

OPTIMAL MAINTENANCE POLICY WITH MINIMAL REPAIR FOR STOCHASTICALLY FAILING SYSTEM

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Abstract We consider a Markovian deteriorating complex system which fails with state- dependent probability. The following two features on cost associated with maintaining such a complex system seem to be reasonable: Replacing a whole system at any failure occasion is more expensive than minimally repairing and using it again. And a replacement at a failure is more costly than a planned replacement of an unfailed system.

This paper develops both countable infinite- and finite-state minimal repair models taking into account the above two features, and discusses a maintenance policy which minimizes the expected total discounted cost. It is shown that the optimal policy is of (i, I) type under some conditions on the transition probabilities and the cost structure.

1. Introduction

In maintaining a deteriorating complex system, the following two features on cost seem to be reasonable: A replacement of a whole system at any failure occasion is more expensive than minimally repairing and using it again. A replacement at a failure occasion is more costly than a planned replacement of an unfailed system.

Barlow and Hunter [1] introduced a minimal repair model taking into account the above situations, and discussed the optimality of so called Policy II (periodic replacement policy with minimal repair). After that, Policy III was proposed to improve Policy II [2], and (t,T)-Policy was studied to generalize Policy II by introducing a breakdown cost [5]. In those models, the age of the system and/or the number of failures were used as a state variable, but no further information on the state of the system was available.

In this paper, we consider a similar minimal repair model, but the infor-

mation on the inner state of the system is assumed to be available. Moreover, the dynamic behavior of the inner state is supposed to be described by an infinite or finite state Markov chain. The infinite state model is intended to present a cumulative discrete damage model. The finite state model is proposed to study the case where the deterioration of performance is classified into finite levels. As a matter of convenience, we will call the underlying state of the system "the deterioration level" in both cases. For a complex system such that a malfunction of any component of a system causes a system's failure, it is preferable to classify the failure states by the deterioration level of the system. We consider the following failure mechanism: At any instant after transition to the deterioration level j , the system fails with a known probability that depends on the level j . Then the state of the system is taken to be in failure with the deterioration level j . Possible maintenance actions are preventive replacement, emergency replacement and minimal repair. There are costs associated with each action. The purpose of this study is to investigate the structure of the maintenance policy minimizing the total expected discounted cost. Under some conditions on the transition probabilities and the cost structure, it is shown that an optimal policy has a form of (i,I) type: Perform an emergency replacement at the first failure after the deterioration level exceeds the level i , and a minimal repair for the failure up to reaching the level i . Perform a preventive replacement at the first transition to the level which is greater than I without failure at any level between i and I .

We will discuss an infinite state model in Sections 2, 3 and 4, and then a finite case in Section 5.

2. The Minimal Repair Model and Formulation

Let $\{0,1,2,\dots\}$ be a set of underlying "deterioration levels of the system, where "0" means that the system is new and "1", "2", \dots represent the increasing deterioration levels of the system. The sequence of successive deterioration levels of the system is assumed to form a discrete Markov chain $\{X_t, t=0,1,\dots\}$ with stationary transition probabilities $\{q_{ij}\}$.

$$\Pr\{X_{t+1}=j|X_t=i\} = \begin{cases} q_{ij}, & i=0,1,\dots, j=i,i+1,\dots, \\ 0, & \text{otherwise.} \end{cases}$$

At the instant after transition from underlying deterioration level i to level $j>i$, the system survives with probability $1-r_j$, and fails with proba-

bility r_j .

Assumption 1. (i) $\sum_{j=k}^{\infty} q_{ij}$ is nondecreasing in i for $k=0,1,\dots$.

(ii) q_{ii} , $i \in S_0$ is nonincreasing in i .

(iii) r_j is nondecreasing in j .

For convenience, we consider a Markov chain $\{Y_t, t=0,1,\dots\}$ instead of underlying process $\{X_t, t=0,1,\dots\}$. The state space $\Omega=S_0 \cup S_f$ is defined as

$$S_0 = \{ 0, 1, 2, \dots \}: \text{Operable state space,}$$

$$S_f = \{ \delta_1, \delta_2, \dots \}: \text{Failure state space,}$$

where a state $i \in S_0$ means that the underlying deterioration level is i and the system is operating, and a state $\delta_i \in S_f$ represents that the level is i and the system is in failure. Transition probabilities are defined as follows: For $i=0,1,\dots$,

