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OPTIMUM INSPECTION POLICIES FOR A STANDBY UNIT

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Abstract An inspection policy for a standby unit is considered by taking a standby electric generator as an example. The expected cost by the time of the electric power supply failure is derived by rising renewal-type equations. We discuss an optimum checking time which minimizes the expected cost. We further obtain a checking time such that the probability that a standby generator has failed at the time of the electric power supply failure is not greater than a prespecified value. Two cases where the failure rate is not disturbed by any inspection and a standby generator may fail during the time of the electric power supply failure are considered. Numerical examples are finally presented.

1. Introduction

System reliability can be improved by providing standby units. Especially, even a single standby unit plays an important role in a case that failure of an operating unit is costly and/or dangerous. A typical example is a case of standby electric generators in hospitals and other public facilities. It is, however, extremely serious if a standby generator fails at the very moment of electric power supply failure. Hence, frequent inspections are necessary to avoid such unfavourable situations. Similar examples can be found in army defence systems, in which all weapons are standing by, and hence, must be inspected.

Several works have been published on the problem of inspecting a single standby unit: Barlow and Proschan [1] summarized schedules of inspections which minimize the two expected costs until detection of failure and per unit of time assuming renewal of a standby unit. Luss and Kander [4] considered a model in which the duration of checking cannot be ignored. Jorgenson and Radner [2], Radner and Jorgenson [7], Luss and Kander [3], and Zacks and Fenske [8] discussed much more complicated systems.

In this paper, we consider an inspection policy for a single standby electric generator. We check a standby generator frequently to guarantee the upper bound of the probability that it has failed at the time of the electric power supply failure, but do not check it too frequently to reduce unnecessary inspection costs. The details of the model are described as follows:

(i) The time to failure of a standby generator has an arbitrary distribution $F(t)$ which is differentiable and $dF(t)/dt \to 0$ as $t \to \infty$, and its failure is detected only by checking.

(ii) A failed standby generator, which was detected by checking, undergoes repair immediately and its repair time has an arbitrary distribution $G(t)$.

(iii) The time required for checking is negligible and a standby generator is as good as new upon inspection or repair.

(iv) The next checking is scheduled at a constant t_0 units of time after either the former inspection or the completion of repair.

(v) Costs c_0 and c_1 are suffered for each checking and repair, respectively, and a cost c_2 is suffered for failure of a generator when the electric power supply fails, where $c_2 > c_1 \geq c_0$.

(vi) The policy terminates with the time of the electric power supply failure, which occurs according to an exponential distribution $(1 - e^{-\alpha t})$.

Under the assumptions above, we consider two optimization problems: (a) An optimum checking time t_1^* which minimizes the expected cost by the time of the electric power supply failure. (b) The largest $\bar{t}_1$ such that the probability that a generator has failed at the time of the electric power supply failure is not greater than a prespecified value ε.

The assumption of "a standby generator is as good as new upon inspection" is not plausible in practice. To overcome this difficulty, we assume that the failure rate of a standby generator remains undisturbed by any inspection, i.e., each checking does not renew a generator and is made only to see whether it is good or not. Then, we can obtain the expected cost and discuss its property briefly.

Further, a standby generator may fail during the time of the electric power supply failure, although it was good when the electric power supply fails. We also consider the case where a standby generator may fail before the time of the electric power supply recovery and obtain the expected cost of the model.

2. Optimization problem

To obtain the expected cost of the inspection model described above, we derive the expected numbers of inspections and of repairs of a standby electric generator and the probability that it has failed at the time of the electric power supply failure.

As an initial condition, we assume for convenience that a generator goes into standby and is good at time 0. Further, for simplicity of equations, we define that

$$A(t) \equiv \begin{cases} 0 & \text{for } t < t_0, \\ 1 & \text{for } t \geq t_0, \end{cases}$$

which is a degenerate distribution at t_0.

