

INFORMATION EXCHANGE BETWEEN DUOPOLISTIC FIRMS

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Abstract We investigate the information exchange between duopolistic firms in a game theoretical framework. In a duopoly model where each of two firms knows his own cost function but not the rival's, we show that he has an incentive to inform the rival of his cost function, and that two firms agree to inform each other of their cost functions.

1. Introduction

In decision problems under uncertainty, information on uncertain events, i.e., knowledge about what events happened or will happen, influences essentially the optimal strategy of a decision maker and his payoff. The more a decision maker knows about which happened or will happen among possible events, the higher payoff he obtains. That is, more information yields higher payoff in one-person decision problems. In game situations with more than one decision maker, is this also true? In zero-sum two-person games, where two players are completely competitive, it remains true. In non-zero-sum games, however, it is not necessarily true, as Levine and Ponssard [4] and other authors (Basar and Ho [1], Ho and Blau [3]) have already pointed out. The profitability of information for a player depends on what and how much he can know from the information, what and how much other players know, and how all players are interdependent.

Levine and Ponssard investigated the value of information about uncertain events depending on nature. In a class of two-person noncooperative games, where players are confronted with the same uncertainty about nature, they showed that each player cannot necessarily obtain higher payoff even if only he knows what the outcome of nature will be.

In this paper, we will consider a problem of communication between two

players in competitive situations where they can have some information respectively. Is it profitable for each player to convey his information to the other? Moreover, do both players exchange their information without any enforcement?

As one of economic models typically describing the situations above, we consider a quantity duopoly model where each of two firms knows his own cost function but not the rival's. The firms produce and supply the same homogeneous commodity noncooperatively. In real economy, the cost functions of firms will be stochastic because their production functions and/or the prices of their materials are fluctuated by some external factor like weather. So, we assume that the marginal costs of the firms are random variables and that each firm can know the realized value of his own marginal cost but not that of the rival's. Since the firms are confronted with uncertainty about each other's cost functions, they must decide their outputs based on their estimates of each other's costs. As the cost function is very important for each firm, he is supposed to keep it secret from the rival. On the contrary to this reasoning, we will show that each firm has an incentive to inform the rival of his cost function, and that two firms agree to inform each other of their cost functions.

In section 2, we describe in detail the duopoly model sketched above. In section 3, we provide and discuss main results. In section 4, we conclude the paper.

2. Duopoly Model

Firms 1 and 2 produce and supply the same homogeneous commodity. The output of firm i ($i=1,2$) is denoted by x_i , where x_i is a nonnegative real number. We assume that there is no limit of capacity.

Firm i has a linear cost function

$$c_i(x_i) = c_i x_i, \quad c_i > 0, \quad i=1,2.$$

For total output $x_1 + x_2$, the price p of the commodity is determined by the linear market demand function

$$p = [a - b(x_1 + x_2)]^+, \quad a, b > 0. \quad 1)$$

Then firm i obtains the profit

1) $[r]^+ = \max(r, 0)$ for a real number r .

$$J_i(x_i, x_j; c_i) = (p - c_i)x_i, \quad i=1,2.$$

Firms decide their outputs noncooperatively to maximize their profits.

In a real situation, the cost functions of firms and the market demand function will be stochastic. In this paper, we consider the stochastic fluctuation of cost functions only. We assume that the marginal cost c_i of firm i ($i=1,2$) is a random variable, of which the range space is denoted by C_i , and that both firms know the joint probability distribution $F(c_1, c_2)$ of c_1 and c_2 . As the information condition, each of firms can know the realized value of his own marginal cost but not that of the rival's, when he decides his output.

To give an example of our duopoly model, two firms produce and sell the identical goods under linear production functions without any fluctuation. But they buy their materials at respective markets in different regions and the price of the material at each market is stochastically fluctuated by the weather in the region. When each firm decides his output, he knows the price of his own material but not that of his rival's. However, they can know the joint probability distribution of prices of the materials at two markets by the data on the past realized prices.

In our model, since the firms are confronted with the uncertainty about each other's cost functions, they must decide their outputs based on their estimates of each other's costs. If one of firms estimates wrongly the other's cost to be too high, he may supply the commodity excessively. Then, his behavior may damage himself as well as the other with the lowering of the price of the commodity. To avoid such a bad result, the firms can negotiate for the exchange of information about each other's cost functions before their marginal costs are realized. In this paper, we do not consider more cooperative behavior of firms, like joint profit maximization, than the information exchange.

The negotiation of firms determines an information structure, which prescribes whether each firm can know his rival's cost function or not. The information structure is simply represented by $U = [U_1, U_2]$, where U_i ($i=1,2$) is a subset of the set $\{1,2\}$. The U_i means the set of firms whose cost functions firm i can know when he decides his output. Let $C^U_i = \prod_{j \in U_i} C_j$ ($i=1,2$).

The C^U_i can be considered to be the set of messages firm i can receive under the information structure U . An element of C^U_i is denoted by $c^U_i = (c_j : j \in U_i)$. We have already assumed $i \in U_i$ ($i=1,2$). Then the following four information structures can be considered:

$$\begin{aligned}
U^0 &= [U_1^0 = \{ 1 \} , \quad U_2^0 = \{ 2 \}] \\
U^1 &= [U_1^1 = \{ 1, 2 \} , \quad U_2^1 = \{ 2 \}] \\
U^2 &= [U_1^2 = \{ 1 \} , \quad U_2^2 = \{ 1, 2 \}] \\
U^{12} &= [U_1^{12} = \{ 1, 2 \} , \quad U_2^{12} = \{ 1, 2 \}] .
\end{aligned}$$

U^0 is the information structure with which two firms are originally confronted. Under $U^i (i=1,2)$, only firm i can know his rival's cost function, i.e., firm j ($\neq i$) conveys his information unilaterally to firm i . Under U^{12} they exchange their information.

