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MATHEMATICAL FRAMEWORK FOR ORGANIZATIONAL RESEARCH

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1. INTRODUCTION

There are many misunderstandings in general organizations theory due to the confusion of two approaches; i.e., the normative approach and the descriptive approach. If we want to remove these misunderstandings and wish to construct an integrated general organizations theory, it is most necessary to develop a framework which can be used as a common starting point for the above-mentioned two approaches. The development of some frameworks has so far been attempted by C.I.Barnard, H.A.Simon and J.G. March, who have played the central roles in the construction of modern organizations theory.

However, their frameworks have some shortcomings in that they are presented in the verbal logics. In order to enhance their operationality and applicability, we reformulate them in the mathematical languages especially, set-theoretic one. And ours may be named "an axiomatic approach". In this approach, we present our discussion in three steps. That is, we will at first formulate principal organizational concepts mathematically, then represent a general organizations model by specifying relationships among these concepts, and lastly develop a framework for organizational research. We use general systems theory as methodology in developing our axiomatic organizations theory. We can describe some developments of our

work as shown in Fig.1.

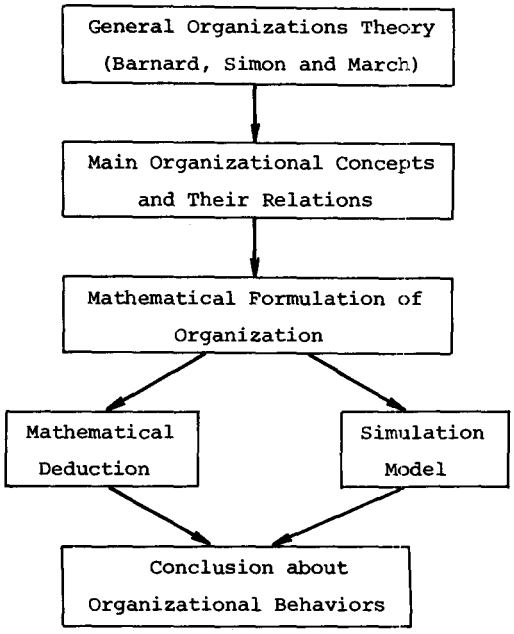

Fig.1. Development process of axiomatic organizations theory.

The structure of this paper is as follows. In Section 2, we abstract principal organizational concepts and reconstruct them in the verbal logics. In Section 3, we formulate organizational concepts abstracted in Section 2 from the mathematical and general systems theoretic viewpoints. In Section 4, we represent a two-level line-organization model, we give a summary and suggest some further researches in this direction in Section 5.

In advance of our discussions, we give some comments on our formulation.

1) We describe organization members as decision-makers; i.e., we use so-called decision model. Hence, ours naturally has benefits and limits of the decision model in comparison to the psychological model.

2) We restrict our attentions upon formal aspects of the organization.

3) We use set theory as our description language, for it enables us to graps general properties of the organization and to introduce more specified mathematical structures on our formulation.†

4) We do not make any value-judgement about organization's "should be" because we are interested in description of the organization.

2 VERBAL LOGICS OF ORGANIZATIONAL CONCEPTS

In this section, we abstract organizational concepts from the studies of C.I.Barnard, H.A.Simon and J.G.March, and reconstruct these concepts using verbal logics. But we restrict our attention only to these concepts which are necessary for the succeeding discussions. Now, we begin with the question as to what the organization is. We have a very famous definition by C.I.Barnard.

DEFINITION 2-1 : The (formal) organization is a system of consiously coordinated activities or forces of two or more persons.††

And he refers to the following condition under which the organization comes into being.

CONDITION 2-1 : Elements of organization

(a) communication

(b) willingness to serve

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† Set-theoretic representations of general systems are given in [8] and [16], however our objects are not general systems but organizations.

†† See Chapter 6 and 7 of [2].

(c) common purpose.†

Meanings of these elements may be clear.

Next, there exist the decision-making processes in which one decides what actions should be taken and how to implement them, in relation to his activities.†† Generally, the decision-making is defined as follows.

DEFINITION 2-2 : *The decision-making process is a process in which one deducts his conclusion from his decision premises,i.e., value premises and factual premises.*†††

Intuitively, we may understand the value premises and the factual premises as the ends and the means of the decision respectively. Here, if we consider that physical activities of any organization are not the essential aspects of it but only the realizing processes of the decisions made, then the organization is properly defined as follows when the Condition 2-1 holds.††††

DEFINITION 2-3 : *The organization is a complex network of decision-makings.*†††††

Now, let us study the common purpose of participants of the organization. Clearly, each participant recieves some inducements from the organization, and makes some contributions to it. We can regard inducements and

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† See Chapter 6 and 7 of [2].

†† Even if decisions are procedures, we can consider these procedures as decision rules or decision processes.

††† See [11], [18] and [19].

†††† C.I.Barnard said that the exclusion of the physical and social environments from the definition of organization for general purposes will on the whole conform to ordinary usage and common sense, and will be accepted without great difficulties as a method of approach to a scientifically useful concepts of organization. See p.68 in [2].

