Journal of the Operations Research
Society of Japan
Vol. 23, No. 4, December 1980

A TWO-UNIT PARALLELED SYSTEM WITH GENERAL DISTRIBUTION

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(Received May 17, 1979; Revised December 22, 1979)

Abstract This paper considers a two-unit paralleled system with general failure time and repair time distributions. The equations governing the behaviour of the system, without using regenerative (or cyclic) nature, have been derived by employing the method of supplementary variable. Then the problems of calculating the Laplace transform of the reliability of the system and the mean time to the first system failure are reduced to that of solved a well-known Fredholm integral equation, depending on the pdf's of the failure and repair time of units. In addition, the probability that repairman is idle at time t, the expected total idle time of repairman during the interval $(0,t]$, and the expected number of repair until the system failure occurs are shown.

1. Introduction

A two-unit redundant system with repair has been studied by many authors to obtain measures of reliability performance. The two-unit paralleled redundant system has previously been treated by using the supplementary variable method [2,6,7,8], semi-Markov processes [10], regenerative properties [3,4,5, 9], and so on. However, the two-unit paralleled system with general failure and repair time distributions has not been analyzed because general failure laws destroy the regenerative (or cyclic) nature of such system. A detailed discussion of the effects that failure and repair time distributions have on the regenerative properties of two-unit redundant systems can be found in [1]. The main purpose of this paper is to investigate the case of a two-unit paralleled system subject to more generalized assumptions.

Suppose two units are arranged in parallel to perform as system. If a unit fails this fact is detected immediately and repair begins. Then the other unit continues doing the job for which the system was intended. If the repair of failed unit is completed when the other unit is still operating then it begins to operate immediately. The total system is considered to fail at the first time both units are in failed condition. We assume that the times to failure and repair times are arbitrarily distributed, respectively.

In the above model the equations governing the behaviour of the system, without using the regenerative nature, have been derived by employing the method of supplementary variable. Then the problems of calculating the Laplace transform of the reliability of the system and the mean time to the first system failure (MTSF) are reduced to that of solved a well-known Fredholm integral equation, depending on the pdf's of the failure and repair time of units. In addition, the probability that repairman is idle at time t, the expected total idle time of repairman during the interval (0,t], and the expected number of repair until the system failure occurs are shown. Finally, two examples are presented in detail, and it is shown that our results include the previous results as special cases.

2. Formulation and Analysis

Consider a system consisting of two identical units working in parallel. We assume that the failure time and repair time have the general continuous pdf $f(t)$ and $g(t)$ respectively. The failure time cdf is denoted by $F(t)$ and the hazard function is $\lambda(t)=f(t)/\bar{F}(t)$, where $F(t)=1-F(t)$. Similarly, the cdf of repair time is $G(t)$ and $\mu(t)=g(t)/\bar{G}(t)$, respectively. When a unit fails it undergoes repair immediately, but once the system fails it remains there. Further we assume that the failure and repair processes for two units are entirely independent, and a unit upon repair is as good as new.

Let us now define $X_i(t)$ $(i=1,2)$ as the age of a unit at time t and set $X_i(0)=0$ $(i=1,2)$. Let $Y(t)$ represent the time that has elapsed (if any) at time t since the beginning of the current repair job. Further let $N(t)$ denote a random variable that assumes values 0,1, or 2. In the model, we shall identify $N(t)=0$ with the event that both units of the system are operating at time t, $N(t)=1$ with the event that one is operating and the other is under repair at time t, and $N(t)=2$ with two units have failed. The inclusion of the variables $N(t)$, $X_i(t)$ $(i=1,2)$, and $Y(t)$ amounts to defining a Markov stochastic process in continuous time.

