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THE BEHAVIOR OF SOME DESIGN FACTORS IN A PARALLEL PRODUCTION LINE

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Abstract The effect of the various design factors on the production rate in a parallel line is discussed by a Markov model and the difference of the effect between parallel lines and series lines is represented. A simple and effective scale is proposed to evaluate the availability of a parallel line and the production rate of a parallel line is approximately estimated by the scale and the relation between the production rates and the buffer capacities in a series line.

1. Introduction

One of the difficult problems for industrial engineers is the design of production lines. When the line is for high volume products, the engineers must estimate particularly the obtainable maximum production rate. Therefore the effects of various design factors on the rate must be considered.

Many papers have been published concerning such problems by Hunt [5], Hillier & Boling [3], [4] and others [1], [2], [6], but most of them discuss only about series lines and there remained many unsolved problems about parallel lines which are designed for line balancing or increasing the production rate.

In this paper we shall consider the fundamental effects of design factors in parallel lines and provide better insight into designing parallel lines. First the effects of buffer capacity and number of stations in each stage on the production rate are discussed and the effectiveness of parallel lines is represented especially by an imaginary buffer capacity. This imaginary capacity is introduced to compare the availability of parallel line with that of series line and to show the effectiveness of parallel lines in a simple form. Secondly the effect of unbalanced operation times on the production rate is discussed. It is very difficult to design a perfect balanced line and an unbalanced line is designed in several cases. From this viewpoint, it is inevitable to study this effect. We shall find out the optimal assignment of the mean operation time which yields the maximum production rate and the range of unbalance which yields the rate as high as or higher than the rate of a balanced line. Thus we educe if the bowl phenomina represented in the series line by Hillier & Boling [3] is also preserved in parallel lines.

2. Model and Formulation

The parallel line to be studied here consists of the buffer storages holding in-process works temporarily and the stages with some stations to operate the work practically, as shown in Fig. 2.1. Each station in one stage has the same operation and operation rate. Then the line is defined by the number of stages, the numbers of stations in each stage, the interstage buffer capacities and the operation rates of stations in each stage, which are denoted by L, S_i, M_j and λ_i $(i=1,2\cdots,L;\ j=1,2,\cdots,L-1)$, respectively. The operation rate λ_i is the reciprocal of the mean operation time a_i of the station in stage i.

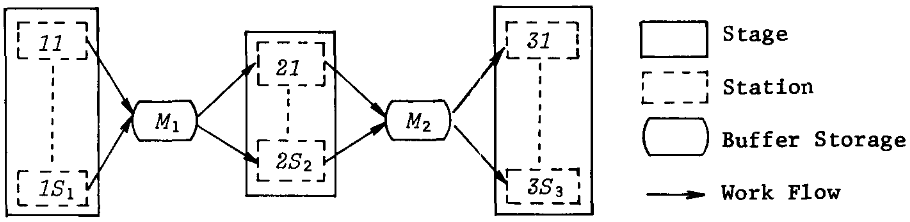

Fig. 2.1 Parallel production line

In the special case in which we show the difference of the effectiveness between the parallel line and the series line, these two lines have to be compared under the condition that their design factors are the same. For this purpose, in Section 3 the production rate yielded by S of the series lines defined by the number of stages L, the interstage buffer capacity M_s and the operation rate of stage λ_{si} is compared with the rate by the equivalent parallel line denoted by $S_i = S$, $M_j = SM_s$ and $\lambda_i = \lambda_{si}$.

Assume that there is always a supply of works ready to be operated at the first stage and there is an infinite buffer capacity just behind the last stage, so that idling due to lack of input works is never occurred in the first stage and blocking is never occurred in the last stage because of ejecting a completed work from the stage. However the other stages except the last one must hold the completed work without beginning to operate next work, if the buffer space is not available and all the stations in the next stage are in operating works. And assume that the variation of the operation time is described by an exponential distribution at each station and the operation times are mutually independent. This means that there is no breakdown at stations or, alternatively, downtime is included in the operation time and that the total time expended in the station to complete a work has an exponential distribution.

Now we formulate the parallel line by a Markov model. The states occurred in each station are W, I and B which denote the state in operating a work, in idling and in blocking respectively. Generally in a parallel line the priority of station must be interested, because the stage has S_i stations and it is required to decide in which station idling or blocking is occurred and released first. For example, when the two works are completed at the same time in the two stations of the first stage and the following buffer has only one work space, we must decide which work is ejected into the buffer and which work blocks the station. We adopt the random priority in the present model, since the stations in one stage have the same operation rate and the priority does not have any effect on the production rate.

The state of the system for the parallel line in a steady state condition is defined by the numbers of stations being in W, I and B respectively in each stage and the numbers of in-process works in each buffer storage.