††††† See p.220 in [19].

contributions as the social values in some sense. Participants decide whether they should participate in it or not by balancing their inducements with their contributions in the organization's process which transforms all contributions into inducements to be allocated. Therefore, the common purpose of participants has a foundation of its existence in this transformation process.† If we take into account Definition 2-3 and the just-mentioned point, we can define the organization as follows.

DEFINITION 2-4 : The organization is a decision-making system which transforms some social values into other social values.

This definition is of importance in organizational research as a starting point. That is, organizational research consists of two steps. Firstly, we must anlyze the transformation process in the organization; secondly, we must investigate decision-makings within it.

By the above definitions of organization, entrepreneurs, employees and customers are included as participants, and this makes our concept of organization broader than conventional concepts. We call the organization as defined above "the organization in a broader sense." But, within "the organization in a broader sense," there exists "an organization in a narrower sense" that is formed by a group of entrepreneurs and a group of employees. This is so-called management organization or administrative organization which poses the central problem of the general organizations theory. Therefore, we will discuss only "the organization in a narrower sense" unless specified otherwise. Now, we describe the means-ends relation and the authority relation, which are the principal concepts that

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† If we concentrate our attention on this transformation process as a whole, we get one problem on economy of inducements-contributions, that is, organizational equilibrium theory.

specify structure of the organization.

DEFINITION 2-5 : *Means-ends relation*

There exists a means-ends relation between two decision-making units in the organization when some parts of value premises of one decision unit consist of some parts of conclusions of the other decision unit.†

Similarly, we can define the authority relation as follows.

DEFINITION 2-6 : *Authority relation*

There exists an authority relation between two decision units in the organization when one decision unit accepts conclusions of the other decision unit as its decision premises.††

Of course, it is a very important problem on what basis this authority relation is established; for example, the existence of sanction mechnism may be a case, but we will not discuss this problem here. By the means-ends relation, authority relation and communication relation between decision units in the organization, we can specify the structure of the organization. At the same time, the authority relation and communication relation come into play when individual decision units are influenced by other units. The process of organizational influence is specified through these relations.

Naturally, in order to make the authority relation effective, it is necessary for indivisuals who participate in the organization at least to accept the following condition.

CONDITION 2-2 : *For individuals, to participate in the organization means to accept the organization's decision premises as their own*

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† See Chapter 4 in [19].

†† See Chapter 7 in [19] and Chapter 12 in [2].

decision premises.†

And we assume the following condition as to how individuals in the organization may behave.

CONDITION 2-3 : *If individuals in the organization are given their decision premises by the organization, they rationally behave at least to that extent.*††

When Condition 2-2 and 2-3 are satisfied, the organization can allocate its decision premises to each individual; that is, it can specify decision environments of each individual. Motivationally, we must investigate the conditions on the individual's identification of his organization with himself and discuss the effects of identification upon organizational behavior, but we neglect these discussions because we are interested in the formal aspects of the organization.

Here we study physical activities of the organization which implement the decisions. In modern organizations, it is very hard to avoid the division of labor or specialization. Considering the fact that decisions are related to physical activities as their inputs and that these physical activities constitute the process of transformation of social values as the organization's ultimate means, we can state the structuring process of the organization as follows: The structuring process of the organization is to connect the organization's common purpose with the transformation process of social values using the sequence of means-ends relation and to establish the authority relation and communication relation between individual decision units.

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† See Chapter 6 in [19].

†† See Chapter 9 and 10 in [19].

Therefore, if the common purpose of the organization can not be connected directly with its transformation process, we may easily expect that the hierarchy of means-ends is observed in the organization. Taking Definition 2-5 into account, this hierarchy means the hierarchy of the value premises.† And if the factual premises are properly allocated to the hierarchy of the value premises, clearly this hierarchy becomes the hierarchy of decision-makings. Thus we get to the problems of the organizational hierarchy.††

3. MATHEMATICAL FORMULATION OF ORGANIZATIONAL CONCEPTS

In Section 2, we discussed definitions of and conditions for the organization using the verbal logics. Since the verbal logics are little operational, we need an operational methodology for further discussions. In this section, we formulate the organizational concepts mathematically. But we do not deal with participants as human beings per se, but abstractly as decision-makers or decision units. We regard those groups of decision-makers, who are more interactive within their groups and are relatively independent from decision-makers of other groups, as decision units. Here we assume that Condition 2-1, 2-2 and 2-3 hold. Let us start with the definition of the decision-making system or decision unit.

DEFINITION 3-1 : *Decision-making system*

The decision-making system D is a system which has each element of a set V of value premises and that of a set F of factual premises as its input and has each element of a set X of conclusions as its out-

† See Chapter 4 in [19]. This intuitively means the hierarchy of sequence of goals.

†† That is, this hierarchy means the division of decisions in the organization.

put, i.e.,

$$D \quad (V \times F) \times X \ .$$

Before investigating the decision-making system in detail, we discuss the decision premises. At first, let us study value premises.

(1) Value premises

Value premises v consist of three elements v_D, v_L and ρ specified as follows.