$$\Pr\{Y_{t+1}=j | Y_t=i\} = p_{ij} = \begin{cases} q_{ij}(1-r_j), & j \geq i+1, \\ q_{ii}, & j=i, \\ 0, & \text{otherwise,} \end{cases}$$

$$\Pr\{Y_{t+1}=\delta_j | Y_t=i\} = q_{i\delta_j} = \begin{cases} q_{ij}r_j, & j \geq i+1, \\ 0, & \text{otherwise,} \end{cases}$$

$$\Pr\{Y_{t+1}=\delta_i | Y_t=\delta_i\} = p_{\delta_i\delta_i} = 1.$$

At the beginning of each period $t=0,1,\dots$, one of possible maintenance actions is taken instantaneously. Either a_0 or a_2 is available on a state $i \in S_0$, where a_0 denotes a waiting action, that is, nothing is changed in the current system, and a_2 denotes the action that preventively replaces the system with a new one. We assume that the failed system has to be repaired or replaced immediately. Then either a_1 or a_2 is taken on a state $\delta_i \in S_f$, where a_1 is a minimal repair action, that is, preserving the system at the same deterioration level i , it only removes the failure, and a_2 is the action that replaces the failed system with a new one. There are costs associated with the action taken: Minimal repair at a state δ_i costs $C_m(i)$. Constant costs C_p and C_e are incurred respectively to a preventive replacement and a replacement of the failed system.

Assumption 2. (i) $\{C_m(i), i \geq 1\}$ is a bounded, nondecreasing sequence of i . (ii) $0 < C_p \leq C_e < \infty$. (iii) $0 \leq C_m(i) \leq C_e$.

Our goal is to find an optimal maintenance policy which minimizes the expected total cost discounted by $\alpha \in [0,1)$ over an infinite planning horizon.

The problem stated above is formulated by a discrete-time Markov decision process. Let $V(i)$ denote the expected total discounted cost incurred when the initial state is i and an optimal policy is employed through the planning horizon. We note that $V(\cdot)$ is a bounded function on the state space because of Assumption 2 and $0 \leq \alpha < 1$. Then $V(i)$ and $V(\delta_i)$ satisfy the following functional equations (see Theorem 6.1 in Ross [4]):

$$(2.1) \quad V(i) = \min\{ \alpha W(i), C_p + \alpha W(0) \}, \quad i \in S_o,$$

$$(2.2) \quad V(\delta_i) = \min\{ C_m(i) + \alpha W(i), C_e + \alpha W(0) \}, \quad \delta_i \in S_f,$$

where

$$(2.3) \quad W(i) = \sum_{j=i}^{\infty} \{ p_{ij} V(j) + p_{i\delta_j} V(\delta_j) \}.$$

3. An Optimal Policy Minimizing the Expected Total Discounted Cost

In this section, we first consider such an n -stage problem that the planning horizon is just n periods. Let define $V_n(i)$ to be the expected total discounted cost when one starts from a state i and employs an optimal policy through the remaining n periods. Similar functional equations to (2.1), (2.2) and (2.3) hold for the case.

$$(3.1) \quad V_n(i) = \min\{ \alpha W_{n-1}(i), C_p + \alpha W_{n-1}(0) \}, \quad i \in S_o,$$

$$(3.2) \quad V_n(\delta_i) = \min\{ C_m(i) + \alpha W_{n-1}(i), C_e + \alpha W_{n-1}(0) \}, \quad \delta_i \in S_f,$$

where

$$(3.3) \quad W_n(i) = \sum_{j=i}^{\infty} \{ p_{ij} V_n(j) + p_{i\delta_j} V_n(\delta_j) \}.$$

Lemma 1. For $n=0,1,\dots$, and $i=1,2,\dots$, $V_n(\delta_i) \geq V_n(i)$.

Proof: From (3.1) and (3.2), there are four possible cases:

$$V(\delta_i) - V(i) = \begin{cases} C_e - C_p, & \text{if } C_e + \alpha W(0) \leq C_m(i) + \alpha W(i) \text{ and } C_p + \alpha W(0) \leq \alpha W(i), \\ C_m(i), & \text{if } C_m(i) + \alpha W(i) \leq C_e + \alpha W(0) \text{ and } \alpha W(i) \leq C_p + \alpha W(0), \\ C_e + \alpha W(0) - \alpha W(i), & \text{if } C_e + \alpha W(0) \leq C_m(i) + \alpha W(i) \text{ and } \alpha W(i) \leq C_p + \alpha W(0), \\ C_m(i) + \alpha W(i) - C_p - \alpha W(0), & \text{if } C_m(i) + \alpha W(i) \leq C_e + \alpha W(0) \text{ and } C_p + \alpha W(0) \leq \alpha W(i). \end{cases}$$