Let $H(t)$ be the distribution of the recurrence time to the state that a standby generator is good upon inspection or completion of repair. Then, we have that

$$(1) \qquad H(t) = \int_0^t \overline{F}(u)dA(u) + [\int_0^t F(u)dA(u)]*G(t),$$

where $\overline{F}(t) \equiv 1 - F(t)$ and the asterisk mark represents the Stieltjes convolution, i.e., $a(t)*b(t) \equiv \int_0^t b(t-u)da(u)$ for any $a(t)$ and $b(t)$. Equation (1) can be explained that the first term on the right-hand side is the probability that a standby generator is good upon inspection in time t, and the second term is the probability that a failed generator is detected by checking and its repair is completed in time t. Further, let $M_0(t)$ and $M_1(t)$ be the expected numbers of inspections of a standby generator and of repairs of a failed generator during $(0, t]$, respectively. Then, the following renewal-type equations are given by

$$(2) \qquad M_0(t) = A(t) + H(t)*M_0(t),$$

$$(3) \qquad M_1(t) = \int_0^t F(u)dA(u) + H(t)*M_1(t).$$

Thus, forming the Laplas-Stieltjes (LS) transforms of (1), (2), and (3), respectively, we have that

$$(4) \qquad h(s) = e^{-st_0}\,\overline{F}(t_0) + e^{-st_0}\,F(t_0)g(s),$$

$$(5) \qquad m_0(s) = e^{-st_0}/[1 - h(s)],$$

(6) $$m_1(s) = e^{-st_0} F(t_0)/[1 - h(s)],$$

where throughout this paper, we denote the LS transform of the function by the corresponding lower case letter, e.g., $h(s) \equiv \int_0^\infty e^{-st} dH(t)$.

Next, let $P(t)$ denote the probability that a standby generator has failed at time t, i.e., a standby generator, which is no good, will be detected by the next checking, or a failed generator, which was detected by the former checking, is now under repair. Then, the probability that a standby generator is good at time t is given by

(7) $$\overline{P}(t) = \overline{F}(t)\overline{A}(t) + H(t)*\overline{P}(t),$$

where $\overline{P}(t) \equiv 1 - P(t)$. Forming the LS transform of $P(t)$, we have that

(8) $$1 - p(s) = \int_0^{t_0} se^{-st} \overline{F}(t)dt/[1 - h(s)],$$

where $h(s)$ is given by (4).

We consider the total expected cost by the time of the electric power supply failure. Note that the inspection model of a standby generator may involve at least the following three costs; the costs incurred by each checking and each repair, and the cost incurred by failure of a standby generator when the electric power supply fails.

Suppose that the electric supply fails at time t. Then, the total expected cost during (0, t] is given by

(9) $$\hat{C}_1(t) = c_0M_0(t) + c_1M_1(t) + c_2P(t).$$

Thus, dropping the condition that the electric supply fails at time t from the assumption (vi), we have the expected cost:

(10) $$C_1(t_0) = \int_0^\infty \hat{C}_1(t)\alpha e^{-\alpha t} dt$$

$$= c_0m_0(\alpha) + c_1m_1(\alpha) + c_2p(\alpha),$$

which is a function of t_0. Using (5), (6), and (8), we have that

$$(11)\qquad C_1(t_0) = \frac{e^{-\alpha t_0}[c_0 + c_1F(t_0)] - c_2\int_0^{t_0} \alpha e^{-\alpha t}\,\overline{F}(t)dt}{1 - e^{-\alpha t_0}[\overline{F}(t_0) + F(t_0)g(\alpha)]} + c_2.$$

It is evident that

$$(12)\qquad C_1(0) \equiv \lim_{t_0\to 0} C_1(t_0) = \infty,$$

$$(13)\qquad C_1(\infty) \equiv \lim_{t_0\to\infty} C_1(t_0) = c_2f(\alpha),$$

where $C_1(\infty)$ represents the expected cost for the case where no inspection is made.

We seek an optimum checking time t_1^* which minimizes the expected cost $C_1(t_0)$. Differentiating log $C_1(t_0)$ with respect to t_0, we have, for large t_0,

$$(14)\qquad \frac{d[\log C_1(t_0)]}{dt_0} \simeq \alpha e^{-\alpha t_0}\{\frac{c_2g(\alpha) - c_0 - c_1}{c_2f(\alpha)} - g(\alpha)\}.$$

Thus, if the bracket on the right-hand side is positive, i.e.,

$$(15)\qquad c_2g(\alpha)[1 - f(\alpha)] > c_0 + c_1,$$

then there exists at least some finite t_0 such that $C_1(\infty) > C_1(t_0)$, and hence, it is better to check a standby generator at finite t_0.