V_D : a set of dimensions of social values which the organization can yield.

$V_L = V_{L_1} \times \cdots \times V_{L_r}$: a value space of the organization, where V_{L_i} is a set of value levels of the i-th value dimension for each i and $r = card\,(V_D)$.

$v_D \in \Pi\,(V_D)$: attended value dimensions (or goals) in a specific decision period.

$v_L \in \Pi\,(V_L)$: standard levels of attended value dimensions (or aspiration levels), which are used to indicate to what extent the goals should be realized.

ρ : a satisficing function, which is an evaluation function for the differences between aspiration levels and expected levels to be realized and may have several forms. For example, we can set this function as follows.

$$\rho : V_L \times V_L \to \{1, 0\},$$

where $\rho\,(v_L', v_L) = 1$ if $v_L' \geq v_L$,

and $\rho\,(v_L', v_L) = 0$ otherwise.

Then, we can write

$$V = (\Pi\,(V_D), \Pi\,(V_L), \{\rho\})$$

and

$$v = (v_D, v_L, \rho) \ . \ \dagger$$

Next, we consider factual premises.

(2) Factual premises

As similar, factual premises f consist of three elements M_0, f_p and f_v.

M_f : a set of all feasible alternatives available for the organization.

$M_0 \in \Pi(M_f)$: an evoked set of alternatives in a specific decision period.

$f_p : M_f \to Y$: a model of production process of outcomes, which is to predict the outcomes of decisions, where Y is a set of outcomes.

$f_v : M_f \times Y \to V_L$: a model of measuring process of values, which is to measure social values from the decisions and their outcomes.

Then, we can write

$$F = (\Pi(M_f), \{f_p\}, \{f_v\})$$

and $f = \{M_0, f_p, f_v\}$. ††

Using (1) and (2), we can specify the decision problem d as follows.

DEFINITION 3-2 : *Decision problem*

If a decision premise $(v_D, v_L, \rho, M_0, f_p, f_v)$ *is given, the problem can be stated as below for a decision-making problem* d. *Find* $m \in M_0$ *such that*

$$\rho(f_v(m, f_p(m)) \mid v_D, v_L) = 1 \ .$$

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† In this paper, *card* (X) and $\Pi(X)$ show cardinality of and a power set of X.

†† In factual premises, if we can assume that f_p and f_v could be appropriately varied by the organization in one decision period, we can define factual premises as $F = (\Pi(M_f), F_p, F_v)$ where $F_p = \{f_p \mid f_p : M_f \to Y\}$ and $F_v = \{f_v \mid f_v : M_f \times Y \to V_L\}$.

Where, $|$ *indicates the restriction operator.*

This definition corresponds to the so-called satisficing decision problem. At the same time this formulation can include so-called optimizing decision problem in general by properly defining the function ρ.

EXAMPLE 3-1 (for Definition 3-1) :

i) Decision-makings studied in [6], [11], [18] and [19] using verbal logics or simulation languages.

ii) Satisficing decision problem in [17], especially see Chapter 10.

iii) Let us suppose a one-dimensional optimization problem such that find $\hat{m} \in M_0$ which maximizes $g(m)$ over M_0. If we fefine

$$\rho : R \times \Pi(R) \to \{1, 0\}$$

so that for any real number r and subset R_0 of R,

$$\rho(r, R_0) = 1 \quad \text{if} \quad (\forall r^1)(r^1 \in R_0 \to r^1 \geq r)$$

and $\rho(r, R_0) = 0$ otherwise,

then we can rewrite the above problem as below.

Find $\hat{m} \in M_0$ such that

$$\rho(g(\hat{m}), g(M_0)) = 1 .$$

In the following discussions, we make the assumption below for the simplcity of discussions.

ASSUMPTION 3-1 : *Each decision-making unit in the organization has the capability and procedures to solve its decision problem once the problem is given.*

This assumption seems to be unrealistic in the real world. That is, there exist many decision problems for which present procedures or algorithm serve little help or none of decision units has capabilities to solve them. However, even in these cases, decision units solve them by one way or the other; for example, by changing present procedures or developing new pro-

cedures or redefining the original problems. Therefore Assumption 3-1 shows that we will not treat with heuristic decision problems. In a decision period, the organization does not give all decision premises, but some parts of decision premises are explicitly or implicitly given to each decision unit in advance.† However we consider that these decision premises given in advance are relatively constant inputs into the decision-making units.

Now, we define the relations between decision units of the organization as follows.

DEFINITION 3-3 : *Relations between decision units*

Let us assume that there are n-decision units $(D_1, \cdots, D_n)$ *in the organization. Then, the following relations ME, A, OR,* $CM \subseteq D_0 \times D_0$ *are called means-ends relation, authority relation, command relation and communication relation, respectively, where* $D_0 = \{D_1, \cdots, D_n\}$.

