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ECONOMIC ASPECTS OF PRODUCTION SCHEDULING SYSTEMS*

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ABSTRACT

This paper considers the problem of scheduling a given number of jobs to be processed on a given number of machines. Due to its combinatorial nature, practical solution procedures to most scheduling problems have not yet been found, even though computational abilities of the electronic data-processing equipments are constantly increasing. With a view to understand the nature of the problem, this paper explores the formulation of the problem and provides economic interpretations of various optimality criteria which are being used for solving the scheduling problem. A general optimization criterion, called minimization of opportunity cost, is proposed for the scheduling problem and it is shown that various optimality criteria currently prevalent in literature represent the components of total opportunity cost.

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Results of a sensitivity analysis of various criteria, performed to determine the effectiveness of various prevalent optimality criteria, are reported according to which none of the criteria minimizes total opportunity cost. These results are then used to establish the importance of each criteria (which reflect the components of total opportunity cost) with respect to the general criterion.

1. INTRODUCTION

Scheduling n jobs on M machines is one of the classical combinational search problems in the fields of Operations Research and Combinational Mathematics. Interest in its mathematical formulation and solution has resulted in a considerable number of reported research efforts. In spite of all these efforts, practical scheduling problems cannot be optimized effectively and efficiently. The reported progress in the field of scheduling theory is not adequate to be of any appreciable use to industry [5, 9].

The lack of success of scheduling theory in solving practical problems may be attributed to the gap between the theoretical developments in scheduling theory and the nature of the practical problems. Theoretical developments in scheduling are often inspired by formal considerations and mathematical elegance, even at the cost of being divorced from the complex realistic situations. Consequently, the elegent formal machinery developed in scheduling is of no appreciable use to practicing schedulers who are thus forced to use old techniques of trial and error [4].

It is, therefore, obvious that, for the success of scheduling theory in solving real-world problems, theoretical developments should be motivated not so much by formal mathematical elegence as by abstractions of real-life features of problems on the basis of practical experience. The purpose of this paper is to discuss some such practical abstractions which may be helpful in the understanding of the scheduling problem.

This paper is a critical examination of one of the basic concepts in scheduling theory, viz: the criterion of optimality. The primary purpose of this research is to provide insight into the economic aspects of scheduling theory. Several important aspects (i.e. costs and usual assumptions) of the scheduling problem, which have been somewhat neglected in the reported research, are investigated and a sensitivity analysis is performed to examine the effectiveness of various criteria of optimality in seeking the optimal schedule.

2. THE SCHEDULING PROBLEM

2.1 Discussion of Existing Work:

The scheduling problem may be defined as [5]:

"Given n jobs to be processed on M machines, the process time of job i on machine m being t_{im}, $(i=1, 2, \cdots, n;\ m=1, 2, \cdots, M)$ and the technological order in which job i is processed on machine m being f_{im}, find the order in which these n jobs should be processed on these M machines to minimize total cost of production."

The above definition of scheduling problem encompasses several types of practical situations. For example, if $f_{im}=m$ for all i and m, then the problem is termed as flowshop scheduling; otherwise it is called job-shop scheduling.

2.1.1 On the Assumptions:

The scheduling problem, as defined above, was first formulated by Johnson [7] as n job, 2 machine problem with $f_{i1}=1$ and $f_{i2}=2$ for all i. Subsequent developments in scheduling theory have been extensions of Johnson's formulation, in that the number of machines is increased to the general case M and f_{im} are allowed to be different.

However, the formulation of the scheduling problem, as an extension of Johnson's is not the general scheduling problem encountered in practice. Thus, the current formulation of scheduling problems has the serious drawback that only a few problems that are found in practice can be similarly formulated [9]. The lack of realism in formulation is the result of the assumptions made in order to define the scheduling problem as the extension of Johnson's two-machine case. Thus, the following assumptions, which were inherent in Johnson's formulation, are present in the current definition of scheduling problem [1—6].

a. all jobs are available simultaneously,
b. all machines are simultaneously available to take up the jobs for processing,
c. all jobs are of equal importance,
d. process times are independent of the sequence.