In general, it is very difficult to compute an optimum checking time t_1^* which minimizes $C_1(t_0)$. However, for specified F(t), G(t), and $1 - e^{-\alpha t}$, $C_1(t_0)$ can be plotted as a function of t_0, using a computer.

In particular, consider the case where $F(t) = 1 - e^{-\lambda t}$ and $G(t) = 1$ for $t \geq 0$, i.e., the failure time is exponential and the repair time is negligible. Then, the resulting cost is

$$(16)\qquad C_1(t_0) = \{e^{-\alpha t_0}[c_0 + c_1(1 - e^{-\lambda t_0})] + c_2[1 - e^{-\alpha t_0} - \frac{\alpha}{\alpha+\lambda}(1 - e^{-(\alpha+\lambda)t_0})]\}/(1 - e^{-\alpha t_0}).$$

Differentiating $C_1(t_0)$ (or log $C_1(t_0)$) wit' respect to t_0 and sett'ng it equal to zero, we have that

$$(17)\qquad c_1 e^{-\lambda t_0}[1 + \frac{\lambda}{\alpha}(1 - e^{-\alpha t_0})] + c_2[1 - e^{-\lambda t_0} - \frac{\lambda}{\alpha+\lambda}(1 - e^{-(\alpha+\lambda)t_0})]$$

$$= c_0 + c_1,$$

where the left-hand side is monotonely increasing in case of $c_2 > [(\alpha+\lambda)/\alpha]c_1$, and conversely, non-increasing in case of $c_2 \leq [(\alpha+\lambda)/\alpha]c_1$. Further, the left-hand side is c_1 as $t_0 \to 0$ and $[\alpha/(\alpha+\lambda)]c_2$ as $t_0 \to \infty$.

Therefore, from the above discussions, if $c_2 > [(\alpha+\lambda)/\alpha](c_1 + c_0)$, then there exists a finite optimum checking time t_1^* which satisfies (17), and $C_1(t_1^*) = c_2 - c_1 - c_0 - \{c_2 - c_1[(\alpha+\lambda)/\alpha]\}\exp(-\lambda t_1^*)$. On the other hand, if $c_2 \leq [(\alpha+\lambda)/\alpha](c_1 + c_0)$, then the optimum time is $t_1^* = \infty$, i.e., no inspection is made, and $C_1(\infty) = c_2[\lambda/(\alpha+\lambda)]$. Note that the inequality of $c_2 > [(\alpha+\lambda)/\alpha] \cdot(c_1 + c_0)$ is also derived from (15).

It is of another interest to make the probability, as small as possible by checking, that a standby generator has failed at the time of the electric power supply failure. If the probability is prespecified, we can compute a checking time $\bar{t}_1$ such that $p(\alpha) \leq \varepsilon$, i.e.,

$$(18)\qquad \frac{\int_0^{t_0} e^{-\alpha t}\, dF(t) - e^{-\alpha t_0} F(t_0)g(\alpha)}{1 - e^{-\alpha t_0}[\bar{F}(t_0) + F(t_0)g(\alpha)]} \leq \varepsilon.$$

For instance, if the repair time is negligible, then the left-hand side in (18) is monotonely increasing in t_0. Hence, there exists a unique checking time $\bar{t}_1$ which satisfies

$$(19)\qquad [\int_0^{t_0} F(t)\alpha e^{-\alpha t}dt]/(1 - e^{-\alpha t_0}) = \varepsilon,$$

for sufficiently small $\varepsilon > 0$.

3. The case that failure rate is not disturbed by inspection

In the first model we have assumed that a standby generator is as good as new upon inspection. Here, we make the same assumptions as the previous ones except that the failure rate of a standby generator remains undisturbed by any inspection. This assumption is more plausible than the previous one in practice, however, the analysis becomes more difficult. We obtain the expected cost by the time of the electric power supply failure and discuss its property briefly.