For each $D_i, D_j \in D_0$

i) $(D_i, D_j) \in ME \leftrightarrow X_i \cap V_j \neq \phi$,

ii) $(D_i, D_j) \in A \leftrightarrow (\exists V_{ji}')(\exists F_{ji}')(\exists X_{ij}')(V_{ji}' \subseteq V_j$ & $F_{ji}' \subseteq F_j$ & $X_{ij}' \subseteq X_i$ & $(V_{ji}' \neq \phi$ or $F_{ji}' \neq \phi)$ & $X_{ij}' \neq \phi$ & $(X_{ij}' \subseteq (V_{ji}' \times F_{ji}')$ or $X_{ij}' \subseteq V_{ji}'$ or $X_{ij}' \subseteq F_{ji}'))$,††

iii) $(D_i, D_j) \in OR \leftrightarrow (D_i, D_j) \in ME$ & $(D_i, D_j) \in A$, *and*

iv) $CM = (A \cup A^{-1}) \cup (ME \cup ME^{-1}) \subseteq D_0 \times D_0$.

EXAMPLE 3-2 (for Definition 3-3) :

i) Usually, the means-ends relation corresponds to the hierarchy of goals of the organization.

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† For example, these imposed decision premises may include the job specification, etc.

†† We call $V_{ji}' \times F_{ji}'$ (or V_{ji}', F_{ji}') the D_j's zone of acceptance of D_i's authority.

ii) The authority relation is essential one in the small group theory in order to study the organizational influence process and is represented by graphs.

iii) When the command relation is represented as a partial order, it corresponds to so-called command system in the organization.

iv) The communication relation corresponds to communication channels among decision units.

Here, we give some comments to Definition 3-3.

a) ME is properly well related to Definition 2-5 and CM is compatible with our conventional concepts.

b) The definition of A seems to be different from Definition 2-6. This difference has its origin in our avoidance of formulating the concept of authority acceptance.

c) The command relation is usually defined as a special case of the authority relation through legitimacy concepts and often forms so-called command system. Since we did not introduce them in our formulation, we could only define OR as above.

d) Since it is apparent from i) and ii) that $ME \subseteq A$, we can write $CM = A \bigcup A^{-1}$ and $OR = ME$.

Main parts of problems in b) and c) may be resolved by the introduction of socio-psychological factors into our formulation. However, they are not so fatal for our discussions if we restrict our attention to formal aspects of the organization, because the legitimacy concepts are mainly reflected on the means-ends relation.

We have briefly discussed the relations between decision units in the organization. Next, we investigate physical activities of the organization. These physical activities compose the process which transform contributions

into inducements. And this process is controlled by decision-making units or systems in the organization. Therefore we can formulate the process as follows.

DEFINITION 3-4 : Transformation process of the organization

The transformation process P of the organization is a system which transforms contributions of participants into their inducements under the controls of the decision-making systems in this organization; that is, $P \subseteq (C \times M) \times I$, whre C is a set of contributions, I and M are a set of inducements and of control or decision variables respectively. When we suppose that C restricts M to the feasible domain M_f, we represent $P \subseteq M_f \times I$ or $P \subseteq (M_f \times \Omega) \times I$ under the presence of an uncertainty set Ω.

In the following discussions, we restrict our attention to the case that P is a functional system; that is, it is represented as $P : M_f \to I$ or $P : M_f \times \Omega \to I$. Now we examine the division of physical activities or the decomposition of the transformation process. Let us assume that P consists of q subprocesses or subactivities. We represent these subprocesses by

$$P_i : M_{fi} \times U_i \to I_i \quad \text{or} \quad P_i : M_{fi} \times U_i \times \Omega_i \to I_i$$

where M_{fi}, U_i, I_i and Ω_i denote a set of decision or control variables, of interactions from other subprocesses produced by the divisionalization, of inducements produced by the i-th subprocess and of uncertainties, respectively. Here we describe each i-th interaction from other subprocesses by the function H_i, that is,

$$H_i : M_f \times I \to U_i .$$

Where $U = U_1 \times \cdots \times U_q$, $M_f = M_{f1} \times \cdots \times M_{fq}$ and $I = I_1 \times \cdots \times I_q$.
Using the natations $H = (H_1, \cdots, H_q)$ and $\vec{P} = (P_1, \cdots, P_q)$, we introduce the following condition.

CONDITION 3-1 : Division of the transformation process

We state that $\overline{P}$ *represents the divisionalized process if and only if the following condition holds.*

$$(\forall m)(\forall i)(m \in M_f \ \& \ i \in I \rightarrow (i = \overline{P}(m, H(m, i)) \leftrightarrow i = P(m)))$$

or

$$(\forall m)(\forall i)(\forall \omega)(m \in M_f \ \& \ i \in I \ \& \ \omega \in \Omega \rightarrow (i = \overline{P}(m, H(m, i), \omega) \leftrightarrow i = P(m, \omega))).$$ †

In the succeeding discussions, we assume that the division of the process satisfies Condition 3-1. Concerning with the connection between these subprocesses and decision units, we suppose that the following condition is satisfied.

CONDITION 3-2 : Relation between process and decision systems

$$(\forall P_i)(\exists D_j)(X_j \cap M_{fi} \neq \phi \ \& \ (I_i \times U_i) \cap Y_j \neq \phi).$$

This condition means that each subprocess is controlled by at least one decision unit and its results or performances are fed back to this decision unit.