The statement of the lemma is obvious in the first two cases from Assumption 2. In the third case, $C_e + \alpha W(0) - \alpha W(i) \geq C_e - C_p \geq 0$ since $\alpha W(i) \leq C_p + \alpha W(0)$. Similarly, $C_m(i) + \alpha W(i) - C_p - \alpha W(0) \geq C_m(i) \geq 0$ in the last case since $C_p + \alpha W(0) \leq \alpha W(i)$. \square

In order to prove Lemma 3 below, we prepare some notations and a lemma. Let $a_{i,j} = 1$ if $i=j$; $a_{i,j} = 0$ if otherwise, and for two functions $h_1(j)$ and $h_2(j)$ put

$$(3.4) \quad G_i(j) = \begin{cases} [1 - (1 - a_{i,j})r_j]h_1(j) + (1 - a_{i,j})r_j h_2(j), & \text{when } j \geq i, \\ 0, & \text{when } -1 \leq j < i, \end{cases}$$

for $i \in S_0$. Let $\Delta_i(k) = G_i(k) - G_i(k-1)$ for $k = 0, 1, \dots$.

Lemma 2. If functions $h_i(j)$, $i = 1, 2$ satisfy the following two properties, (i) $h_i(j)$ is nondecreasing in j ($i = 1, 2$), (ii) $0 \leq h_1(j) \leq h_2(j)$ for $j = 1, 2, \dots$, then for any fixed i (a) $\Delta_i(k) = \Delta_{i+1}(k)$ for $k \geq i+3$, (b) $\Delta_i(k) \geq 0$ for all k .

Proof: (a) is obvious. As for (b), it suffices to prove that $\Delta_i(k) \geq 0$ for $k \geq i$, since $\Delta_i(k) = 0$ for $k < i$. Let us begin with $k = i$ and $i+1$. It easily follows that $\Delta_i(i) \geq 0$ and $\Delta_i(i+1) \geq 0$ from the properties (i) and (ii). Next for $k \geq i+2$,

$$\begin{aligned} \Delta_i(k) &= (1 - r_k)h_1(k) + r_k h_2(k) - (1 - r_{k-1})h_1(k-1) - r_{k-1} h_2(k-1) \\ &\geq (r_{k-1} - r_k)h_1(k-1) + (r_k - r_{k-1})h_2(k-1) \\ &= (r_k - r_{k-1})(h_2(k-1) - h_1(k-1)) \geq 0. \end{aligned}$$

The first inequality follows from the property (i). Assumption 1-(ii) and the property (ii) guarantee the second inequality. \square

Lemma 3. For any $n = 0, 1, \dots$, $W_n(i)$ is a monotone nondecreasing function of $i \in S_0$.

Proof: Using an induction, we shall prove that $W_n(i)$ as well as $V_n(i)$ and $V_n(\delta_i)$ is nondecreasing in i .

When $n = 0$, the lemma is true since $W_0(i) = V_0(i) = V_0(\delta_i) = 0$ for all i . We assume that $W_{n-1}(i)$ is nondecreasing in i . Considering the equations (3.1), (3.2) and Lemma 1, it follows that $V_n(i)$ and $V_n(\delta_i)$ satisfy the two properties required for $h_1(i)$ and $h_2(i)$ in Lemma 2. Let us define

$$G_{i,n}(j) = \begin{cases} [1 - (1 - a_{i,j})r_j]V_n(j) + (1 - a_{i,j})r_j V_n(\delta_j), & j \geq i, \\ 0, & -1 \leq j < i, \end{cases}$$

and $\Delta_{i,n}(k) = G_{i,n}(k) - G_{i,n}(k-1)$, then two assertions of Lemma 2 hold for $\Delta_{i,n}(k)$.

