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INTEGER PROGRAMMING WITH IRREGULAR DATA SPACE

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Abstract This work presents a different computational way in finding optimal solutions of Integer-Programming problems under irregular data spaces. In addition, a real application on advertisement problems is given, where the maximum advertisement result is achieved with the minimum value of the risk probability of our campaign under limited resources. Graph theory, Metric Spaces, Taxonomy theory, Boolean algebra and principles of the General Catastrophe theory are used.

INTRODUCTION

Usually in Operations Research our data is modified within a limited period of time $\Delta t=t_2-t_1$, $(t_2>t_1)$. Thus, if it is necessary to make a decision for the instant of time t_2, based on our data which is held at the instant of time t_1, then it has been observed that very often this decision is not optimal at all at t_2. Certainly if we can know how data change within Δt then we can make our system mathematically controlled using Statistical control methods or corresponding Simulation models [1]. But if the data space is an irregular one, then it is very often impossible to find "how data change" using any known mathematical technique. However, this problem is very difficult to be solved in a general way. Thus, in paper [2] a computational method which solves our problem in the case of Plant Location Analysis is given. In this paper, the general case of integer programming is studied and especially, a special type of integer advertisement problem is researched; in addition a real application by a computer is given. Finally, it must be noted that some preliminary knowledge on the principles of the General Catastrophe theory is required (see appendix).

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CATASTROPHE INTEGER PROGRAMMING

The possibility of change of our data into Δt, presupposes the existence of a certain set of catastrophe situations which can act upon our data space and consequently they are able to change at least one of the values of our data. Broadly speaking, catastrophe situation can be a war, a revolution, pre-electoral situations, a strike, etc, etc. More specifically, the advertisement space is occasionally an irregular one, e.g. technical difficulties hampering the showing of the film on T.V. at t_2 or the unexpected appearance of an interesting Broadcasting on a different channel at t_2 are sufficient to ruin our plans. In this paper the above situations are called forces and the data spaces irregular data spaces. On the other hand, the hypothesis of irregular data space means that the economic space A is a Force Field,

$$A=\{a_1,a_2,\dots,a_n\}$$

where a_i, $i=1,2,\dots,n$ are the factors which we must take into consideration in order to make an optimal decision. However, this alteration implies the existence of a Potential into A regarding the given set of forces F by the period of time Δt, i.e. into system $(A,F,\Delta t)$,

$$F=\{f_1,f_2,\dots,f_m\}$$

If B is the set of profit which corresponds to A at the instant of time t_1, $B=\{b_1,b_2,\dots,b_n\}$, then we define as catastrophe point of $(A,F,\Delta t)$ every element $a_i\varepsilon A$ which changes the $b_i\varepsilon B$ within Δt. The catastrophe set of $(A,F,\Delta t)$ contains every catastrophe point and will be symbolized by $G(A,F,\Delta t)$. Consider the set $C=\{c_1,c_2,\dots,c_n\}$ of risk probabilities of the elements of the system $(A,F,\Delta t)$; $c_i\varepsilon C$ is the risk probability of $a_i\varepsilon A$, i.e. $c_i\varepsilon[0,1]$ is the probability associated with the change of profit $b_i\varepsilon B$ within Δt. Therefore, c_i is the probability to hold $a_i\varepsilon G(A,F,\Delta t)$. Consequently the Average Intensity of the system $(A,F,\Delta t)$ can be represented by $E=(\sum_i c_i)/|A|$, $i=1,2,\dots,n$. Thus, the value $c_i\varepsilon C$ will give the intensity of $(A,F,\Delta t)$ at the point $a_i\varepsilon A$. We distinguish two kinds of force fields: (i) Homogeneous force field, where for each $(a_i a_j)\varepsilon A^2$ it holds $c_i=c_j$, and (ii) Heterogeneous force field, where there exist at least two points of A with different intensities. In this work we consider only the case (ii) as it is more comprehensive. Certainly, if we can estimate the probabilities c_i, then we can easily get a stohastic integer program. But this is impossible in the case of irregular data space, especially when we have to solve a real world problem. However, we cannot estimate the average intensity, but it is possible to know the existence of it. On the other hand, we seek to find a solution which corresponds to a maximum profit, subject to constraints of the problem, so that the solution to has the minimum possible number of catastrophe points, i.e. the minimum number of changes until t_2. Therefore, if we define the matrix $X=(x_i)$, $i=1,2,...,n$ so that x_i is integer-valued and corresponds to the element $a_i \varepsilon A$, then the following integer-programming model arises:

$$\text{Maximize } K(x) = \sum_{a_i \varepsilon A^*} b_i x_i, \quad A^* \subset A \tag{1}$$

subject to,

(a) The usual constraints (2)

(b) $K(x) \geq K(x^o) - e$ (3)

(c) Minimize $|\{a_i \varepsilon A^*: \; x_i \neq 0 \text{ and } a_i \varepsilon G(A,F,\Delta t)\}|$ (4)

where $K(x^o)$ is the maximum solution of (1) under the condition (2) with $A^*=A$; i.e. $K(x^o)$ is the maximum value of the objective function when the potential does not exist. The parameter e is a non negative integer number, giving the information of how much profit at most is permitted to be lost in order to get the maximum possible confidence of our solution.

As we have already explained, we cannot know a priori the catastrophe set $G(A,F,\Delta t)$; owing to this, we must replace condition (c) by a weaker restriction. Utilizing a certain information, it is possible to find the forces and the possibility of their action upon the elements of A within Δt. Consequently, we can create the <u>catastrophe matrix</u> G of the system $(A,F,\Delta t)$:

$$G=(g_{ij}), \quad i=1,2,...,n \qquad j=1,2,...,m$$

where,

$$g_{ij}=1, \text{ if } f_i \text{ can act upon } a_i \text{ at } t_2$$
$$=0, \text{ otherwise.}$$

Define the function $\delta(a_u,a_v) \equiv \delta_{uv} = \sum_j |g_{uj}-g_{vj}|$, $j=1,2,...,m$ $\quad u,v=1,2,...,n$.

It is well known, that δ by its definition is a distance into the set A and so, (A,δ) is a metric space. Consequently, the matrix Δ is created:

$$\Delta=(\delta_{uv}), \qquad u,v=1,2,...,n$$

After that, we propose instead of condition (4), the following restriction:

$$\text{Maximize } U(A^*)=(2/|\bar{A}^*|\cdot(|\bar{A}^*|-1)). \sum_{u,v} \delta_{uv}\cdot\bar{x}_u\cdot\bar{x}_v, \quad a_u,a_v \varepsilon A^* \tag{5}$$

where $\bar{x}_u=1$ if $x_u>0$ and $\bar{x}_u=0$ otherwise, and $|\bar{A}^*|=\sum_u \bar{x}_u$, $a_u \varepsilon A^*$. Condition (5) means that the elements of the solution have the maximum difference of the probability to be catastrophe points. So far, our experimental results show that the proposed measure U(A*) is a criterion very "close" to restriction (4). However, an optimal solution to problem (1), (2), (3), (5) is a "good" heuristic solution to the initial problem (1), (2), (3), (4). On the other hand, the problem (1), (2), (3), (5) requires not only the estimation of the unknown vector x but also to find the subset A*. Therefore, this problem cannot be solved by any known mathematical method even when a fast computer is used; because it is necessary to be solved w integer programs using Branch-and-Bound:

$$w=\binom{A}{1}+\binom{A}{2}+\ldots+\binom{A}{n}=2^n$$

Owing to this, the conclusion is drawn that, if TB is the time required for the Branch-and-Bound, then the program runs in time $O(TB*2^n)$. Therefore we have to create some new method based on which it would be possible to get only the subsets A* which satisfy the conditions (3) and (5).

THE METHOD

We give an initial value to the positive number $\Theta\varepsilon\{0,1,2,\ldots, \max \delta_{uv}\}$. Any how the application of the method itself will help fix the final value of Θ.

