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BIVARIATE SHOCK MODELS: NBU AND NBUE PROPERTIES, AND POSITIVELY QUADRANT DEPENDENCY

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Abstract In the reliability theory the joint life distribution of units is usually derived under the assumption of stochastic independence for the behavior of units. But it is seen in the practical situation that the stochastic dependence arises by the cause of putting the units in a same environment and so on. A shock model is used to formulate the joint life distribution in the existence of correlation. For example bivariate exponential distribution function and bivariate Erlang distribution function are ones formulated using the shock model. In the present paper the wider class of bivariate distribution function which is called bivariate shock model is defined, and some properties are discussed. Using these properties the practically observable characteristic trend is explained and some bounds for the distribution function are given.

1. Introduction.

Reliability of a system is derived usually under the assumption that the units are stochastically independent, which construct the system. But it is seen in the practical situation that unit B fails by the failure of unit A, units A and B fail simultaneouslly by the same outer cause and so on. In the existence of the correlated failure the joint failure time distribution function of units A and B, i.e., $\bar{H}(x,y)=\Pr[X_1>x,\ X_2>y]$, is defined using the shock models as the following. Three stochastically independent shock processes, $\{N_1(t),t\geqq 0\}$, $\{N_2(t),t\geqq 0\}$ and $\{N(t),t\geqq 0\}$ are considered. A shock from $\{N_1(t),t\geqq 0\}$ is to the unit A only, a shock from $\{N_2(t),t\geqq 0\}$ is to the unit B only and a shock from $\{N(t),t\geqq 0\}$ is to both units simultaneously. Units A and B have the joint probability $\bar{P}(k_1,k_2)$ of surviving the first k_1 shocks and k_2 shocks respectively. Then $\bar{H}(x,y)=\Sigma_{k_1=0}^{\infty}\Sigma_{k_2=0}^{\infty}\bar{P}(k_1,k_2)\Pr[N_1(x)+N(x)=k_1,\ N_2(y)+N(y)=k_2]$. If $\{N_1(t),t\geqq 0\}$, $\{N_2(t),t\geqq 0\}$ and $\{N(t),t\geqq 0\}$ are Poisson processes with respective parameter λ_1, λ_2 and λ, $\bar{H}(x,y)$ is called bivariate shock models which is a bivariate extention of shock models defined by J.D.ESARY, A.W.MARSHALL and F. PROSCHAN [3]. If $\bar{P}(k_1,k_2)=1$ $(k_1=k_2=0)$, $\bar{P}(k_1,k_2)=0$ (otherwise), i.e., each shock is fatal to units, $\bar{H}(x,y)$ is bivariate exponential distribution function defined by A.W.MARSHALL and I.OLKIN [7]. Letting $\bar{P}(k_1,k_2)=1$ $(k_1\leqq l_1,\ k_2\leqq l_2)$ and $\bar{P}(k_1,k_2)=0$ (otherwise), $\bar{H}(x,y)$ is bivariate Erlang distribution function defined by T.ITOI, M.MURAKAMI, M.KODAMA and T.NISHIDA [4]. F.OHI and T.NISHIDA [9] define $\bar{H}(x,y)$ assuming $\bar{P}(k_1,k_2)=\bar{P}^{(1)}_{k_1}\bar{P}^{(2)}_{k_2}$. This means that the strength of units A and B are independent mutually. But it is seen that the way to construct the system makes the units A and B be dependent, then $\bar{P}(k_1,k_2)$ is more practical than $\bar{P}^{(1)}_{k_1}\bar{P}^{(2)}_{k_2}$.

The correlated failure makes the redundancy technique little effect, which is used to make the system high reliable. And in general the reliability of series system is higher than that under the assumption of independency and the unreliability of parallel system is higher than that under the same assumption. This means the existence of positive dependency between X_1 and X_2. Several definitions of positive dependency is seen in E.L.LEHMAN [6], R.E.BARLOW and F. PROSCHAN [1] and F.OHI and T.NISHIDA [9]. In the section 1 some concepts of positive dependency which will be needed in this paper are summarized and the theorem 4.2 in the section 4 gives the conditions under which $\bar{H}(x,y)$ has positive dependency. The above mentioned general tendency is explained by this theorem.

