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A DETERMINISTIC INPUT-OUTPUT MODEL TO FACILITATE MANAGEMENT OF A HOSPITAL SYSTEM*

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Abstract

In a period of inflation, the cost of hospital services is rising more rapidly than costs in other areas of the economy. It has been estimated that by 1975, hospital costs will consume almost 7 1/2–8% of the American gross national product. In this situation it is imperative that hospital administrations are able to function with maximum effectiveness. In order to be able to do this, administrations need all the help and assistance which they can get. And in this regard, the techniques of operations research are of primary importance. Prior to this time input-output analysis has been restricted to dealing with national economic problems. This paper deals with a new application of the technique of input-output analysis to the operation of a hospital-health services system.

In this paper a general mathematical model for a hospital system is

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developed and its use is illustrated with the help of a numerical example.

This paper is organized into five sections: Section 2 gives the assumptions and the nomenclature used in the development of the model; Section 3 gives the derivation of the general mathematical model; Section 4 presents numerical examples to illustrate the use of the model; Section 5 gives the conclusions and the summary of this paper.

1. Introduction

In a period of inflation, the cost of hospital services is rising more rapidly than costs in other areas of the economy. It has been estimated that by 1975, hospital costs will consume almost 7 1/2–8% of the American gross national product [5]. In this situation it is imperative that hospital administrations are able to function with maximum effectiveness. In order to be able to do this, administrations need all the help and assistance which they can get. And in this regard, the techniques of operations research are of primary importance. Prior to this time input-output analysis has been restricted to dealing with national economic problems. This paper deals with a new application of the technique of input-output analysis to the operation of a hospital-health services system.

The technique of input-output analysis has some special features which make it of the utmost utility in its application to a hospital-health services system.

In this paper a general mathematical model for a hospital system is developed and its use is illustrated with the help of a numerical example.

This paper is organized into five sections: Section 2 gives the assumptions and the nomenclature used in the development of the model; Section 3 gives the derivation of the general mathematical model; Section 4 presents numerical examples to illustrate the use of the model; Section 5 gives the conclusions and the summary of this paper.

2. Assumptions and Nomenclature

The basic assumptions of the model are the following:

1. The elements of the flow matrix are known and constant. This means that they do not vary over the period of analysis.
2. The unit of measurement of the various inputs and the sectional outputs can be different.
3. Interdependence of various activities is allowed.
4. The input proportions within an activity need not be fixed for (a) intersectional flows (b) basic inputs.
5. New innovations and ideas are allowed.
6. Production functions can be nonlinear.

The symbols used in this paper including those referred to in the tables are defined as follows:

N=the number of sections into which the services offered by the hospital-health services system are classified.

X=the $(N\times 1)$ vector which represents the outputs of the different sections of the system.

Y=the $(N\times 1)$ vector which represents the outputs delivered outside the system in order to satisfy the demand upon the system by the community which it serves.

X_j=total output of section j.

Y_i=output delivered outside the system by the ith section.

x_{ij}=amount of output of section i absorbed by section j.

B=the $(h\times 1)$ vector which represents the total basic inputs to the system.

b_{lm}=amount of basic input of type l used by section m.

B_l=total amount of basic input of type l.

3. Formulation and Application of the General Model

The formulation of the general deterministic model is organized in the following manner:

1. Choosing measures of effectiveness.

2. Construct the equations for the flow of activities in the different sections and present them in the form of an input-output table.

3. Construct the equations involving the interdependence coefficients.

4. Express the total output vector in terms of the final demand vector.

5. Express the total basic input vector in terms of the total output vector.

6. Illustrate the use of the model.

These steps are discussed in turn below:

1. Choosing measures of effectiveness for the model to be constructed consists essentially of choosing the units of measurement for the inputs and the outputs of the different sections of the hospital system.