The strategy of firm $i (i=1,2)$ under the information structure U can be considered to be his supply plan which assigns his output to each of his receivable messages under U . The supply plan $x_i [U]$ of firm i under the information structure U is a real-valued bounded measurable function on the message space $C^U i$. The set of all $x_i [U]$ is denoted by $X_i [U]$. If no confusion arises, we omit the U in $x_i [U]$ and $X_i [U]$. For a supply plan pair $x=(x_1, x_2)$ of firms under the information structure U , the expected profit $EJ_i(x)$ of firm i is defined by

$$EJ_i(x) = \int_{C_1} \int_{C_2} J_i(x_1(c_1^U), x_2(c_2^U) ; c_i) dF(c_1, c_2) .$$

We assume that the firms have the risk-neutral utility functions with respect to profit. Then they maximize their expected profits.

Definition. A supply plan pair $x^* [U] = (x_1^* [U], x_2^* [U])$ under the information structure U is called an *equilibrium point under U* if and only if

$EJ_i(x^*) \geq EJ_i(x^*/x_i)$ for all $x_i \in X_i [U]$ and all i , where $x^*/x_i = (x_i, x_j^*) (i \neq j)$. The x_i^* and $EJ_i(x^*)$ are called the *equilibrium supply plan* and the *expected equilibrium profit of firm i under U* , respectively.

We will investigate how the changes of the information structure affect the equilibrium supply plans and the expected equilibrium profits of firms.

Remark 1. Our duopoly model can be formally represented as an extensive game with two stages. In the first stage, two firms negotiate for the information exchange. As a result, an information structure U of the second stage is determined. The second stage has the following moves:

- (1) A chance move selects firms' marginal costs, c_1 and c_2 , according to the probability distribution F .
 - (2) Firm $i (i=1,2)$ decides his output $x_i [U] (c^U i)$, receiving a message $c^U i$.
- The second stage belongs to the class of games with incomplete information

introduced by Harsanyi [2]. Then, the supply plan x_i [U] of firm i is his pure strategy in the second stage with the information structure U . The equilibrium point under U is a Nash equilibrium point in this stage.

3. Main Results

We introduce some notations for two random variables, x and y : $F(x)$, the marginal probability distribution of x ; $F(x|y)$, the conditional probability distribution of x , given y ; $E(x)$, the expectation of x ; $E(x|y)$, the conditional expectation of x , given y ; $V(x)$, the variance of x ; $\text{Cov}(x,y)$, the covariance of x and y .

We assume the following:

$$(A) \quad 0 < K_i < a/2, \text{ where } C_i = [0, K_i], \quad i=1,2.$$

$$(B) \quad (\text{Linear Regression})$$

$$E(c_i|c_j) = \alpha_j c_j + \beta_j, \quad i, j=1,2 \quad (i \neq j),$$

$$\text{where } |\alpha_j| \leq K_i/K_j \text{ and } [-\alpha_j]^+ \leq \beta_j/K_j \leq K_i/K_j - [\alpha_j]^+.$$

The assumption (A) implies that each firm supplies a positive quantity in equilibrium under any information structure. We need the assumption (B) for providing the equilibrium point under U^0 .

Lemma 1. A supply plan pair $x = (x_1, x_2)$ under U^0 is an equilibrium point under U^0 if and only if x satisfies, for $i, j=1, 2 \quad (i \neq j)$,

$$(1) \quad x_i(c_i) = [(a-c_i)/2b - E(x_j|c_i)/2]^+ \quad (\text{a.e.}) \quad 2)$$

Proof: $EJ_i(x)$ can be rewritten as

$$\begin{aligned} EJ_i(x) &= \int_{C_i} \int_{C_j} J_i(x_i(c_i), x_j(c_j); c_i) dF(c_j|c_i) dF(c_i) \\ &= \int_{C_i} E(J_i(x)|c_i) dF(c_i). \end{aligned}$$

Then, for some fixed x_j , x_i maximizes $EJ_i(x)$ if and only if $x_i(c_i)$ maximizes $E(J_i(x)|c_i)$ almost everywhere on C_i .

Now, let us put

$$L_i(m, x_j; c_i) = \int_{C_j} J_i(m, x_j(c_j); c_i) dF(c_j|c_i),$$

- 2) The "a.e." (almost everywhere) means that the equation holds at all points of the space with the possible exception of points of a measure zero set with respect to the relevant probability distribution.

where m is a non-negative real number. Then, noting that

$$[r_1]^+ - [r_2]^+ \leq [r_1 - r_2]^+ \quad \text{for all } r_1, r_2,$$

we obtain

$$\begin{aligned} & I_i(m, x_j; c_i) - I_i(m', x_j; c_i) \\ & \leq \int_{C_j} [(a-bm)m - (a-bm')m' - b(m-m')x_j(c_j)]^+ dF(c_j|c_i) - c_i(m-m'). \end{aligned}$$

Let $a/2b < m \leq a/b$ and $m' = a/b - m (< a/2b)$. Then, we have

$$\begin{aligned} & I_i(m, x_j; c_i) - I_i(m', x_j; c_i) \\ & \leq \int_{C_j} [-b(m-m')x_j(c_j)]^+ dF(c_j|c_i) - c_i(m-m') \\ & = -c_i(m-m') < 0 \quad (c_i > 0), \end{aligned}$$

since $(a-bm)m = (a-bm')m'$.