In a two-stage production line, the state probabilities of the system are represented by the following notations;

$P(iB_1 \mid M_1)$: i stations in Stage 1 are in blocking, the others in Stages 1 and 2 in operating and M_1 in-process works are in the buffer $(1 \leqq i \leqq S_1)$,

$P(W \mid m_1)$: all the stations in Stages 1 and 2 are in operating and m_1 in-process works are in the buffer $(0 \leqq m_1 \leqq M_1)$,

$P(iI_2 \mid 0)$: i stations in Stage 2 are in idling, the others in Stages 1 and 2 in operating and no in-process work is in the buffer $(1 \leqq i \leqq S_2)$.

In these notations, the affixed number to B and I denotes the stage number. The total number N of the states of the system is

(2.1) $$N = S_1 + S_2 + M_1 + 1.$$

In a three-stage production line, the state probabilities of the system are represented in the same way as follows;

$P(iB_1 \mid M_1,m_2)$	:	$1 \leqq i \leqq S_1$, $0 \leqq m_2 \leqq M_2$
$P(iB_2 \mid m_1,M_2)$	:	$1 \leqq i \leqq S_2$, $0 \leqq m_1 \leqq M_1$
$P(iB_1,jB_2 \mid M_1,M_2)$	:	$1 \leqq i \leqq S_1$, $1 \leqq j \leqq S_2$
$P(W \mid m_1,m_2)$	:	$0 \leqq m_1 \leqq M_1$, $0 \leqq m_2 \leqq M_2$
$P(iI_2 \mid 0,m_2)$	:	$1 \leqq i \leqq S_2$, $0 \leqq m_2 \leqq M_2$
$P(iI_3 \mid m_1,0)$	:	$1 \leqq i \leqq S_3$, $0 \leqq m_1 \leqq M_1$
$P(iI_2 \cdot jI_3 \mid 0,0)$	:	$1 \leqq i \leqq S_2$, $1 \leqq j \leqq S_3$
$P(iB_1,jI_3 \mid M_1,0)$	:	$1 \leqq i \leqq S_1$, $1 \leqq j \leqq S_3$
$P(iI_2jB_2 \mid 0,M_2)$	:	$1 \leqq i \leqq S_2-1$, $1 \leqq j \leqq S_2-i$

In these notations the variables or constants after "|" denote the numbers of in-process works in the buffers between stages 1 and 2, and between stages 2 and 3. For example, $P(iB_1 \mid M_1,m_2)$ means the state probability that i stations in stage 1 are in blocking, all the other stations in stages 1, 2 and 3, i.e. $(S_1-i) + S_2 + S_3$ stations, in operating and M_1, m_2 in-process works are in the buffers between stages 1 and 2, and between stages 2 and 3 respectively. The total number of the states of the system is

$$N = (1+M_1)(1+M_2) + (S_1+S_2)(1+M_2) + (S_2+S_3)(1+M_1) + (S_1S_2+S_2S_3+S_3S_1) + S_2(S_2-1)/2. \tag{2.2}$$

Consider the steady state probability equations. These equations are given by the above state probabilities and the transition matrix. In a two-stage production line, they are

$$\begin{aligned}
S_1\lambda_1P(S_2I_2 \mid 0) &= \lambda_2P((S_2-1)I_2 \mid 0)\\
(i\lambda_2+S_1\lambda_1)P((S_2-i)I_2 \mid 0) &= S_1\lambda_1P((S_2-i+1)I_2 \mid 0) + (i+1)\lambda_2\\
&\qquad P((S_2-i-1)I_2 \mid 0) \quad (1 \leqq i \leqq S_2-2)\\
((S_2-1)\lambda_2+S_1\lambda_1)P(1I_2 \mid 0) &= S_1\lambda_1P(2I_2 \mid 0) + S_2\lambda_2P(W \mid 0)\\
(S_2\lambda_2+S_1\lambda_1)P(W \mid 0) &= S_1\lambda_1P(1I_2 \mid 0) + S_2\lambda_2P(W \mid 1)\\
(S_2\lambda_2+S_1\lambda_1)P(W \mid m_1) &= S_1\lambda_1P(W \mid m_1-1) + S_2\lambda_2P(W \mid m_1+1)\\
&\qquad (1 \leqq m_1 \leqq M_1-1)\\
(S_2\lambda_2+S_1\lambda_1)P(W \mid M_1) &= S_1\lambda_1P(W \mid M_1-1) + S_2\lambda_2P(1B_1 \mid M_1)\\
(S_2\lambda_2+(S_1-1)\lambda_1)P(1B_1 \mid M_1) &= S_1\lambda_1P(W \mid M_1) + S_2\lambda_2P(2B_1 \mid M_1)
\end{aligned} \tag{2.3}$$

$$(S_2\lambda_2+(S_1-i)\lambda_1)P(iB_1 \mid M_1) = (S_1-i+1)\lambda_1 P((i-1)B_1 \mid M_1) + S_2\lambda_2 P((i+1)B_1 \mid M_1) \qquad (2 \leqq i \leqq S_1-1)$$

$$S_2\lambda_2 P(S_1B_1 \mid M_1) = \lambda_1 P((S_1-1)B_1 \mid M_1)$$

Normalizing, so all state probabilities sum to one, gives the actual probabilities, and as a result the production rate R_p of the parallel line and the mean number L_q of in-process works in the buffer are represented by

