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ADAPTIVE SEARCH AND STOP WITH UNCERTAIN SENSOR CAPABILITY

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Abstract A search and stop problem is studied where the detection capability of the search sensor as well as existence of the object in the given area is assumed uncertain. Two stopping rules based on the posterior probability of the object being in the area are examined. Our particular concern is to evaluate the effectiveness of utilizing "dummies" to obtain extra information which is expected to give us better estimate of the posterior distribution of the sensor capability. The efficiency of two rules are compared numerically, and some interesting results are obtained.

1. Introduction

Consider a situation described below. An object is supposed to be, but not surely, in an area, say A, and a search for the object is started. If the object is not detected in spite of a certain duration of search, the searcher becomes uneasy to continue his search. There are at least two kinds of uncertainties. First, he is not sure that the object is really in A. His estimate of the probability p that the object is in A was less than unity. Second, he estimated his sensor capability, made a plan, and started the search, but he knows from his past experiences that his estimate of his sensor capability often fails. The searcher, therefore, thinks of stopping the search when some time has elapsed without any detection.

Search and stop models are not new. The necessity of taking the uncertainty of the sensor effectiveness into account was learnt during the search operation for the wrecked submarine Scorpion in 1968 [1], [3] and [4]. H. R. Richardson and B. Belkin [2] investigated the search and stop problem assuming that the prior object-location distribution and the prior sweep width distribution are known, and derived a formula of the optimal stopping time from the viewpoint of trade-off between the search cost and the object value.

In our paper, a similar search and stop problem based on the posterior probability of the object being in the area is dealt with. A new idea presented in this paper is to put "dummies" for search randomly in the search area before the start of search operation, the detection of which will enable us to estimate the posterior sensor capability. To improve the prior distributions of the object position and the sensor capability, Richardson and Belkin uses only the information that the object has not been found out up to the present time. We add an extra information, the number of detections of "dummies", to improve the prior distributions.

We introduce two different stopping rules and discuss the structure of the stopping rules in the next sections. In section 4, the prior distribution of sensor capability is assumed to be of gamma, and the efficiency of utilizing "dummies" is investigated numerically.

2. Definitions and General Assumptions

The following model is considered in this paper: An object is supposed to be, but not surely, in a given area A. Let E be the event that the object really exists in the area A. It is assumed that the possible location of the object is equally probable at any point in the area A, and the prior probability of its existence in A, $P\{E\}=p$, is less than unity.

The detection capability of the search sensor is constant through the whole operation but not known exactly in advance. It is assumed that the searcher only knows its prior probability distribution, $F(\alpha)=P\{Z\le\alpha\}$, where random variable Z denotes the detection capability of the sensor.

Finally, we assume the random search, that is to say,

$$P\{ D\le t \mid E, Z=\alpha \} = 1-\exp(-\alpha t), \tag{2.1}$$

where random variable D represents the first time from the start of search to the detection of the object. Under this assumption, α is the reciprocal of the expected detection time when the object is really in A.

3. Formulations and Structure of Stopping Rules

Suppose the search is started. If the object is detected, the operation comes to the end. On the contrary, if a long time elapses without any detection, the search should be stopped, since the object might not be in the area. Then, when to stop? In this paper, we present stopping rules based on the posterior probability.

If the object is not detected in some duration of search, the posterior probability of the object being in A decreases. Let us adopt a rule that the search operation is stopped when the object is not detected and its posterior probability goes down below a prescribed value $\gamma \in (0,p)$. The critical value γ represents the probability that the object is really in A when the searcher gives up to persist in searching.

In this section, we will formulate two stopping rules, referred to as Rule I and Rule II, and then discuss the structure of the stopping rules.

3.1. Rule I

In estimating the probability of existence of the object, let the searcher utilize only the information that no detection has occurred up to time t. The stopping rule based on the posterior probability is named as Rule I.

Let $p_t = P\{E|D>t\}$, the posterior probability of the object being in A on the condition that the object has not been detected up to time t. Then,

$$p_t = \frac{p\int_0^\infty \exp(-\alpha t)\,dF(\alpha)}{1-p+p\int_0^\infty \exp(-\alpha t)\,dF(\alpha)} . \tag{3.1}$$

Let τ_1 be the stopping time under Rule I.