A. Simultaneous Availability of Jobs:

There are very few practical situations where it may be assumed that the jobs are available simultaneously. However, even if it were so, simultaneous availability of jobs does not imply that the shop starts processing all the jobs simultaneously. The management of the firm, for various reasons, may not procure all the raw-materials at the same time. For example, if the planning section of the firm can ascertain the optimal schedule for a given set of jobs, and determine the times at which various jobs are to be started; the management may plan the inventory in such a fashion that the raw-materials are procured only when they are required in the shop, provided the optimal ordering and inventory policies demand it to be so. Thus, the availability of jobs depends not only on the characteristics of scheduling but on other allied functions in industry.

B. Simultaneous Availability of Machines:

The processing of jobs in a shop is a continual process. Thus, once the shop is in operation, at any one given period of time, all the machines cannot be expected to be free. There are back-logs of work at any given time. Thus, the assumption that all the machines are simultaneously available to take up the jobs is not a realistic assumption.

C. Equal Importance of Jobs:

Jobs are usually of unequal importance in practice. A manufacturer usually delives goods (jobs) to different customers and the terms and conditions of supply are different. For example, in some contracts there may be a penalty clause for delayed supply, whereas there may not be any such clause in other contracts.

D. Independence of Process Times:

Generally, process times are made up of two parts, viz: set up times and operation times. Since setup times depend on the preceding and succeeding jobs in sequence, the sum of the process time and setup time, which is represented as process time, cannot be treated independent of the sequence.

2.1.2 Cost of Production

The above discussion of the assumptions indicate that the formulation of the scheduling problem as an extension of Johnson's two-machine case is inadequate to represent the scheduling situation. However, in the current formulation of scheduling problem, the greatest difficulty is encountered in the quantification and definition of cost of production. Production cost is usually composed of more than one type of cost which may be conflicting in nature (i.e. decrease in one particular cost function may cause an increase in some other cost functions), but the research models seem to employ one component of production cost as representative of the total cost of production. Once again, as an attempt to generalize Johnson's formulation, the most commonly used measure of production cost is the make-span (defined as the time period in which all the n jobs are processed on all the M machines). Use of such a simplified measure, as will be seen subsequently, results in sub-optimization and thus creates a reluctance on the part of business executives to accept the scheduling techniques for practical use.

The analysis of the scheduling problem, as revealed in the literature, suggests the following components of production cost:

i. Operation Cost

ii. Job waiting (or inprocess inventory) cost, if jobs wait during processing
iii. Machine idle cost, if machines are idle, and
iv. Penalty cost of jobs, if jobs are delivered later than their promised due dates.

Each of the above four cost components may be visualized to be made of two parts: the controllable and uncontrollable costs. The controllable costs depend on the schedule, and by a suitable selection of the schedule, these costs may be lowered and thus minimized. The uncontrollable costs are independent of the schedule(s) and hence the selection of a schedule has no effect in increasing or decreasing these costs. Thus, the scheduling techniques can be used to minimize only the controllable costs. Since these controllable costs reflect the opportunity loss as compared to an ideal situation, these costs are termed as the *opportunity costs* [5].

2.2 Formal Redefinition of Scheduling Problem:

The general optimization criterion employed in defining the problem should be the minimization of total opportunity cost. Thus, the scheduling problem may be redefined as [5]:

"Given n jobs to be processed on M machines, the process time of job i, available for processing at time A_i, on machine m, available to take up the job for processing at time B_m, being t_{im}, $(i=1, 2, \cdots, n;\ m=1, 2, \cdots, M)$; the technological order in which job i is processed on machine m being f_{im}, and if job k precedes job i, the setup time on machine m for job i being s_{kim}; find the order in which these n jobs should be processed on the M machines to minimize *total opportunity cost.*"

3. DESCRIPTION OF COST COMPONENTS

This section discusses the four components of the production cost and gives explicit expressions for the opportunity costs due to each of the four components stated in the previous section.

3.1 Operation Cost:

The component of cost representing the cost incurred in actual production which may be treated as the processing cost of the jobs at all the machines is defined as operation cost.

Let H_{im} represent *the machine hour rate* of machine m for processing job i. Then the operation cost for job i, α_i, in schedule S, may be represented as:

$$(1) \qquad \alpha_i = \sum_{m=1}^{M} [t_{im}H_{im} + h_{im} \cdot s_{kim}]$$

where h_{im} is the setup cost per unit time and s_{kim} is the setup time of job i when job k precedes it at machine m in schedule S. Then total operation cost of a schedule S, $\alpha'(S)$, is:

$$(2) \qquad \alpha'(S) = \sum_{i=1}^{n} \sum_{m=1}^{M} t_{im} \cdot H_{im} + \sum_{i=1}^{n} \sum_{m=1}^{M} h_{im} \cdot s_{kim} \ .$$

The quantity $\sum_{i=1}^{n} \sum_{m=1}^{M} t_{im}H_{im}$ remains constant and does not depend on the sequence.