$$(2.4) \qquad R_p = \sum_{i=1}^{S_2} (S_2-i)\lambda_2 P(iI_2 \mid 0) + S_2\lambda_2\left(1-\sum_{i=1}^{S_2} P(iI_2 \mid 0)\right)$$

$$= KS_1\lambda_2\phi \sum_{i=0}^{S_2-2} \frac{1}{i!}(S_1\phi)^i + S_2\lambda_2\left(1-K\sum_{i=0}^{S_2-1} \frac{1}{i!} (S_1\phi)^i\right),$$

$$(2.5) \qquad L_q = \sum_{m_1=1}^{M_1} m_1 P(W \mid m_1) + M_1 \sum_{i=1}^{S_1} P(iB_1 \mid M_1)$$

$$= K \left\{ \frac{S_1^{S_2}}{S_2!} \phi^{S_2} \sum_{i=1}^{M_1} i\left(\frac{S_1\phi}{S_2}\right)^i + M_1 \frac{S_1!}{S_2!} \frac{S_1^{S_2+M_1}}{S_2^{S_1+M_1}} \phi^{S_1+S_2+M_1} \sum_{i=1}^{S_1} \frac{S_2^{S_1-i}}{(S_1-i)!} \frac{1}{\phi^{S_1-i}} \right\},$$

where

$$K = \left[\sum_{i=0}^{S_2-1} \frac{1}{i!} (S_1\phi)^i + \frac{1-(S_1\phi/S_2)^{M_1+1}}{1-(S_1\phi/S_2)} \frac{S_1^{S_2}}{S_2!} \phi^{S_2} + \frac{S_1!}{S_2!} \frac{S_1^{S_2+M_1}}{S_2^{S_1+M_1}} \phi^{M_1+S_1+S_2} \sum_{i=0}^{S_1-1} \frac{1}{i!} \left(\frac{S_2}{\phi}\right)^i \right]^{-1},$$

$$\phi = \lambda_1/\lambda_2 .$$

In a three-stage production line, the steady state probability equations are given in the same way. However there are numerous states and it is very difficult to solve the actual probabilities and the rate R_p in a general form. In this paper their numerical values are calculated by Gaussian Elimination Method to apply to the following study.

3. The Effect of Buffer Storages

3.1 In a two-stage production line

We discuss what effects the buffer capacity and the number of stations have on the production rate and demonstrate the availability of paralleling. This availability and the effectiveness of paralleling are shown by the imaginary buffer capacity which is introduced to compare the parallel line with the original series line.

Consider the parallel line which is designed by paralleling S of the series lines. When each of them has the buffer capacity M_s, this parallel line is defined by $S_i=S$ and $M_j=SM_s$, and the production rate R_p is from (2.4)

$$R_p = KS\lambda_1 \sum_{i=1}^{S-2} \frac{(S\phi)^i}{i!} + S\lambda_2 \left\{ 1-K \sum_{i=0}^{S-1} \frac{(S\phi)^i}{i!} \right\}, \tag{3.1}$$

where

$$K = \left[\sum_{i=0}^{S-1} \frac{(S\phi)^i}{i!} + \frac{(S\phi)^S}{S!} \frac{1-\phi^{SM_s+1}}{1-\phi} + \phi^{S(2+M_s)} \sum_{i=0}^{S-1} \frac{1}{i!}\left(\frac{S}{\phi}\right)^i \right]^{-1} \qquad (\phi \neq 1)$$

$$= \left[2 \sum_{i=0}^{S-1} \frac{S^i}{i!} + \frac{S^S}{S!}(1+SM_s) \right]^{-1} \qquad (\phi = 1).$$

The values of the mean production rate per station R_p/S for various S and M_s are given in Table 3.1, where the values for $S=1$ represent the production rates of the original series lines.

On the other hand, the production rate R_s of the series line is presented by Hunt [5] as follows;

$$R_s = \lambda_1\lambda_2 \frac{\lambda_1^{M_s+2} - \lambda_2^{M_s+2}}{\lambda_1^{M_s+3} - \lambda_2^{M_s+3}} \qquad (\lambda_1 \neq \lambda_2)$$

$$= \lambda_2 \frac{M_s+2}{M_s+3} \qquad (\lambda_1 = \lambda_2).$$

Therefore the increase $\Delta R_p/S$ of the production rate by paralleling is represented by the difference between the mean production rate per station R_p/S and the production rate R_s of the original series line, and it is for the balanced line, i.e. $\lambda_1=\lambda_2$, (3.2) $$\Delta R_p/S = \frac{R_p}{S} - \lambda_2 \frac{M_s+2}{M_s+3}$$

$$= \lambda_2 \frac{M_s(S-1)-2+2\frac{S!}{S^S}\sum_{i=0}^{S-1}\frac{S^i}{i!}}{(M_s+3)(SM_s+1+2\frac{S!}{S^S}\sum_{i=0}^{S-1}\frac{S^i}{i!})}.$$