We define the following state probabilities:

$P_0(t,x)dx=\Pr[N(t)=0,\ 0\leq N(t')\leq 1\ (t'\leq t),\ X_1(t)=X_2(t),\ x<X_1(t)<x+dx \mid N(0)=0]$,

$P_1(t,x,y)dxdy=\Pr[N(t)=0,\ 0\leq N(t')\leq 1\ (t'\leq t),\ x<X_i(t)<x+dx,\ y<X_{3-i}(t)<y+dy$

$| \; N(0)=0] \; (x>y) \; (i=1 \text{ or } 2),$

$P_2(t,x,y)dxdy = Pr[N(t)=1, \; 0 \leq N(t') \leq 1 \; (t' \leq t), \; x < X_i(t) < x+dx, \; y < Y(t) < y+dy$

$| \; N(0)=0] \; (x>y) \; (i=1 \text{ or } 2).$

For tractability, let us introduce the supplementary variable x into probability that at time t operating time of both units is t. That is, $P_0(t,x)$. By connecting the above state probabilities at time t+h with those at time t and taking limits as $h \to 0$, we get the following differential equations governing the behaviour of the system:

(1)
$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 2\lambda(x)\right]P_0(t,x)=0,$$
$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \lambda(x) + \lambda(y)\right]P_1(t,x,y)=0,$$
$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \lambda(x) + \mu(y)\right]P_2(t,x,y)=0.$$

These equations are to be solved under the following boundary and initial conditions:

Boundary conditions

(2)
$$P_1(t,x,0)= \int_0^x P_2(t,x,u)\mu(u)du,$$
$$P_2(t,x,0)= 2\lambda(x)P_0(t,x) + \int_0^x P_1(t,x,u)\lambda(u)du + \int_x^t P_1(t,u,x)\lambda(u)du,$$
$$P_0(t,0)= P_1(t,0,y)= P_2(t,0,y)= 0.$$

Initial conditions

(3)
$$P_0(0,0)=1,$$
$$P_1(0,x,y)=P_2(0,x,y)=0.$$

Taking Laplace transform of equations (1) and (2) with respect to t and using initial conditions, we have

(4-a) $\left[s + \frac{\partial}{\partial x} + 2\lambda(x)\right]P_0^*(s,x)=0,$

(4-b) $\left[s + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \lambda(x) + \lambda(y)\right]P_1^*(s,x,y)=0,$

(4-c) $\left[s + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \lambda(x) + \mu(y)\right]P_0^*(s,x,y)=0,$

(5-a) $P_0^*(s,0)=1,$

(5-b) $P_1^*(s,x,0)= \int_0^x P_2^*(s,x,u)\mu(u)du,$

(5-c) $P_2^*(s,x,0)=2\lambda(x)P_0^*(s,x) + \int_0^x P_1^*(s,x,u)\lambda(u)du + \int_x^\infty P_1^*(s,u,x)\lambda(u)du,$

where denote by $P_0^*(s,x)$ etc. the Laplace transform of $P_0(t,x)$etc..

The solution of equation (4-a), using (5-a), is given by

(6) $$P_0^*(s,x)= \overline{F}(x)^2 e^{-sx} .$$

The partial differential equation (4-b) is Lagrange's linear equation. Thus Lagrange's auxiliary equations are given by

$$\frac{dx}{1} = \frac{dy}{1} = \frac{-dP_1^*(s,x,y)}{[s+\lambda(x)+\lambda(y)]P_1^*(s,x,y)} \quad . \qquad (x>y).$$

Solving these equations, we have

$$x=y+c_1 \ ,$$

$$P_1^*(s,x,y)=c_2\overline{F}(x)\overline{F}(y)e^{-sy}/\overline{F}(c_1),$$

where c_1 and c_2 are arbitrary constants. Therefore the general solution of equation (4-b) is given by

(7) $$P_1^*(s,x,y)=H_1(s,x-y)\overline{F}(x)\overline{F}(y)e^{-sy},$$

where $H_1(s,x-y)$ is an arbitrary function. Similarly the general solution of equation (4-c) is given by

(8) $$P_2^*(s,x,y)=H_2(s,x-y)\overline{F}(x)\overline{G}(y)e^{-sy},$$

where $H_2(s,x-y)$ is an arbitrary function.