2. An input-output flow table (shown in Table 1) summarizes the observed relationships between the inputs and outputs associated with the set of activities comprising the hypothetical hospital-health services system. These activities are divided into two classes. One class consists of intermediate activities which are produced and used within the hospital but are not directly demanded by the community; rather they are used as inputs by other intermediate activities and by final activities. The other class consists of final activities or services which are those directly demanded and used by the community.

The sections of the system are listed in two ways in the table; in a row of sections (numbered 1 to N) which produce services and in a column of sections (numbered 1 to N) which absorb services.

By referring to the input-output flow table and making use of the property

$$\sum_{j=1}^{N} x_{ij}+Y_i=X_i \qquad \text{for} \quad i=1,\cdots,N$$

we can develop the following equation

$$AX=Y \tag{3.1}$$

Table 1. Nomenclature for the Deterministic Model.

Basic Input	Type 1	b_{11}	b_{12}	—	—				b_{1N}	0	B_1
	Type 2	b_{21}	b_{22}	—	—				b_{2N}	0	B_2
	—	—	—						—	—	—
	Type h	b_{h1}	b_{h2}	—	—				b_{hN}	0	B_h

	Absorbing Sectors / Producing Sectors	Section 1	Section 2	|	|				Section N	Final Demand	Total Output
Intermediate	Section 1	x_{11}	x_{12}	—	—				x_{1N}	Y_1	X_1
	Section 2	x_{21}	x_{22}	—	—				x_{2N}	Y_2	X_2
	—	—	—						—	—	—
	—	—	—						—	—	—
FA											
	Section N	x_{N1}	x_{N2}	—	—				x_{NN}	Y_N	X_N

where,

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & & & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix}$$

where

$$a_{ij}=-\frac{x_{ij}}{X_j} \qquad \text{for all } i \neq j$$

$$=1-\frac{x_{ij}}{X_j} \qquad \text{for all } i=j$$

Similarly it can be shown that the equation for basic inputs is

$$CX = B \tag{3.2}$$

where the matrix

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1N} \\ \vdots & & & \vdots \\ c_{h1} & c_{h2} & \cdots & c_{hN} \end{bmatrix}$$

where

$$c_{lm} = \frac{b_{lm}}{X_m} \qquad l = 1, \cdots, h;\ m = 1, \cdots, N$$

and the matrix

$$B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_h \end{bmatrix}$$

The use of the equations (3.1) and (3.2) is illustrated by applying them to the data collected from Shoitz Memorial Hospital, Waterloo, Iowa.

4. Numerical Example

The appropriate starting point in the application of input-output analysis is to quantify the elements of flow Table 2. This table, which represents Shoitz Memorial Hospital, contains the sum total of all activities which comprise the system. The main components of Table 2 consist of:

a. *Basic inputs*: The whole system, consisting of 17 sections described above, uses the basic inputs which can include the following:
   Supplies
   Labor
   Plant & Equipment

Table 2. Given Flow Matrix.

	Basic Inputs	1	2	3	4	5
1	Supplies	47,048	38,137	105,956	14,059	17,155
2	Labor	221,887	167,796	18,274	18,356	0
3	Plant & Equipment	14,123	19,037	844	767	0
	Intermediate Activities					
1	Laboratory	0	0	0	0	0
2	X-ray	0	0	0	0	0
3	Pharmacy	0	0	0	10,512	0
4	Inhalation Therapy	0	0	0	0	0
5	Anesthesiology	0	0	0	0	0
6	Physical Therapy	0	0	0	0	0
7	Administration	17,191	10,525	2,807	3,859	0
8	Plant	10,701	5,707	1,239	1,051	0
9	Laundry & Linen	102	1,026	0	112	0
10	Housekeeping	3,594	2,763	271	271	0
11	Dietary	3,528	2,193	572	763	0
	Final Demands					
12	Medical Records	0	0	0	0	0
13	Surgery	0	0	0	0	0
14	Medical	0	0	0	0	0
15	Pediatrics	0	0	0	0	0
16	Obstetrics	0	0	0	0	0
17	Extended Therapy	0	0	0	0	0