Hence, if $x_i(c_i)$ maximizes $E(J_i(x)|c_i)$ for the fixed x_j almost everywhere on C_i , we must have $x_i(c_i) \leq a/2b$ (a.e.). Together with the definition of an equilibrium point, this shows that $x=(x_1, x_2)$ is an equilibrium point under U^0 if and only if, for $i, j=1, 2$ ($i \neq j$),

$$\begin{aligned} (2) \quad & x_i(c_i) \leq a/2b \quad (\text{a.e.}), \\ & E(J_i(x)|c_i) = \max_{0 \leq m \leq a/2b} I_i(m, x_j; c_i) \quad (\text{a.e.}). \end{aligned}$$

Since

$$\begin{aligned} I_i(m, x_j; c_i) &= \int_{C_j} \{[a - b(m + x_j(c_j))]^+ m - c_i m\} dF(c_j|c_i) \\ &= \{a - c_i - bm - bE(x_j|c_i)\} m \end{aligned}$$

if $0 \leq m, x_j(c_j) \leq a/2b$ (a.e.), we can easily show that (2) is equivalent to (1).

Q.E.D.

Lemma 2. The unique equilibrium point under U^0 is given by

$$(3) \quad x_i^*(c_i) = \frac{\alpha_i - 2}{b(4 - \alpha_1 \alpha_2)} (c_i - Ec_i) + \frac{a - 2Ec_i + Ec_j}{3b}, \quad i, j=1, 2 \quad (i \neq j). \quad 3)$$

The expected equilibrium profit of firm i under U^0 is given by

3) Supply plans which take the same value almost everywhere are identified.

$$(4) \quad E J_i(x^*) = \frac{(a - 2Ec_i + Ec_j)^2}{9b} + \frac{1}{b} \left(\frac{\alpha_i - 2}{4 - \alpha_1 \alpha_2} \right)^2 V c_i, \quad i, j=1, 2 (i \neq j).$$

Proof: First, we prove that the term in the bracket of (1) is positive for almost all c_i . From (1) and $E(x_i | c_j) \geq 0$ (a.e.),

$$(5) \quad E(x_j | c_i) \leq a/2b \quad (\text{a.e.}) .$$

Then, from (5) and the assumption (A),

$$\begin{aligned} (a - c_i)/2b - E(x_j | c_i)/2 &\geq (a - c_i)/2b - a/4b \\ &\geq (a/2 - K_i)/2b > 0 \quad (\text{a.e.}) . \end{aligned}$$

Hence, from Lemma 1, an equilibrium point $x^* = (x_1^*, x_2^*)$ satisfies

$$(6) \quad x_i(c_i) = (a - c_i)/2b - E(x_j | c_i)/2 \quad (\text{a.e.}), \quad i, j=1, 2 (i \neq j) .$$

Secondly, we prove that (6) has the unique solution. Let us define a mapping T from $X = X_1 \times X_2$ to itself by

$$(T_1 x)(c_1) = (a - c_1)/2b - E(x_2 | c_1)/2$$

$$(T_2 x)(c_2) = (a - c_2)/2b - E(x_1 | c_2)/2$$

where $T(x) = (T_1 x, T_2 x)$. Let $x, \bar{x} \in X$. Then

$$\begin{aligned} (7) \quad |(T_1 x)(c_1) - (T_1 \bar{x})(c_1)| &= |E(x_2 | c_1) - E(\bar{x}_2 | c_1)|/2 \\ &\leq E(|x_2 - \bar{x}_2| | c_1)/2 \quad (\text{a.e.}) \end{aligned}$$

where $|x_2 - \bar{x}_2| (c_2) = |(x_2 - \bar{x}_2)(c_2)|$ for all c_2 .

We introduce the L^1 -norm

$$|| x_i || = \int_{C_i} |x_i(c_i)| \, dF(c_i), \quad i=1, 2 ,$$

on the space X_i . Then, from (7),

$$(8) \quad || T_1 x - T_1 \bar{x} || \leq || x_2 - \bar{x}_2 || / 2 .$$

Similarly we obtain

$$(9) \quad || T_2 x - T_2 \bar{x} || \leq || x_1 - \bar{x}_1 || / 2 .$$

Now, if x and \bar{x} are solutions of (6), $Tx=x$ and $T\bar{x}=\bar{x}$. Then, from (8) and (9),

$$\begin{aligned} ||x_1 - \bar{x}_1|| &= ||T_1 x - T_1 \bar{x}|| \leq ||x_2 - \bar{x}_2|| / 2 \\ &= ||T_2 x - T_2 \bar{x}|| / 2 \leq ||x_1 - \bar{x}_1|| / 4. \end{aligned}$$

Hence $||x_1 - \bar{x}_1|| = 0$, i.e., $x_1 = \bar{x}_1$. Similarly we obtain $x_2 = \bar{x}_2$.

This shows that (6) has the unique solution.