Two elements $a_i, a_j \varepsilon A$ will be 'similar' if, and only if, it holds $\delta_{ij} \leq \Theta$, $i,j=1,\ldots,n$. Consequently we get the matrix:

$$H=(h_{ij}),\ i,j=1,\ldots,n$$
$$h_{ij}=1, \text{ if } \delta_{ij}\leq\Theta$$
$$=0 \text{ otherwise.}$$

Based on H the undirected graph (A,H) with value on the edge (a_i,a_j) the value δ_{ij}, can be constructed.

Clique of an undirected graph is defined every subgraph in which for each couple of points there exists the corresponding arc [3].

Maximum clique of an undirected graph is defined every clique of it that (i) is not included in any other clique of the graph and (ii) has the maximum possible cardinal number. We find all the maximum cliques of (A,H) solving the following Boolean system [4]:

$$(\bar{H} * \vec{a}) \cdot \vec{a} = \vec{0} \qquad (6)$$

where:

$\bar{H}$ is the Boolean negation of H,

* is the Boolean multiplication between matrix and vector,

$\cdot$ is the Boolean multiplication between two vectors,

$\vec{a}=(a_1,a_2,\ldots,a_n)$ and $\vec{0}=(0,0,\ldots,0)$.

Based on the maximum cliques we can find by computational methods all the equivalent partitions of A with the minimum cardinal number [5]. From all these partitions we are interested in finding the optimal, whose classes give the maximum dissimilarity; this means that its classes have the maximum difference of the probability to be subsets of $G(A,F,\Delta t)$. After that, in order to solve program (1) subject to (2), (3) and (5), we give an algorithm whose steps are ordered as follows:

STEP 0. Set $\Theta=0$. Open the master list ML. Find the value $K(x^o)$ and the matrix Δ. Compute the number $U(A)$. Keep into ML the $K(x^o)$, x^o, and $U(A)$.

STEP 1. Set instead of $\Theta,\Theta+1$. Find the matrix H and solve the system (6).

STEP 2. Find the minimum partitions Γ_ϕ. If there exists only one minimum partition then go to step 5. Otherwise for every Γ_ϕ compute the distance:

$$Q_\phi(L_i^\phi,L_i^\phi)=(2/|L_i^\phi|\cdot|L_j^\phi|)\cdot\sum_{\lambda,s}\delta_{\lambda,s}$$

where $(a_\lambda,a_s)\varepsilon(L_i^\phi X L_j^\phi)$ and $L_i^\phi \neq L_j^\phi \quad \varepsilon\ \Gamma_\phi$.

STEP 3. Find the numbers Φ_ϕ:

$$\Phi_\phi=\sum_{i,j} Q_\phi(L_i^\phi,L_i^\phi),\quad i\neq j=1,2,\ldots,|\Gamma_\phi|$$

STEP 4. Find the partition which corresponds to the maximum value of Φ_ϕ; let Γ_ϕ^o be this partition:

$$\Gamma_\phi^o=\{L_1^\phi,L_2^\phi,\ldots,L_k^\phi\}$$

STEP 5. Solve by Branch-and-Bound every program (1) subject to (2) which is created as follows:

$$A^*=\{a_{i_{\phi 1}},a_{i_2},\ldots,a_{i_k}\}$$

where $a_{i_1}\varepsilon L_1,a_{i_2}\varepsilon L_2^\phi,\ldots,a_{i_k}\varepsilon L_k^\phi$.

STEP 6. Compute the $U(A^*)$ for each solution which satisfies the condition (3). Keep into ML only this solution that has the maximum $U(A^*)$.

If there exist equivalent solutions then keep this one which gives the maximum $k(x)$.

STEP 7. If there exists only one solution in ML then go to step 8. Otherwise keep into ML only this solution which gives the maximum $U(A^*)$.

STEP 8. If $\Theta=\max \delta_{uv}$ go to step 9. Otherwise go to step 1.