Univariate life distribution function F is said to be NBU or NBUE if $\bar{F}(x+y)\leqq\bar{F}(x)\bar{F}(y)$ or $\int_0^\infty\bar{F}(x+y)dx\leqq\bar{F}(y)\int_0^\infty\bar{F}(x)dx$ respectivelly. Relating to the replacement problem of one unit these concepts are defined and applied by A.W. MARSHALL and F.PROSCHAN [8]. The extention of these concepts to the bivariate case is done and precisely discussed by F.OHI and T.NISHIDA [10]. The definitions of bivariate NBU (BNBU) and bivariate NBUE (BNBUE) distribution functions are presented in the section 2 and theorems 2.1-2.4 give BNBU distribution functions some bounds. If the conditions under which $\bar{H}(x,y)$ is BNBU are presented, then these bounds may be used to evaluate $\bar{H}(x,y)$.

Throughout this paper we assume that $\bar{F}(x,y)=\Pr[X>x,\ Y>y]$, where $F(x,y)$ is joint distribution function of random variables X and Y. Marginal distribution functions of $F(x,y)$ are $F_1(x)=F(x,+\infty)$ and $F_2(y)=F(+\infty,y)$, or $\bar{F}_1(x)=\bar{F}(x,0)$ and $\bar{F}_2(y)=\bar{F}(0,y)$. Distribution functions dealt with in this paper are assumed to satisfy the following condition, i.e., $F(x,y)=0$ $(x\leqq 0$ or $y\leqq 0)$.

2. Definitions.

Definition 2.1. $N_1(t)$, $N_2(t)$ and $N(t)$ are mutually independent Poisson processes with parameter λ_1, λ_2 and λ respectively. $P(k_1, k_2)$ is discrete bivariate distribution function. Bivariate shock model (BVSM) is the bivariate distribution function $H(x,y)$ as the following.

$$\bar{H}(x,y) = \Sigma_{k_1=0}^{\infty}\Sigma_{k_2=0}^{\infty} \bar{P}(k_1,k_2)\Pr[N_1(x)+N(x)=k_1,\ N_2(y)+N(y)=k_2]$$

Remark 2.1. Since $N_1(t)$, $N_2(t)$ and $N(t)$ have stationary independent increments respectively,

$$\Pr[N_1(x+t)+N(x+t)=k_1,\ N_2(y+t)+N(y+t)=k_2]$$
$$=\Sigma_{l_1=0}^{k_1}\Sigma_{l_2=0}^{k_2} \Pr[N_1(x)+N(x)=k_1-l_1,\ N_2(y)+N(y)=k_2-l_2]\cdot\Pr[N_1(t)+N(t)=l_1,\ N_2(t)+N(t)=l_2].$$

Definition 2.2. Bivariate distribution function G is said to be :

(i) BNBUS (BNWUS) if

$$\bar{G}(x+t,y+t)\leq(\geq)\bar{G}(t,t)\bar{G}(x,y),$$
$$\bar{G}_i(x+t)\leq(\geq)\bar{G}_i(t)\bar{G}_i(x) \qquad (i=1,2),$$

for any $x>0$, $y>0$ and $t>0$.

(ii) BNBUP (BNWUP) if

$$[1-G(x+t,y+t)]\leq(\geq)[1-G(t,t)][1-G(x,y)],$$
$$\bar{G}_i(x+t)\leq(\geq)\bar{G}_i(t)\bar{G}_i(x) \qquad (i=1,2),$$

for any $x>0$, $y>0$ and $t>0$.

BNBUS (BNWUS) properties mean that the life length of a two units series system is stochastically greater (less) than that of a two units series system composed of two unfailed items of age x and y. BNBUP (BNWUP) properties tell us similar physical meaning for a two units parallel system.

Theorem 2.1. Let F be BNBUS. Then

(i) $\bar{F}(x,y)\geq[\bar{F}(t,t)]^{1/k}$ on $[t/(k+1)<x\leq t/k$ and $y\leq t/k]$ or $[t/(k+1)<y\leq t/k$ and $x\leq t/k]$ $(k=1,2,\ldots)$.

(ii) $\bar{F}(x,y)\leq[\bar{F}(t,t)]^{l}[\bar{F}_1(t)]^{k-l}$ if $k\geq l$,

$\bar{F}(x,y)\leq[\bar{F}(t,t)]^{k}[\bar{F}_2(t)]^{l-k}$ if $l>k$,

on $[kt<x\leq(k+1)t$ and $lt<y\leq(l+1)t]$ $(k,l=0,1,\ldots)$.