Thirdly, we determine the constants A, B, \bar{A} and \bar{B} so that

$$(10) \quad x_1(c_1) = Ac_1 + B$$

$$(11) \quad x_2(c_2) = \bar{A}c_2 + \bar{B}$$

is the solution of (6). From the assumption (B),

$$(12) \quad E(c_i | c_j) = \alpha_j c_j + \beta_j, \quad i, j=1, 2 \quad (i \neq j).$$

Taking the expectation of both sides of (12), we have

$$(13) \quad E(E(c_i | c_j)) = Ec_i = \alpha_j Ec_j + \beta_j.$$

By eliminating β_j in (12) and (13), we have

$$(14) \quad E(c_i | c_j) = \alpha_j (c_j - Ec_j) + Ec_i.$$

From (6) for $i=2$, (10) and (14), we have

$$(15) \quad x_2(c_2) = (a - c_2)/2b - \{A\alpha_2(c_2 - Ec_2) + AEc_1 + B\} / 2.$$

From (6) for $i=1$, (14) and (15), we have

$$(16) \quad x_1(c_1) = \frac{-2 + \alpha_1 + bA\alpha_1\alpha_2}{4b} c_1 + \frac{1}{2} \left\{ \frac{a}{2b} + \frac{AEc_1 + B}{2} - \frac{1 + bA\alpha_2}{2b} \alpha_1 Ec_1 + \frac{Ec_2}{2b} \right\}.$$

Comparing (16) with (10), we must have

$$(17) \quad \begin{aligned} A &= \frac{-2 + \alpha_1 + bA\alpha_1\alpha_2}{4b} \\ B &= \frac{1}{2} \left\{ \frac{a}{2b} + \frac{AEc_1 + B}{2} - \frac{1 + bA\alpha_2}{2b} \alpha_1 Ec_1 + \frac{Ec_2}{2b} \right\}. \end{aligned}$$

We can show that (17) has the unique solution,

$$(18) \quad \begin{aligned} A &= \frac{\alpha_1 - 2}{b(4 - \alpha_1\alpha_2)} \\ B &= \frac{a - 2Ec_1 + Ec_2}{3b} + \frac{2 - \alpha_1}{b(4 - \alpha_1\alpha_2)} Ec_1. \end{aligned}$$

From (10) and (18), we can obtain (3) for $i=1$. Similarly we can obtain (3) for $i=2$.

Finally, from (6), for the equilibrium point x^* ,

$$\begin{aligned} E(J_i(x^*)|c_i) &= \{a - c_i - bE(x_j^*|c_i) - bx_i^*(c_i)\} x_i^*(c_i) \\ &= bx_i^{*2}(c_i) \quad (\text{a.e.}). \end{aligned}$$

Then

$$(19) \quad EJ_i(x^*) = b \int_{c_i} x_i^{*2}(c_i) dF(c_i) .$$

(4) follows from (3) and (19).

Q.E.D.

Lemma 3. The unique equilibrium point under U^1 is given by

$$(20) \quad x_1^*(c_1, c_2) = (a - 2c_1 + c_2)/3b + \{c_1 - E(c_1|c_2)\} / 6b$$

$$(21) \quad x_2^*(c_2) = \{a - 2c_2 + E(c_1|c_2)\} / 3b .$$

The expected equilibrium profits of firms are given by

$$(22) \quad \begin{aligned} EJ_1(x^*) &= (a - 2Ec_1 + Ec_2)^2/9b + \\ &\quad \{9Vc_1 + 4Vc_2 + 7V(E(c_1|c_2)) - 16\text{Cov}(c_1, c_2)\} / 36b , \end{aligned}$$

$$(23) \quad \begin{aligned} EJ_2(x^*) &= (a - 2Ec_2 + Ec_1)^2/9b + \\ &\quad \{4Vc_2 + V(E(c_1|c_2)) - 4\text{Cov}(c_1, c_2)\} / 9b . \end{aligned}$$

Proof: Similarly to Lemma 1 and the proof of Lemma 2, we can show that an equilibrium point $x^* = (x_1^*, x_2^*)$ under U^1 satisfies

$$(24) \quad x_1(c_1, c_2) = (a - c_1)/2b - x_2(c_2)/2 \quad (\text{a.e.})$$

$$(25) \quad x_2(c_2) = (a - c_2)/2b - E(x_1|c_2)/2 \quad (\text{a.e.}) .$$

Substituting (24) into (25), we obtain

$$(26) \quad x_2(c_2) = (a - c_2)/2b - [\{a - E(c_1|c_2)\} / 2b - x_2(c_2)/2] / 2 .$$

(21) follows from (26). (20) follows from (21) and (24).

Similarly to the proof of Lemma 2, we obtain

$$(27) \quad EJ_1(x^*) = b \int_{c_1} \int_{c_2} x_1^{*2}(c_1, c_2) dF(c_1, c_2) .$$

Substituting (20) into (27) and arranging it, we obtain

$$(28) \quad EJ_1(x^*) = \frac{1}{36b} \int_{c_1} \int_{c_2} \{2(a - 2Ec_1 + Ec_2) - 3(c_1 - Ec_1) + 2(c_2 - Ec_2) + (Ec_1 - E(c_1|c_2))\}^2 dF(c_1, c_2) .$$

Since

$$(29) \quad \int_{c_1} \int_{c_2} (Ec_1 - E(c_1|c_2)) dF(c_1, c_2) = Ec_1 - Ec_1 = 0 ,$$

$$(30) \quad \int_{c_1} \int_{c_2} (c_1 - Ec_1)(Ec_1 - E(c_1|c_2)) dF(c_1, c_2) = -V(E(c_1|c_2)) ,$$

and

$$(31) \quad \int_{c_1} \int_{c_2} (c_2 - Ec_2)(Ec_1 - E(c_1|c_2)) dF(c_1, c_2) = -Cov(c_1, c_2) ,$$

we obtain (22) from (28). Similarly we can obtain (23).

Q.E.D.

Remark 2. (21) can be rewritten as

$$(32) \quad x_2^*(c_2) = (\alpha_2 - 2)(c_2 - Ec_2)/3b + (a - 2Ec_2 + Ec_1)/3b .$$

This equation makes it ease for us to compare $x_2^*[U^1]$ with $x_2^*[U^0]$.