Let us introduce the imaginary buffer capacity M_I in order to evaluate the effectiveness of paralleling. The imaginary capacity is the capacity which is required to yield the same production rate R_p/S in the original series line, and by substituting R_p/S to Hunt's equation we have

(3.3)

$$M_I = \frac{1}{Log\phi} Log \frac{R_p/S - \lambda_1}{R_p/S - \lambda_2} - 3$$

$$= [\frac{1}{Log\phi} Log \frac{1-(1-\phi)\frac{S!}{(S/\phi)^S}\sum_{i=0}^{S}\frac{(S/\phi)^i}{i!}}{1-(1-1/\phi)\frac{S!}{(S\phi)^S}\sum_{i=0}^{S}\frac{(S\phi)^i}{i!}} - 4] + SM_s \qquad (\phi \neq 1)$$

$$M_I = \frac{\lambda_2}{\lambda_2 - R_p/S} - 3$$

$$= 2[\frac{S!}{S^S}\sum_{i=0}^{S-1}\frac{S^i}{i!} - 1] + SM_s \qquad (\phi = 1).$$

These imaginary buffer capacities are calculated for various M_s and S, and are represented in Table 3.2.

Now we discuss the important results presented by the above equations. The most important one is that we can expect the increase of the production rate by paralleling. This is demonstrated by (3.2), since $\Delta R_p/S$ is always positive for $S \geqq 2$. The others are as follows. The production rate R_p/S of the parallel line monotonously increases as the buffer capacity M_s increases, but the increase of the rate is not so large as that in the series line and decreases as the number of stations S increases. And R_p/S also monotonously increases as S increases, and the increase of the rate becomes smaller as M_s increases. Therefore the effectiveness of paralleling is especially large and we can expect the availability of paralleling when the buffer capacity is small. The imaginary buffer capacity also suggests us useful facts and presents the quatitative effect of paralleling in a simple form. First it is that paralleling gives the higher production rate to each of the original series lines than the rate which is represented by the series line monopolizing the total buffer capacity SM_S because the first term in square brackets in (3.3) is always positive for $S \geqq 2$. Second it is that the imaginary capacity is estimated by the sum between SM_S and the imaginary capacity for $M_S = 0$ because the first term in square brackets in (3.3) is independent of M_S. Consequently the imaginary buffer capacity by paralleling is described simply as follows;

$$(3.4) \qquad M_I = M_I \mid_{M_S=0} + SM_S$$

and only the imaginary buffer capacity for $M_S = 0$ must be solved to estimate M_I and as a result to estimate the production rate for various M_S and S. Thus we can use the imaginary buffer capacity as the scale to evaluate the effectiveness of paralleling.

3.2 In a three-stage production line

Consider the three-stage parallel line which is designed by paralleling S of the balanced series lines. When this parallel line has the operation rate $\lambda_i = \lambda$ and the buffer capacity $M_j = M$, the production rates R_p/S and the imaginary buffer capacities M_I for various M and S are shown in Tables 3.3 and 4. The imaginary capacity is calculated by applying the Hillier & Boling's numerical results for the three-stage series line [4] to Newton's forward interporation formula, since the exact formula for the production rate of the general three-stage series line has not been obtained and the approximate formula by Knott [6] is not appropriate to estimate the imaginary buffer capacity within a small error.

These results show that the effects of the buffer capacity M and the number of stations S are the same with the effects in the two-stage parallel line and that the effectiveness of paralleling can be expected in the three-stage line too. And as shown in Table 3.4 this effectiveness is that the each original series line can yield the higher production rate than the rate presented by the series line monopolizing the buffer capacity M. Therefore it is the same with the effectiveness in the two-stage line. The effects of the buffer capacity and the number of stations on the imaginary buffer capacity are as follows. From the comparison between the imaginary capacities in Table 3.2 and 4, it is appeared that the imaginary capacity for the three-stage parallel line is a little smaller than the imaginary capacity for the two-stage parallel line if the buffer capacity is small and is almost equal to it if $M \geqq 5$. And these differences of the imaginary capacities are very small and only about 6% at most. Therefore we can educe that the imaginary capacity for the three-stage line is approximately equal to the imaginary capacity for the two-stage line and estimated by (3.3). This indicates that the effects of the buffer capacity and the number of stations on the imaginary buffer capacity are approximately the same with the effects in the two-stage line and are represented by (3.4). Consequently the production rate of the three-stage parallel line is able to be estimated by (3.3) and the relation between the buffer capacity and the production rate for the original three-stage series line within a small error. And at a same time it appears that the imaginary buffer capacity for the parallel line with many stages becomes less dependent on the number of stages as the buffer capacity M increases. This result is important because the imaginary buffer capacity and the production rate for the parallel line with many stages and the large buffer capacity will be approximately estimated like the three-stage parallel line.

4. Various Paralleling in a Two-Stage Production Line

When we consider line balancing under the situation that the operations assigned to the stations or the productivities of the stations are unequal, various parallel lines with unequal numbers of stations in each stage will be designed. In this section we study what effect the number of stations have on the production rate in the basic two-stage parallel line and find out the optimal assignment of the stations which yields the maximum production rate under the condition that the total number of stations assigned to the stages is constant.