In order to find the functions $H_1(s,x)$ and $H_2(s,x)$ in (7) and (8), we proceed as follows; the probability densities along x axis in (7) and (8), setting y=0, are given by

(7') $$P_1^*(s,x,0)=H_1(s,x)\overline{F}(x),$$

(8') $$P_2^*(s,x,0)=H_2(s,x)F(x).$$

Substituting (8) and (7') into both sides of (5-b) yield the following integral equation

(9) $$H_1(s,x)= \int_0^x H_2(s,x-u)g(u)e^{-su}du.$$

Similarly substituting (6), (7), and (8') into both sides of (5-c) yield the following integral equation

(10) $$H_2(s,x)=2f(x)e^{-sx} + \int_0^x H_1(s,x-u)f(u)e^{-su}du + \int_x^\infty H_1(s,u-x)f(u)e^{-sx}du.$$

Further, substituting (9) into the right hand side of (10) yield the following equation

(11) $$H_2(s,x)=2f(x)e^{-sx} + \int_0^\infty K(s,x,u)H_2(s,u)du,$$

where

$$K(s,x,u)=\int_0^\infty [f(x-u-y)e^{-sy} + f(x+u+y)e^{-s(x+u)}]g(y)dy,$$

and let us define $f(t)=0$ for $t<0$. Equation (11) is the well-known Fredholm integral equation. Once we have obtained $H_2(s,x)$, and then $P_1^*(s,x,y)$, and $P_2^*(s,x,y)$ by (7), (8), and (9), we can compute various operating characteristics.

3. Operating Characteristics

In the preceding section we have derived various state probabilities (densities) when both units are $X_i(0)=0$ $(i=1,2)$ initially. In this section we shall obtain the operating characteristics viz. the reliability of the system, the probability that repairman is idle at time t, the expected total idle time of repairman during the interval $(0,t]$, and the expected number of repair started by time t, when the system failure doesn't occur in $(0,t]$. These are of great important in reliability theory.

(3.1) The reliability of the system.

Consider the reliability of the system when both units are $X_i(0)=0$ $(i=1,2)$ initially. Now let $R(t)$ denote the reliability of the system and T denote time to the first system failure. Then we have

$$R(t)=\Pr[T>t]$$
$$=\Pr[0\leqq N(u)\leqq 1 \quad (u\leqq t) \mid X_i(0)=0 \quad (i=1,2)].$$

From the definition of $P_0(t,x)$ and $P_i(t,x,y)$ $(i=1,2)$ integrating these functions and adding, the reliability of the system can be written as,

$$R(t)=\int_0^t [P_0(t,x) + \int_0^x \{P_1(t,x,y) + P_2(t,x,y)\}dy]dx.$$

Then using (6)-(8) we obtain the Laplace transform of the reliability of the system

$$(12) \quad R^*(s)= \int_0^\infty [\overline{F}(x)^2 e^{-sx} + H_1(s,x)A_1(s,x) + H_2(s,x) A_2(s,x)]dx,$$

where

$$A_1(s,x)=\int_0^\infty \overline{F}(x+u)\overline{F}(u)e^{-su}du,$$
$$A_2(s,x)=\int_0^\infty \overline{F}(x+u)\overline{G}(u)e^{-su}du,$$

and the mean time to the first system failure is given by

$$(13) \quad E[T]= \lim_{s\to 0} R^*(s)$$
$$=\int_0^\infty [\overline{F}(x)^2 + H_1(x)A_1(x) + H_2(x)A_2(x)]dx,$$

where $A_i(x)=A_i(0,x)$ and $H_i(x)=H_i(0,x)$ $(i=1,2)$. Then integral equation (11) is given by

(13') $H_2(x)=2f(x) + \int_0^\infty K(x,u)H_2(u)du,$

where

$$K(x,u)=\int_0^\infty [f(x-u-y) + f(x+u+y)]g(y)dy.$$

The above integral equation can also be written as

$$H_2(x)=2f(x) + 2\sum_{n=1}^{\infty} \int_0^\infty K_n(x,u)f(u)du,$$

where

$$K_1(x,u)=K(x,u),$$

$$K_n(x,u)=\int_0^\infty K(x,y)K_{n-1}(y,u)dy, \quad (n=2,3,\ldots).$$

Thus using the equation (9), we have

$$H_1(x)= 2\int_0^x f(x-u)g(u)dx + 2\sum_{n=1}^{\infty} \int_0^x\int_0^\infty K_n(x-u,y)f(y)g(u)dydu.$$

(3.2) The probability that repairman is idle at time t.

