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TANK-TANKER PROBLEM
OPTIMIZATION OF TANK CAPACITY AND DETERMINATION OF TANKER SCHEDULE

**SABURO SUGANAMI, YOICHI SORIMACHI
and AKIO SONOBE**

Mitsubishi Research Institute

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Abstract

Recently, tankers carrying crude oils tend to grow in their sizes, and it is even said that in the near future 400,000 to 500,000 ton tankers may appear. With such a tendency in view, tanks for storing crude oils are also becoming larger and larger.

In Japan, where a variety of tankers whose tonnages vary from 20,000 to 200,000 are used to carry different kinds of oils mainly from such distant countries as Kuwait and Saudi Arabia spending forty days for a round trip, the problem of optimizing

a) The tanker schedule, or the determination of ports for which tankers are bound

b) Tank capacity at the base

becomes important matters.

In addition to this, the following factors will also have to be considered:

1) Oil-processing pattern at the refineries
2) Demurrage on tankers for unscheduled overtime stay at ports
3) Fluctuation of interarrival periods

We formulate a mathematical model for finding the minimum feasible tank capacity by taking the above conditions into account. We obtained some workable solutions to this problem in the following two cases

I) case-1: one port loading and one port discharge
II) case-2: more than one port loading and/or more than one port discharge (the computation is carried out for the case that the total number of ports in both sides is 3.)

A tanker schedule is determined as a sequence of decisions which minimizes the feasible tank capacity.

We consider that solutions based on our formulation of the problem can be used to establish a general rule for tanker allocation.

We developed a computer program, which is called 'Otasche' (oil tanker schedule; in Japanese it means 'HELP') for the IBM S/360-75J. This single program solves the problem under different combinations of conditions by simply changing input data.

1. Preface

The import of crude oil of Japan in 1969 has exceeded 165,000,000k*l*. This fact is extraordinary, as compared with that of 10,000,000 k*l* of about ten years ago. Accordingly, the rate of imported energy over the total energy consumed in Japan has gone up from 1/4 to 4/5 during the above ten years.

Most of imported crude oil, surprisingly summing up to 90%, comes from the Middle East District, that is, Iran, Iraq, Kuwait, Saudi Arabia, *etc.*

Since the Middle East is over 10,000 km far from Japan and about 40 days are required for a round trip by oil tanker, it could be easily seen that the transportation cost has a serious influence on the energy cost in Japan, hence even on the Japanese industry itself.

Countries in the world have made a rush for constructing gigantic tankers to reduce the transportation cost. In 1950, the whole world was surprised to know that Japan had 28,000 tonnage tankers, but 20 years later we are now having 300,000 tonnage tankers. It is even said, that we shall have 500,000 tonnage tankers in the near future.

These huge tankers consequently require large storage tank at the base for storing crude oils arriving in batch. So, it could be justified that, since the construction of a tank requires a great deal of investment, the establishment of a tanker schedule which is appropriate to the process pattern of the refinery and the determination of the most economical tank capacity are most important problems for oil companies.

2. Miniature Model

We start to analyze the problem with the following simplest miniature model:

i) Condition of the process pattern at refinery

There is only one refinery with two plants I and II, working separately. Figs. 1 and 2 show the process patterns of crude oils A, B, C processed at plant I and II, respectively.

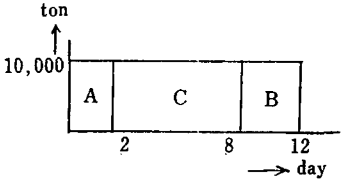

Fig. 1. Refinery process pattern of plant I.

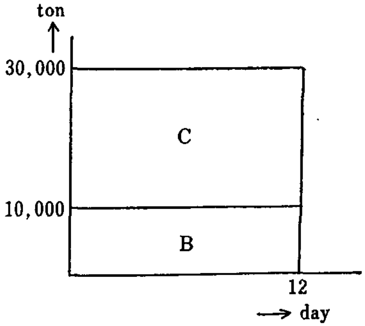

Fig. 2. Refinery process pattern of plant II.