EXAMPLE 3-3 (for Definition 3-4, Condition 3-1, 3-2) :

i) In economics, P consists of production function and cost function or P is represented as production possibility set. While, in general organizations theory, it consists of the above two functions and accounting function as cited by H.A.Simon [17].

ii) In (mathematically formulated) decentralized organizations theory, many of its discussions start with models based on Condition 3-1 and 3-2.

We call a sequence of decision units, in which each decision unit can

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† This condition is originally described by M.D.Mesarović and others [13].

be connected with some units by the means-ends relation, the sequence of means-ends. It is very natural to impose the following condition on it.

CONDITION 3-3 : *Condition of organizational hierarchy*

(a) $(\forall D_i)(\forall D_j)((D_i, D_j) \in ME \rightarrow (D_j, D_i) \notin ME)$.

Then we define the relation $\preccurlyeq \subseteq D_0 \times D_0$ *as follows.*

$$D_i \preccurlyeq D_j \leftrightarrow ((D_j, D_i) \in ME \text{ or } (\exists D_k, D_l, \cdots, D_s)((D_j, D_k) \in ME \ \& \ (D_k, D_l) \in ME \ \& \cdots \& (D_s, D_i) \in ME) \text{ or } D_i = D_j) .$$

(b) $(\exists \tilde{D}_0)(\forall D_i)(D_i \preccurlyeq \tilde{D}_0 \ \& \ (\forall \tilde{\tilde{D}}_0)(\exists D_j)(\tilde{D}_0 \neq \tilde{\tilde{D}}_0 \rightarrow \tilde{\tilde{D}}_0 \not\preccurlyeq D_j))$.

(c) $\neg(\forall D_i)(\exists D_k, \cdots, D_s)((D_i, D_k) \in ME \ \& \cdots \& (D_s, D_i) \in ME)$.

(a) means one-directionality of the means-ends relation, that is, its asymmetry. This implies that it is impossible that two decision units are influenced their value premises by each other. And we can deduce its anti-reflexibility from *(a)*. The relation $\preccurlyeq$ could be named the infimal-supremal relation in the organization. *(b)* means that there exists only one most supremal decision unit in the organization. Lastly, *(c)* means that we exclude the possibility of the means-ends cycle. The relation $\preccurlyeq$ which satisfies Condition 3-3 is a partial order, which can be shown easily. Thus, we need Condition 3-3 to describe decision units in the organization hierarchically.

EXAMPLE 3-4 (for Condition 3-3) :

i) $\preccurlyeq$ corresponds to so-called organization chart or organizational hierarchy very well. (See iii) in Example 3-2.)

ii) Using Condition 3-3, we can construct a hierarchy of decision units in any organization even if it is not represented explicitly.

iii) If Condition 3-3 holds and any decision unit is involved in some sequence of means-ends in an organization, we call this organization a multi-level organization. Moreover, if any sequence of means-

ends can reach some subprocess by Condition 3-2 in a multi-level organization, then we call it a multi-level line-organization which will be studied in detail in the next section.†

With definitions, conditions and assumptions as mentioned above, we are able to represent the organization O by a set D_O of decision units, relations ME, A and CM and the division of process $\overline{P}$ and interactions H within $\overline{P}$ as $O = (D_O, ME, A, CM, \overline{P}, H)$ or $O = (D_O, \preccurlyeq, A, \overline{P}, H)$. This representation of the organization is very general and includes diverse forms of organizations, for example, Taylor's functional organization, decentralized organization, etc., provided that we properly define the elements of O. And it is possible to study principles of management theory or administration theory as conditions that should be added to the above framework.†† Furthermore, we can also study other organizational concepts that were not refered in to in our framework.

4. TWO-LEVEL LINE-ORGANIZATION

A line-organization to be described here is imposed the following conditions and, therefore, may be useful as a model of the large-scale organization or multi-divisional multi-level organization which is often analyzed in decentralized organizations theory. We can point out reasons why we concentrate our attention to this line-organization.

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† We consider that the staff-organization supplies the line-organization only with factual premises. Hence, our definition of the multi-level organization corresponds usual notions of the line-staff organization.

†† For example, we can represent the principle of unity of command as follows.
a) The relation ME satisfies Condition 3-3.
b) $(\forall D_i)(\forall D_j)(\forall D_k)(D_i \neq D_0 \ \& \ (D_j, D_i) \in ME \ \& \ (D_k, D_i) \in ME \rightarrow D_j = D_k)$.

(1) The ultimate foundation of the organization lies in its transformation process of contributions into inducements, and the line-organization has a function as a decision-making system which controls this transformation process. That is, the essential organizational behaviors appear in the line-organization.

(2) Since the staff organization is oriented to support the line-organization, we can evaluate it as a system which should be added to the line-organization after the formulation of this line-organization.

We state about the two-level line-organization as below. At first, we suppose the following assumption.

ASSUMPTION 4-1 :

(a) Participants of the organization completely accept decision premises of decision units, in which they participate, as their own decision premises.