However, since s_{kim} depends on the sequence, the quantity $\sum_{i=1}^{n} \sum_{m=1}^{M} h_{im} \cdot s_{kim}$ represents the opportunity cost component for operation cost due to the sequence of the jobs. This component needs to be included in the total opportunity cost relation. To be explicit, this component, $\alpha(S)$, is given by:

$$(3) \qquad \alpha(S) = \sum_{i=1}^{n} \sum_{m=1}^{M} h_{im} \cdot s_{kim} \ .$$

3.2 Job Waiting Cost:

This cost, which is also known as the in-process inventory cost, reflects the opportunity cost due to the waiting of the semi-finished jobs in the shop for processing by some machines. The job which waits in the shop is a form of capital tied up in the shop. This capital could have been utilized to produce additional return on capital.

Let the raw-material cost for job i be R_i and the value added by machine m be v_{im}. Then the value of job i at machine m', $R_{im'}$, is:

$$(4) \qquad R_{im'} = R_i + \sum_{m=1}^{m'-1} v_{im}, \quad m' = 1, 2, 3, \cdots, M .$$

If the expected return for this invested capital (tied up in-process inventory) is r per unit time then the opportunity cost loss for job i, β_i, is:

$$(5) \qquad \beta_i = \sum_{m'=1}^{M} \{r(R_i + \sum_{m=1}^{m'-1} v_{im})\, y_{im'}\}$$

where $y_{im'}$ is the time for which job i waits before machine m'.

Thus, the component of opportunity cost of schedule S, due to job waiting: $\beta(S)$, is given by:

$$(6) \qquad \beta(S) = \sum_{i=1}^{n} \sum_{m'=1}^{M} \{r(R_i + \sum_{m=1}^{m'-1} v_{im})\, y_{im'}\}$$

where $y_{i1} = 0 = v_{i0}$. Let,

$$(7) \qquad r(R_i + \sum_{m=1}^{m'-1} v_{im}) = w_{im'} .$$

where $w_{im'}$ may be defined as the waiting cost of job i at machine m per unit time of waiting.

Then:

$$(8) \qquad \beta(S) = \sum_{i=1}^{n} \{\sum_{m=1}^{M} y_{im} \cdot w_{im}\}$$

Since y_{im} depends on the schedule followed, this cost depends on the alternative schedule selected and can be controlled by selecting a suitable schedule. In other words, $\beta(S)$ represents the component of opportunity cost due to waiting of jobs in the shop.

3.3 Machine Idle Cost:

When machines are idle, some opportunity is lost because, by utilizing this idle capacity of the machines, some return on machine capital could be obtained. Determination of machine idle cost may be divided into two categories:

1. The idle time of the machines can be utilized to perform some other work which may not be as profitable as the existing work.
2. The idle time of the machines cannot be utilized to perform any other useful work.

For Case (1), the machine idle cost is the difference between the expected rate of return from the machine and the actual rate of return from the machine which is obtained by utilizing the idle capacity of the machine on a subordinate job. For Case (2), the idle cost of the machine is the expected rate of return.

Let the idle time at machine m before job i starts processing be X_{im} and let the net loss of the rate of return be r_m per unit time. Then the machine idle cost for machine m, γ_m, is:

$$\gamma_m = \sum_{i=1}^{n} X_{im} r_m \tag{9}$$

and the component of the opportunity cost in schedule S, due machine idleness, $\gamma(S)$, is:

$$\gamma(S) = \sum_{i=1}^{n} \sum_{m=1}^{M} X_{im} r_m \,. \tag{10}$$

Since the machine idle time depends on the schedule, $\gamma(S)$ depends on the schedule selected and, therefore, is included in the total cost equation.

3.4 Penalty Cost of Jobs:

If the jobs are not completed by their due dates, certain costs are incurred. Following Gere [3], these costs include: (1) direct dealing with the customer's paperwork, telephone calls, executive time taken up; (2) penalty clause in the contract, if any; (3) loss of good will resulting in an increased probability of losing the customer for some or all of his future business; or perhaps a damaged reputation which will turn customers away; and (4) expediting because the jobs may have to be moved quickly through the shop at an extra setup cost and inefficient use of workmen and machinery.