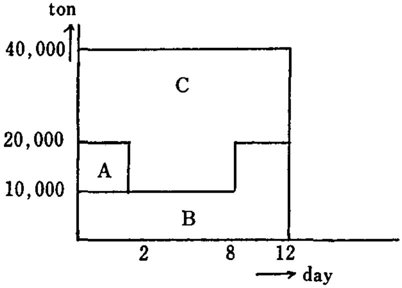

Fig. 3. Total refinery process pattern.

They are processed sequentially with a 12 day cycle. Consequently, the compound process pattern of the refinery may be shown as Fig. 3.

ii) Available tankers and their schedule

Four tankers (30, 200, 150, 100 kilo-tonnage) are sailing around in this sequence with an equal interval[1] of 3 days, consequently having a 12 day cycle each. (Five days for one way, one day for loading and one day for discharge.) This tonnage is set so that the balance in one cycle may be reached. $((10+30)\times 12=30+200+150+100$ kilo-ton.)

We assume that the arrival interval of tankers is fixed when

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[1] In general, an equal interval is not needed.

we determine the tank capacity. Possible stockout of crude oil at the discharging port due to actual fluctuations of the arrival interval will be covered by the principle of safety stock.

iii) Other conditions

(a) Assuming that the cost of tank construction is monotone increasing with increasing tank capacity, it would be sufficient to minimize the total inventory of the oils.

In the above, it is assumed that a tank is adequately divided into smaller tanks of any capacity to guarantee that the total tank capacity is independent of the assignment of tanks to specific kind of oils at the base.

(b) We do not treat the case of mixed loading, but only treat the one-port-loading, one-port-discharge case.

(c) We neglect the difference in shipping charges arising from different loading places.

iv) Analysis

Take the arrival interval of tankers as the time unit. Assume the incoming tanker arrives at the n-th time, where the 0-th time represents the time of termination of our model.

Roughly speaking, we could express that the maximum inventory $F(x)$ with the optimum selection of crude oils satisfies the functional equation:

$$F(n)=\min_{(D)} \max [I, F(n-1)] .$$

Here, 'I' indicates the oil inventory at the n-th time (see Fig. 4) and D corresponds to a decision of the selection of crude oils.

For the precise description of the problem, let us start with the definitions as follows.

$\tilde{X}_n$: vector of which component i indicates the inventory of crude oil i at time n just before a tanker comes in

$\langle \tilde{X}_n \rangle$: sum of $\tilde{X}_n$ components $\sum_{n_i} \tilde{X}_{n_i}$ showing the total inventory at time n

$\tilde{d}(n, n-1)$: vector of which component i indicates the consumption of crude oil i during time n and $n-1$

$\langle \tilde{d}(n, n-1) \rangle$: sum of $\tilde{d}(n, n-1)$ components which is equivalent to the total consumption of crude oils i during period $[n, n-1]$

$I_D(\tilde{X}_n)$: vector of which component i indicates the inventory of crude oil i at time $n-1$, after the discharge of crude oil i from the incoming tanker at time $n-1$, with decision D at time $n-1$, under the condition that the inventory at time n is $\tilde{X}_n$.

$\langle I_D(\tilde{X}_n) \rangle$: sum of $I_D(\tilde{X}_n)$ components

$\tilde{T}_n$: vector of which component i indicates the quantity of crude oil i carried by the incoming tanker at time n

$\tilde{\alpha}_n$: equal to $\tilde{T}_n - \tilde{d}(n+1, n)$

$F(n-1, \tilde{X}_n)$: the minimum feasible tank capacity with the sequence of optimal decisions during preceding $(n-1)$ periods, starting with the inventory $\tilde{X}_n$ at time n