(b) The authority relation among decision units is constrained into the means-ends relation and its inverse; that is, $A \subseteq ME \cup ME^{-1}$. *Therefore,* $CM = ME \cup ME^{-1}$. *And, for each decision unit, all communications among decision units are restricted within the zones of acceptance of authority.*

(c) For each decision unit, the dimension of values is one; that is, each decision unit has only one goal.

(d) The supremal (decision) unit does not control the transformation process directly. This means the so-called indirect intervention or control.

(e) Each infimal (decision) unit controls only one subprocess.

(a) is assumed in order to avoid problems of human being himself in the organization. *(b)* implies that there are no communication among infimal

decision units. We assume *(c)*, *(d)* and *(e)* for the simplicity and operationality of our model.

REMARK 4-1 (on Assumption 4-1) :

i) *(a)* is very controversial from the socio-psychological viewpoints. This fact makes our formulation a "machine" model rather than a "decision" model, although we have discussed the organization on the basis of the latter one. However, *(a)* is implicitly assumed in models of decentralized organizations theory, team theory, classical organizations theory and traditional management theory.†

ii) *(b)* can be imposed on our model whenever *(a)* holds. About the former half of it, we can hypothetically consider that all the communications among the infimal decision units are transfered through the supremal decision unit. Moreover, R.L.Ackoff [1] and M.D.Mesarović [13] point out that communications among the infimal decision units could not necessarily increase the efficiency of overall decisions.

iii) About *(c)*, if we can deal with the multi-dimensionality of the value premises for example, by transforming them into one-dimensional value or linear pre-order, then we have no problem about them except for technical ones. About *(c)* and *(d)*, there exist no problem for us.

iv) Assumption 4-1 is satisfied in (mathematically formulated) decentralized organizations theory in general. In team theory, *(b)* and *(d)* are not satisfied because it does not specify the relations

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† About team theory, see Chapter 9, 10 and 11 in [12].

among decision units and we can not find a counterpart to the supremal unit in it (except for its application to resource allocations). L.Hurwicz's model of the large-scale economic organizations satisfies *(a)*, *(c)*, *(d)* and *(e)* but does not *(b)* in part, for it does not specify the means-ends relation among decision units but deals with communication through the market mechanism (as a counterpart to the supremal unit).† Although we can not expect that real organizations are structured systematically as the above assumption, we suppose a line-organization based on it as an ideal-type of line-organizations.

Here we study the roles of the supremal unit. Since we deal with only the routine-decision-making problems as mentioned in Section 3, the roles of the supremal unit are mainly to resolve conflicts between infimal units, and to lead infimal units to the achievement of the organization's overall goal or objective.†† These activities are called the organizational coordination, which is an essential activity so as to maintain the harmony or order of the organization. Then, we will continue discussions on the organizational coordination.

What relation does exist between the objective of the supremal unit and the organization's overall objective? Of course, we can assume that two objectives are equal to each other, but this assumption reduces the importance of the division of decision-making. Because infimal units aim to achieve their own local objectives in order to contribute the overall objective, so as to maintain the benefit of the divisionalization of pro-

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† About Hurwicz's model, see Chapter 14 in [12].

†† Fundamentally, the supremal unit should play roles as a leader in the organization, but these roles do not appear explicitly in our model from Assumption *(a)* and *(b)*.

cess and decision, it is sufficient that decision units are structured so that if the supremal unit achieves its own objective then the overall object is also realized. Therefore, the objective of the supremal unit, when it coordinates infimal units, could be different from the overall objective. Now, in order to formulate this idea, we use the following notations.†

$p(x, d) \equiv$ true if and only if d is a decision problem and x is a solution of it.

$\tilde{d}_0$: overall decision problem of the organization.

d_0 : decision problem of the supremal unit.

$d_{1r}(\gamma)$: decision problem of the i-th infimal unit when the coordination input from the supremal unit is γ, where γ is a solution of d_0 at the same time. And

$$d_1(\gamma) = (d_{11}(\gamma), \cdots, d_{1q}(\gamma)) .$$

x : solution of the infimal decision problem $d_1(\gamma)$.

Using these notations, we define the conflict in the organization.

DEFINITION 4-1 : *Organizational conflict*

(a) Vertical conflict.

When γ is a coordination input from the supremal unit and x is a solution of $d_1(\gamma)$, there exists a vertical conflict in this organization if and only if $Q_1(\gamma, x) \equiv$ true, where $Q_1(\gamma, x)$ is defined as follows.

$$(\forall\gamma)(\forall x)(\gamma \in \Gamma \ \& \ x \in X \rightarrow (Q_1(\gamma, x) \leftrightarrow \neg(p(\gamma, d_0) \ \& \ p(x, d_1))))),$$

where Γ is a set of coordination inputs and X is a set of solutions for the infimal decision problems.

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† We apply notations and concepts described in [14] to our model here.

(b) Lateral conflict.