Proof: (i) Since $\bar{F}(mt/k,mt/k)\leq\bar{F}((m-1)t/k,(m-1)t/k)\cdot\bar{F}(t/k,t/k)$, then $\bar{F}(t/k,t/k)\geq[\bar{F}(mt/k,mt/k)]^{1/m}$ inductively. Hence $\bar{F}(x,y)\geq\bar{F}(t/k,t/k)\geq[\bar{F}(t,t)]^{1/k}$.
(ii) If $k\geq l$, $\bar{F}(x,y)\leq\bar{F}(kt,lt)\leq[\bar{F}(t,t)]^{l}\bar{F}_1((k-l)t)\leq[\bar{F}(t,t)]^{l}[\bar{F}_1(t)]^{k-l}$. In the case of $l>k$, the proof is similar. ||

Theorem 2.2. Let F be BNBUP. Then

(i) $1-F(x,y)\geq[1-F(t,t)]^{1/k}$ on $[t/(k+1)<x\leq t/k$ and $y\leq t/k]$ or $[t/(k+1)<y\leq t/k$ and $x\leq t/k]$ $(k=1,2,\ldots)$.

(ii) $1-F(x,y)\leq[1-F(t,t)]^{l}[1-F_1(t)]^{k-l}$ if $k\geq l$,

$1-F(x,y)\leq[1-F(t,t)]^{k}[1-F_2(t)]^{l-k}$ if $l>k$,

on $[kt<x\leq(k+1)t$ and $lt<y\leq(l+1)t]$ $(k,l=0,1,2,\ldots)$.

Proof: Corresponding $1-F(x,y)$ to $\bar{F}(x,y)$, this theorem is proved similarly as theorem 2.1. ||

Theorem 2.3. Let F be BNWUS. Then

(i) $\bar{F}(x,y)\leq[\bar{F}(t,t)]^{1/(k+1)}$ on $[t/(k+1)<x\leq t/k$ and $t/(k+1)<y]$ or $[t/(k+1)<x$ and $t/(k+1)<y\leq t/k]$ $(k=1,2,\ldots)$.

(ii) $\bar{F}(x,y)\geq[\bar{F}(t,t)]^{l+1}[\bar{F}_1(t)]^{k-l}$ if $k\geq l$,

$\bar{F}(x,y)\geq[\bar{F}(t,t)]^{k+1}[\bar{F}_2(t)]^{l-k}$ if $l>k$,

on $[kt<x\leq(k+1)t$ and $lt<y\leq(l+1)t]$ $(k,l=0,1,2,\ldots)$.

Proof: (i) $\bar{F}(x,y)\leq\bar{F}(t/(k+1),t/(k+1))\leq[\bar{F}(t,t)]^{1/(k+1)}$.

(ii) If $k\geq l$, we have $\bar{F}(x,y)\geq\bar{F}((k+1)t,(l+1)t)\geq[\bar{F}(t,t)]^{l+1}[\bar{F}_1(t)]^{k-l}$. In the case of $l>k$, the proof is similar. ||

Theorem 2.4. Let F be BNWUP. Then

(i) $1-F(x,y)\leq[1-F(t,t)]^{1/(k+1)}$ on $[t/(k+1)<x<t/k$ and $t/(k+1)<y]$ or $[t/(k+1)<x$ and $t/(k+1)<y\leq t/k]$ $(k=1,2,\ldots)$.

(ii) $1-F(x,y)\geq[1-F(t,t)]^{l+1}[1-F_1(t)]^{k-l}$ if $k\geq l$,

$1-F(x,y)\geq[1-F(t,t)]^{k+1}[1-F_2(t)]^{l-k}$ if $l>k$,

on $[kt<x\leq(k+1)$ and $lt<y\leq(l+1)$] $(k,l=0,1,2,\ldots)$.

Proof: Corresponding $1-F(x,y)$ to $\bar{F}(x,y)$, this theorem is proved similarly as theorem 2.3. ||

Definition 2.3. Bivariate distribution function G is said to be:

(i) BNBUES (BNWUES) if

$$\int_0^\infty\bar{G}(x+t,y+t)dt\leq(\geq)\bar{G}(x,y)\int_0^\infty\bar{G}(t,t)dt,$$

$$\int_0^\infty\bar{G}_i(x+t)dt\leq(\geq)\bar{G}_i(x)\int_0^\infty\bar{G}_i(t)dt \quad (i=1,2)$$

for any $x>0$ and $y>0$.