Then, the equation

$$(2.1) \qquad F(n-1, \tilde{X}_n) = \min_{D} \max(\tilde{X}_n,\ I_D(\tilde{X}_n),\ F(n-2, I_D(\tilde{X}_n))$$

is introduced with the initial condition

$$(2.2) \qquad F(0, \tilde{X}_1) = \langle \tilde{X}_1 - \tilde{d}(1, 0) + \tilde{T}_0 \rangle$$
$$= \langle \tilde{X}_1 + \tilde{\alpha}_0 \rangle$$

where the function F is defined on the region of $\tilde{X}_n$ and D satisfying

$$(2.3) \qquad \tilde{X}_n \geq \tilde{d}(n, n-1)$$

$$(2.4) \qquad I_D(\tilde{X}_n) \geq \text{M.F. } \tilde{X}_{n-1}$$

Here, M.F. $\tilde{X}_{n-1}$ indicates the $\tilde{X}_{n-1}$ which minimizes $\langle \tilde{X}_{n-1} \rangle$ on a set of $\tilde{X}_{n-1}$ which guarantees the feasibility for preceding $(n-1)$ periods.

The set of conditions (2.3) and (2.4) assures the feasibility of the problem in such a sense that the stockout never occurs at any time during time n and time 0.

Now, in order to see the nature of equation (2.1) we had better follow the equation in case of $n=2$, 3:

$$\begin{aligned}F(1, \tilde{X}_2)&=\min_{D} \max[\langle\tilde{X}_2\rangle, \langle I_D(\tilde{X}_2)\rangle, F(0, I_D(\tilde{X}_2))]\\&=\min_{D} \max[\langle\tilde{X}_2\rangle, \langle\tilde{X}_2+\tilde{\alpha}_1\rangle, \langle\tilde{X}_2+\tilde{\alpha}_1+\tilde{\alpha}_0\rangle]\\&=\min_{D}[\langle\tilde{X}_2\rangle+\max[0, \langle\tilde{\alpha}_1\rangle+\max[0, \langle\tilde{\alpha}_0\rangle]]],\\F(2, \tilde{X}_3)&=\min_{D}[\langle\tilde{X}_3\rangle+\max[0, \langle\tilde{\alpha}_2\rangle+\max[0, \langle\tilde{\alpha}_1\rangle\\&\quad+\max[0, \langle\tilde{\alpha}_0\rangle]]]].\end{aligned}$$

Accordingly, the relation in the general case would be deduced as follows:

$$(2.5)\qquad F(n-1, \tilde{X}_n)=\min_{D}[\langle\tilde{X}_n\rangle+\max[0, \langle\tilde{\alpha}_{n-1}\rangle+\max[0, \langle\tilde{\alpha}_{n-2}\rangle +\cdots+\max[0, \langle\tilde{\alpha}_0\rangle]\cdots]].$$

From equation (2.5), it can be seen that, in case of one-port-loading, one-port-discharge, $\langle\tilde{\alpha}_n\rangle$ itself is decision free, thus the feasible minimum tank capacity $F(n, \tilde{X}_{n-1})$ depends only upon the initial inventory $\langle\tilde{X}_n\rangle$.

The vector $\tilde{X}_n$ should not be arbitrary but must be feasible in a sense as is mentioned above, and $F(n-1, \tilde{X}_n)$ attains its minimum value when $\langle\tilde{X}_n\rangle$ is minimum.

3. Numerical Example (I)

The vector $\tilde{d}$, in accordance with the process pattern of Fig. 3, and the order of incoming tankers are given in Fig. 4.

$$\tilde{\alpha}_0=\tilde{T}_0-\tilde{d}(1, 0)$$

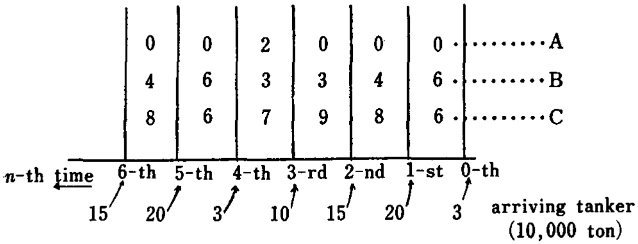

Fig. 4. Process pattern and arriving tanker.