Similarly, there exists a lateral conflict in the organization if and only if $Q_2(\gamma, x) \equiv$ true, *where* $Q_2(\gamma, x)$ *is defined as follows.*

$$(\forall\gamma)(\forall x)(\gamma\in\Gamma \ \&\ x\in X \leftrightarrow (Q_2(\gamma,x) \leftrightarrow \neg(p(\gamma, d_1(\gamma)) \ \&\ p(x, \tilde{d}_0)))).$$

In the above definition, the meaning of *(a)* is clear, while the meaning of *(b)* seems to be different from our usual image of the lateral conflict. But, if we take into account the fact that an essential aspect of the lateral conflict is strictly observed on the incompatibility of the individual infimal decisions, *(b)* is very natural.

When we regard the resolution of conflict as the organizational coordination, we can point out two possibilities of the conflict resolution or coordinabilities.

DEFINITION 4-2 : *Organizational coordinability*

(a) Coordinability for vertical conflict.

We state that the organization has the coordinability for vertical conflict if and only if there exist γ *and* x *which satisfy the following condition.*

$$(\exists\gamma)(\exists x)(\gamma\in\Gamma \ \&\ x\in X \ \&\ \neg Q_1(\gamma,x)).$$

(b) Coordinability for lateral conflict.

As similar, we state that the organization has the coordinability for lateral conflict if and only if there exist γ *and* x *which satisfy the following condition.*

$$(\exists\gamma)(\exists x)(\gamma\in\Gamma \ \&\ x\in X \ \&\ \neg Q_2(\gamma,x)).$$

REMARK 4-2 (on Definition 4-1, 4-2) :

i) H.A.Simon and J.G.March [10] define conflict as "a breakdown in the standard mechanisms of decision-making", which has properly

well correspondence with our definition. They study two types of the organizational conflict, i.e., individual conflict and intergroup conflict, and point out several factors affecting these conflicts systematically. In our notations, the individual conflict can be defined as $\neg p(x, d_{1i}(\gamma))$ for any infimal unit i and $\neg p(\gamma, d_0)$ for the supremal unit. They investigate the intergroup conflict restricting their attentions mainly upon the lateral one. Moreover, they show the organizational reaction (coordination in our terminologies) to the organizational conflict.

ii) In the socio-psychological approach to the organizational conflict, its attentions are focused on the vertical conflict at relatively lower levels of the organization. Such motivational factors (in human relations school), communication patterns (in small group theory) and leadership patterns (in modern socio-psychology school), etc., are pointed out as those affecting the organizational conflict.

iii) R.M.Cyert and J.G.March chiefly refer to the lateral conflict by concepts of coalition and of quasi-resolution of conflict.

iv) In the above-mentioned studies, only one of two conflicts is usually studied assuming that the other is constant or relatively stable. However, Assumption 4-1 is not always imposed on their models.

However, the above two coordinabilities are not necessarily independent, we can combine them into one logical predicate when we represent the consistency or harmony of the organization. The state in the harmony of the organization is that where each decision unit solves its own decision problem for the corresponding coordination and this solution composes the

solution of the overall decision problem. In this situation, the resolution of vertical conflict implies the resolution of lateral conflict.

DEFINITION 4-3 : Consistency of organization

We state that the organization has the consistency of organization if and only if the following condition holds.

a) $(\forall\gamma)(\forall x)(\gamma\in\Gamma \ \& \ x\in X \rightarrow (\ \urcorner Q_1(\gamma, x) \rightarrow \urcorner Q_2(\gamma, x)))$,

or equivalently

a') $(\forall\gamma)(\forall x)(\gamma\in\Gamma \ \& \ x\in X \rightarrow (p(x, d_1(\gamma)) \ \& \ p(\gamma, d_0) \rightarrow p(x, \tilde{d}_0)))$.

REMARK 4-3 (on Definition 4-3 and this section) :

i) In general, *a')* rather than *a)* is often used to represent the consistency of organization. In classical organizations theory, it is assumed that the consistency must be held in any management organization. However, we do not make the value judgement whether any organization must satisfy it or not, for we are interested in what organizations do satisfy it or not.

ii) Differences among (mathematically formulated) organization models are summarized in Table 1.

5. SUMMARY AND FURTHER RESEARCH

We have abstracted the essential organizational concepts of general organizations theory from descriptive organizations theory by C.I.Barnard, H.A.Simon and J.G.March and formulated them mathematically. If the ultimate purpose of general organizations theory is to present an integrated and systematic framework to specific organizations theories, then this paper presents it in a relatively simple form.

The areas that need further researches are as follows.

Table.1. Comparison of organization models.

Item \ Theory	Decentralized Organizations Theory	Team Theory	Hurwicz's Theory
Assumption 4-1	satisfied	(*a*), (*c*), (*e*)	satisfied except for (*b*)
Overall problem	given a priori	given a priori	not exist or hypothetically to achieve Pareto-satisfactoriness
Roles of supremal unit	coordination as described in § 4	not exist in general or hypothetically to select optimal decision functions and information functions	calculation of prices of resources
Representation of the consistency of organization	form *a'*) in Definition 4-3	principle of person-by-person satisfactoriness	principle of Pareto-satisfactoriness
Other characteristics	interested in internal mechanisms of the organization	studied under uncertainties	privacy respecting mechanism

(1) To investigate satisficing decision problems, especially under uncertainties.