Lemma 4. The unique equilibrium point under U^{12} is given by

$$(33) \quad x_i^*(c_1, c_2) = (a - 2c_i + c_j)/3b , \quad i, j=1,2 \ (i \neq j) .$$

The expected equilibrium profit of firm i is given by

$$(34) \quad EJ_i(x^*) = (a - 2Ec_i + Ec_j)^2/9b + \{4Vc_i + Vc_j - 4Cov(c_1, c_2)\}/9b .$$

Proof: Similarly to (6), (24) and (25), we can show that an equilibrium point $x^* = (x_1^*, x_2^*)$ satisfies

$$(35) \quad x_i(c_1, c_2) = (a - c_i)/2b - x_j(c_1, c_2)/2 \quad (a.e.), \quad i, j=1,2 \ (i \neq j) .$$

It can be easily seen that (35) has the unique solution (33).

Similarly to (19) and (27), we have

$$(36) \quad EJ_i(x^*) = b \int_{c_1} \int_{c_2} x_i^{*2}(c_1, c_2) dF(c_1, c_2) .$$

Then, (34) follows from (33) and (36).

Q.E.D.

From Lemmas 2,3 and 4, we can see how the changes of the information structure affect the equilibrium supply plans of firms. In equilibrium, the average outputs $Ex_i^*[U]$ of firm i ($i=1,2$) are identical for all information structures U , as we can see from (3), (20), (21) and (33). That is, $Ex_i^*[U] = (a - 2Ec_i + Ec_j)/3b$. But, the actual output assigned by his

equilibrium supply plan depends on the information structure U as follows, when a pair of firms' marginal costs is realized.

Under U^0 : Since firm i cannot know the realized value of firm j 's marginal cost c_j ($j \neq i$), firm i cannot decide the output x_i so as to maximize his profit $J_i(x_i, x_j; c_i)$ when $x_j = x_j^*(c_j)$. That is, his actual output $x_i^*(c_i)$ is not optimal to firm j 's output $x_j^*(c_j)$. As we can see from (2), $x_i^*(c_i)$ maximizes his profit when $x_j = E(x_j^* | c_i)$.

Under U^1 : Since firm 1 can know the realized value of c_1 and c_2 , he can always decide his output $x_1^*(c_1, c_2)$ optimal to firm 2's output $x_2^*(c_2)$, as we can see from (24).

Under U^{12} : Both firms can always decide the outputs optimal to each other's outputs, because of perfect information about c_1 and c_2 . Hence, the pair of their outputs, $(x_1^*(c_1, c_2), x_2^*(c_1, c_2))$, is the usual "Cournot equilibrium".

Let ρ be the correlation coefficient between c_1 and c_2 . In the following theorem, we exclude the trivial cases that $\rho = \pm 1$ or $\text{Var } c_1 \cdot \text{Var } c_2 = 0$, in which all or some information structures are essentially same.

Theorem. The changes of the information structure affect the expected equilibrium profits of firms as follows.

- (i) $EJ_1(x^*[U^{12}]) > EJ_1(x^*[U^1]), EJ_1(x^*[U^0]).$
- (ii) $EJ_1(x^*[U^0]) > EJ_1(x^*[U^1])$ if and only if

$$2(16 - \rho^2) / (40 - 7\rho^2) < \rho \sqrt{\text{Var } c_1 / \text{Var } c_2} < 2.$$
- (iii) $EJ_2(x^*[U^{12}]) > EJ_2(x^*[U^1]) \geq EJ_2(x^*[U^0])$, where the equality holds if and only if $\rho \sqrt{\text{Var } c_1 / \text{Var } c_2} = 2$.

Proof: From (14), we have

$$\begin{aligned}
 (37) \quad V(E(c_i | c_j)) &= \int_{c_i} \int_{c_j} \{E(c_i | c_j) - E c_i\}^2 dF(c_i, c_j) \\
 &= \alpha_j^2 \int_{c_i} \int_{c_j} (c_j - E c_j)^2 dF(c_i, c_j) \\
 &= \alpha_j^2 \text{Var } c_j.
 \end{aligned}$$

On the other hand, from (14) and (30), we have

$$\begin{aligned}
 (38) \quad V(E(c_i | c_j)) &= \int_{c_i} \int_{c_j} (c_i - E c_i)(E(c_i | c_j) - E c_i) dF(c_i, c_j) \\
 &= \alpha_j \int_{c_i} \int_{c_j} (c_i - E c_i)(c_j - E c_j) dF(c_i, c_j) \\
 &= \alpha_j \text{Cov}(c_1, c_2) .
 \end{aligned}$$

From (37) and (38), we obtain

$$(39) \quad V(E(c_i | c_j)) = \text{Cov}(c_1, c_2)^2 / V c_j = \rho^2 V c_i ,$$

$$(40) \quad \alpha_j = \text{Cov}(c_1, c_2) / V c_j = \rho \sqrt{V c_i / V c_j} .$$

Substituting (39) and (40) into (4), (22), (23) and (34) and arranging them, we obtain

$$\begin{aligned}
 (41) \quad E J_1(x^*[U^1]) - E J_1(x^*[U^0]) \\
 = \frac{(2\sqrt{V c_1} - \rho \sqrt{V c_2})^2}{b} \left\{ \frac{1}{9} - \frac{1}{(4 - \rho^2)^2} \right\} + \frac{1 - \rho^2}{36b} (4V c_2 - 7V c_1) ,
 \end{aligned}$$

$$(42) \quad E J_2(x^*[U^1]) - E J_2(x^*[U^0]) = \frac{(2\sqrt{V c_2} - \rho \sqrt{V c_1})^2}{b} \left\{ \frac{1}{9} - \frac{1}{(4 - \rho^2)^2} \right\} ,$$

$$\begin{aligned}
 (43) \quad E J_1(x^*[U^{12}]) - E J_1(x^*[U^1]) &= 7\{V c_1 - V(E(c_1 | c_2))\} / 36b \\
 &= 7(1 - \rho^2) V c_1 / 36b ,
 \end{aligned}$$

$$\begin{aligned}
 (44) \quad E J_2(x^*[U^{12}]) - E J_2(x^*[U^1]) &= \{V c_1 - V(E(c_1 | c_2))\} / 9b \\
 &= (1 - \rho^2) V c_1 / 9b .
 \end{aligned}$$

We can prove (i) and (iii) from (41), (42), (43) and (44).