There are three possible cases of crude oil discharge at time 0:

case 1) A tanker laden with crude oil A arrives, namely,

$$\bar{\alpha}_0=\begin{pmatrix}3\\0\\0\end{pmatrix}-\begin{pmatrix}0\\6\\6\end{pmatrix}=\begin{pmatrix}3\\-6\\-6\end{pmatrix}$$

case 2) A tanker laden with crude oil B arrives, namely,

$$\bar{\alpha}_0=\begin{pmatrix}0\\3\\0\end{pmatrix}-\begin{pmatrix}0\\6\\6\end{pmatrix}=\begin{pmatrix}0\\-3\\-6\end{pmatrix}$$

case 3) A tanker laden with crude oil C arrives, namely,

$$\bar{\alpha}_0=\begin{pmatrix}0\\0\\3\end{pmatrix}-\begin{pmatrix}0\\6\\6\end{pmatrix}=\begin{pmatrix}0\\-6\\-3\end{pmatrix}.$$

Write the above three equations together in a matrix form.

$$\overset{1)}{[\bar{\alpha}_0]}=\begin{array}{c}\begin{array}{ccc}\mathrm{A} & \mathrm{B} & \mathrm{C}\end{array}\\ \begin{pmatrix}3 & 0 & 0\\ -6 & -3 & -6\\ -6 & -6 & -3\end{pmatrix}\end{array}$$

$\langle\bar{\alpha}_0\rangle$ is obviously -9 regardless of what kind of oil the tanker transprots to the refinery.

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1) [ ]: matrix expression of which column indicates the vector $\bar{\alpha}_n$ or $\tilde{X}_n$ which corresponds to the possible cases of crude oils carried.

From (2.3),

$$\tilde{X}_1 \geq \begin{pmatrix} 0 \\ 6 \\ 6 \end{pmatrix}.$$

Accordingly, from (2.5), we obtain

$$\begin{aligned} F(0,\ \tilde{X}_1) &= \min\langle\tilde{X}_1\rangle + \max\ [0,\ \langle\tilde{\alpha}_0\rangle] \\ &= 12 + \max[0,\ -9] \\ &= 12\,. \end{aligned}$$

Thus, at time $n=1$, we have

$$[\tilde{\alpha}_1] = 20\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 8 \end{pmatrix}(1,\ 1,\ 1) = \left(\begin{array}{ccc} \text{A} & \text{B} & \text{C} \\ 20 & 0 & 0 \\ -4 & 16 & -4 \\ -8 & -8 & 12 \end{array}\right).$$

On the other hand from (2.3), we have at least

$$\tilde{X}_2 \geq \begin{pmatrix} 0 \\ 4 \\ 8 \end{pmatrix}, \tag{3.1}$$

whereas from (2.4),

$$I_D(\tilde{X}_2) = \tilde{X}_2 - \tilde{d}(2,\ 1) + \tilde{T}_1 = \tilde{X}_2 + \tilde{\alpha}_1 \geq \text{M.F. } \tilde{X}_1$$

where M.F. $\tilde{X}_1$ means Minimum Feasible $\tilde{X}_1$ which is equal to

$$\begin{pmatrix} 0 \\ 6 \\ 6 \end{pmatrix}.$$

The latter inequality leads to

$$\begin{aligned} [\tilde{X}_2] &\geq [\text{M.F. } \tilde{X}_1] - [\tilde{\alpha}_1] \\ &\geq \begin{pmatrix} 0 \\ 6 \\ 6 \end{pmatrix}\cdot(1,\ 1,\ 1) - \begin{pmatrix} 20 & 0 & 0 \\ -4 & 16 & -4 \\ -8 & -8 & 12 \end{pmatrix} \\ &= \left(\begin{array}{ccc} \text{A} & \text{B} & \text{C} \\ -20 & 0 & 0 \\ 10 & -10 & 10 \\ 14 & 14 & -6 \end{array}\right). \end{aligned} \tag{3.2}$$

Combining (3.1) and (3.2), we have

$$(3.3) \qquad [\tilde{X}_2] \geq \begin{array}{c} \begin{array}{ccc} \mathrm{A} & \mathrm{B} & \mathrm{C} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 \\ 10 & 4 & 10 \\ 14 & 14 & 8 \end{pmatrix} \end{array}$$