First, we derive the expected numbers of inspections and of repairs of a standby generator during (0, t]. The probability that a failed generator is detected by checking is

$$(20)\qquad \sum_{k=1}^{\infty} \int_0^t \int_0^u A^{(k-1)}(v)dF(v)dA^{(k)}(u),$$

where $A^{(k)}$ represents the k-fold convolution of A with itself and $A^{(0)}(t) = 1$ for $t \geq 0$, and 0 for $t < 0$. For, after a standby generator was checked exactly at the (k - 1) times, it failed at time v and its failure is detected by the k^{th} checking at time u. It is also written by

$$(21)\qquad \sum_{k=1}^{\infty} \int_0^t \int_0^u [F(u) - F(v)]dA^{(k-1)}(v)dA(u-v).$$

Thus, the distribution of the recurrence time to the state that a standby generator is good upon completion of repair is

$$(22)\qquad H(t) = \{\sum_{k=1}^{\infty} \int_0^t \int_0^u [F(u) - F(v)]dA^{(k-1)}(v)dA(u-v)\}*G(t),$$

and its LS transform is given by

$$(23)\qquad h(s) = g(s) \sum_{k=1}^{\infty} e^{-skt_0} \{F(kt_0) - F[(k-1)t_0]\}.$$

Note that a standby generator does not become new upon inspection in this case.

To derive the total expected number $M_0(t)$ of inspections during (0, t], we consider the three cases:

(i) A standby generator has not failed during (0, t]. In this case, the expected number of inspections is

$$(24)\qquad \sum_{k=1}^{\infty} k[A^{(k)}(t) - A^{(k+1)}(t)]\overline{F}(t).$$

(ii) A standby generator failed but its failure is not yet detected. In this case, after the last k^{th} checking was made at time v, a standby generator failed at time u and its failure is not detected until t;

$$(25)\qquad \sum_{k=1}^{\infty} k \int_0^t \int_0^u \overline{A}(t-v)dA^{(k)}(v)dF(u).$$

(iii) A failed standby generator was detected by checking and undergoes repair. In this case, upon completion of repair, a standby generator becomes new and repeats the same inspection policy;

(26) $$\sum_{k=1}^{\infty} k \int_0^t \int_0^u A^{(k-1)}(v)dF(v)dA^{(k)}(u) + H(t)*M_0(t).$$

Therefore, by adding (24) - (26), we have the renewal-type equation:

(27) $$M_0(t) = \sum_{k=1}^{\infty} k\{\int_0^t \overline{F}(u)\overline{A}(t-u)dA^{(k)}(u) + \int_0^t \int_0^u A^{(k-1)}(v)dF(v)dA^{(k)}(u)\} + H(t)*M_0(t).$$

Solving (27), we have the LS transform:

(28) $$m_0(s) = \{\sum_{k=1}^{\infty} e^{-skt_0} \overline{F}[(k-1)t_0]\}/[1 - h(s)].$$

Further, from (21) and (22), the expected number $M_1(t)$ of repairs of a failed generator is given by the following renewal-type equation:

(29) $$M_1(t) = \sum_{k=1}^{\infty} \int_0^t \int_0^u [F(u) - F(v)]dA^{(k-1)}(v)dA(u-v) + H(t)*M_1(t),$$

and the LS transform is given by

(30) $$m_1(s) = \sum_{k=1}^{\infty} e^{-skt_0} \{F(kt_0) - F[(k-1)t_0]\}/[1 - h(s)].$$

Next, derive the probability that a standby generator has failed at time t. The probability that a standby generator is good at time t is

(31) $$\overline{P}(t) = \overline{F}(t) + H(t)*\overline{P}(t),$$

and solving (31), the LS transform is given by

(32) $$1 - p(s) = [1 - f(s)]/[1 - h(s)]^{(*)}.$$

By an argument similar to (10), the expected cost by the time of the electric power supply failure is

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(*) For simplicity of analysis, we put $\overline{P}(0) = \overline{F}(0) = 1$ so as easily to form the LS transform of (31).

$$C_2(t_0) = \frac{c_0 \sum_{k=1}^{\infty} e^{-\alpha k t_0} \bar{F}[(k-1)t_0] + c_1 \sum_{k=1}^{\infty} e^{-\alpha k t_0}[\bar{F}((k-1)t_0) - \bar{F}(kt_0)] - c_2[1 - f(\alpha)]}{1 - g(\alpha) \sum_{k=1}^{\infty} e^{-\alpha k t_0} [\bar{F}((k-1)t_0) - \bar{F}(kt_0)]} + c_2. \tag{33}$$