Theorem 1. Under Rule I, the stopping time $\tau_1 = \min\{t : p_t \le \gamma\}$ is given by the unique root of the equation:

$$\int_0^\infty \exp(-\alpha t)\,dF(\alpha) = C, \tag{3.2}$$

where $C = (1-p)\gamma/(p(1-\gamma))$.

Proof: It is easily seen that p_t given by (3.1) is continuous, and monotone decreasing in t since $\int_0^\infty \exp(-\alpha t)dF(\alpha)$ is monotone decreasing in t. In addition to the above, the facts, $\lim_{t\to\infty} p_t = 0$ and $\lim_{t\to 0} p_t = p$, suffice to prove the theorem.

3.2. Rule II

In carrying out the search operation effectively, it is important to take posterior estimates of the sensor capability into consideration. In the following, we propose an idea for gaining the information of the sensor capability to make stopping search more effective.

Before initiating the search operation the searcher randomly scatters n false-objects (we call them "dummies" hereafter) having the same signal characteristics as the true object in the area A. Then the searcher starts the search. If a dummy is detected at some time, the searcher counts one, and then restores the dummy into A. It is assumed that the searcher loses the position of the restored dummy immediately, and so the distribution of n dummies is always uniform in A. Let $N(t)$ be the total number of counts up to time t. Then, from the above-mentioned assumption together with the assumption of random search, $N(t)$ is a Poisson random variable, i.e.,

(3.3) $$\mathrm{P}\{N(t)=k|Z=\alpha\} = (\mathrm{n}\alpha t)^k \exp(-\mathrm{n}\alpha t)/k! \ , \ k=0,1,2,\cdots,$$

and is independent of D if conditioned on Z, i.e.,

(3.4) $$\mathrm{P}\{N(t)=k, D>t|Z=\alpha\} = \mathrm{P}\{N(t)=k|Z=\alpha\}\mathrm{P}\{D>t|Z=\alpha\}.$$

We will adopt $N(t)$ into our stopping rule as an additional information, and will call the stopping rule based on the posterior probability which takes $N(t)$ into account as Rule II.

Let $p_{t,k}=\mathrm{P}\{\mathrm{E}|D>t,N(t)=k\}$, and using the relation (3.4),

(3.5) $$p_{t,k} = \frac{\mathrm{p}\int_0^\infty e^{-(\mathrm{n}+1)\alpha t}\alpha^k \mathrm{d}F(\alpha)}{\int_0^\infty (1-\mathrm{p}+\mathrm{p}e^{-\alpha t})\alpha^k e^{-\mathrm{n}\alpha t}\mathrm{d}F(\alpha)} \ .$$

Note that for n=0 and k=0, (3.5) reduces to (3.1).

To show the existence of the stopping time τ_2 under this rule, we need the next lemmas.

Lemma 1. For any fixed k,

(i) $$h(t,k) \geq (1+\mathrm{n}^{-1})h((1+\mathrm{n}^{-1})t,k),$$

(ii) $$\frac{\partial}{\partial t} h(t,k) \leq (1+\mathrm{n}^{-1})\frac{\partial}{\partial t} h((1+\mathrm{n}^{-1})t,k),$$

where $h(t,k)=\int_0^\infty \alpha^k e^{-\mathrm{n}\alpha t}\mathrm{d}F(\alpha)$.

Proof: For each fixed k, the relations,

$$\frac{\partial}{\partial t} h(t,k-1) = -\mathrm{n}h(t,k),$$

$$\frac{\partial}{\partial t} h((1+\mathrm{n}^{-1})t,k-1) = -(\mathrm{n}+1)h((1+\mathrm{n}^{-1})t,k),$$

are easily derived. Since $\partial^2 h(t,k)/\partial t^2 \geq 0$ for all t,

$$\partial h(t,k-1)/\partial t \leq \partial h((1+\mathrm{n}^{-1})t,k-1)/\partial t.$$

Then (i) follows.

Similarly, considering that $\partial^2 h(t,k-1)/\partial t^2=-\mathrm{n}\partial h(t,k)/\partial t$, $\partial^2 h((1+\mathrm{n}^{-1})t, k-1)/\partial t^2=-(\mathrm{n}+1)\partial h((1+\mathrm{n}^{-1})t,k)/\partial t$, and $\partial^3 h(t,k-1)/\partial t^3 \leq 0$ for all t, (ii) will be easily proven.