Since $W_n(i)$ is rewritten as $W_n(i) = \sum_{j=i}^{\infty} a_{i,j} G_{i,n}(j)$, we have

$$\begin{aligned}
 W_n(i+1) - W_n(i) &= \sum_{j=i+1}^{\infty} q_{i+1j} \sum_{k=0}^j \Delta_{i+1,n}(k) - \sum_{j=i}^{\infty} q_{ij} \sum_{k=0}^j \Delta_{i,n}(k) \\
 &= \sum_{k=i+1}^{i+2} \Delta_{i+1,n}(k) \sum_{j=k}^{\infty} q_{i+1j} - \sum_{k=i}^{i+2} \Delta_{i,n}(k) \sum_{j=k}^{\infty} q_{ij} \\
 &\quad + \sum_{k=i+3}^{\infty} \Delta_{i,n}(k) \sum_{j=k}^{\infty} (q_{i+1j} - q_{ij}) \\
 &\geq \Delta_{i+1,n}(i+2) \sum_{j=i+2}^{\infty} q_{i+1j} - \Delta_{i,n}(i+1) \sum_{j=i+1}^{\infty} q_{ij} \\
 &\quad - \Delta_{i,n}(i+2) \sum_{j=i+2}^{\infty} q_{ij} + \Delta_{i+1,n}(i+1) - \Delta_{i,n}(i) \\
 &= \{G_{i+1,n}(i+2) - G_{i,n}(i+1)\} \sum_{j=i+2}^{\infty} (q_{i+1j} - q_{ij}) + \{V_n(i+1) \\
 &\quad - V_n(i)\} q_{ii} + r_{i+1} \{V_n(\delta_{i+1}) - V_n(i+1)\} (q_{ii} - q_{i+1i+1}) \\
 &\geq 0.
 \end{aligned}$$

The first inequality results from $\sum_{k=i+3}^{\infty} \Delta_{i,n}(k) \sum_{j=k}^{\infty} (q_{i+1j} - q_{ij}) \geq 0$. The second inequality is easily seen from Lemmas 1, 2 and Assumption 1-(ii). Thus, $W_n(i)$ is nondecreasing in i for all n . □

Let the set A_o and A_f be such that

$$(3.5) \quad A_o = \{i \in S_o : \alpha[W(i) - W(0)] > C_p\},$$

$$(3.6) \quad A_f = \{\delta_i \in S_f : \alpha[W(i) - W(0)] > C_e - C_m(i)\}.$$

Considering the fact that $\lim_{n \rightarrow \infty} W_n(i) = W(i)$ (see, for example, Sec.6 in Ross [4]), the next theorem directly follows from Lemma 3.

Theorem 1. Let i^* and I^* be the number such that $I^* = \inf\{i : i \in A_o\}$ and $i^* = \inf\{i : \delta_i \in A_f\}$. We use the convention that i^* (or I^*) = ∞ if A_o (or A_f) is empty. Then under Assumptions 1 and 2, an optimal maintenance policy is to be of the form:

- For $i \in S_o$, "do nothing if $0 \leq i < I^*$ ",
- "replace the system with a new one if $i \geq I^*$ ".
- For $\delta_i \in S_f$, "minimally repair it if $0 \leq i < i^*$ ",
- "replace it with a new one if $i \geq i^*$ ". □

Now, we will investigate the relation between the cost structure and the form of an optimal (i, I) type policy. For the sake of simplicity, we restrict our concern to the case where $C_m(i) = C_m$ for all i .

case I: $i^* = 1$ if $C_m = C_e$. (Simple preventive replacement case)

This follows from (2.2) and Lemma 3.

Case II: $i^* < I^*$ if $C_e - C_p < C_m$.

To prove the above, it suffices to show that $k \notin A_o$ for any k such that $\delta_k \notin A_f$. From the definition of A_f , $\alpha W(k) \leq C_e - C_m + \alpha W(0)$ for any $\delta_k \notin A_f$. Then, we have

$\alpha W(k) < C_p + \alpha W(0)$, i.e., $k \notin A_0$,

Case III: $I^* = I^*$ if $C_e - C_p = C_m$.

If $\delta_k \notin A_f$, then $\alpha W(k) \leq C_p + \alpha W(0)$, i.e., $k \in A_0$. On the other hand, if $k \in A_0$, then $C_m + \alpha W(k) \leq C_p + \alpha W(0)$, i.e., $\delta_k \in A_f$. Thus, $I^* = I^*$.

Case IV: $I^* = \infty$ if $C_e = C_p$.

Let us define π to be the policy that takes action a_0 up to the first failure and thereafter coincides an optimal policy. Letting $Q_\pi^n(i, \delta_j)$ be the probability that the process first enters a state $\delta_j \in S_f$ at the n -th step after starting from $i \in S_0$ when one uses the policy π , then the total expected discounted cost $V_n(i)$ is given by

$$\begin{aligned} V_\pi(i) &= \sum_{j=i+1}^{\infty} \sum_{n=1}^{\infty} \alpha^n \cdot Q_\pi^n(i, \delta_j) \cdot V(\delta_j) \\ &\leq \{C_p + \alpha W(0)\} \sum_{j=i+1}^{\infty} \sum_{n=1}^{\infty} \alpha^n \cdot Q_\pi^n(i, \delta_j) \\ &< C_p + \alpha W(0). \end{aligned}$$

On the other hand, $V(i) \leq V_\pi(i)$ since the policy π is not optimal. Hence, we have $V(i) < C_p + \alpha W(0)$ for all $i \in S_0$, that shows $I^* = \infty$.