At time $n=2$, $F(1, \tilde{X}_2)$ is calculated by (2.5) as follows.

$$\begin{aligned} F(1, \tilde{X}_2) &= \langle \mathrm{M.F.}\ \tilde{X}_2 \rangle + \max [0, \langle \tilde{\alpha}_1 \rangle + \max [0, -9]] \\ &= 18 + \max [0, 8+0] \\ &= 26\,. \end{aligned}$$

For time steps n, $n \geq 3$, we apply the same procedure recursively as illustrated above.

We take the summation of $F(n-1, \tilde{X}_n) = \langle \mathrm{M.F.}\ \tilde{X}_{n-1} \rangle + a(n-1)$ for a sufficiently large n as a solution to our Minimum Feasible Tank Capacity problem.

For computational purpose, the following property is worth to mention. Define $a(n-1)$, the second term of the right hand side (2.5), as

$$(3.4) \qquad a(n-1) = \max [0, \langle \tilde{\alpha}_{n-1} \rangle + \max [0, \langle \tilde{\alpha}_{n-2} \rangle + \cdots + \max [0, \langle \tilde{\alpha}_0 \rangle] \cdots]$$

It can be proved that the value of $a(n)$ cycles.
Actually

$$\begin{aligned} a(0) &= 0 \\ a(1) &= 8 \\ a(2) &= 11 \\ a(3) &= 9 \\ a(4) &= 0 \\ a(5) &= 8 \\ &\vdots \end{aligned}$$

for the above case.

4. Comment on the Multi-Refinery Case

We have discussed the case of one refinery above. Now we shall make a brief comment on the multi-refinery case.

The notation used in the following discussion are the same as those of one-refinery case, with the only exception that an index k is introduced to denote the refinery sites.

For the multi-refinery case, we have equation (4.1) which corresponds to equation (2.1) for the one-refinery case:

$$(4.1)\qquad \sum_k F^{(k)}(n-1,\ \tilde{X}_n^{(k)})$$
$$=\min_D \sum_k [\langle \tilde{X}_n^{(k)}\rangle,\ \langle I_D(\tilde{X}_n^{(k)})\rangle,\ F^{(k)}(n-2,\ I_D(\tilde{X}_n^{(k)})].$$

The right hand side of (4.1) is rewritten as

$$(4.2)\qquad =\min_D [\sum_k \langle \tilde{X}_n^{(k)}\rangle + \sum_k \langle a^{(k)}(n-1)\rangle].$$

with the use of $a^{(k)}(n-1)$, similarly to (3.4).

Our program 'OTASCHE', which is written in FORTRAN with about 400 statements, contains both algorithms one for the case of one port loading, two port (refinery) discharging, and the other for the case of two port loading, one port discharging.

In either of these algorithms, we divide the decision D of (4.2) as

$$(4.3)\qquad D=D_p \overset{1)}{\times} D_c.$$

Here, the D_p denotes the decision on the determination of the refineries at which crude oils are to be discharged. The D_c denotes the decision on the determination of the kind of crude oils to be loaded and, in case of more than one port loading, the ratio of crude oils to be loaded.

Considering the fact that the second term of (4.2), $\sum_k a^{(k)}(n-1)$, is independent of D_c and only dependent upon D_p, we can deduce the

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[1] X indicates the direct product of the decision spaces.

following approximate formula to (4.2).

$$(4.4)\qquad \sum_k F^{(k)}(n-1,\ \tilde{X}_n^{(k)})$$
$$=\min_D [\sum_k \langle\tilde{X}_n^{(k)}\rangle+\sum_k \langle a^{(k)}(n-1)\rangle]$$
$$=\min_{D_p\times D_c} [\sum_k \langle\tilde{X}_n^{(k)}\rangle+\sum_k \langle a^{(k)}(n-1)\rangle]$$
$$=\min_{D_p} \sum_k \langle a^{(k)}(n-1)\rangle+\min_{D_c} \sum_k \langle\tilde{X}_n^{(k)}\rangle\,.$$

The process of optimization is divided into two phases as mentioned above; that is,

Phase I: determination of D_p that makes $\langle\sum_k a^{(k)}(n-1)\rangle$ minimal

Phase II: determination of D_c that makes $\langle\sum_k \tilde{X}^{(k)}(n-1)\rangle$ minimal

In phase I, we determine the refinery (or the discharging port) by the similar calculation as in the one-refinery case.