These basic inputs are used by the intermediate and final activities in the manner shown in Table 2.

b. *Intermediate activities*: Sections 1 through 11 shown in Table 2 are activities produced and used within the hospital, which are not directly demanded by the community. These activities are:

Laboratory	Physical Therapy
X-ray	Administration
Pharmacy	Plant
Inhalation Therapy	Laundry & Linen
Anesthesiology	Housekeeping
	Dietary

Table 2. *Continued.*

	6	7	8	9	10	11	12
1	2,696	52,621	87,379	21,765	21,309	175,159	3,354
2	68,961	238,411	62,807	59,518	133,278	204,008	36,521
3	4,645	23,841	11,456	8,390	2,167	26,061	2,004
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
7	11,578	0	8,771	14,384	36,487	55,784	8,771
8	6,383	10,889	0	5,688	1,727	20,557	1,840
9	5,954	0	0	0	0	2,865	0
10	2,686	0	0	0	0	11,691	444
11	2,384	8,773	1,812	2,956	7,533	11,540	1,812
12	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0

c. *Final activities*: Sections 12 through 17 in Table 2 are activities demanded directly by the community and they are therefore defined as final activities.

These are:

Medical Records

Surgery

Medical

Pediatrics

Obstetrics

Extended Therapy

d. *Final demands*: The last but one column of the table shows

Table 2. *Continued.*

	13	14	15	16	17	Final Demand*	Total Output
1	49,968	17,486	3,909	2,755	9,167		
2	457,383	308,423	41,263	135,304	89,768		
3	19,262	17,852	3,697	7,831	8,016		
1	147,789	159,448	0	35,662	0		391,795
2	122,999	132,699	0	29,679	0		353,624
3	108,270	116,718	0	22,217	65,713		226,013
4	0	75,899	0	0	0		75,899
5	43,396	5,221	0	0	0		48,607
6	151,310	0	0	0	0		151,310
7	62,155	57,638	11,744	13,913	35,233		304,654
8	38,055	26,696	8,651	16,203	31,896		159,520
9	34,892	23,657	11,418	14,358	6,858		86,202
10	58,883	45,711	10,810	27,744	28,366		147,785
11	131,098	116,841	24,692	70,465	85,536		473,958
12	15,900	17,146	3,240	3,838	9,720		49,844
13	0	0	0	0	0	27,490	27,490
14	0	0	0	0	0	25,493	25,493
15	0	0	0	0	0	6,148	6,148
16	0	0	0	0	0	5,264	5,264
17	0	0	0	0	0	15,461	15,461

* All figures in the table are dollar values except the final demand. These are patient day figures. They are easily converted to dollar values by multiplying by the daily hospital room rate.

the final demand by the community upon the hospital system. These final demands constitute the total outflow of work from the hospital to the community. The numbers in the final demand column are the patient days. They can be easily converted to dollar values by multiplying by the daily hospital room rate.

e. *Total outputs*: The last column of the table lists the total flows or outputs of each section of the hospital. These are called the total outputs.

Explanation of Table 2: The figures in this table represent the interactions between the various sections of the hospital. The table should be interpreted in the following way: Consider for example, the Administration which has a total output of 304,654, dollars per year. The Administration does not supply a service which is directly used by the community and therefore the final demand for this section is zero cost units. However, many sections of the hospital require the services of the Administration, for example; as shown in Table 2, Dietary takes 8,773 dollars of these services per year. These services plus all those being used by the other sections add up to 304,654 dollars per year which figure constitutes the total output of the Administration. Examination of Table 2 shows the total outputs, final demands and interdepartmental flows of all the other sections of the hospital system. Similar reasoning may be applied to the basic inputs. For example, 167,796 dollars per year is spent on the labor force by X-ray. It should be noted that there is no final demand for labor.

Table 3. New Estimated Final Demands.