(ii) BNBUEP (BNWUEP) if

$$\int_0^\infty[1-G(x+t,y+t)]dt\leq(\geq)[1-G(x,y)]\int_0^\infty[1-G(t,t)]dt,$$

$$\int_0^\infty\bar{G}_i(x+t)dt\leq(\geq)\bar{G}_i(x)\int_0^\infty\bar{G}_i(t)dt \quad (i=1,2)$$

for any $x>0$ and $y>0$.

BNBUES (BNWUES) properties mean that the mean life length of a two units series system is stochastically greater (less) than that of a two units series system composed of two unfailed items of age x and y. BNBUEP (BNWUEP) properties tell us similar physical meaning for a two units parallel system. Precise discussions on BNBUS and BNBUP distribution functions are shown in F.OHI and T. NISHIDA [10].

Definition 2.4. Given nonnegative real valued random variables S and T and joint distribution function G, we say the following.

(i) S and T are *positively quadrant dependent* (PQD(S,T)) if

$\Pr[S\leq s, T\leq t]\geq \Pr[S\leq s]\Pr[T\leq t]$ for all $s\geq 0$, $t\geq 0$.

In this case G (or $\bar{G}$) is said to be PQD.

(ii) T is *left tail decreasing* in S (LTD(T|S)) if

$\Pr[T\leq t|S\leq s]\downarrow_{s\geq 0}$ for all $t\geq 0$.

If LTD(T|S) and LTD(S|T) hold, G (or $\bar{G}$) is called LTD.

(iii) T is *right tail increasing* in S (RTI(T|S)) if

$\Pr[T>t|S>s]\uparrow_{s\geq 0}$ for all $t\geq 0$.

If RTI(T|S) and RTI(S|T) hold, G (or $\bar{G}$) is called RTI.

If S and T are mutually independent, the conditions of definition 2.4 are satisfied. PQD and LTD are mentioned by E.L.LEHMANN [6], and RTI is presented by J.D.ESARY and F.PROSCHAN [2]. Some other concepts of positive dependency are seen in R.E.BARLOW and F.PROSCHAN [1] and F.OHI and T.NISHIDA [9].

Definition 2.5. Bivariate function K is said to be *totally positive in order* 2 (TP_2) if

$$\begin{vmatrix} K(x_1,y_1) & K(x_1,y_2) \\ K(x_2,y_1) & K(x_2,y_2) \end{vmatrix} \geq 0 \qquad \text{for any } x_1<x_2 \text{ and } y_1<y_2.$$

Precise discussions on TP_2 functions will be seen in S.KARLIN [5].

3.NBU and NBUE Properties of BVSM.

This section presents the conditions under which $\bar{H}$ is BNBU or BNBUE. If these conditions are verified, we may evaluate $\bar{H}$ using the bounds given in the previous section.

Theorem 3.1. (i) BVSM is BNBUS (BNWUS) if

$$\bar{P}(k_1,k_2)\bar{P}(l_1,l_2)\geq(\leq)\bar{P}(k_1+l_1,k_2+l_2) \qquad (k_1,k_2,l_1,l_2=0,1,2,\ldots).$$

(ii) BVSM is BNBUP (BNWUS) if

$$[1-P(k_1,k_2)][1-P(l_1,l_2)]\geq(\leq)1-P(k_1+l_1,k_2+l_2) \qquad (k_1,k_2,l_1,l_2=0,1,2,\ldots).$$

Proof: Let $f(x,k_1;y,k_2)=\Pr[N_1(x)+N(x)=k_1, N_2(y)+N(y)=k_2]$.