Among the most difficult factors to be ascertained in the penalty cost are the goodwill losses. For this purpose, the manager's judgment may be used to compute the probabilistic trend of the losses. This probabilistic trend may be treated as a risk and a cost assigned to the risk involved.

Let the per unit time penalty cost for job i be p_i and the time by which job i is late be d_i. Then the penalty cost for job i in schedule S, δ_i, is given by:

$$\delta_i = p_i d_i \tag{11}$$

and the total penalty cost of schedule S, $\delta(S)$, is:

$$\delta(S) = \sum_{i=1}^{n} p_i d_i \ . \tag{12}$$

The value of d_i depends on the completion time of job i which in turn depends on the schedule. Thus, the selection of a schedule determines the amount of loss due to penalty or job lateness. Hence equation (12) represents the component of the opportunity cost due to job lateness.

3.5 Total Opportunity Cost:

The total opportunity cost of a schedule S is the sum of the components of the opportunity cost described above. Thus, if $TC(S)$ represents the total opportunity cost of schedule S; then:

(13) $$TC(S)=\alpha(S)+\beta(S)+\gamma(S)+\delta(S)$$

on substitution and simplification, $TC(S)$ is given by:

(14) $$TC(S)=\sum_{i=1}^{n}\{\sum_{m=1}^{M}[h_{im}s_{kim}+y_{im}w_{im}+X_{im}r_m]+p_id_i\} \ .$$

3.6 Optimal Schedule:

The above discussion of the various cost components and opportunity cost directly leads to the definition of an optimal schedule. Thus, an optimal schedule is that feasible schedule which minimizes the total opportunity cost given by equation (14) above.

4. CRITERIA OF OPTIMALITY

This section describes the four most prevalent criteria for scheduling problems. These criteria are the minimization of the cost components (cf. section 2 above) viz: (1) make-span (maximum flow-time); (2) job waiting cost; (3) job penalty cost; and (4) machine idle cost.

4.1 Make-span Criterion:

This criterion minimizes the total completion time of the jobs. The logic behind minimization of total completion time is that it maximizes the rate of production. But does a maximum rate of production mean producing maximum quantity at the minimum cost?

The obvious answer to this question is that maximum rate of production does not imply production at the minimum cost. All maximum rate of production minimizes is the overhaed costs. The direct cost of production may in fact increase and hence offset the benefits of the maximum rate of production.

Turning to the cost aspects of the jobs being produced, one notes that minimization of maximum flow-time considers the effect of the machine idle times at the last machine only. From equation (14) it is seen that the make-span criterion assumes that all cost co-efficients except r_M are zero. This implies that if there are some intermediate machines which are more valuable than the last machine minimization of maximum flow-time may result in additional idle time of some intermediate machines thus increasing total cost. In such a case, the criterion of minimizing maximum flow-time does not reflect the idleness on intermediate machines. Also, in-process inventory cost is not reflected in this criterion. Further, this criterion assumes that all jobs are of equal importance.

4.2 Job Waiting Cost Criterion:

There are some papers in the literature which consider the waiting time and/or cost to be important for scheduling the jobs on the machines. This criterion assumes that the waiting cost is to be minimized. Minimization of waiting cost may increase some other costs, such as the machine idle cost and/or the penalty cost of the jobs. Thus, the total cost of production may in fact be increasing instead of decreasing while minimizing the waiting cost of jobs.

4.3 Machine Idle Cost Criterion:

It was remarked earlier that machine idle cost of the intermediate machines is usually neglected in the formulation of the problem. The existing literature does not explicitly define that the idle cost of the machines is an important factor.

An industrial survey conducted by the Sequencing Research Group at Texas Tech University indicated that the percentage idleness of the machines varies from 15% to 35%. This survey shows the need of including machine idle cost in the objective function. However, machine idle cost by itself may not be suitable to find the optimal schedule.

4.4 Penalty Cost Criterion:

Penalty cost of jobs forms an important part of the scheduling problem, because it is highly unlikely that all jobs are of equal importance in the shop. Apart from the fact that there are due dates assigned to the jobs, the contract clauses of various jobs are different.

Minimization of penalty cost also results in a lower job-waiting cost. In general, there is no relation between the makespan criterion and the penalty cost criterion. The logic behind using penalty cost as the criterion seems to be the fact that customer satisfaction and goodwill are a most important integral part of any business.

