation-time Rule I, and $\{T_k, k \geq 0\}$ as Operation-time Rule II.

4.2. Numerical examples for gamma detection capability

In this subsection, we discuss the effectiveness of utilizing dummies, i.e., that of Operation-time Rule II in comparison with Rule I, numerically. We first assume that the detection capability Z has a gamma distribution and the density function $f(\alpha)$ is given by (3.18). Then the necessary condition (4.5) is represented by

$$(4.7) \quad p\{(n+1)T_k + \beta\} + (1-p)\{(n+1)T_k + \beta\}^{k+\lambda+1} (nT_k + \beta)^{-k-\lambda} - \mu p(k+\lambda) = 0, \quad k=0,1,\dots,$$

and the detection probability of the object (4.2) is rewritten as

$$(4.8) \quad p\left\{1 - \frac{1}{n+1} \sum_{k=0}^\infty \sum_{j=0}^k \binom{n}{n+1} \frac{j(k-j+\lambda-1)!}{(k-j)!(\lambda-1)!} \frac{\beta^\lambda (nT_k)^{k-j}}{\{(n+1)T_k + \beta\}^{k-j+\lambda}} \right\} = \gamma.$$

Thus, from (4.7) and (4.8), Operation-time Rule II $\{T_k, k=0,1,\dots\}$ and μ are determined. Finally we have to check whether $\{T_k, k \geq 0\}$ satisfies Assumption 3. If it does, we have the optimal $\{T_k^*, k=0,1,\dots\}$. If not, we repeat the same procedure for another set of parameters.

In the computation of our examples, however, μ is treated as a parameter as well as p, λ, β and n . When these parameters are given, T_k for each k is calculated from (4.7). Then we compute the expected operation time

$$E[S(t)] = \frac{\beta}{n(\lambda-1)} \sum_{k=0}^{\infty} \sum_{j=k+1}^{\infty} p \left(\frac{n}{n+1}\right)^{k+1} \frac{(\lambda+j-2)!}{(\lambda-2)!j!} \frac{\beta^{\lambda-1} \{(n+1)T_k\}^j}{\{(n+1)T_k+\beta\}^{\lambda+j-1}}$$

$$+ \frac{\beta}{n(\lambda-1)} \sum_{k=0}^{\infty} \sum_{j=k+1}^{\infty} (1-p) \frac{(\lambda+j-2)!}{(\lambda-2)!j!} \frac{\beta^{\lambda-1} (nT_k)^j}{(nT_k+\beta)^{\lambda+j-1}}$$

and the detection capability given by the left-hand side of (4.8). To illustrate the effectiveness of utilizing the dummies, we also calculate the expected operation time for Operation-time Rule I.

$$E[S(\bar{\tau}_1)] = \frac{p\beta}{\lambda-1} \{ 1 - (1+\bar{\tau}_1/\beta)^{-\lambda+1} \} + (1-p)\bar{\tau}_1,$$

where $\bar{\tau}_1$ is given by the root of the equation:

$$(1+\bar{\tau}_1/\beta) \{ 1 + \frac{1-p}{p} (1+\bar{\tau}_1/\beta)^\lambda \} - \mu\lambda/\beta = 0.$$

Some examples are shown in Tables 1 and 2. It appears that they have similar tendency to those discussed in [2]; that is, Operation-time Rule II becomes more efficient than Rule I in the following cases. Case 1: μ is large, i.e., the preassigned detection probability γ is large. Case 2: The coefficient of variation $v=1/\sqrt{\lambda}$ is large, i.e., the degree of uncertainty of the sensor capability is large. As for Assumption 3 which implies the IIA

Table 2. Expected operation time ($p=0.5, \lambda=2, \beta=1$)

μ_1	μ_2	γ	$E[S(\bar{\tau}_1)]$	$E[S(t^*)]$
9559	1000	0.4993	13.26	4.273
306.1	100	0.4930	4.167	2.664
112.1	50	0.4862	2.922	2.190
12.24	10	0.4357	1.215	1.158
1.970	1	0.2335	0.3198	0.3149

*

Table 2. Expected operation time ($p=0.5, \lambda=4, \beta=1$)

	μ_1	μ_2	γ	$E[S(\bar{\tau}_1)]$	$E[S(t^*)]$
n=1 n=2	209.6 249.5	100	0.4977 0.4980	1.586 1.663	1.204 1.163
n=1 n=2	91.00 105.3	50	0.4955 0.4960	1.282 1.336	1.048 1.023
n=1 n=2	12.88 14.11	10	0.4779 0.4795	0.7408 0.7629	0.6931 0.6936
n=1 n=2	2.046 2.071	2	0.3911 0.3923	0.3454 0.3479	0.3445 0.3461

*
*

condition, Fig. 1 shows again the region in which the condition is satisfied, when c/V is replaced by $1/u$. In Tables 1 and 2, the cases marked "*" violate the condition.

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