Lemma 2. (i) For any fixed k, $p_{t,k}$ is monotone decreasing in t.
(ii) For any fixed t, $p_{t,k}$ is monotone decreasing in k.

Proof: Rearranging (3.5), Lemma 2 is equivalent to the statement that $\phi(t,k)=h(t,k)/h((1+n^{-1})t,k)$ is monotone decreasing in each of t and k. Differentiating $\phi(t,k)$ directly with respect to t, we can prove (i) readily from Lemma 1. Property (ii) is equivalent to the inequality: For any fixed t,

$$\frac{h(t,k-1)}{h((1+n^{-1})t,k-1)} \leq \frac{h(t,k)}{h((1+n^{-1})t,k)} .$$

To prove this, we need somewhat tedious procedure, and we do not present it here.

Now, using Lemma 2 we can establish the next theorem. Let τ_2 be the stopping time under Rule II.

Theorem 2. Under Rule II, a stopping time τ_2 exists (including $\tau_2=\infty$) and is given by

(3.6) $$\tau_2 = \inf \{ t: p_{t,N(t)} \leq \gamma \},$$

that is, when $N(t)=k$, τ_2 is the unique root, if it exists, of the following equation:

$$\phi(t,k) = h(t,k)/h((1+n^{-1})t,k) = 1/C.$$

If otherwise, τ_2 is defined as ∞.

3.3. Structures of the stopping rules

In this paragraph, some structures of the stopping rules are investigated. Let the random variable S_i, $i=1,2$ denote the time at which the search operation terminates under Rule I, II respectively. That is, $S_i=\min\{D,\tau_i\}, i=1,2$.

First, suppose the searcher adopts Rule I. Rule I tells the searcher to continue searching until detection of the object or the prescribed stopping time τ_1, whichever comes first. Since $S_1=\min\{D,\tau_1\}$,

(3.7) $$P\{S_1=D|Z=\alpha\} = p(1-e^{-\alpha\tau_1}).$$

(3.8) $$P\{S_1=\tau_1|Z=\alpha\} = 1-p+pe^{-\alpha\tau_1}.$$

Next, the structure of the stopping Rule II is discussed. It is more complicated than the above case in the sense that τ_2 is not uniquely determined in advance, but is regarded as a random variable dependent on the value of $N(t)$. Let t_k denote

(3.9) $$t_k = \inf\{\ t:\ p_{t,k} \le \gamma\ \},\ k=0,1,2,\cdots.$$

From Lemma 2.(i), for each fixed k, t_k is uniquely determined (including ∞) similarly to τ_1. In Fig.1, solid line curves correspond to $p_{t,k}$, $k=0,1,\cdots$, and the intersections with the horizontal line with ordinate γ give $t_1,t_2,\cdots$, and $t_0=\infty$. The fact that $\{t_k,\ k=0,1,\cdots\}$ is an ordered set, $t_{k+1}<t_k$ for all k, follows from Lemma 2.

Since the index k of t_k represents the value of the random variable $N(t)$ at time t, Rule II indicates that, if the searcher cannot find out the object up to time t, then his decision at time t is

continue the search if $t < t_{N(t)}$,

stop it if $t \ge t_{N(t)}$.

In other words, if $N(t)=k$, $(k=0,1,2,\cdots)$, this rule tells the searcher to continue searching until whichever of the following three events occurs first: (1) detection of the object, (2) the $(k+1)$st capture of the dummy, (3) arrival of the time t_k. The search process then comes to the end except for the case (2) with the $(k+1)$st capture time earlier than t_{k+1}. In the latter case, the state goes over to $k+1$; the way of continuing search is again subject to the above-mentioned rule with k replaced by $k+1$.