5. SENSITIVITY ANALYSIS OF VARIOUS CRITERIA OF OPTIMALITY

The above analysis implies that the various criteria of optimization used in the current literature are but components of the total opportunity cost and hence do not represent any total measure of performance which can be used in seeking the optimal solution to the scheduling problem. For optimizing the system all the costs should be included in the objective function as shown by equation (14).

This section reports the results of an investigation conducted to examine the effect of various cost components on the total opportunity cost and determine if there is anyone single critical cost component which may be used as the objective function without affecting the total cost significantly.

Deciding on the criterion of optimality is not a question of mathematical programming theory. In programming theory, all that can be done is to formulate the given problem and then seek methods for its solution. The optimality criterion to be used is formulated before the application of programming techniques. The sensitivity analysis reported in this section attempts to answer the question "what criterion of optimality can be used for the formulation of the problem?"

The sensitivity analysis reported here studies the effect of adopting one particular cost component as the criterion of optimality versus any other cost component. This effect is reported in the form of a percent deviation in the total opportunity cost of the schedule obtained by following certain criterion of optimality from the overall total optimal opportunity cost. This percentage deviation, in the present study may be considered as the *precisional efficiency.*

5.1 Optimality Criteria:

The optimality criteria used in the sensitivity analysis are cost components described in Section 3. However, in order to include all the possible optimality criteria considered in literature, all possible combinations are also considered. Specifically, the criteria of optimality used are minimizing:

1. Total opportunity cost as given by equation (14).
2. Job waiting cost+penalty cost.
3. Job waiting cost+machine idle cost.
4. Penalty cost+machine idle cost.
5. Job waiting cost.
6. Penalty cost.
7. Machine idle cost.
8. Make-span.

In the subsequent analysis, these criteria of optimality are referenced by the above numbers instead of the detailed components.

5.2 Precisional Efficiency:

Precisional efficiency of each criterion is defined as the percentage deviation of the cost of the optimal schedule obtained by using that criterion of optimality from the overall optimal cost found by considering the total opportunity cost as the criterion of optimality.

Let C_{i0} be the total cost of the optimal schedule found by using the i-th criterion. Then the precisional efficiency of the i-th criterion, Δ_i, is given by:

$$\Delta_i = \frac{C_{i0} - C_{10}}{C_{10}} \times 100 ; \; i=2, 3, \cdots, 8 \; . \tag{15}$$

The Δ_i value for any given problem reflects the increase in the cost because of choosing i as the criterion of optimality rather than 1 (i.e., total opportunity cost) while determining the optimal schedule.

The total opportunity cost of the optimal schedules according to two criteria may be the same. If the opportunity costs for the first and the i-th criteria are the same, then $C_{i0}=C_{10}$. The precisional efficiency in this case, for the i-th criterion, therefore, is zero. This implies that the effectiveness of the criterion under consideration is 100%. If ξ_i represents the effectiveness of the i-th criterion, then ξ_i is given by:

$$(16) \qquad \xi_i=100-\Delta_i \ .$$

The disadvantage of omitting certain cost components while determining the optimal schedule is reflected in the value of the precisional efficiency. The greater the value of the precisional efficiency, the smaller the effectiveness of the i-th criterion.

For the sensitivity analysis, therefore, the precisional efficiency values are used to determine the effectiveness of various criteria of optimality.

5.3 Method of Analysis:

The only method available to compare the effectiveness and/or precisional efficiency of various criteria of optimality is to collect some sample data and draw conclusions based on the data so gathered. Since no real scheduling problem has been reported in the literature, simulated shop conditions are used for the sensitivity analysis.

5.3.1 Generation of Problems:

The real-life scheduling problems may be approximated to two types of problems: random matrix problems and ordered distribution problems.

A. Random Problems:

The scheduling problems which are referred to as random problems are those problems where the process times of the jobs do not bear any relation to one another. Thus, for simulating these problems, the process times are drawn from the uniform distribution. One hundred and eightly (180) flow shop problems are generated from a random uniform distribution between the limits 0 and 999.

Various cost component rates are also generated randomly. Special care is taken while generating the unit waiting costs of the jobs. The values of w_{im} can be calculated as indicated in equation (7). However, for the sensitivity analysis the values of w_{im} are generated randomly (with the restriction that $w_{im+1} \geqq w_{im}$) since w_{im} are the variables which are of interest in the analysis.