Consider the probability that repairman is idle at time t when the system failure doesn't occur in (0,t]. Denote by q(t) such probability when both units are $X_i(0)=0$ $(i=1,2)$ initially. Then we have

$$q(t)=Pr[N(t)=0 \mid X_i(0)=0 \quad (i=1,2)].$$

Thus from the definitions of $P_0(t,x)$ and $P_1(t,x,y)$, the Laplace transform of q(t) is obtained by integrating (6) and (7) over x and y.

(14) $q^*(s)=\int_0^\infty [\overline{F}(x)^2e^{-sx} + H_1(s,x)A_1(s,x)]dx.$

Next consider the expected total idle time of a repairman during the interval (0,t] (t<T). Denote by Q(t) the expected total idle time of a repairman during (0,t] when both units are $X_i(0)=0$ $(i=1,2)$ initially. Then the Laplace transform of Q(t) is given by

$$Q^*(s)=q^*(s)/s,$$

and the expected total idle time of a repairman prior to the system failure

(15) $Q= \lim_{s\to 0} sQ^*(s)$

$$=\int_0^\infty [\overline{F}(x)^2 + H_1(x)A_1(x)]dx.$$

(3.3) The expected number of repair during (0,t].

Consider the expected number of repair during the interval (0,t] (t<T). Then using the definition of $P_2(t,x,y)$, the expected number of repair is

is given by

$$M(t)=\int_0^t\int_0^x P_2(x,u,0)dudx.$$

Thus the Laplace transform of M(t), using (8), is obtained

$$(16)\quad M^*(s)=\frac{1}{s}\int_0^\infty H_2(s,x)\bar{F}(x)dx,$$

and the expected number of repair prior to the system failure

$$(17)\quad M=\lim_{s\to 0} sM^*(s)$$
$$=\int_0^\infty H_2(x)\bar{F}(x)dx.$$

Example 1.

Consider the case where times to failure obey a k-Erlang distribution and arbitrarily distributed. Then the pdf of the failure time is given by

$$(18)\quad f(t)=\frac{\lambda^k t^{k-1}}{(k-1)!}e^{-\lambda t},\qquad \lambda>0.$$

We shall not solve the Fredholm integral equation (11), but will obtain the operating characteristics using the relation of integral equations (9) and (10). Inserting equation (18) into equations (9) and (10), we have

$$(19)\quad H_1(s,x)=\int_0^x H_2(s,x-u)g(u)e^{-su}du,$$

$$(20)\quad H_2(s,x)=\frac{2\lambda^k x^{k-1}}{(k-1)!}e^{-(s+\lambda)x}+\frac{\lambda^k}{(k-1)!}e^{-(s+\lambda)x}\int_0^x H_1(s,u)(x-u)^{k-1}e^{(s+\lambda)u}du$$
$$+\frac{\lambda^k}{(k-1)!}e^{-(s+\lambda)x}\int_0^\infty H_1(s,u)(x+u)^{k-1}e^{-\lambda u}du.$$

In order to obtain the operating characteristics, let us now define $H_{ij}(s)$

$$H_{ij}(s)=\int_0^\infty H_i(s,x)x^j e^{-\lambda x}dx.\qquad i=1,2.,\ j=0,1,\ldots,k-1.$$

Then multiplying equations (19) and (20 by $x^j e^{-\lambda x}$ and integrating we obtain

$$(21)\quad H_{1j}(s)=\sum_{n=0}^{j}\binom{j}{n}(-1)^{j-n}g^{(j-n)}(s+\lambda)H_{2n}(s)$$

$$(22)\quad H_{2j}(s)=\frac{(k+j-1)!}{(k-1)!}\frac{2\lambda^k}{(2\lambda+s)^{k+j}}+\sum_{n=0}^{j}\binom{j}{n}\frac{(k+n-1)!}{(k-1)!}\frac{\lambda^k}{(2\lambda+s)^{k+n}}H_{1j-n}(s)$$
$$+\sum_{n=0}^{k-1}\binom{k-1}{n}\frac{(n+j)!}{(k-1)!}\frac{\lambda^k}{(2\lambda+s)^{n+j+1}}H_{1k-n-1}(s),$$

where

$$g^{(j)}(s)=\int_0^\infty(-x)^j g(x)e^{-sx}dx,\qquad j=0,1,\ldots,k-1.$$

The above equation is a set of 2k linear equation in 2k variables $H_{ij}(s)$ (i=1, 2., j=0,1,...,k-1) with known coefficients.