Now, for $t\in(t_{k+1},t_k]$, the probability conditioned on $Z=\alpha$ with which the search operation terminates in $(t,\ t+\Delta t)\subset(t_{k+1},t_k]$ by detecting the object/stopping the search is given by

$$\Delta P_D(t|\alpha) = \mathrm{p}\mathrm{e}^{-\alpha t}\sum_{j=0}^{k}\frac{(\mathrm{n}\alpha t)^j}{j!}\mathrm{e}^{-\mathrm{n}\alpha t}\cdot\alpha\Delta t\ ,$$

$$\Delta P_{\tau_2}(t|\alpha) = \begin{cases}(1-\mathrm{p}+\mathrm{p}\mathrm{e}^{-\alpha t})\dfrac{(\mathrm{n}\alpha t)^k}{k!}\mathrm{e}^{-\mathrm{n}\alpha t}\cdot\mathrm{n}\alpha\Delta t\ , & t\in(t_{k+1},t_k),\\[2ex] (1-\mathrm{p}+\mathrm{p}\mathrm{e}^{-\alpha t_k})\dfrac{(\mathrm{n}\alpha t_k)^k}{k!}\mathrm{e}^{-\mathrm{n}\alpha t_k}\ , & t=t_k.\end{cases}$$

Since $S_2=\min\{D,\tau_2\}$, then we get

(3.10) $$\mathrm{P}\{S_2=D|Z=\alpha\} = \sum_{k=0}^{\infty}\int_{t_{k+1}}^{t_k}\mathrm{d}P_D(t|\alpha),$$

(3.11) $$\mathrm{P}\{S_2=\tau_2|Z=\alpha\} = \sum_{k=0}^{\infty}\left[\int_{t_{k+1}}^{t_k}\mathrm{d}P_{\tau_2}(t|\alpha)+P_{\tau_2}(t_k|\alpha)\right]\ .$$

We note that (i) $\mathrm{P}\{t<S_2<t+\Delta t|Z=\alpha\} = \Delta P_D(t|\alpha)+\Delta P_{\tau_2}(t|\alpha)$, (ii) $\mathrm{P}\{S_2\le\infty|Z=\alpha\}=1$, (iii) (3.10) represents the probability of detecting the object before τ_2 comes, and (3.11) gives the probability of stopping the search before D comes.

Now we are in a position to examine whether and in what cases Rule II is more efficient compared with Rule I. However, it is difficult to present an analytical consideration, and so we will deal with the problem numerically in the next section.

4. Comparison of Efficiency of Two Rules

Hereafter, it is assumed that $F(\alpha)$ is a gamma distribution. Let $f(\alpha)$ denote its density function,

$$(4.1)\qquad f(\alpha) = \begin{cases} \dfrac{\beta^{\lambda}}{(\lambda-1)!}\,\alpha^{\lambda-1}\exp(-\beta\alpha), & \alpha>0, \\ 0, & \alpha\le 0, \end{cases}$$

where $\lambda>1$ is an integer. Then $E[Z]=\lambda/\beta$, $V[Z]=\lambda/\beta^2$, and hence the coefficient of variation is given by $v=1/\sqrt{\lambda}$. Substituting (4.1) into (3.1) and (3.5), we obtain

$$(4.2)\qquad P\{E|D>t\} = \frac{1}{1+\dfrac{1-p}{p}\left(1+\dfrac{t}{\beta}\right)^{\lambda}},$$

and

$$(4.3)\qquad P\{E|D>t,\ N(t)=k\} = \frac{1}{1+\dfrac{1-p}{p}\left(1+\dfrac{t}{nt+\beta}\right)^{k+\lambda}}.$$

Thus, by (3.2) and (3.9),

$$(4.4)\qquad \tau_1 = \beta(\,1/C_0 - 1\,),$$

$$(4.5)\qquad t_k = \beta/(\,C_k/(1-C_k) - n\,),\quad k=0,1,2,\cdots,$$

when C_k given below is larger than $n/(n+1)$. If otherwise, t_k is defined as ∞.

$$(4.6)\qquad C_k = C^{1/(k+\lambda)},\qquad C = \gamma(1-p)/(p(1-\gamma)).$$

Note that (i) $t_0=\tau_1$ when $n=0$. (ii) letting $\lambda\to\infty$ with $\lambda/\beta=\alpha_0$ fixed constant, then we have $\tau_1\to\tau_0$ and also $t_k\to\tau_0$ for any k, where τ_0 is being given by (4.13).

Let $\pi_i=P\{S_i=D\}$ denote the probability that the object is detected before τ_i, $i=1,2$. From (3.2), i.e., $\int_0^{\infty}\exp(-\alpha\tau_1)dF(\alpha)=C$, we have

$$(4.7)\qquad \pi_1 = p\int_0^{\infty}(1-e^{-\alpha\tau_1})dF(\alpha) = \frac{p-\gamma}{1-\gamma}.$$

Note that τ_1 does not depend on the distribution of the detection capability $F(\alpha)$, although τ_1 depends on $F(\alpha)$.