B. Ordered Problems:

These problems differ from the random problems in the nature of their process times. Such problems reflect the situations for which the process times of the jobs are not independent of each other, but have some relationship [9]. Specifically, the process times of these problems are generated in a special manner. The process times of the first job on all the machines and the process times of all jobs on the first machine are generated from a random uniform distribution between the limits 0 and 999. Subsequently, the process time of job i at machine m, t_{im}, is calculated by the following relation:

$$(17) \qquad t_{im} = \frac{t_{i1} \cdot t_{1m}}{t_{11}}\,;\ i=2, 3, \cdots, n\,;\ m=2, 3, \cdots, M\,.$$

The number of ordered problems generated equals 180. The various cost component rates of these problems are generated as for the random problems.

The number of jobs and the number of machines for both types of problems varies from 4 to 6.

5.4 Analysis Procedure:

Enumeration is used to find the six best schedules according to each of the eight measures of performance described in Section 5.1.

Two types of precisional efficiencies are noted: one which reflects the effectiveness of the criterion when only one optimal schedule is generated, and the other which indicates the effectiveness of the techniques used which can generate $k(=6)$ best schedules, out of which a schedule with a minimum total opportunity cost can be picked.

Let η_{1i} and η_{2i} be the precisional effectiveness as described above. Then according to equation (15), η_{1i} and η_{2i} are given by:

(18) $$\eta_{1i}=\frac{C_{i0}-C_{10}}{C_{10}}\times 100,\ i=2,3,\cdots,8$$

(19) $$\eta_{2i}=\frac{C_{i0}'-C_{10}}{C_{10}}\times 100,\ i=2,3,\cdots,8$$

where:

(20) $$C_{i0}'=\min_{j}(C_{ij}),\ i=2,\cdots,8;\ j=1,2,\cdots,k\ .$$

The η_{1i} and η_{2i} values thus obtained are converted into their frequency distributions and cumulative distributions by using a 5% interval.

Let $f_1(i,x)$ and $f_2(i,x)$ be the frequency distributions of the precisional efficiencies η_{1i} and η_{2i}, respectively where x is the upper limit of the percentage interval. Further let the corresponding cumulative distributions be denoted by $F_1(i,x)$ and $F_2(i,x)$. Then:

(21) $$F_1(i,x)=\sum_{x_1\leqq x} f_1(i,x_1)$$

and

(22) $$F_2(i,x)=\sum_{x_1\leqq x} f_2(i,x_1)\ .$$

The values of $f_1(i,x)$, $f_2(i,x)$, $F_1(i,x)$, and $F_2(i,x)$ are used to draw conclusions from the analysis.

5.5 Results of Analysis:

Tables I through IV show the values of $f(i,x)$ and $F(i,x)$ for 180 random and 180 ordered flowshop problems respectively.

Table I. η_{1i} for 180 random flowshop problems

i		Number of Problems in Interval Indicated																
		0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65	65-70	70-75	75-80	>80
2	f	125	20	14	4	6	3	3	0	1	1	0	0	0	1	0	0	2
	F	125	145	159	163	169	172	175	175	176	177	177	177	177	178	178	178	180
3	f	62	14	20	19	3	11	5	10	6	6	4	6	0	5	2	1	8
	F	62	76	94	113	116	127	132	142	148	154	158	164	164	169	171	172	180
4	f	80	17	21	15	11	6	3	2	6	4	5	3	3	0	0	0	5
	F	80	97	119	134	145	151	154	156	162	166	169	172	175	175	175	175	180
5	f	45	15	13	16	9	4	10	12	11	6	7	7	4	2	2	0	17
	F	45	60	73	89	98	102	112	124	135	141	148	155	159	161	163	163	180
6	f	92	8	19	12	9	5	2	4	3	2	3	3	1	0	2	1	4
	F	92	110	129	141	150	155	157	161	164	166	169	172	173	173	175	176	180
7	f	2	1	1	1	0	5	4	7	4	8	2	8	10	4	8	3	112
	F	2	3	4	5	5	10	14	21	25	33	35	43	53	57	65	68	180
8	f	23	10	15	7	14	12	6	13	7	5	12	2	7	7	5	3	32
	F	23	33	48	55	69	81	87	100	107	112	124	126	133	140	145	148	180