STEP 9. The solution which is existed in ML is the optimal solution to the program (1) subject to (2), (3) and (5). END.

If we have two fast subroutines, one for the system (6) and the other for the minimum partitions Γ_ϕ, then the method which we have already presented gives the optimal solution by a computer in a few minutes. Certainly the use here of the subroutine LIFO-BB (Last In, First-out, Branch-and-Bound) is a prerequisite [6]. Finally it must be noted that the proposed algorithm is a general one on pure integer programming under catastrophe conditions and it is possible to fix this technique for special cases or to extend it for linear programming using instead of LIFO-BB method, the SIMPLEX subroutine; but this extension is given as an opened question as well as the corresponding linear program.

THE APPLICATION OF THE METHOD TO THE SOLUTION OF ADVERTISEMENT PROBLEMS

How the optimal solution may be carried out in practice or the fixed of the proposed algorithm in real applications, may be explained by giving the following real problem:

"The Firm Y wishes to have the maximum advertisement result of its good y with the minimum value of risk probability of its advertisement system under limited resources and irregular data space".

This problem was actually more difficult because another similar good y' of the firm Y' had been sold two years before. At first, we found that the Television had to be used as the main medium; this was found using the theory of games and a computer program in WATFIV-FORTRAN [7], [8]. At the instant of time t_2 the following broadcastings were available:

$$A=\{a_1,\dots,a_8\}$$

Using statistical data we found the following set of profit at t_1:

$$B=\{100,90,85,79,75,68,65,62\}$$

where the a_i had $(b_i \cdot 1000)$ auditors, $i=1,\dots,8$. In addition, the corresponding set of forces and the set of cost were as follows:

$F=\{f_1,\ldots,f_{10}\}$, $W=\{w_1,\ldots,w_8\}=\{15,16,14,12,13,11,14,12\}$

where $w_i \varepsilon W$ is the cost which corresponds to $a_i \varepsilon A$. The available sum for the exploitation of A was $\Theta=52$ and $e=20$. After that the following special program arises:

$$\underset{x,A^*}{\text{Maximize}}\ K(x) = \sum_{a_i \varepsilon A^*} b_i \cdot x_i,\ A^* CA$$

subject to,

$$\sum_{a_i \varepsilon A^*} w_i \leq 52,\ K(x) \geq K(x^o)-20$$

$$\text{Maximixe } U(A^*)=(2/|A^*|\cdot(|A^*|-1)).\ \sum_{u,v} \delta_{uv}, a_u, a_v \varepsilon A^*.$$

$x_i=1$, if $a_i \varepsilon A^*$

$=0$ otherwise.

We see that in this special case, in fact, we must find only the unknown A*, because the vector x is defined directly from the subset A*. On the other hand the catastrophe matrix G was the following one:

1	0	1	1	1	0	0	1	1	1
1	1	1	1	1	1	0	1	1	1
0	1	0	1	1	0	0	1	1	1
1	1	1	0	1	1	0	0	1	1
1	1	1	0	1	0	1	0	1	1
0	1	0	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1	0	0
0	1	1	0	1	1	1	0	1	1

Consequently the matrix Δ is as follows:

0	2	3	4	4	7	5	6
2	0	3	2	4	5	3	4
3	3	0	5	5	4	6	5
4	2	5	0	2	7	5	2
4	4	5	2	0	7	5	2
7	5	4	7	7	0	4	5
5	3	6	5	5	4	0	5
6	4	5	2	2	5	5	0

Thus, we get:

STEP 0. $\Theta=0$; $K(x^o)=332, x^o=(x_1,x_3,x_4,x_6)$, $U(A)=30/6$. We keep this solution into the master list ML.

STEP 1. $\Theta=1$; it is valid $H=(h_{ij})=0$. The maximum solution of (6) is $\{a_i,\ldots,a_8\}$.

STEP 2. There exists only one partition, $\{\{a_1\},\ldots,\{a_8\}\}$.

STEP 5. We see that $A^*=A$; Step 6, step 7, step 8. Go to step 1.