No.	Section	New Final Demand
1	Laboratory	0.0
2	X-ray	0.0
3	Pharmacy	0.0
4	Inhalation Therapy	0.0
5	Anesthesiology	0.0
6	Physical Therapy	0.0
7	Administration	0.0
8	Plant	0.0
9	Laundry & Linen	0.0
10	Housekeeping	0.0
11	Dietary	0.0
12	Medical Records	0.0
13	Surgery	25045.5
14	Medical	23551.4
15	Pediatrics	6408.7
16	Obstetrics	4832.3
17	Extended Therapy	16693.0

Table 4. New Flow Matrix (Basic Inputs).

i \ j		1	2	3	4	5
1	Supplies	47386.6	33170.1	101812.9	11743.2	14610.9
2	Labor	184274.4	156864.8	18304.6	18921.2	0.0
3	Plant & Equipment	11447.1	20457.4	801.8	755.2	0.0

	6	7	8	9	10	11	12
1	2353.5	53383.0	82132.8	20799.2	20899.5	186767.8	3112.6
2	56881.6	222724.9	58421.5	55357.6	112056.3	193136.8	31996.9
3	3856.9	20214.1	11384.7	8303.7	1898.2	23002.7	1956.2

	13	14	15	16	17	Final Demand	Total Output
1	42952.5	18791.5	3669.9	3070.7	9530.8	0.0	656187.1
2	364277.7	333779.6	47830.0	130564.6	88611.3	0.0	2074000.0
3	16029.4	18268.2	4370.2	7757.0	8067.6	0.0	158570.1

Statement of the problem and of the method to solve it: Because the problem is to respond to the increased (or changed) final demand of the community for the services of the hospital system, it is necessary to estimate quantitatively these new final demands and in turn, use the input-output model developed earlier, to compute: (1) The new interdepartmental flows (2) The new basic inputs and (3) The new total outputs. A set of new estimated final demands for all the sections is shown in Table 3. Having estimated the x_{ij}'s and Y_i's (Table 2) and new final demands, we can determine the new levels of basic inputs, total outputs and intersectional flows. The new flow matrix is shown in Tables 4 and 5.

A computer program has been developed which incorporates the following basic steps:

a. Quantify the elements of the matrices, A, Y and X.
b. Estimate the changed final demand (Y).
c. Use the model $Y=AX$ as follows to find the new basic inputs, intersectional flows and total outputs.

Table 5. New Flow Matrix (Intermediate Activities and Final Demands).

		1	2	3	4	5
1	Laboratory	0.0	0.0	0.0	0.0	0.0
2	X-ray	0.0	0.0	0.0	0.0	0.0
3	Pharmacy	0.0	0.0	0.0	9726.8	0.0
4	Inhalation Therapy	0.0	0.0	0.0	0.0	0.0
5	Anesthesiology	0.0	0.0	0.0	0.0	0.0
6	Physical Therapy	0.0	0.0	0.0	0.0	0.0
7	Administration	15553.5	8771.3	2958.7	3683.5	0.0
8	Plant	8453.0	5285.1	1107.4	923.9	0.0
9	Laundry & Linen	83.7	1013.2	0.0	115.1	0.0
10	Housekeeping	3468.7	2610.1	265.9	220.9	0.0
11	Dietary	3521.3	2087.1	536.0	731.7	0.0
12	Medical Records	0.0	0.0	0.0	0.0	0.0
13	Surgery	0.0	0.0	0.0	0.0	0.0
14	Medical	0.0	0.0	0.0	0.0	0.0
15	Pediatrics	0.0	0.0	0.0	0.0	0.0
16	Obstetrics	0.0	0.0	0.0	0.0	0.0
17	Extended Therapy	0.0	0.0	0.0	0.0	0.0

	6	7	8	9	10	11	12
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	9743.0	0.0	8809.4	13544.6	37119.0	53983.9	8464.6
8	5123.8	9457.2	0.0	5331.2	1421.7	20028.7	1631.7
9	5218.7	0.0	0.0	0.0	0.0	2707.2	0.0
10	2146.8	0.0	0.0	0.0	0.0	10287.4	384.8
11	2056.7	8217.6	1633.5	2525.4	6490.2	10930.6	1924.9
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 5. *Continued.*