Table II. η_{2i} for 180 random flowshop problems

i		Number of Problems in Interval Indicated																
		0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45–50	50–55	55–60	60–65	65–70	70–75	75–80	>80
2	f	175	1	3	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	F	175	176	179	180	180	180	180	180	180	180	180	180	180	180	180	180	180
3	f	139	10	15	3	2	3	2	2	1	1	0	0	0	1	0	0	1
	F	139	149	164	167	169	172	174	176	177	178	178	178	178	179	179	179	180
4	f	150	8	6	5	3	1	0	4	1	0	0	0	1	0	0	0	1
	F	150	158	164	169	172	173	173	177	178	178	178	178	179	179	179	179	180
5	f	115	18	18	8	7	3	4	2	0	2	0	0	0	2	0	0	1
	F	115	133	151	159	166	169	173	175	175	177	177	177	177	179	179	179	180
6	f	150	8	8	5	3	1	1	2	0	1	0	0	0	0	0	0	1
	F	150	158	166	171	174	175	176	178	178	179	179	179	179	179	179	179	180
7	f	14	2	8	6	6	7	12	10	12	8	6	10	9	5	9	11	180
	F	14	16	24	30	36	43	55	65	77	85	91	101	110	115	124	135	180
8	f	72	14	20	10	10	10	5	10	5	3	4	2	2	3	4	0	6
	F	72	86	106	116	126	136	141	151	156	159	163	165	167	170	174	174	180

Table III. η_{1i} for 180 ordered flowshop problems

i		Number of Problems in Interval Indicated																
		0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65	65-70	70-75	75-80	>80
2	f	109	12	17	12	6	3	4	6	1	0	3	1	0	1	1	0	4
	F	109	121	138	150	156	159	163	169	170	170	173	174	174	175	176	176	180
3	f	68	20	16	7	12	10	3	11	6	6	6	3	3	1	1	0	9
	F	68	88	104	111	123	133	136	147	153	157	163	166	169	170	171	171	180
4	f	77	12	18	13	12	11	6	6	2	3	2	5	3	2	0	0	10
	F	77	39	107	120	130	141	147	153	155	158	160	165	168	170	170	170	180
5	f	45	16	24	13	12	9	6	13	6	3	4	4	2	2	5	0	16
	F	45	61	85	98	110	119	125	138	144	147	151	155	157	159	164	164	180
6	f	65	18	16	13	10	11	8	6	3	3	7	3	2	3	0	2	10
	F	65	83	99	112	122	133	141	147	150	153	160	163	165	168	168	170	180
7	f	5	0	0	3	4	3	5	2	4	6	7	10	8	10	7	3	7
	F	5	5	5	8	12	15	20	22	26	32	39	49	57	63	70	73	180
8	f	8	5	8	8	7	8	6	3	8	5	7	4	6	3	5	5	84
	F	8	13	21	29	36	44	50	53	61	66	73	77	83	86	91	96	180

Table IV. η_{2i} for 180 ordered flowshop problems

i		Number of Problems in Interval Indicated																
		0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65	65-70	70-75	75-80	>80
2	f	162	3	5	6	0	1	0	0	0	0	1	0	0	0	1	0	1
	F	162	165	170	176	176	177	177	177	177	177	178	178	178	178	179	179	180
3	f	126	14	9	4	5	5	2	4	3	2	1	0	1	0	1	0	3
	F	126	140	149	153	158	163	165	169	172	174	175	175	176	176	177	177	180
4	f	138	7	9	5	2	6	3	1	3	3	1	0	1	0	0	0	1
	F	138	145	154	159	161	167	170	171	174	177	178	178	179	179	179	179	180
5	f	113	16	12	11	7	3	1	7	3	2	2	0	0	0	0	0	2
	F	113	129	142	153	160	160	163	164	171	174	176	178	178	178	178	178	180
6	f	145	5	15	4	1	4	3	2	0	2	2	1	0	0	0	0	1
	F	145	150	160	164	165	169	172	174	174	176	178	179	179	179	179	179	180
7	f	14	7	4	7	10	13	4	5	9	7	6	8	7	7	4	3	65
	F	14	21	25	32	42	55	59	64	73	80	86	94	101	108	112	115	180
8	f	46	6	6	10	7	7	3	7	4	2	8	5	3	8	1	8	49
	F	46	52	58	68	75	82	85	92	96	98	106	111	114	122	123	131	180

For the sensitivity analysis, the values to be observed from Tables I through IV are the values of $f(i, 5)$ since these frequencies illustrate the number of problems for which the percentage deviations are less than or equal to 5%.