(i) Noticing the remark 2.1,

$$\begin{aligned}\bar{H}(x,y)\bar{H}(t,t)&=[\Sigma_{k_1=0}^{\infty}\Sigma_{k_2=0}^{\infty}\bar{P}(k_1,k_2)f(x,k_1;y,k_2)][\Sigma_{l_1=0}^{\infty}\Sigma_{l_2=0}^{\infty}\bar{P}(l_1,l_2)f(t,l_1;t,l_2)]\\ &\geq(\leq)\Sigma_{j_1=0}^{\infty}\Sigma_{j_2=0}^{\infty}\bar{P}(j_1,j_2)\Sigma_{k_1=0}^{j_1}\Sigma_{k_2=0}^{j_2}f(x,k_1;y,k_2)f(t,j_1-k_1;t,j_2-k_2)\\ &=\Sigma_{j_1=0}^{\infty}\Sigma_{j_2=0}^{\infty}\bar{P}(j_1,j_2)f(x+t,j_1;y+t,j_2)\\ &=\bar{H}(x+t,y+t).\end{aligned}$$

From the assumption, $\bar{P}_i(k)\bar{P}_i(l)\geq(\leq)\bar{P}_i(k+l)$ $(i=1,2)$ hold. Then using the theorem 3.1 of J.D.ESARY, A.W.MARSHALL and F.PROSCHAN [3], it follows that $\bar{H}_i(x)\cdot$

$\bar{H}_i(t) \geq (\leq) \bar{H}_i(x+t) \quad (i=1,2)$.

(ii) $1-H(x,y)=\Sigma_{k_1=0}^{\infty}\Sigma_{k_2=0}^{\infty}[1-P(k_1,k_2)]f(x,k_1;y,k_2)$. Then corresponding $1-P(k_1, k_2)$ to $\bar{P}(k_1,k_2)$, the proof is done similarly as (i). ||

Let $A(l_1,l_2)=\int_0^{\infty}\Pr[N_1(t)+N(t)=l_1, N_2(t)+N(t)=l_2]dt$

$$=\Sigma_{k=0}^{\min(l_1,l_2)}\lambda^k\lambda_1^{l_1-k}\lambda_2^{l_2-k}(l_1+l_2-k)!/(\lambda+\lambda_1+\lambda_2)^{l_1+l_2-k+1}k!(l_1-k)!(l_2-k)!$$

$A(l_1,l_2)$ means the expected time that the system is in the state (l_1,l_2), i.e., unit i survives l_i shocks $(i=1,2)$

Theorem 3.2. (i) BVSM is BNBUES (BNWUES) if

$$\bar{P}(k_1,k_2)\Sigma_{l_1=0}^{\infty}\Sigma_{l_2=0}^{\infty}A(l_1,l_2)\bar{P}(l_1,l_2) \geq (\leq) \Sigma_{l_1=0}^{\infty}\Sigma_{l_2=0}^{\infty}A(l_1,l_2)\bar{P}(k_1+l_1,k_2+l_2) \quad (k_1,k_2=0,1,2,\ldots)$$

and $\quad \bar{P}_i(k)\Sigma_{l=0}^{\infty}\bar{P}_i(l) \geq (\leq) \Sigma_{l=0}^{\infty}\bar{P}_i(k+l) \quad (k=0,1,2,\ldots,\ i=1,2)$.

(ii) BVSM is BNBUEP (BNWUEP) if

$$[1-P(k_1,k_2)]\Sigma_{l_1=0}^{\infty}\Sigma_{l_2=0}^{\infty}A(l_1,l_2)[1-P(l_1,l_2)] \geq (\leq) \Sigma_{l_1=0}^{\infty}\Sigma_{l_2=0}^{\infty}A(l_1,l_2)[1-P(k_1+l_1,k_2+l_2)] \quad (k_1,k_2=0,1,2,\ldots)$$

and $\quad \bar{P}_i(k)\Sigma_{l=0}^{\infty}\bar{P}_i(l) \geq (\leq) \Sigma_{l=0}^{\infty}\bar{P}_i(k+l) \quad (k=0,1,2,\ldots,\ i=1,2)$.

Proof: (i) From theorem 3.1 of J.D.ESARY, A.W.MARSHALL and F.PROSCHAN [3], we have $\bar{H}_i(x)\int_0^{\infty}\bar{H}_i(t)dt \geq (\leq) \int_0^{\infty}\bar{H}_i(x+t)dt \quad (i=1,2)$.

$$\int_0^{\infty}\bar{H}(x+t,y+t)dt=\Sigma_{k_1=0}^{\infty}\Sigma_{k_2=0}^{\infty}\Pr[N_1(x)+N(x)=k_1,N_2(y)+N(y)=k_2]\Sigma_{l_1=0}^{\infty}\Sigma_{l_2=0}^{\infty}A(l_1,l_2)\cdot\bar{P}(k_1+l_1,k_2+l_2).$$

$$\bar{H}(x,y)\int_0^{\infty}\bar{H}(t,t)dt=\Sigma_{k_1=0}^{\infty}\Sigma_{k_2=0}^{\infty}\Pr[N_1(x)+N(x)=k_1,N_2(y)+N(y)=k_2]\bar{P}(k_1,k_2)\cdot\Sigma_{l_1=0}^{\infty}\Sigma_{l_2=0}^{\infty}A(l_1,l_2)\bar{P}(l_1,l_2).$$

Then the result is evident.