From (3.10) and (4.1), π_2 is derived after somewhat tedious work:

$$\pi_2 = p\left[1 - \frac{1}{n+1}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\left(\frac{n}{n+1}\right)^j \frac{(k-j+\lambda-1)!}{(k-j)!(\lambda-1)!}\frac{\beta^\lambda (nt_k)^{k-j}}{\{\beta+(n+1)t_k\}^{k-j+\lambda}}\right]. \tag{4.8}$$

When n=0 (k=0), (4.8) is reduced to (4.7), and π_2 is monotone decreasing to 0 from p in $\gamma\in[0,p]$.

Next, we will give the expected duration of the search operation under Rule I and Rule II. Note that the operation comes to the end either by detecting the object or by stopping. Thus, if Rule I is employed, $S_1\le\tau_1$, where the equality is valid if and only if the object is not detected up to time τ_1. Thus,

$$E[S_1] = \int_0^\infty \int_0^{\tau_1} t\,dP\{D\le t|Z=\alpha\}dF(\alpha) + \tau_1 P\{D>\tau_1\}.$$

After substituting (2.1) and (4.1) into the above equation, it is easily derived that

$$E[S_1] = \frac{p\beta}{\lambda-1}\left(1 - C_0^{\lambda-1}\right) + (1-p)\tau_1 , \tag{4.9}$$

where τ_1 is given by (4.4).

Similarly, we derive $E[S_2]$ by using the relation $\Delta P\{S_2=t|Z=\alpha\}=\Delta P_D(t|\alpha) + \Delta P_{\tau_2}(t|\alpha)$ which have been already shown in the previous section. Then, $E[S_2]$ is given by the sum of $E[S_2;D<\tau_2]$ and $E[S_2;D\ge\tau_2]$. The notation $E[X;A]$ with a random variable X and an event A refers to $Pr\{A\}\cdot E[X|A]$. From (3.10) and (3.11),

$$E[S_2=D|Z=\alpha] = \sum_{k=0}^{\infty}\int_{t_{k+1}}^{t_k} t\, pe^{-\alpha t}\sum_{j=0}^{k}\frac{(n\alpha t)^j}{j!}e^{-n\alpha t}\cdot\alpha\cdot dt,$$

$$E[S_2=\tau_2|Z=\alpha] = \sum_{k=0}^{\infty}\left[\int_{t_{k+1}}^{t_k} t(1-p+pe^{-\alpha t})\frac{(n\alpha t)^k}{k!}e^{-n\alpha t}\cdot n\alpha\cdot dt + t_k(1-p+pe^{-\alpha t_k})\frac{(n\alpha t_k)^k}{k!}e^{-n\alpha t_k}\right],$$

and they are easily integrated by t. After simplifying the results, the conditional expectation of S_2 is,

$$\frac{p}{\alpha}\left[1 - \sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{n^k}{(n+1)^{j+1}}\frac{(\alpha t_k)^{k-j}}{(k-j)!}e^{-(n+1)\alpha t_k}\right] + \frac{1-p}{n\alpha}\sum_{k=0}^{\infty}\left[1 - \sum_{j=0}^{k}\frac{(n\alpha t_k)^j}{j!}e^{-n\alpha t_k}\right].$$

Finally, we obtain the unconditional expectation of S_2 as follows:

(4.10)
$$E[S_2] = \frac{\beta}{n(\lambda-1)} \sum_{k=0}^{\infty} \sum_{j=k+1}^{\infty} \frac{(\lambda+j-2)!}{(\lambda-2)!j!} \times \left[p\left(\frac{n}{n+1}\right)^{k+1} \frac{\beta^{\lambda-1}\{(n+1)t_k\}^j}{\{\beta+(n+1)t_k\}^{\lambda+j-1}} + (1-p)\frac{\beta^{\lambda-1}(nt_k)^j}{(\beta+nt_k)^{\lambda+j-1}} \right].$$

When n=0 (k=0), (4.10) reduces to (4.9).