(2) To study the coordination processes in the multi-level line-organization whose decision problems are specified as satisficing decisions.

(3) To represent the organization as a dynamical system.

(4) To construct the relation between the "organization in a broader sence" and the "organization in a narrower sense". This means to construct the organizational equilibrium theory. The organizational equilibrium should be studied from two standpoints corresponding to the above two organizations; that is, the organizational external-equilibrium and the organizational internal-equilibrium.

(5) To introduce the organizational slacks into our framework in order to represent more behavioral decision-making processes and study the stability of the organizational equilibrium.

Then the structure of the axiomatic organizations theory may be shown as Fig.2.

In this structure, we may expect to cover many organizational concepts. Now, the benefits of the axiomatic organizations theory are as follows.

(1) For descriptive organizations theory, it will give one starting point when we analyse the organization from the decision-making viewpoint.

(2) For normative organizations theory, it will clarify the conditions implicitly assumed, and will give a useful foundation for the construction of organizations models.

Consequently, we believe that our approach will be valuable for the organizational researches.

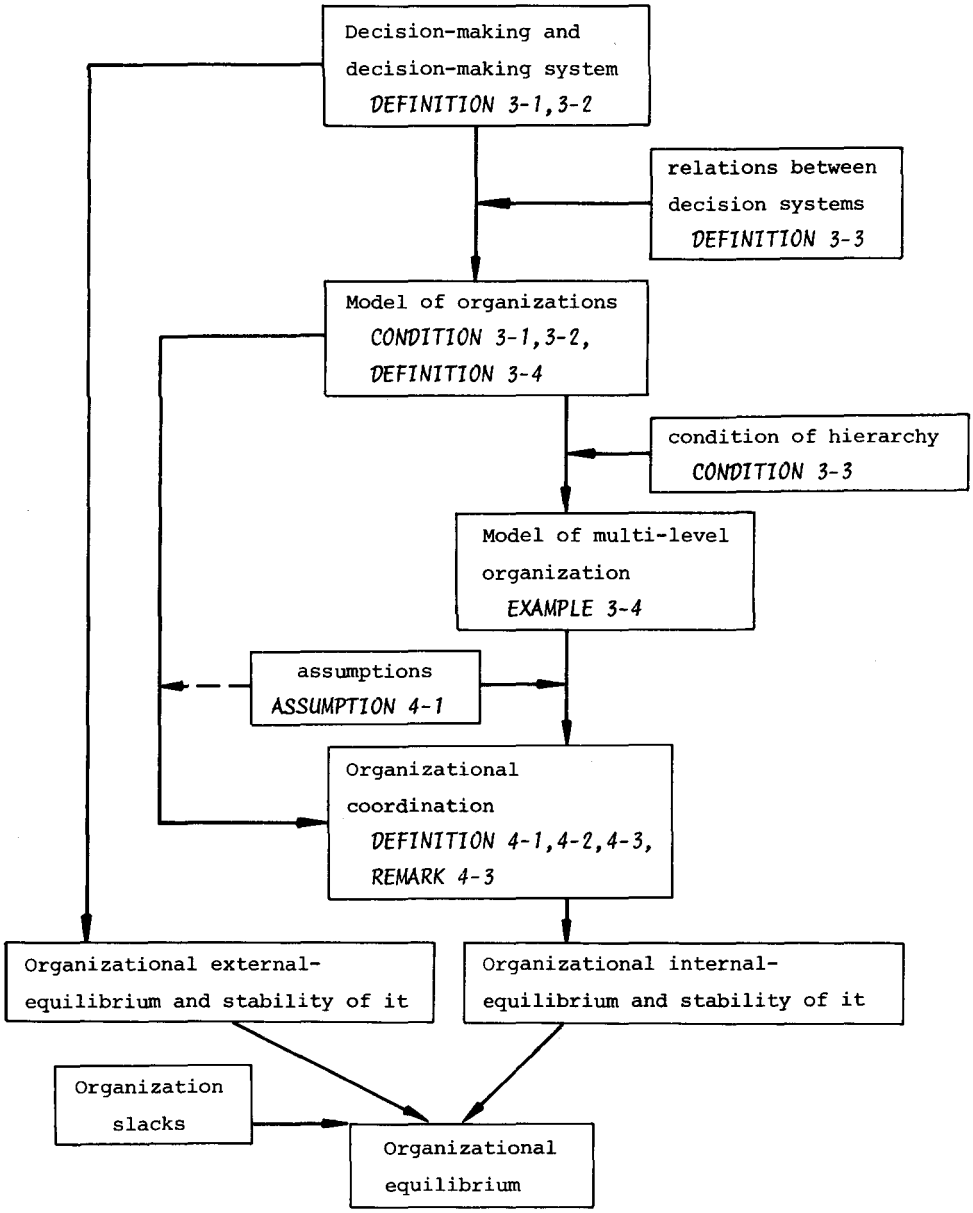

Fig.2. Structure of the axiomatic organizations theory.

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