It is evident that $C_2(0) = \infty$ and $C_2(\infty) = c_2 f(\alpha)$, which agree with (12) and (13), respectively. Further, for large t_0,

$$\frac{d[\log C_2(t_0)]}{dt_0} \simeq \alpha e^{-\alpha t_0}\{ \frac{c_2 g(\alpha) - c_0 - c_1}{c_2 f(\alpha)} - g(\alpha)\}. \tag{34}$$

Thus, if

$$c_2 g(\alpha)[1 - f(\alpha)] > c_1 + c_0, \tag{35}$$

then there exists at least some finite t_0 such that $C_2(\infty) > C_2(t_0)$.

It is very difficult to obtain an optimum checking time t_2^* which minimizes the expected cost $C_2(t_0)$ in (33). It is noted, however, that the expected cost $C_2(t_0)$ in (33) agrees with (11) in special case of $F(t) = 1 - e^{-\lambda t}$.

4. The case that a generator fails during the electric power supply failure

We consider the case where a standby generator fails during the time of the electric power supply failure and obtain the expected cost by the time of the supply recovery again.

In addition, we make the following assumptions:

(i) The failure time of a standby generator in operation has the same failure time distribution $F(t)$ as it is in standby.

(ii) A standby generator which fails during the time of the electric power supply failure undergoes no repair.

(iii) The inspection of a standby generator is not made during the time of the electric power supply failure even if the time for inspection comes.

(iv) A cost c_2 is suffered for failure of a standby generator during the time of the electric power supply failure, which is the same as the cost for failure when the electric power supply fails.

Suppose that x is the required time from the time of the electric power supply failure to its recovery and is previously given. Let $\bar{P}(x,t)$ be the probability that a standby generator is good at time t and will continue to operate for an interval of time x. Then, we easily have that

$$(36)\qquad \bar{P}(x,t) = \bar{F}(t+x)\bar{A}(t) + H(t)*\bar{P}(x,t).$$

Dropping the condition that the electric power supply fails at time t,

$$(37)\qquad \bar{p}(x,\alpha) \equiv \int_0^\infty \alpha e^{-\alpha t}\,\bar{P}(x,t)dt$$

$$= [\int_0^{t_0} \alpha e^{-\alpha t}\,\bar{F}(t+x)dt]/[1 - h(\alpha)],$$

where $h(\alpha)$ is given by (4). It is evident that $\bar{p}(x,\alpha)$ becomes $1 - p(\alpha)$ in (8) as $x \to 0$.

Using (5), (6), and (37), we have the expected cost by the time of the electric power supply recovery:

$$(38)\qquad C_3(t_0,x) = c_0 m_0(\alpha) + c_1 m_1(\alpha) + c_2[1 - \bar{p}(x,\alpha)]$$

$$= \frac{e^{-\alpha t_0}[c_0 + c_1 F(t_0)] - c_2\int_0^{t_0} \alpha e^{-\alpha t}\,\bar{F}(t+x)dt}{1 - e^{-\alpha t_0}[\bar{F}(t_0) + F(t_0)g(\alpha)]} + c_2.$$

It is evident that $C_3(0,x) = \infty$, $C_3(\infty,x) = c_2\int_0^\infty \alpha e^{-\alpha t}\,F(t+x)dt$. Further, we have, for large t_0,

$$(39)\qquad \frac{d[\log C_3(t_0,x)]}{dt_0} \simeq \alpha e^{-\alpha t_0}\{\frac{c_2 g(\alpha) - c_0 - c_1}{c_2\int_0^\infty \alpha e^{-\alpha t}\,F(t+x)dt} - g(\alpha)\ .$$

Thus, if

$$(40)\qquad c_2 g(\alpha)\int_0^\infty \alpha e^{-\alpha t}\,\bar{F}(t+x)dt > c_1 + c_0,$$

then there exists at least some finite t_0 such that $C_3(\infty,x) > C_3(t_0,x)$.