In our model, the uncertainty of the sensor capability is of much concern, and the stopping rules are sought which are expected to be efficient in the case of uncertain sensor capability. If the sensor capability is surely known, i.e., $P\{Z=\alpha_0\}=1$, Rule II reduces to Rule I itself, and in this case, the detection probability and the expected duration of the search operation are found to be,

(4.11)
$$\pi_0 = P\{D\le\tau_0\} = p(1-C) = (p-\gamma)/(1-\gamma),$$

(4.12)
$$E[S_0] = p\int_0^{\tau_0} t\,d(1-e^{-\alpha_0 t})+(1-p+pe^{-\alpha_0\tau_0})\tau_0 = \pi_0/\alpha_0+(1-p)\tau_0,$$

where τ_0 represents the stopping time in this case and is given by

(4.13)
$$\tau_0 = -\alpha_0^{-1}\ln C = -\alpha_0^{-1}\ln(1-\pi_0/p).$$

These formulae are utilized in the following in comparing the efficiency of the two stopping rules. It should be pointed out that $\pi_0=\pi_1$, which is as expected since the expression for π_1 is distribution-free.

Parameters involved in our model are v, p and n. The scale parameter $1/\beta$ is chosen as unity through the calculations without any loss of generality. When v, p and n are given, then π_i and $E[S_i]$, i=0,1,2, are easily calculated for each stopping criterion γ using the above formulae.

To compare the efficiency of the stopping rules, $E[\hat{S}_0]/E[\hat{S}_i]$, i=1,2, are tentatively adopted in our study. $E[\hat{S}_i]$, i=1,2 are calculated not with a common value γ of the stopping criteria, but with a common value π of π_i's. The stopping criteria γ_i in Rule i (i=I, II) are chosen so as to give π_i the common value π, and the expected times of search operation $E[\hat{S}_i]$ under Rule i with γ_i are calculated. As to the additional case of certain sensor capability, similar calculations are carried out and the expected time is denoted as $E[\hat{S}_0]$, where α_0 is chosen as equal to $E[Z]=\lambda/\beta=\lambda$, i.e., $\alpha_0=v^{-2}$.

Fig.2 shows two typical examples among all the cases examined.

There are two important factors in making comparison of the efficiency of the rules; (i) degree of the requirement for complete search, i.e., π/p, the probability that the searcher has to detect the object divided by the prior

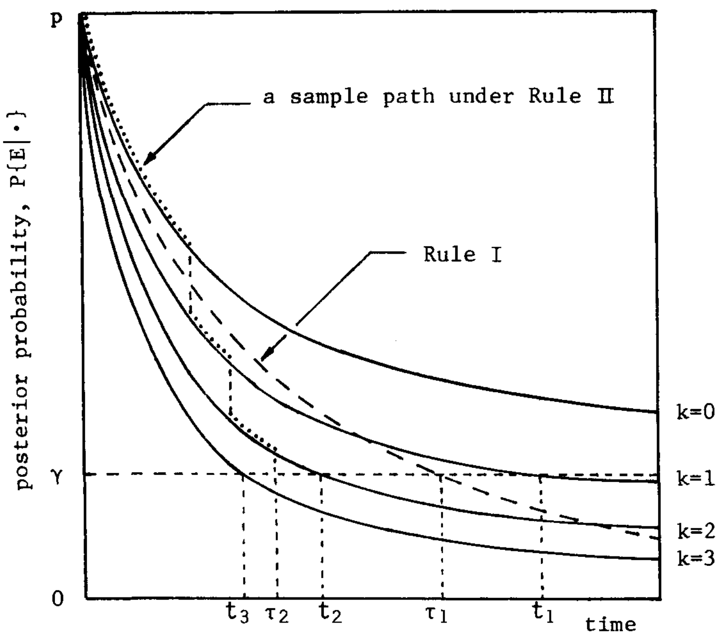

Fig.1. Structures of the Stopping Rules.