Consider the parallel line which is defined by the numbers of stations S_i and the buffer capacity M_1. Then the production rate R_p for the balanced line in which each stage has the equal operation rate, i.e. $\lambda_1 S_1 = \lambda_2 S_2$, is

$$(4.1) \qquad R_p = S_2\lambda_2 \{ 1 - [\frac{S_1!}{S_1^{S_1}} \sum_{i=0}^{S_1-1} \frac{S_1^{i}}{i!} + \frac{S_2!}{S_2^{S_2}} \sum_{i=0}^{S_2-1} \frac{S_2^{i}}{i!} + M_1+1]^{-1} \} . \qquad (S_1, S_2 \geqq 1)$$

This equation is given by substitution of $\phi = \lambda_1/\lambda_2 = S_2/S_1$ into (2.4). And the imaginary buffer capacity M_I is from the above equation and Hunt's equation

(4.2) $$M_I = \left[\frac{S_1!}{S_1^{S_1}} \sum_{i=0}^{S_1-1} \frac{S_1^i}{i!} + \frac{S_2!}{S_2^{S_2}} \sum_{i=0}^{S_2-1} \frac{S_2^i}{i!} - 2 \right] + M_1 .$$

In this case the imaginary buffer capcity is the capacity which is required to yield the above production rate in the corresponding balanced series line with the same operation rate $S_1\lambda_1$.

These equations show that R_p for the balanced parallel line with unequal numbers of stations in each stage is always higher than the production rate for the corresponding series line. This is demonstrated by the fact that the term in square brackets in (4.1) is always larger than $M_1 + 3$ if $S_1 + S_2 \geqq 3$. And these equations show that R_p is proportional to the operation rate of the stage like the series line and is independent of the operation rate of the station in each stage. The effect of the number of stations on the production rate is appeared by the imaginary buffer capacity. First M_I is the symmetrical and monotone increasing function with respect to S_1 and S_2. Therefore the change of the stages has no effect on the production rate and the increase of the numbers of stations always increases the production rate. This suggests us that installing the higher productive machines may decrease the production rate because it decreases the number of stations. Second M_I is the function of the numbers of stations and the buffer capacity, and is estimated by the sum between the buffer capacity M_1 and the imaginary capacity for $M_1 = 0$. And as shown in (4.2) the numbers of stations S_1 and S_2 have the independent effect on the imaginary buffer capacity in the same functional form. Consequently M_I is estimated by the imaginary capacity for $M_1 = 0$ and $S_2 = 1$, the imaginary capacity for $M_1 = 0$ and $S_1 = 1$, and the buffer capacity M_1 , and is represented by

(4.3) $$M_I = M_I \big|_{M_1=0,S_2=1} + M_I \big|_{M_1=0,S_1=1} + M_1$$

$$= \left[\frac{S_1!}{S_1^{S_1}} \sum_{i=0}^{S_1-1} \frac{S_1^i}{i!} - 1 \right] + \left[\frac{S_2!}{S_2^{S_2}} \sum_{i=0}^{S_2-1} \frac{S_2^i}{i!} - 1 \right] + M_1 .$$

Now find out the optimal assignment of the stations. This problem is discussed under the balancing condition, i.e. $\lambda_1 S_1 = \lambda_2 S_2$, and $S_1 + S_2$ = constant. From (4.3) it is appeared that the first and the second term in square brackets are monotone increasing functions with respect to S_1 and S_2 respectively and the increase monotonously decreases as S_1 and S_2 increase. Therefore the optimal assignment is given by

(4.4) $S_1 = S_2 = \frac{C}{2}$ if C = even number

$S_1, S_2 = [\frac{C}{2}]$, $[\frac{C}{2}] + 1$ or $[\frac{C}{2}] + 1$, $[\frac{C}{2}]$ if C = odd number

where C is the total number of stations and $[C/2]$ is the integral part of $C/2$. From this it is concluded that the maximum production rate can be obtained by balancing the number of stations in each stage. However it is also appeared from (4.3) that the total number of stations and the way of assignment have the large effect on the production rate if M_1 is small but have little effect if M_1 is large.

5. The Effect of Unbalanced Operation Times

5.1 In a two-stage production line

One of the important design factors in a parallel line is the mean operation time and this factor will have particularly large effect when the unbalanced line is designed. In this section we shall discuss what effect the unbalance of the mean operation times has on the production rate. This is discussed under the condition that $S_i = S$, $M_j = SM_s$ and $a_1 + a_2$ = constant in order to find out the optimal assignment of the total mean operation time.

The effect of unbalanced operation times in the parallel line is immediately derived from (3.1). The various representative results for the unbalanced parallel lines and the comparison of the effects between the series lines and the parallel lines are shown in Fig. 5.1.