In phase II, we determine the optimal ratio of different kinds of crude oils to be loaded.

This ratio can be obtained easily from the optimal solution of the following LP model.

$$\min \sum_i X_i^{(k)}\,,$$

subject to

$$\begin{cases} X_i^{(k)}\geqq d_i^{(k)}(n,\ n-1) & i=1,\ \cdots,\ m \\ X_i^{(k)}-d_i^{(k)}(n,\ n-1)+Z_i\geqq Y_i^{(k)}(n-1) & i=1,\ \cdots,\ m \\ \sum_{i=1}^{m} Z_i=T, & \end{cases}$$

where Z_i denotes the quantity of crude oils to be loaded at the loading port i, which corresponds to crude oil i, and $Y_i^{(k)}(n)$ is defined by the following three equations.

$$(4.5)\qquad Y_i^{(k)}(n)=X_i^{(k)}\qquad i=1,\ \cdots,\ m$$

for the refinery k, where a tanker comes in at time n,

$$(4.6)\qquad Y_i^{(k)}(n)=Y_i^{(k)}(n-1)+d_i^{(k)}(n,\ n-1),\quad i=1,\ \cdots,\ m$$

for the refinery k, where a tanker does not come in at time n, and

Table 1. Result of computation for the case of one port loading, one port discharging.

N $N(F)$	21 350	22 350	23 350	24 350	25 350	26 350	27 350	28 350	29 350	30 350	31 350
ZAIKO YURYO											
A	0.5	0.3	0.3	0.0	0.5	0.0	0.0	0.0	0.1	0.2	0.3
	40	10	10	10	30	0	0	0	20	20	20
	40	10	10	10	30	0	0	0	20	20	20
B	0.2	0.3	0.5	0.1	0.1	0.4	0.4	0.2	0.2	0.2	0.4
	80	140	180	60	90	150	190	70	100	130	170
	80	140	180	60	90	150	190	70	100	130	170
C	0.2	0.3	0.1	0.2	0.2	0.3	0.1	0.2	0.2	0.3	0.1
	140	200	80	170	140	200	80	170	140	200	80
	140	200	80	170	140	200	80	170	140	200	80
KETTEI YURYO	*A				*A						
	B		*B		B		*B		*B		*B
	C	*C		*C	C	*C		*C	C	*C	
TANKER	30	200	150	100	30	200	150	100	30	200	150
SEISEI YURYO											
A		0	0	0	20	0	0	0	20	0	0
B		60	40	30	30	60	40	30	30	60	40
C		60	80	90	70	60	80	90	70	60	80

$$Y_i^{(k)}(0)=0 \qquad i=1,\cdots,m \tag{4.7}$$

for all refineries.

5. Numerical Example (II)

Table 1 shows the result of calculation by our computer program OTASCHE, for the miniature model discussed above.

In Table 1;

$F(N)$: gives the minimum tank capacity at time N.

ZAIKO YURYO: gives the amounts of crude oils A, B, C, which are stored.

KETTEI YUSHU: gives the kinds of crude oils which are decided to be loaded.

TANKER: denotes the tanker to be arrived.

SEISEI YURYO: gives the amount of crude oils to be processed at refineries.

The computing time for a problem of 200 periods was 7 seconds by the S/360-75J.

Program constraints.

Our current program has the following constraints:

(1)	number of refineries	2
(2)	number of crude oils to be processed at a refinery	5
(3)	number of periods in one process pattern cycle	30
(4)	number of tankers	13
(5)	simulation length (number of periods)	400.

6. Acknowledgements

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