Then we have the operating characteristics from the solution of 2k linear equations. In order to obtain the reliability R(t), inserting (18) into (12) we have

(23) $$R^*(s)=\sum_{j=0}^{k-1}\sum_{j=0}^{k-1}\binom{i+j}{i}\frac{\lambda^{i+j}}{(2\lambda+s)^{i+j+1}}+\sum_{i=0}^{k-1}\sum_{j=0}^{k-1}\sum_{n=0}^{j}\binom{i+j-n}{i}\frac{\lambda^{i+j}H_{1n}(s)}{n!(2\lambda+s)^{i+j-n+1}}$$

$$+\sum_{i=0}^{k-1}\sum_{n=0}^{i}\frac{\lambda^{i}}{n!}\left[\frac{1}{(s+\lambda)^{i-n+1}}-\sum_{m=0}^{i-n}\frac{(-1)^m g^{(m)}(s+\lambda)}{m!(s+\lambda)^{i-n-m+1}}\right]H_{2n}(s).$$

Thus inserting a solution of the set of linear equation into (23) we find the reliability $R^*(s)$, and the mean time to the first system failure by setting $s=0$. Similarily we obtain the other operating characteristics;

(24) $$Q^*(s)=\sum_{i=0}^{k-1}\sum_{j=0}^{k-1}\binom{i+j}{i}\frac{\lambda^{i+j}}{s(2\lambda+s)^{i+j+1}}+\sum_{i=0}^{k-1}\sum_{j=0}^{k-1}\sum_{n=0}^{j}\binom{i+j-n}{i}\frac{\lambda^{i+j}H_{1n}(s)}{n!s(2\lambda+s)^{i+j-n+1}},$$

(25) $$M^*(s)=\sum_{i=0}^{k-1}\frac{\lambda^{i}}{i!s}H_{2i}(s).$$

The above reliability $R^*(s)$ have been obtained by using the regenerative properties [4]. However, the other operating characteristics are new.

Next consider a two-stage Erlang failure distribution as special case. Then the pdf of failure time is

$$f(x)=\lambda^2 xe^{-\lambda x}\qquad \lambda>0.$$

Hence the solution of linear equations (21) and (22) is

$$H_{10}(s)=2\lambda^2(2\lambda+s)g(s+\lambda)D,$$

$$H_{11}(s)=2\lambda^2[2g(s+\lambda)-(2\lambda+s)g^{(1)}(s+\lambda)]D,$$

$$H_{20}(s)=2\lambda^2(2\lambda+s)D,$$

$$H_{21}(s)=4\lambda^2 D,$$

where

$$D^{-1}=(2\lambda+s)[(2\lambda+s)(2\lambda+s+\lambda^2 g^{(1)}(s+\lambda)-4\lambda^2 g(s+\lambda)].$$

$$g(s+\lambda)=g^{(0)}(s+\lambda).$$

Thus inserting $H_{ij}(s)$ $(i=1,2.,j=0,1.)$ into equation (23), we find the reliability $R^*(s)$:

$$R^*(s)=\frac{10\lambda^2+6\lambda s+s^2}{(2\lambda+s)^3}+\frac{2\lambda^2 D}{(2\lambda+s)^2}[(16\lambda^2+8\lambda s+s^2)g(s+\lambda)-\lambda(6\lambda^2+5\lambda s+s^2)g^{(1)}(s+\lambda)]$$

$$+\frac{2\lambda^2 D}{(s+\lambda)^2}[(6\lambda^2+6\lambda s+s^2)(1-g(s+\lambda))+\lambda(2\lambda^2+3\lambda s+s^2)g^{(1)}(s+\lambda)].$$

and the mean time to the first system failure

$$E[T]=\frac{5}{4\lambda}+\frac{12-4g(\lambda)+g^{(1)}(\lambda)}{4\lambda[2(1-g(\lambda))+\lambda g^{(1)}(\lambda)]}$$