This figure appears that the effect of unbalanced operation times is the same with the effect in the series line, i.e. the mean operation times of the stations in each stage a_1 and a_2 have the symmetrical effect on the production rate and the rate is maximized when the line is balanced. This symmetrical effect is also demonstrated directly from (3.1). Furthermore it is appeared that the decrease of the production rate R_p / S by unbalancing is promoted as S and M_s increase. This indicates that as S and M_s increase the effect of idling and blocking by the variation of the operation time is reduced and the production rate is importantly affected by the slowest mean operation time. Therefore, if S and M_s are large, the unbalanced assignment must be avoided in the parallel line.

5.2 In a three-stage production line

The effect of unbalanced operation times is remarkably appeared in a three-stage parallel line. In this case we study it under the condition that $S_i = S$, $M_j = SM_3$ and $a_1 + a_2 + a_3 =$ constant.

The various representative results for $a_1 + a_2 + a_3 = 3.0$ are shown in Fig.s 5.2∿4. In these figures the production rates R_p / S for the unbalanced line are represented by the ratios to the rate for the balanced line and the mean operation times to be assigned to the stations in stage i and j are represented by a_i and a_j. Fig. 5.2 shows the relation between the stage where the minimum operation time is assigned and the production rate, and appears where the minimum operation time should be assigned to yield the maximum production rate. In this case the production rate R_p/S are also presented in Table 5.1. Fig.s 5.3 and 4 show what effect the buffer capacity M_s and the number of stations S have on the production rate R_p/S in the unbalanced parallel line.

These results demonstrate that the unbalanced parallel line can yield the higher production rate than the rate of the balanced parallel line and that the maximum rate is obtained by assigning a little smaller mean operation time to the stations in the middle stage of the line than to the stations on the both ends. Consequently there is some range of unbalance which can yield the production rate as high as or higher than the rate of the balanced line and there is flexibility in assigning the total operation time. In other words the unbalanced line instead of the balanced line may be designed in the above range of unbalance if balancing is very difficult. As shown in Table 5.1 the effect of the mean operation times a_1 and a_3 is as follows. These mean times a_1 and a_3 have not the symmetrical effect on the production rate R_p/S unlike the series line presented by Hillier & Boling [3] and have a little different effect, but this difference is very small so that we can consider that a_1 and a_3 have almost the symmetrical effect on R_p/S. The effect of the buffer capacity M_s and the number of stations S is that as M_s or S increases the maximum production rate by unbalancing approaches the rate of the balanced line and the range of unbalance which can yield the rate no less than the rate of the balanced line is narrowed. And the production rate sharply decreases as the unbalance increases. Therefore the availability of unbalancing can not be obtained and unbalancing of the parallel line should be avoided when the buffer capacity and the number of stations are large. This appears that the variation of the operation time is absorbed by the buffer storages and the maximum mean operation time becomes the primary limitation on the production rate. The same thing is appeared about the increase of the number of stations, since the increase of the number of stations imaginarily have the same effect as increasing the buffer capacity.

These effects are the same with the effects presented in the series line by Hillier & Boling and it is concluded that the bowl phenomina is preserved in the parallel line.

6. Conclusion

We have discussed the effects of various design factors in the parallel lines by a Markov model. The important conclusion is that the production rate in the parallel lines is estimated by the imaginary buffer capacity and the production rate of the series lines and that the bowl phenomina represented in the series line is preserved in the parallel line too.

However there remained many unsolved problems for the large scale parallel lines and the network lines. These problems are important for the industrial engineers and will be a subject for near future research.

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Table 3.1 R_p/S for two-stage parallel lines ($\lambda_1 = \lambda_2 = 1.0$)

M_s \ S	1	2	3	4	5
0	0.6667	0.7500	0.7907	0.8161	0.8339
1	0.7500	0.8333	0.8714	0.8940	0.9093
2	0.8000	0.8750	0.9072	0.9256	0.9376
3	0.8333	0.9000	0.9274	0.9427	0.9524
4	0.8571	0.9167	0.9404	0.9534	0.9616
5	0.8750	0.9286	0.9494	0.9607	0.9678

Table 3.2 M_I for two-stage parallel lines ($\lambda_1 = \lambda_2$)

M_s \ S	1	2	3	4	5
0	0	1	1.778	2.438	3.021
1	1	3	4.778	6.438	8.021
2	2	5	7.778	10.438	13.021
3	3	7	10.778	14.438	18.021
4	4	9	13.778	18.438	23.021
5	5	11	16.778	22.438	28.021

Table 3.3 R_p/S for three-stage parallel lines ($\lambda_1 = \lambda_2 = \lambda_3 = 1.0$)

M \ S	1	2	3	4	5
0	0.5641	0.6657	0.7176	0.7505	0.7739
1	0.6705	0.7323	0.7666	0.7895	0.8063
2	0.7340	0.7761	0.8006	0.8176	0.8304
3	0.7767	0.8072	0.8258	0.8389	0.8490
4	0.8075	0.8307	0.8452	0.8557	0.8639
5	0.8308	0.8490	0.8607	0.8693	0.8760
6	0.8490	0.8637	0.8733	0.8805	0.8861
7	0.8637	0.8758	0.8838	0.8899	
8	0.8757	0.8859	0.8927		