	13	14	15	16	17	Final Demand	Total Output
1	128968.7	148430.9	0.0	36057.5	0.0	0.0	313457.4
2	99142.1	117512.3	0.0	32230.8	0.0	0.0	248885.7
3	85558.3	109247.4	0.0	21431.9	72549.9	0.0	298514.4
4	0.0	71933.4	0.0	0.0	0.0	0.0	71933.4
5	31892.7	4823.5	0.0	0.0	0.0	0.0	36716.1
6	130588.9	0.0	0.0	0.0	0.0	0.0	130588.9
7	47208.6	58955.7	13937.9	14561.1	37122.7	0.0	334417.8
8	32736.4	26419.9	8326.7	15452.3	31380.9	0.0	173080.0
9	32783.8	20157.0	11915.5	11958.8	6901.5	0.0	92854.4
10	47095.7	46297.5	10316.6	32796.8	26715.8	0.0	182607.4
11	98950.5	128307.3	29885.3	75549.9	79465.4	0.0	452813.6
12	13910.7	16001.3	3196.0	3817.5	10333.4	0.0	47258.9
13	0.0	0.0	0.0	0.0	0.0	25045.5	25045.5
14	0.0	0.0	0.0	0.0	0.0	23551.4	23551.3
15	0.0	0.0	0.0	0.0	0.0	6408.7	6408.7
16	0.0	0.0	0.0	0.0	0.0	4832.3	4832.3
17	0.0	0.0	0.0	0.0	0.0	16693.0	16693.0

(i) $X^{\text{new}} = A^{-1} Y^{\text{new}}$

(ii) $x_{ij}^{\text{new}} = a_{ij}^{\text{old}} X_j^{\text{new}}$

(iii) $B^{\text{new}} = C^{\text{old}} X^{\text{new}}$

(iv) $b_{lm}^{\text{new}} = b_{lm}^{\text{old}} X_m^{\text{new}}$

5. Conclusions and Summary

1. Because the demands upon the hospital facilities have changed, the values of the elements of the matrix Y are changed. In order to meet this new level of demand both the basic inputs and total outputs of each of the effected sections will require readjustment. Furthermore the changed total output of each section will alter the intersectional flows between each section. The model gives quantitative values for the new levels of basic inputs, total outputs and intersectional flows. These new values are shown in Tables 4 and 5. The figures shown in Tables 4 and 5 help management to formulate policies, which involve problems of

resource allocation, in a quantitative manner.

2. Because input-output analysis replaces qualitative planning and decision making with quantitative planning and decision making, cost effectiveness is increased throughout the whole system. This factor also applies when management is considering new innovations in the system.

3. The input-output technique can be usefully applied to subsections, sections, departments, whole institutions or even to groupings of institutions with corresponding benefits.

4. Because most of the raw data required for input-output analysis is usually already available within current accounting procedures, the extra cost of using the model program should be well within the budget of the system being considered.

5. In this paper, one specific hospital problem has been dealt with, namely the effect of the changing of the final demand on the hospital upon the basic inputs to and the intersectional flows in the hospital. The mathematical model which has been discussed here can also be used to deal with a variety of other problems. For example (a) Sensitivity analysis of each cell of matrix A with respect to a specific output (b) Design and reallocation of resources to minimize total cost per unit output.

6. The deterministic input-output technique is not a panacea. For example, the model present in this paper has two disadvantages. Firstly, it is a static model. This means that it ignores variations of a_{ij}'s with time. Secondly, it is a deterministic model. This means that the x_{ij}'s are constant and do not fluctuate over the period of analysis. Most if not all real systems are dynamic and probabilistic.

7. More work is in progress in the following areas: (a) Probabilistic input-output model (b) Dynamic input-output model (c) Sensitivity analysis of input-output model.

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