STEP 1. $\Theta=2$. The matrix H is as follows,

1	1	0	0	0	0	0	0
1	1	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	1	1	0	0	0
0	0	0	1	1	0	0	1
0	0	0	0	0	1	0	0
0	0	0	0	0	0	1	0
0	0	0	1	1	0	0	1

The maximum solutions of system (6) are,

$\{a_1,a_2\},\{a_2,a_4\},\{a_5,a_8\}$.

STEP 2. There exist the following minimum partitions:

$\Gamma_1=\{\{a_1,a_2\},\{a_5,a_8\},\{a_3\},\{a_4\},\{a_6\},\{a_7\}\}$

$\Gamma_2=\{\{a_2,a_4\},\{a_5,a_8\},\{a_1\},\{a_3\},\{a_6\},\{a_7\}\}$

STEP 3. $\Phi_1=216$, $\Phi_2=218$.

STEP 4. $\Gamma^o_\phi=\Gamma_2$.

STEP 5. There exist the following subsets:

$A^*_1=\{a_2,a_5,a_1,a_3,a_6,a_7\}$

$A^*_2=\{a_2,a_8,a_1,a_3,a_6,a_7\}$

$A^*_3=\{a_4,a_5,a_1,a_3,a_6,a_7\}$

$A^*_4=\{a_4,a_5,a_1,a_3,a_6,a_7\}$

In this special case, it is easy to see that it is not necessary to apply the LIFO-BB method for each of the subsets above, because it is obvious that no one satisfies the condition $\sum_{a_i\in A^*} w_i\leq 52$.

STEP 6, step 7, step 8. Go to step 1.

STEP 1. $\Theta=3$. The matrix H is as follows,

1	1	1	0	0	0	0	0
1	1	1	1	0	0	1	0
1	1	1	0	0	0	0	0
0	1	0	1	1	0	0	1
0	0	0	1	1	0	0	1
0	0	0	0	0	1	0	0
0	1	0	0	0	0	1	0
0	0	0	1	1	0	0	1

We get the following maximum solutions of system (6):

$$\{a_1,a_2,a_3\},\quad \{a_4,a_5,a_8\}$$

STEP 2. There exist the following minimum partitions:

$$\Gamma_3=\{\{a_1,a_2,a_3\},\{a_4,a_5,a_8\},\{a_6\},\{a_7\}\}$$

$$\Gamma_4=\{\{a_1,a_3\},\{a_2,a_7\},\{a_4,a_5,a_8\},\{a_6\}\}$$

STEP 3. $\Phi_3=59\ 5/9$, $\Phi_4=58\ 2/3$.

STEP 4. $\Gamma^o_\phi=\Gamma_3$.

STEP 5. There exist the following subsets:

$$A^*_5=\{a_1,a_4,a_6,a_7\}\ ,\ A^*_6=\{a_1,a_5,a_6,a_7\}$$

$$A^*_7=\{a_1,a_8,a_6,a_7\}\ ,\ A^*_8=\{a_2,a_4,a_6,a_7\}$$

$$A^*_9=\{a_2,a_5,a_6,a_7\}\ ,\ A^*_{10}=\{a_2,a_8,a_6,a_7\}$$

$$A^*_{11}=\{a_3,a_4,a_6,a_7\}\ ,\ A^*_{12}=\{a_3,a_5,a_6,a_7\}$$

and $A^*_{13}=\{a_3,a_8,a_6,a_7\}$

We can easily see that only the subsets A^*_5,A^*_7 and A^*_{13} satisfy the condition $\sum_{a_i\varepsilon A^*} w_i\leq 52$. The solutions of these subsets are,

$$K(x^5)=312,\quad x^5=(x_1,x_4,x_6,x_7)$$

$$K(x^7)=295,\quad x^7=(x_1,x_6,x_7,x_8)$$

$$K(x^{13})=290,\quad x^{13}=(x_3,x_6,x_7,x_8)$$

STEP 6. The only solution which satisfies the condition $K(x)\geq 332-20$ is the solution $K(x^5)$; $U(A^*_5)=32/6$. We keep into ML this solution.