Finally, let us prove (ii). (41) can be rewritten as follows.

$$\begin{aligned}
 (45) \quad E J_1(x^*[U^1]) - E J_1(x^*[U^0]) \\
 = \frac{(1 - \rho^2) V c_2}{9b} \left\{ \left(\frac{2\sqrt{t} - \rho}{4 - \rho^2} \right)^2 (7 - \rho^2) + \frac{4 - 7t}{4} \right\}
 \end{aligned}$$

where $t = V c_1 / V c_2$. The sign of the left side of (45) is identical to the sign of

$$\begin{aligned}
D(\rho, t) &= \left(\frac{2\sqrt{t} - \rho}{4 - \rho^2} \right)^2 (7 - \rho^2) + \frac{4 - 7t}{4} \\
&= \frac{\rho\sqrt{t} - 2}{4(4 - \rho^2)^2} \{ (40 - 7\rho^2) \rho\sqrt{t} - 2(16 - \rho^2) \}.
\end{aligned}$$

It can be easily seen that $D(\rho, t) < 0$ if and only if

$$2(16 - \rho^2)/(40 - 7\rho^2) < \rho\sqrt{t} < 2.$$

This proves (ii).

Q.E.D.

Remark 3. Since U^2 is symmetric to U^1 , we can obtain the inequalities as to firms' expected equilibrium profits under U^0 , U^2 and U^{12} , interchanging 1 and 2 in the inequalities of Theorem.

Putting $\rho = 0$ in Theorem, we can obtain the following result in the case that c_1 and c_2 are independent.

Corollary. If firms' marginal costs c_1 and c_2 are independent, the following inequalities hold for $i = 1, 2$:

$$EJ_i(x^*[U^{12}]) > EJ_i(x^*[U^j]) \quad (j=1,2) > EJ_i(x^*[U^0]) .$$

Our theorem shows the following results. Each of duopolistic firms has an incentive to inform the other of his cost function, and moreover that they can obtain the highest expected equilibrium profits when they exchange their information about cost functions. However, as we can see from (ii) in Theorem, information about each firm's cost function is not always profitable for the other. The profitability of the information depends on both ρ and Vc_i/Vc_j , or equivalently both ρ and $\alpha_{i,j}$. Ranges of ρ and Vc_1/Vc_2 which determine the inequalities between $EJ_i(x^*[U^i])$ and $EJ_i(x^*[U^0])$ are illustrated in Figure 1. In Region 1 including the case that c_1 and c_2 are independent ($\rho=0$), information about each firm's cost function is profitable for the other. In Region 2, however, information about firm 2's cost function is not profitable for firm 1. For example, when $Vc_1/Vc_2 < 0.83$, i.e., the variance of firm 2's marginal cost is relatively larger than that of firm 1's marginal cost, the information is profitable for firm 1, because he confronts with high uncertainty about firm 2's cost function unless he acquires it. On the other hand, when Vc_2 is smaller than Vc_1 to some extent, the information is not profitable for firm 1 in a wide range of ρ . Range 3 is symmetric to Range 2. In Range 4, where ρ and Vc_1/Vc_2 are close to 1, the unilateral conveyance of information is not profitable for either firm receiving it.