The above result agrees with that of Kodama et al. [4]. The other operating characteristics are

$$Q^*(s)=\frac{10\lambda^2+6\lambda s+s^2}{s(2\lambda+s)^3}+\frac{2\lambda^2 D}{s(2\lambda+s)^2}[(16\lambda^2+8\lambda s+s^2)g(s+\lambda)-\lambda(6\lambda^2+5\lambda s+s^2)g^{(1)}(s+\lambda)],$$

$$Q=\frac{5}{4\lambda}+\frac{8g(\lambda)-3\lambda g^{(1)}(\lambda)}{4\lambda[2(1-g(\lambda))+\lambda g^{(1)}(\lambda)]},$$

$$M^*(s)=2\lambda^2(4\lambda+s)D/s,$$

$$M=2/[2(1-g(\lambda))+\lambda g^{(1)}(\lambda)].$$

Example 2.

Consider the case where times to failure are distributed uniformly in the interval (0,1) and repair times are constant, i.e.

$$F(x)=\begin{cases} x & 0\leq x<1,\\ 1 & 1\leq x,\end{cases}$$

(26)

$$G(x)=\begin{cases} 0 & 0\leq x<1/2,\\ 1 & 1/2\leq x .\end{cases}$$

In example 1, we obtained the operating characteristics by solving the 2k linear equations instead of the Fredholm integral equation. In this example, we solve explicitly integral equation and obtain the mean time to the first system failure, the expected total idle time of a repairman prior to the system failure, and the expected number of repair prior to the system failure.

First let us solve the integral equation (13'). For $0\leq x<1$, inserting (26) into (13'), we have

$$(27)\quad H_2(x)=\begin{cases} 2+\int_0^x H_2(u)du & 0\leq x<1/4,\\ 2+\int_0^{-x+1/2} H_2(u)du & 1/4\leq x<1/2,\\ 2+\int_0^{x-1/2} H_2(u)du & 1/2\leq x<1.\end{cases}$$

For $0 \leqq x < 1/4$, differentiating both sides of integral equation (27) with respect to x yields

$$\frac{d}{dx} H_2(x) = H_2(x)$$

Thus using $H_2(0)=2$, the solution of the above differential equation is given by

$$(28)\quad H_2(x) = 2e^{-x}$$

For $1/4 \leqq x < 1/2$, using (28), the solution of the integral equation (27) is given by

$$(29)\quad H_2(x) = 2e^{-x+1/2}$$

Similarly for $1/2 \leqq x < 1$, using (28)-(29), we have

$$(30)\quad H_2(x) = \begin{cases} 2e^{x-1/2} & 1/2 \leqq x < 3/4, \\ 2(2e^{1/4} - e^{1-x}) & 3/4 \leqq x < 1. \end{cases}$$

Furthermore using the relation of equation (9), we have

$$(31)\quad H_2(x) = \begin{cases} 0 & 0 \leqq x < 1/2, \\ 2e^{-x-1/2} & 1/2 \leqq x < 3/4, \\ 2e^{1-x} & 3/4 \leqq x < 1. \end{cases}$$

Next let us calculate $A_1(x)$ and $A_2(x)$. From equation (26) we have

$$(32)\quad A_1(x) = \begin{cases} (1-x)^2(2+x)/6 & 0 \leqq x < 1, \\ 0 & 1 \leqq x, \end{cases}$$

$$A_2(x) = \begin{cases} (3-4x)/8 & 0 \leqq x < 1/2, \\ (1-x)^2/2 & 1/2 \leqq x < 1, \\ 0 & 1 \leqq x. \end{cases}$$

Then inserting (28)-(32) into (13), the mean time to the first system failure E[T] is given by

$$E[T] = (123e^{1/4} - 141)/24 .$$

Similarly, the other operating characteristics are given by

$$Q = (299e^{1/4} - 348)/96 ,$$

$$M = (49e^{1/4} - 64)/8 .$$

4. Conclusions and Remarks

The reliability of two-unit parallel redundant system with general distribution is obtained by solving the integral equation, depending the pdf's of failure and repair time. However, the integral equation is not easily solved. The procedure developed in this paper is, in principle, applicable to other models such that the process which describes the behavior of the system has not regeneration states. The other techniques (see, for example, regenerative properties [4,9] and Markov-renewal processes [10]) are not applicable to such system.

Acknowledgement

The authors are grateful to the referees for their valuable comments on this paper.

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