STEP 7. There exist the following solutions in ML:
$K(x^o)=332$, $x^o(x_1,x_3,x_4,x_6)$, $U(A)=30/6$

$K(x^5)=312$, $x^5=(x_1,x_4,x_6,x_7)$, $U(A_5^*)=32/6$.

we keep into ML the solution $K(x^5)$.

STEP 8. Go to step 1.

STEP 1. $\Theta=4$, etc, etc.

The program until the value $\Theta=7$ did not give any other better solution than the $K(x^5)$. Consequently the optimal solution is the $K(x^5)$. In other words, this means that firm Y must cover the broadcastings a_1, a_4, a_6 and a_7 with profit 312,000 auditors.

BLACK ECONOMICAL HOLES

As far as we can tell, it is possible to solve integer-programming problems under catastrophe conditions, so that the existing potential to create a 0-1 catastrophe matrix. However it would be very important if we could get a matrix $G=(g_{ij})$ such that $g_{ij}\varepsilon[0,1]$, i.e. the g_{ij} be the probability of acting of f_j upon a_i at t_2. This is the Stochastic-Catastrophe integer-programming which is given as an opened question. An other important composite opened question is the extension of irregular data-spaces on Investment theory, Theory of Games and Plant Location analysis in three-dimensional space [9], [13].

Black Economical Hole (BEH) is defined the subset A^*CA so that to exist a force $f_{j_o}\varepsilon F$ such that for every $a_i\varepsilon A^*$ to hold $g_{ij}=1$. For instance, the solution $K(x^o)$, $x^o=(x_1,x_3,x_4,x_6)$ of the presented application (with potential equal to zero), corresponds to a BEH, because there exists the force f_{10} such that:

$$g_{ij}=1,\ j=10,\ a_i\varepsilon A^*=\{a_1,a_3,a_4,a_6\}$$

Consequently the force f_{10} is able to ruin all our plans. In our application the force f_{10} described the following situation: "firm Y' makes the decision to begin a new campaign for the good y' at t_2". Because of this, the conclusion is drawn that it would be better to keep into master list ML not only one solution per each iteration but the R best solutions, R=2,3,....

Finally, our opinion is that the theory presented in this paper as well as the proposed method give some new realistic ideas and possibilities to Mathematical programming, especially in the case where it is impossible for us to estimate the risk probabilities c_i.

APPENDIX

Catastrophe Theory is one of the most modern and important theories of the last decase [10], [11]. We distinguish the General Catastrophe Theory (G.C.T.)

and the Specific Catastrophe Theory (S.C.T.) Whereas we can say so much about the S.C.T. (which in our case is almost unnecessary), however we know only a few on G.C.T., because it is only comparatively recently that the subject of G.C.T. has attracted the serious attention of researchers of applied sciences.

The G.C.T. signifies a space, whose elements are under certain forces and consequently we get some changes during the time; owing to this we can say, generally, that the space must be a force field. As a result, some of the elements (at least one) create interruption points into the force field. These elements are called catastrophe points. Usually, these points are found by determining the minimum point or the local minimum points of the field, because these points have the maximum probability to become catastrophe points during a period of time. In addition, the G.C.T. presupposes the existence of a certain potential, which is not necessarily like the well known potential of physical sciences; the potential of the G.C.T. can be something more general and it means that our system is not a static space and, consequently, at least one point will be changed within a certain period of time. In addition, it must be noted that the G.C.T. presupposes the existence of a potential, but does not expand on it.

Many researchers define seven simple kinds of catastrophe and they assert that every other kind is a combination of them. But this idea is not a new one, because a more general idea was first developed by Lamaism; Lamaism partitioned the set of all energies into seven classes and called the first of them "Catastrophe Ray" which was also partitioned into seven simple classes [12].

Finally, it is our opinion that the principles of G.C.T. have a significant role to play in the basic ideas of Operational research.

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