4. Expected Average Cost Criterion

In this section, we demonstrate that an optimal maintenance policy minimizing the expected average cost exists in the class of (I,I) type policies under Assumptions 1 and 2.

Let $\Phi_\pi(i)$ be the expected average cost when a policy π is employed with the initial state i .

Theorem 2. Under Assumptions 1 and 2, there exists an optimal maintenance policy π^* which minimizes the expected average cost in the class of (I,I) type policies.

Proof: From (2.1), (2.2) and the fact $V(0) = \alpha W(0)$, we have inequalities:

$$0 \leq V(i) - V(0) \leq C_p \quad \text{for all } i \in S_0 \text{ and } \alpha \in [0, 1),$$

$$0 \leq V(\delta_i) - V(0) \leq C_e \quad \text{for all } \delta_i \in S_f \text{ and } \alpha \in [0, 1),$$

which assure the application of Theorem 1.1 in Ross [3] to our problem. By virtue of the theorem, there exists a constant g and an increasing functions $U(i)$ and $U(\delta_i)$ such that

$$(4.1) \quad g + U(i) = \min\{ \tilde{W}(i), C_p + \tilde{W}(0) \}, \quad i \in S_0,$$

$$(4.2) \quad g + U(\delta_i) = \min\{ C_m(i) + \tilde{W}(i), C_e + \tilde{W}(0) \}, \quad \delta_i \in S_f,$$

where $\tilde{W}(i) = \sum_{j=i}^{\infty} q_{ij} \{ [1 - (1 - a_{ij})r_j]U(j) + (1 - a_{ij})r_j U(\delta_j) \}$. Moreover, Theorem 1.2 in Ross [3] ensures that an optimal policy for the expected average cost is to be the policy which chooses the minimizing action in the right hand side of (4.1) and (4.2). Thus, similarly to the expected total discounted cost case, it can be shown that the structure of the optimal policy is to be (i,I) type. \square

In the expected average cost case, the elementary renewal theorem is helpful to obtain an optimal (i,I) policy and the expected average cost under the policy. Let Π denote the class of (i,I) type policies (where $i \leq I$). The expected average cost when one employs $\pi \in \Pi$ and initiates with a new system is given by

$$\Phi_{\pi}(0) = E_{\pi}R/E_{\pi}N,$$

where $E_{\pi}N$ represents the expected time interval between replacements under the policy π , and $E_{\pi}R$ denotes the expected cumulative cost incurred during the interval. According to Theorem 2, an optimal expected average cost when one starts with a new system, $\Phi^*(0)$, is given by

$$\Phi^*(0) = \inf_{\pi \in \Pi} E_{\pi}R/E_{\pi}N.$$

5. Minimal Repair Model with Finite States

We consider the same minimal repair model stated in Section 2 except for the state space. "Deterioration levels" of the system's performance can be classified into one of $0, 1, \dots, n-1$. Let us define $\delta_1, \delta_2, \dots, \delta_n$ to represent failure states. $\delta_i, i=1, 2, \dots, n-1$, represents that the system is in failure at level i . δ_i stands for a complete failure state, and thus, it is assumed that the system is replaced at this state without any consideration. Then we construct a Markov chain $\{\bar{Y}_t, t=0, 1, \dots\}$ with state space $\bar{\Omega} = \bar{S}_0 \cup \bar{S}_f$, where $\bar{S}_0 = \{0, 1, \dots, n-1\}$ and $\bar{S}_f = \{\delta_1, \delta_2, \dots, \delta_n\}$. Let us assume that the transition probabilities of $\{\bar{Y}_t, t=0, 1, \dots\}$ are given by

$$\Pr\{\bar{Y}_{t+1}=j | \bar{Y}_t=i\} = \begin{cases} q_{ij}(1-r_j), & i=0, 1, \dots, n-2; j=i+1, i+2, \dots, n-1, \\ q_{ii}, & i=0, 1, \dots, n-2; j=i, \\ 1-r_n, & i=j=n-1, \\ 0, & \text{otherwise,} \end{cases}$$

$$\Pr\{\bar{Y}_{t+1}=\delta_j | \bar{Y}_t=i\} = \begin{cases} q_{ij}r_j, & i=0, 1, \dots, n-2; j=i+1, i+2, \dots, n-1, \\ r_n, & i=n-1; j=n, \\ 0, & \text{otherwise.} \end{cases}$$

Assumption $\bar{1}$. (i) $\sum_{j=k}^{n-1} q_{ij}$ is nondecreasing in i for $k=0,1,\dots,n-1$.