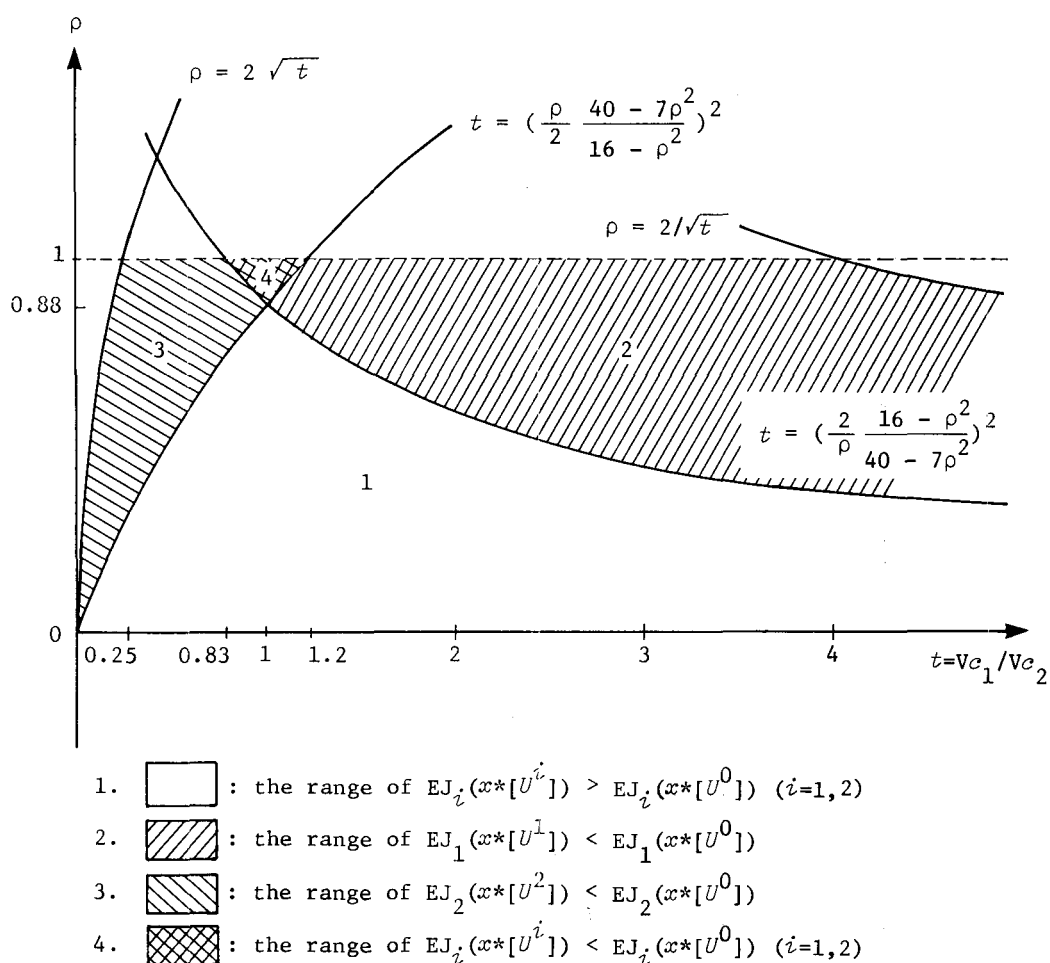


Fig. 1

We discuss the results above with the help of Figure 2. First, we see how a pair of firms' actual outputs assigned by their supply plans changes in equilibrium under each information structure as their marginal costs c_1, c_2 change from the expected values $E c_1, E c_2$ respectively. For simplicity, in Figure 2, we consider the special case that c_1 and c_2 are independent. The lines R_i, R'_i , ($i=1,2$) are the firm i 's "reaction curves" for his marginal cost at the levels of $E c_i, c_i$ respectively, which designate his output optimal to each of firm j 's outputs. The intersection points C of R_1 with R_2 , X^{12}

of R_1' with R_2' , are the "Cournot equilibrium" for firms' corresponding marginal costs. The I_i ($i=1,2$) is the iso-profit curve of firm i through x^{12} . When each firm i 's marginal cost is at the level of Ec_i ($i=1,2$), his actual output is the average output $Ex_i^* = (a - 2Ec_i + Ec_j)/3b$ in equilibrium under every information structure, as we can see from (3), (20), (21) and (33). So, in that case, the firms' output pair is at C in equilibrium under every information structure.

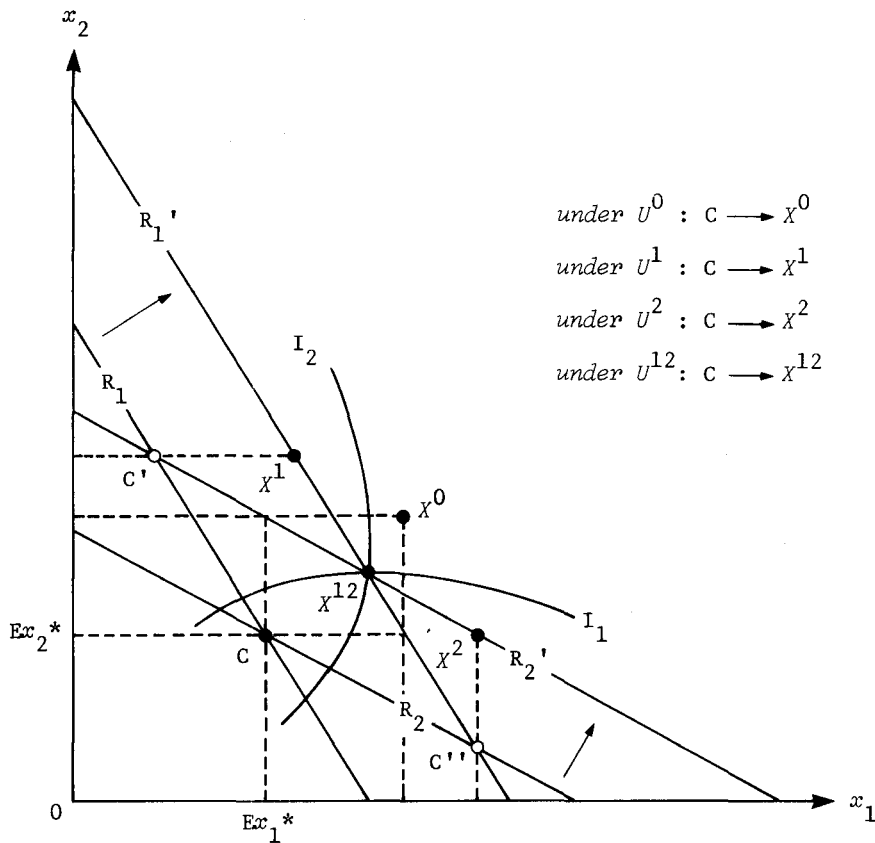


Fig. 2. The change of a pair of firms' outputs in equilibrium under the information structure U