Table 3.4 M_I for three-stage parallel lines ($\lambda_1 = \lambda_2 = \lambda_3$)

M \ S	1	2	3	4	5
0	0	0.94	1.70	2.35	2.92
1	1	1.97	2.73	3.38	3.96
2	2	2.98	3.75	4.40	4.98
3	3	3.99	4.76	5.42	6.00
4	4	5.00	5.77	6.43	7.02
5	5	6.00	6.78	7.44	8.03
6	6	7.00	7.78	8.46	9.03
7	7	8.00	8.79	9.43	
8	8	9.00	9.75		

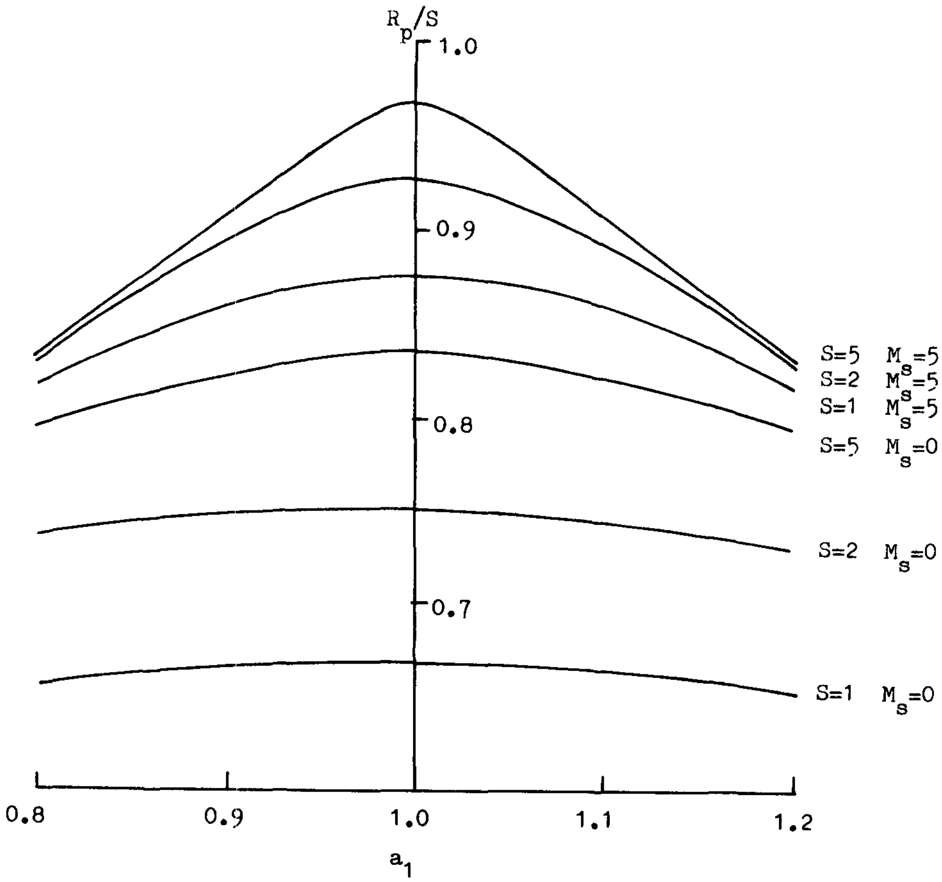

Fig. 5.1 R_p/S for two-stage unbalanced series lines and parallel lines ($a_1 + a_2 = 2.0$)

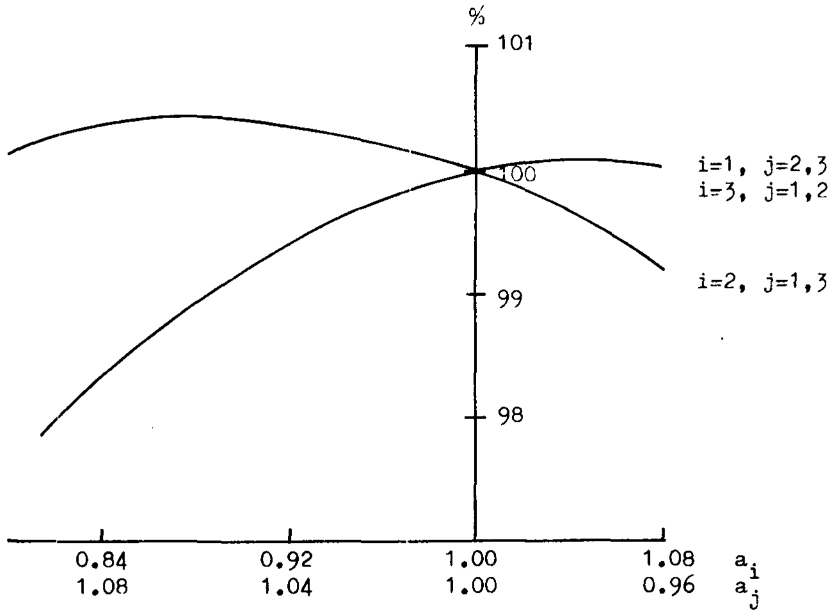

Fig. 5.2 R_p/S for three-stage unbalanced parallel lines without buffer (S = 2, M_s = 0)