Next, suppose that x is a random variable with a distribution $B(y)$. Then, from (38), we have the expected cost:

$$(41)\qquad C_3(t_0) = \int_0^\infty C_3(t_0,y)dB(y)$$

$$= \frac{e^{-\alpha t_0}[c_0 + c_1 F(t_0)] - c_2 \int_0^\infty [\int_0^{t_0} \alpha e^{-\alpha t} \overline{F}(t+y)dt]dB(y)}{1 - e^{-\alpha t_0}[\overline{F}(t_0) + F(t_0)g(\alpha)]} + c_2.$$

Thus, we can make similar discussions to the above ones.

Further, if the failure rate of a standby generator remains undisturbed by any inspection then

(42) $$\overline{P}(x,t) = \overline{F}(t+x) + H(t)*\overline{P}(x,t),$$

and

(43) $$\overline{p}(x,\alpha) = [\int_0^\infty \alpha e^{-\alpha t} \overline{F}(t+x)dt]/[1 - h(\alpha)],$$

where $h(\alpha)$ is given by (23). Thus, by the similar method of obtaining $C_3(t_0,x)$ in (38), we can obtain the expected cost for this case, although we omit it here.

We have considered the inspection model under the assumption (i). This assumption might not be true for some cases. For, the failure rates of a standby generator in standby and in actual operation may be different from each other, and in general, the failure rate in operation would be greater than that in standby. If the failure times in operation and in standby are independent of each other, it is easy to obtain the expected cost. Otherwise, it might be difficult.

6. Conclusions and examples

We have obtained the expected costs of the following three inspection models:

(1) A standby generator is as good as new upon inspection or repair.

(2) The failure rate of a standby generator remains undisturbed by any inspection.

(3) A standby generator fails during the time of the electric power supply failure.

We have shown a sufficient condition for each model that there exist finite checking times t_i ($i = 1, 2, 3$). This could help us to compute optimum times t_i^* numerically.

It is very difficult to derive optimum inspection policies analytically which minimize the expected costs except some particular cases. We give the numerical examples where $\overline{F}(t) = (1 + \lambda t)e^{-\lambda t}$ and $\overline{G}(t) = (1 + \mu t)e^{-\mu t}$, both of

Table 1. Dependence of the mean failure time $2/\lambda$ and the cost c_2 in optimum checking times t_1^* and t_2^* hrs when c_0 = \$3, c_1 = \$30, $1/\mu$ = 12 hrs, and $1/\alpha$ = 1,460 hrs.

mean failure time $2/\lambda$ hrs	c_2 = 150 t_1^*	c_2 = 150 t_2^*	c_2 = 250 t_1^*	c_2 = 250 t_2^*	c_2 = 350 t_1^*	c_2 = 350 t_2^*
1200	292	480	249	308	224	241
1600	368	535	311	354	279	280
2000	439	594	369	399	330	318
2400	507	656	424	445	379	356
2800	572	720	477	491	425	393
3200	635	783	528	537	469	430
3600	697	848	578	582	512	467
4000	757	914	626	628	554	503

Table 2. Dependence of the failure time x of the electric power supply and the cost c_2 in optimum cheking time t_3^* hrs when c_0 = \$3, c_1 = \$30, $2/\lambda$ = 2,000 hrs, $1/\mu$ = 12 hrs, and $1/\alpha$ = 1,460 hrs.

failure time x	c_2 = 150	c_2 = 250	c_2 = 350
0.1	439	369	330
0.5	439	370	330
1.0	439	370	331
2.0	439	370	331
5.0	440	371	332
10.0	442	372	333
20.0	446	376	337

which are the gamma distributions with a shape parameter 2.

Table 1 shows the optimum checking times t_1^* and t_2^* of the models (1) and (2), respectively, for the mean failure time $2/\lambda$ and the cost c_2 when we assume c_0 = 3 dollars, c_1 = 30 dollars, $1/\mu$ = 12 hours, and $1/\alpha$ = 1,460 hours, i.e., the electric power supply fails 6 times a year on the average. It has been shown that both of checking times are increasing if $2/\lambda$ is increasing and are decreasing if c_2 is increasing. Further, t_1^* becomes greater than t_2^* when c_2 and $2/\lambda$ are large enough.

Table 2 shows the optimum checking time t_3^* of the model (3), for the failure time x of the electric power supply and the cost c_2 under the same assumptions as ones in Table 1 when $2/\lambda$ = 2,000. We can see from this table that the optimum checking time is little changed even if x is increasing.

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