(ii) Corresponding $1-P(k,l)$ to $\bar{P}(k,l)$, the proof is done similarly as (i). ||

4. Positive Dependency of BVSM.

In this section the following result is given. If $\bar{P}$ is PQD, then $\bar{H}$ is PQD. This result explains the general tendency in practical situation which is mentioned in section 1.

Theorem 4.1. For any fixed $x \geq 0$ and $y \geq 0$ $\Pr[N_1(x)+N(x)=k_1, N_2(y)+N(y)=k_2]$ is TP_2 in $k_1 \geq 0$ and $k_2 \geq 0$.

Proof: See F.OHI and T.NISHIDA [9]. ||

From theorem 4.1 it is easily seen that $PQD(N_1(x)+N(x), N_2(y)+N(y))$ holds for any fixed $x \geq 0$ and $y \geq 0$.

Theorem 4.2. BVSM is PQD, if $\bar{P}$ is PQD.

Proof:

$$\bar{H}(x,y)=\Sigma_{k_1=0}^{\infty}\Sigma_{k_2=0}^{\infty}\bar{P}(k_1,k_2)\Pr[N_1(x)+N(x)=k_1,N_2(y)+N(y)=k_2]$$
$$\geqq\Sigma_{k_1=0}^{\infty}\Sigma_{k_2=0}^{\infty}\bar{P}_1(k_1)\bar{P}_2(k_2)\Pr[N_1(x)+N(x)=k_1,N_2(y)+N(y)=k_2]$$
$$=\Sigma_{k_1=0}^{\infty}\Sigma_{k_2=0}^{\infty}p_1(k_1)p_2(k_2)\Pr[N_1(x)+N(x)<k_1,N_2(y)+N(y)<k_2]$$
$$\geqq\Sigma_{k_1=0}^{\infty}\Sigma_{k_2=0}^{\infty}p_1(k_1)p_2(k_2)\Pr[N_1(x)+N(x)<k_1]\Pr[N_2(y)+N(y)<k_2]$$
$$=\bar{H}_1(x)\bar{H}_2(y) ,$$

where $p_i(k_i)$ is a mass function of discrete distribution function $P_i(k_i)$, i.e., $p_i(k_i)=P_i(k_i)-P_i(k_i-1)$ $(k_i=1,2,\ldots)$ $p_i(0)=0$ $(i=1,2)$. ||

From theorem 4.2 if $\bar{P}$ is PQD, it follows that $\bar{H}(t,t)\geqq\bar{H}_1(t)\bar{H}_2(t)$ and $H(t,t)\geqq H_1(t)H_2(t)$. The former inequality tells us that the reliability of a two units series system of which joint survival probability for the life lengths is described by BVSM is higher than that under the assumption of independency. The latter inequality means the unreliability of a two units parallel system is higher than that under the assumption of independency.

5. Concluding Remarks.

The joint distribution function for two units having stochastic correlation has been formulated to be called bivariate shock models. This bivariate distribution function may be thought to reflect various practical phenomenon. If the nature of $\bar{P}(k_1,k_2)$ descrived in theorem 3.1 is verified, the bounds given in theorem 2.1-2.4 are able to be used. In the practical situation the approximation would be used sufficiently. It is more easy to estimate $\bar{F}(t,t)$, $\bar{F}_1(t)$ and $\bar{F}_2(t)$ for some fixed t than to determine the exact distribution function.

The bivariate shock models make us explain reasonably the observable phenomenon relating to two units parallel and series systems, which is mentioned in section 1. We showed that if $\bar{P}$ is PQD, then $\bar{H}$ is PQD. By analogy, it is seen that there is some correspondence between the positive dependency of $\bar{P}$ and that of $\bar{H}$, i.e., if $\bar{P}$ is RTI, then $\bar{H}$ is RTI, and so on. These are remained as a open problem.

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