Table 5.1 R_p/S for three-stage unbalanced parallel lines without buffer (S = 2, M_s = 0)

Mean Operation Times		R_p/S	Percent	R_p/S	Percent	R_p/S	Percent
a_i	a_j	i = 2	j = 1,3	i = 1	j = 2,3	i = 3	j = 1,2
1.08	0.96	0.6606	99.2	0.6657	100.0	0.6655	100.0
1.04	0.98	0.6635	99.7	0.6662	100.1	0.6661	100.1
1.00	1.00	0.6657	100.0	0.6657	100.0	0.6657	100.0
0.96	1.02	0.6672	100.2	0.6643	99.8	0.6643	99.8
0.92	1.04	0.6680	100.3	0.6618	99.4	0.6620	99.4
0.88	1.06	0.6681	100.4	0.6585	98.9	0.6587	98.9
0.84	1.08	0.6675	100.3	0.6543	98.3	0.6545	98.3
0.80	1.10	0.6661	100.1	0.6494	97.6	0.6496	97.6

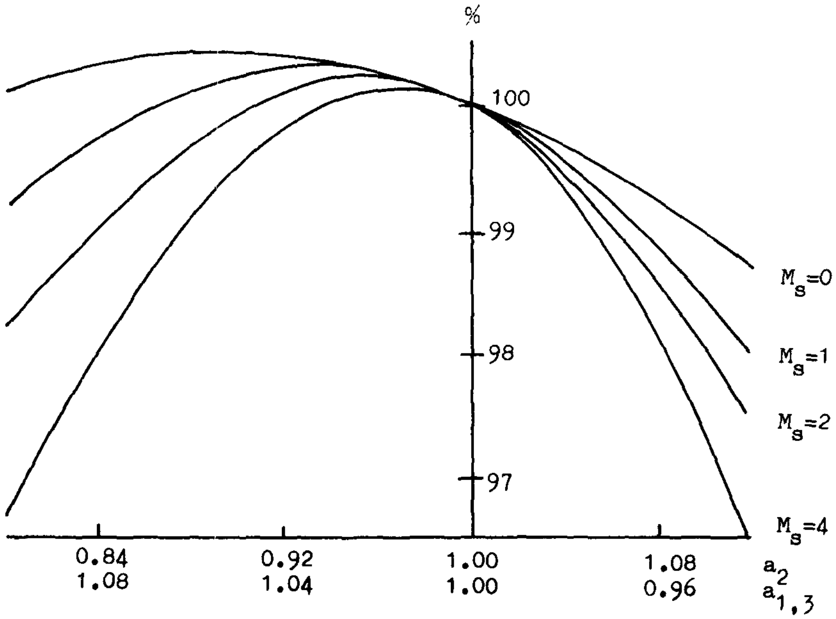

Fig. 5.3 The effect of M_s for three-stage unbalanced parallel lines (S = 2)

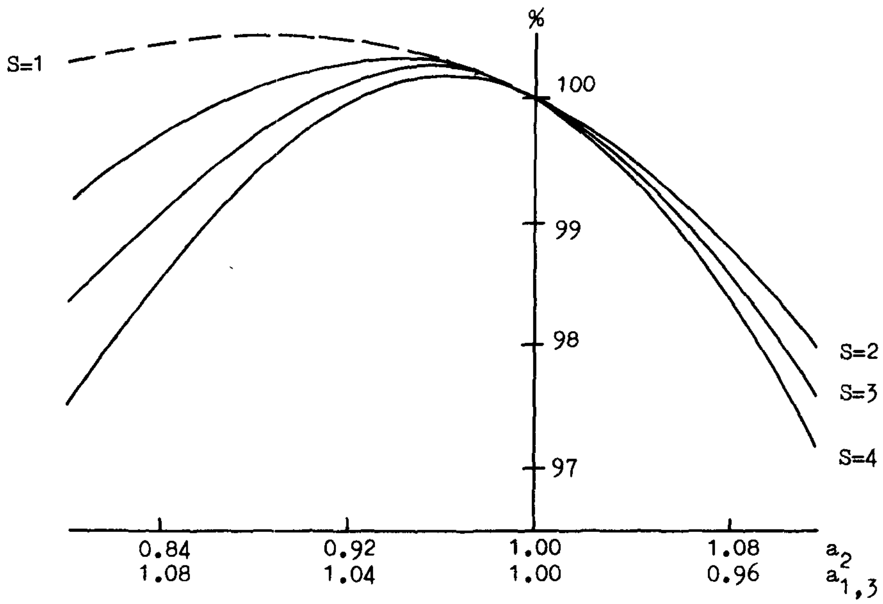

Fig. 5.4 The effect of S for three-stage unbalanced parallel lines (M_s = 1)

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