6. DISCUSSION AND CONCLUSIONS

The above sensitivity analysis is now used to draw conclusions. Specifically, three types of conclusions are presented, viz: conclusions concerning (1) the selection of single-dual cost component(s) as the optimization criterion (2) the ranking of various criteria and (3) the use of heuristic rules for finding the optimal or the near optimal schedules.

6.1 Optimization Criterion:

Tables I though IV indicate that none of the criteria gives optimal results (considering the total opportunity cost) for all problems. Theoretically, therefore, it is not satisfactory to use any single component or combination of components less than total opportunity cost as the criterion of optimality.

However, from a practical viewpoint, some deviations from the optimal solutions may be permitted. If a precisional efficiency of 5% is allowed, then the second criterion (i.e., job waiting cost+penalty cost) seems to yield the best results. About 65% of the problems can be solved by this criterion to get the results with a precisional efficiency of less than or equal to 5%. This criterion represents the sum of two cost components of the total opportunity cost. However, the second criterion is the sum of two criteria which have been used in the literature. A question may now be asked: "Why use this criterion instead of the total opportunity cost?"

The analysis of the scheduling problem in Sections 2 and 3 indicates that the calculations involved in computing the job waiting cost are almost the same as those needed to calculate the machine idle cost.

Thus, while using the second criterion, some information, which is generated while calculating the job waiting cost, is being lost and is not used to determine the machine idle cost. This information, if included, will generate total opportunity cost.

Even from the viewpoint of developing an optimizing algorithm, it seems that any mathematical development which can incorporate job waiting cost as well as the penalty cost can also incorporate machine idle cost in the development without causing any appreciable additional computations.

Thus, the conclusion of the present work is that there is no single or dual cost component(s) which can be used as the optimizing criterion to obtain the overall optimal results. If overall optimal results are desired, then the only criterion which can be used is the total opportunity cost.

6.2 Ranking of Various Criteria:

The above analysis may also be used to rank the various criteria of optimality in ascending orders of $f_1(i, 5)$ and $f_1(i, 5)$. From Tables I and III, it is observed that there is a general pattern of $f_1(i, x)$ and $F_1(i, x)$ with the changes in i. Accordingly, various criteria may be ranked depending on the values of $f_1(i, 5)$ because $f_1(i, 5)$ represents the percentage number of problems which are either optimized or have precisional efficiencies less than or equal to 5%. This ranking for the random problems is:

1. Job waiting cost+penalty cost.
2. Penalty Cost.
3. Penalty cost+machine idle cost.
4. Job waiting cost+machine idle cost.
5. Job waiting cost.
6. Make-span.
7. Machine idle cost.

The ranking for the ordered problem is;

1. Job waiting cost+penalty cost.
2. Penalty cost+machine idle cost.
3. Job waiting cost+machine idle cost.
4. Penalty cost.
5. Job waiting cost.
6. Make-span.
7. Machine idle cost.

6.3 Use of Heuristic Rules:

Heuristic rules are simple rules derived to generate optimal or near optimal schedules. In the development of these rules, some simplified assumptions are made, which from a rigorous mathematical viewpoint are not necessarily valid, but result in a simplified solution technique to the problem [5].

The sensitivity analysis reported above indicates that even if optimizing techniques which generate first six best schedules could be developed, none of the criteria, in general, can generate the overall optimal schedule which minimizes total opportunity cost. Thus, even if a heuristic rule or algorithm for minimizing the maximum flow-time, the penalty cost, the job waiting cost or the machine idle cost could generate the six best schedules, there is no guarantee that one of these schedules would prossess the optimal total opportunity cost.

Thus, it is clear that the use of heuristic rules for any optimization criterion other than the total opportunity cost will not, in general, result in an optimal or near optimal schedule with respect to total opportunity cost.

6.4 Some General Remarks:

The sensitivity analysis of the various criteria of optimality clearly indicates that the use of make-span as the optimality criterion proposed by Johnson [7] and subsequently defended by Manne [8] is not cost minimizing In fact, the sensitivity analysis above indicates that the make-span criteria ranks almost last even for near optimal solutions. These economic aspects of production scheduling systems indicate the need to generalize the scheduling problem. Any simplifications made in the study of scheduling problem, then, can be viewed from the generalized model so generated and their implications examined.

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