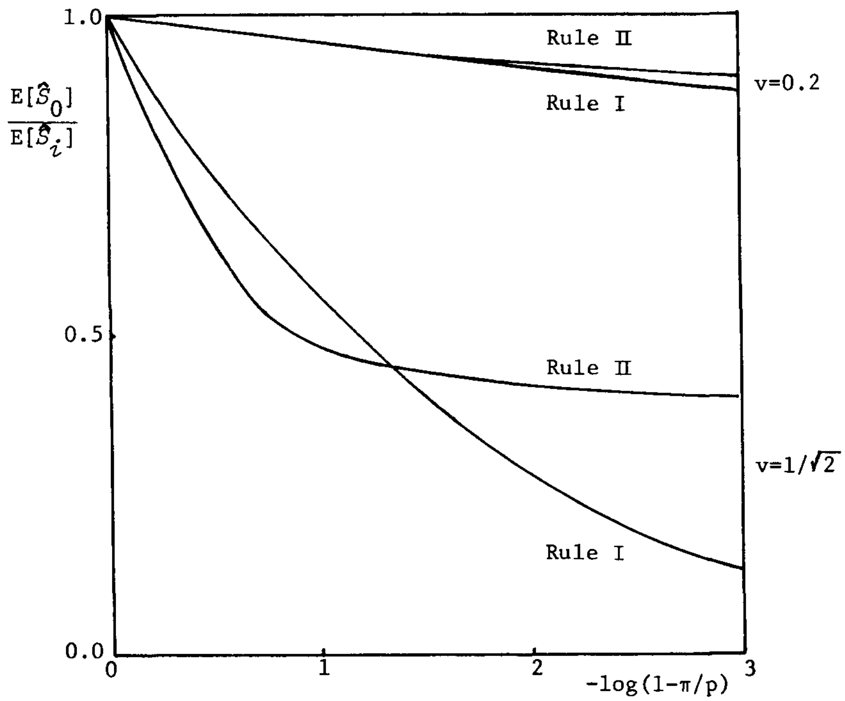

Fig.2. The Efficiency of Rule I and Rule II $(p=0.5,\ \alpha_0=\lambda=v^{-2},\ n=1,\ \beta=1)$.

probability, and (ii) the quality of the prior information about the sensor capability, namely, the coefficient of variation v.

Generally speaking, the stronger the request for detection, the longer the searcher must continue to search. It is very interesting to note that $E[\hat{S}_2]$ increases gradually with increasing π/p while $E[\hat{S}_1]$ increases rapidly. In Fig.2, for example, when $v=1/\sqrt{2}$, the value $E[\hat{S}_1]/E[\hat{S}_2]$, the efficiency of Rule II relative to Rule I, is around 1 when $\pi/p=0.950$, but the value is about 1.5 for $\pi/p=0.990$. When the requirement is comparatively weak, however, the efficiency of our Rule II with more information for decision is found to be slightly inferior to that of Rule I. (In Fig.2, this happens to occur when π/p is smaller than about 0.95.) This apparently unreasonable property is not unexpected because Rule II is not a rule devised so that the expected operation time is minimized for a given value of π. We will deal with the latter optimization problem separately.

It is pointed out that the less certain the prior knowledge about sensor capability (in other words, the larger the coefficient of variation), the better the efficiency of Rule II relative to Rule I. It is easily seen from Fig.2 that when $\pi/p=0.999$, the value $E[\hat{S}_1]/E[\hat{S}_2]$ for $v=0.2$ is approximately 1. On the other hand, the value for $v=1/\sqrt{2}$ is as large as 3.

Finally, we should give a comment on the number of dummies n. Roughly speaking, when the number of dummies is increased, the efficiency of Rule II improves, but only slightly. Therefore, it seems not wise to use so many dummies. When the number is small, however, the detected dummy should be carefully restored in the given area, because our model assumes the equally probable distribution of dummies in the area.

5. Conclusion and Acknowledgement

In this paper, it is assumed that the prior information about the detection capability of the search sensor is uncertain, and we investigated the effectiveness of a stopping rule which utilizes an additional information obtainable by use of dummies placed in advance in the search area. The result is qualitatively stated as follows.

Utilize Rule II with dummies when the sensor capability is rather uncertain, and if the object to be detected is so valuable that overlooking is by no means permitted.

Our present study is of preliminary nature, and optimization concept is never involved in the model. The optimal stopping rules under suitable objective functions would be studied in the near future.

In our model, the detected dummy is assumed to be picked up and restored. But in practice, the procedure of restoration requires time and labor, and so a search and stop procedure without any restoration of detected dummies is preferable from the viewpoint of real operation. Search and stop rule without restoration of detected dummies should also be investigated.

Finally, we wish to express our gratitude to Dr. Y. Tabata for his useful discussions on our work.

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