(ii) q_{ii} is nonincreasing in i .

(iii) r_j is nondecreasing in j .

Let us define $\bar{V}(\cdot)$ which corresponds to $V(\cdot)$ in the infinite model, then

$$(5.1) \quad \bar{V}(i) = \min\{ \alpha\bar{W}(i), C_p + \alpha\bar{W}(0) \}, \quad i \in \bar{S}_0,$$

$$(5.2) \quad \bar{V}(\delta_i) = \min\{ C_m(i) + \alpha\bar{W}(i), C_e + \alpha\bar{W}(0) \}, \quad i=1,2,\dots,n-1,$$

$$(5.3) \quad \bar{V}(\delta_n) = C_e + \alpha\bar{W}(0),$$

where

$$(5.4) \quad \bar{W}(i) = \begin{cases} \sum_{j=i}^{n-1} q_{ij} \{ [1 - (1 - \alpha_{ij})r_j] \bar{V}(j) + (1 - \alpha_{ij})r_j \bar{V}(\delta_j) \}, & i=0,1,\dots,n-2, \\ (1 - r_n) \bar{V}(n-1) + r_n \bar{V}(\delta_n), & i=n-1. \end{cases}$$

We are much concerned about the structure of optimal policies; that is, "Is there an (i,I) type optimal policy in the finite state model?". Our answer is affirmatively given by the next theorem.

Theorem 3. Under Assumptions $\bar{1}$ and 2, an optimal policy is to be an (i,I) type policy.

Proof: It suffices to prove that $\bar{W}(i)$ is nondecreasing in i . Employing the same way as the proof of Lemma 3, it is easily shown that $\bar{W}(i+1) \geq \bar{W}(i)$ for $i=0,1,\dots,n-2$. We will prove $\bar{W}(n-1) \geq \bar{W}(n-2)$ by restricting the planning horizon. The notations used here are similar to those defined in the proof of Lemma 3. Obviously, $\bar{W}_0(n-1) = \bar{W}_0(n-2) = 0$. We assume that $\bar{W}_{m-1}(n-1) \geq \bar{W}_{m-1}(n-2)$, then $\bar{V}_m(n-1) \geq \bar{V}_m(n-2)$ from (5.1). When the remaining periods are m ,

$$\begin{aligned} \bar{W}_m(n-1) - \bar{W}_m(n-2) &= (1 - r_n) \bar{V}_m(n-1) + r_n \bar{V}_m(\delta_n) - q_{n-2, n-2} \bar{V}_m(n-2) \\ &\quad - q_{n-2, n-1} [(1 - r_{n-1}) \bar{V}_m(n-1) + r_{n-1} \bar{V}_m(\delta_{n-1})] \\ &\geq (r_n - q_{n-2, n-1} r_{n-1}) [\bar{V}_m(\delta_{n-1}) - \bar{V}_m(n-1)] \geq 0, \end{aligned}$$

since $\bar{V}_m(\delta_n) \geq \bar{V}_m(\delta_{n-1}) \geq \bar{V}_m(n-1)$ and $q_{n-2, n-2} = 1 - q_{n-2, n-1}$. Considering the fact that $\lim_{m \rightarrow \infty} \bar{W}_m(i) = \bar{W}(i)$ for all $i \in \bar{S}_0$, then the theorem follows. \square

6. Concluding Remarks

In this paper, we discussed the structure of optimal policy for the Markovian deteriorating system with countable infinite or finite state space. As a result, some sufficient conditions under which an optimal policy is to be the form of (i,I) type are Assumptions 1 and 2. We note that these assup-

tions seem to be reasonable in many real situations,

It should be mentioned about the transition from state i to state δ_i . Owing to our modeling of failure mechanism, such a transition is not allowed in our model. One may, however, assert that the failure of the system will occur without regard to the transition of the underlying deterioration level. If the transition from i to δ_i with positive probability is taken into account, then the condition, nonincreasing q_{ii} in i , is omitted from the sufficient condition for the optimality of (i,I) type policy.

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