Now suppose that each firm i 's marginal cost decreases from Ec_i to c_i ($i=1,2$). Then, his reaction curve changes from R_i to R_i' . Under U^{12} , firms can decide the Cournot equilibrium outputs because of perfect information, as we have already discussed. So, in equilibrium under U^{12} , their output pair changes from C to X^{12} and they increase their outputs. Under U^1 , since firm 2 does not know the true value c_1 of firm 1's marginal cost, he estimates it at Ec_1 (or $E(c_1|c_2)$, if c_1 and c_2 are dependent). Then, he decides his output $x_2^*[U^1](c_2)$ at C' on the assumption that the firm 1's reaction curve remains unchanged. On the other hand, since firm 1 can know the true reaction curve R_2' of firm 2, he can predict this output of firm 2 and decides his output $x_1^*[U^1](c_1, c_2)$ optimal to it. So, in equilibrium under U^1 , their output pair changes from C to X^1 . Similarly in equilibrium under U^2 , their output pair changes from C to X^2 . Under U^0 , since each firm i does not know the true reaction curve of firm j ($j \neq i$), he can not predict firm j 's output precisely. Firm i only decides his output optimal to firm j 's average output Ex_j^* . So, in equilibrium under U^0 , their output pair changes from C to X^0 . In Figure 2, both firms decide higher outputs at X^0 than at X^{12} and they obtain lower profits.

In general, the output pairs X^0 , X^1 , X^2 and X^{12} under respective information structures U^0 , U^1 , U^2 and U^{12} , are random variables fluctuating around C . Then firms' expected equilibrium profits under these information structures are in proportion to the means square of respective random variables, as we can see from (19), (27) and (36). Since U^2 is symmetric to U^1 , we consider information structures except U^2 . Firms obtain the highest expected equilibrium profits under U^{12} , although they may happen to obtain higher profits under U^0 or U^1 than under U^{12} . Furthermore, as to firm 2, since $|x_2^*[U^0](c_2) - Ex_2^*| \leq |x_2^*[U^1](c_2) - Ex_2^*|$ from (3) for $i=2$ and (32), the mean square of $x_2^*[U^1](c_2)$ is not less than that of $x_2^*[U^0](c_2)$. Hence $EJ_2(x_2^*[U^1]) \geq EJ_2(x_2^*[U^0])$. Finally, we discuss the condition in (ii) of Theorem. This condition is equivalent to $2 > \alpha_2 > 2(16 - \rho^2)/(40 - 7\rho^2) (> 0.8)$. Suppose again that firms' marginal costs decrease. If $0 < \alpha_1 < 1/2$ and $\alpha_2 > 2$, firm 1 may wrongly estimate under U^0 that his marginal cost will decrease more greatly than firm 2's. Then, he may oversupply the commodity with a bullish feeling. His such behavior is considered to cause $EJ_1(x^*[U^0]) < EJ_1(x^*[U^1])$. At the boundary condition $\alpha_2 = 2$, we can easily show that $x_1^*[U^0](c_1) = x_1^*[U^1](c_1, c_2)$ and $x_2^*[U^0](c_2) = x_2^*[U^1](c_2)$. Hence, $EJ_i(x^*[U^0]) = EJ_i(x^*[U^1])$ ($i=1,2$). If α_2 is at the lower level, firm 1 may undersupply the commodity for the contrary reason to the case of $\alpha_2 > 2$. Especially, if $\alpha_1 > 2$ and $0 < \alpha_2 < 1/2$, firm 1 may be so timid as to decrease his output, although

his marginal cost decreases. His such behavior is also considered to cause $EJ_1(x^*[U^0]) < EJ_1(x^*[U^1])$.

4. Conclusion

We have investigated the information exchange between duopolistic firms in a game theoretical framework. We have shown that each firm has an incentive to inform the other of his cost function, and moreover that they can obtain the highest expected equilibrium profits when they exchange their information about cost functions. From these results, we can say that two firms agree to inform each other of their cost functions.

To conclude this paper, we make some remarks about our model. First, in our model, two firms negotiate for information exchange before their marginal costs are realized. As we have shown, they agree to make the rule that, whichever marginal costs might be realized, obliges them to inform each other of those realized costs. Besides exchanging their information completely like this, it may happen that they do partially, in a sense that each firm informs the other of only some realized values of his marginal cost. For example, when a realized cost of, say, firm 1 is high, firm 2 may wrongly estimate firm 1's cost to be lower than the realized value, if he is not informed of it. Then, based on his wrong estimate, firm 2 will supply the commodity at a low level, so that firm 1 will obtain higher profit. Thus, firm 1 may not disclose his weak point that his realized cost is high. That is, each firm may want to inform the other of his marginal cost when it is low, and may not when it is high. In order to make our investigation more detail like this, we will need to reformulate our model so that firms negotiate for information exchange after they know their own marginal costs. Secondly, we have not considered the possibility that a firm may reveal false information to the other after the information exchange agreement is reached. But, his betrayal may be detected by the other in the long run if our model is repeated, and the agreement will be annulled. So, the betrayal does not seem to be profitable for him. Thirdly, in an oligopoly with more than two firms, it seems to be possible that each firm exchanges information with some of others, and thus that some exclusive groups of firms are formed for the purpose of the information exchange. This problem of group formation of oligopolistic firms for the information exchange will be investigated in future papers. Finally, we must note that, if we consider information about demand function, the situation is quite different from ours. Basar and Ho [1] and Ponsard [5] have

shown that each firm obtains the highest expected equilibrium profit when only he is informed of the demand function. Hence, in this case, each firm never has an incentive to communicate his information to the other. These results together with ours show that the problem of information in non-zero-sum games is much more complicated than in one-person decision problems and zero-sum two-person games. The game theoretical viewpoint will be useful to investigate this problem, especially the strategic aspect of information.

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